RADIO ENGINEERING

VOLUME TWO

E. K. SANDEMAN

RADIO ENGINEERING

This is a textbook for beginners and a reference book for experienced engineers and designers of radio equipments and circuits.

It is designed to educate the novice to the level of an expert, and to provide engineers with a series of properly designed tools ready for immediate use.

The method of approach is fundamental and general, and the specialised aspects of allied arts such as radar and television are not covered. Practically the whole of the subject matter is, however, an essential part of such arts.

For details of the scope of the work reference should be made to the Preface.

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PREFACE

THE origin of this book was an instruction written primarily for maintenance engineers at B.B.C. transmitting stations. It was designed so that new-comers with no knowledge of radio technique could acquire familiarity with working principles in the shortest possible time. Subsequently it was extended so as to constitute a book of reference for experienced engineers.

The reader should have a working knowledge of elementary algebra, and should preferably understand logarithms. The necessary elements of trigonometry are stated, while complex algebra is developed from first principles. With these qualifications, it is true to say that in the main body of the book information is imparted in a logical sequence so that a novice who conscientiously reads the book from the beginning always finds the subject-matter within his grasp. He is, on occasions, asked to accept formulae which are sometimes, but not always, proved in later chapters. This is consistent with the policy which has been pursued throughout, of providing the maximum useful practical information with a minimum of effort on the part of the reader. It is in many cases quite unnecessary for a practical engineer to be familiar with the fundamental theory underlying the derivation of formulae.

The scope of the treatment and the method of presentation are specifically related to practical ends, either in the form of technical facts, designs or methods, or else in the form of essential formulae.

The subject-matter is in advance of that in existing textbooks in the following respects:

Practical methods of lining up class C amplifiers and modulated amplifiers, including inverted amplifiers, are developed from fundamental principles, and presented in the form of explicit instructions.

Instructions are given for the practical operation of transmitters.

The practical methods of adjusting different types of capacity neutralizing circuits are given, and the elimination of parasitic oscillations is dealt with thoroughly.

A simple, easily applied and up-to-date theory of oscillators replaces the rather involved discussions of the past.

The mode of operation of multi-vibrators is described in detail.

New and rapid precision methods are given for the design of coupled circuits, whether serving as inter stages or output couplings of transmitters or receivers, or as impedance matching devices in lines. These enable coupling circuits to be designed either as band-pass filters or to secure a required kVA/kW ratio in chosen elements.

Practical methods are given for lining up short, medium and long wave aerial circuits, including those using power factor correction, as, for instance, in the long wave aerial at the B.B.C. Transmitting Station at Droitwich.

Feedback is dealt with fundamentally without evading any of the practical issues.

The basic principles of filter design are presented in such simple fashion that all the more common types of filter can be designed in a minimum of time after a brief perusal of the instructions.

The harmonic analysis of a number of standard waves is given, while methods of application of both graphical and analytical methods of harmonic analysis are described explicitly.

The treatment of resonant circuits leads to easily applied formulae and is specifically related to their proper use in transmitters and receivers.

Specific instructions are given for equalizer design which lead directly to all the practical alternative forms in which a structure of described characteristics can be realized. The characteristics of a number of basic structures are given.

Line transmission theory is developed without the use of hyperbolic functions, and the resultant technique is simple to understand and easy to apply.

The design of peak voltmeters for radio frequency measurements is reduced to an exact science.

Vector and matrix algebra are developed from consideration of the physical effects concerned, instead of being described in terms of abstract symbols divorced from reality.

The concept of impedance is developed from physical considerations without the use of calculus.

Information which is scamped or ignored in most text-books is included: e.g. R.F. resistance, inductance of straight wires, impedance characteristics of feeders, curves for finding the spectra of frequency modulated waves.

The author is indebted to so many sources of information that it is impossible to make full acknowledgement. K. S. Johnson, F. E. Terman, W. L. Everitt, A. E. Kennelly, T. E. Shea, E. H. Armstrong, P. P. Eckersley, B. Hague, Hans Roder, B. Van der Pol, A. Russell, I. B. Crandall constitute a few only of those whose

publications have been used. B.B.C. engineering practice has been largely drawn upon and discussed and a large number of the author's associates in the B.B.C. have also contributed both information and suggestions. Many of the instruments described have been developed by one or other of these engineers. In particular, the information on dividers, multipliers and multi-vibrators is due to W. E. C. Varley, R. Calvert and others. The section on stress diagrams in aerial arrays was written in its entirety by L. W. Turner. The description of the Chireix transmitter is due to R. L. Fortescue. In addition, other engineers have assisted in carrying out experiments to establish the mode of operation of certain equipment : among these are H. F. Humphries, S. A. Williams, L. F. Ivin and D. R. Cowie. The aerial power density diagrams were calculated by H. Page and others, including the author.

The sections on the aerial feeders are largely based on work and discussions with Group-Captain W. P. Wilson, N. W. D'Arcy, E. W. Hayes and A. N. Thomas.

Discussions with H. L. Kirke, A. B. Howe, H. S. Walker, H. B. Rantzen, A. R. A. Rendall, D. C. Birkenshaw, H. V. Griffiths, E. W. Hayes, H. Page, C. G. Mayo, R. Calvert, G. R. Tomlinson, H. Ellis, D. B. Weigall, R. D. A. Maurice and D. R. Cowie have contributed to the clarification of obscure and debatable points.

The section on A.V.C. circuits is largely abstracted from a memorandum by C. W. J. Hill and J. Wardley-Smith. Information on valves has been obtained from H. S. Walker and J. Tomlinson ; the section on the operation and maintenance of valves is based on a memorandum by G. R. Tomlinson. A number of items have been included at the suggestion of C. W. Skinner and E. Bonong.

I am especially indebted to Group-Captain W. P. Wilson for initiating me into B.B.C. methods and also for convincing me, very much against my will, of the value of matrix algebra. The sections on matrix algebra are based on a translation by W. P. Wilson and F. M. G. Murphy of an article by F. Strecker and R. Feldtkeller in *E.N.T.*, Vol. 6, No. 9, " A Basis for a General Four Terminal Line Theory ".

In the face of such extensive acknowledgements, the author hopes that it will be apparent that in all other cases the treatments are developed *ab initio*. Also many of the treatments constitute original work.

The inception of this book is due to L. Hotine, the Senior Superintendent Engineer in the B.B.C., and its appearance is due to his help and encouragement.

PREFACE

My thanks are due to the following engineers for thoroughly checking all calculations and technical statements: S. W. Amos, D. C. Birkenshaw, J. L. Bliss, C. Buckle, J. C. Gallagher, A. B. Howe, D. Maurice, C. G. Mayo, H. Page, Dr. A. R. A. Rendall, L. W. Turner, H. S. Walker and W. H. Ward.

I am indebted to H. Bishop, Controller (Engineering) of the B.B.C., for permission to include information concerning B.B.C. practice, which adds so much to the scope of the book.

These bare acknowledgements are very inadequate to express the wealth of good will with which all information and help has been freely extended, and I take this opportunity of saying how fully I have appreciated the pleasant atmosphere in which the results of a twenty years' study of broadcasting have been recorded.

E. K. S.

SPECIAL ACKNOWLEDGEMENT

IT is with considerable pleasure that the author expresses his appreciation of the generosity with which he has been given permission to include copyright matter of various kinds. A few of the names of those to whom he is indebted are given below.

The Editor of the American Institute of Radio Engineers; the Secretary of the Institute of Electrical Engineers; the Editor of *Electronics* for use of Circle Diagram and extensive information on Radio Frequency Resistance; Messrs. John Wiley & Sons, for comprehensive Conversion Tables between Units extracted from their *Electrical Engineers' Handbook* by H. Pender and I. McIlwain; the Editor of *The Wireless Engineer* for extensive quotations from the author's articles on "Coupling Circuits as Band Pass Filters" and "Feedback"; Messrs. the General Electric Company, for very full details and circuit of their "Overseas Ten" Receiver; Messrs. Standard Telephones and Cables, for quotations from the author's article in *Electrical Communication* on "Generalized Network Characteristics", and from C. E. Strong's article in the same journal on "The Inverted Amplifier", and for various sheets of valve characteristics; Messrs. Marconi's Wireless Telegraph Company for their Floating Carrier Circuit and various valve characteristics; Messrs. Mullards Radio Valve Company and Messrs. the Edison Swan Electric Company, for various valve characteristics; Dr. B. Hague and the Physical Society for his valuable curves of Bessel Functions of Integral Order; Messrs. The Western Electric Company, for circuit of a Grid Modulated Doherty Amplifier and valve characteristics; the Bell Telephone Laboratories for their Reactance Chart and various other figures; Messrs. D. van Nostrand for the figures giving extensive details of Filter Characteristics; Messrs. Sir Isaac Pitman's, the London Electric Wire Company and W. T. Glover & Co. for Wire Resistance Tables; the Editor the Bell System Technical Journal, for various figures, Mr. J. B. Birks and the Director of Communications Development, Ministry of Supply (Air), for information on the characteristics of Dielectric Materials.

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All figures are in the text except the following, which are *folding plates* :—

VOLUME TWO

CHAPTER XVII

BALANCED AND UNBALANCED CIRCUITS

REGARDLESS of its length or purpose, an electrical circuit for conveying power from one piece of apparatus to another may be balanced or unbalanced. Such a circuit consists of one or two wires and is called a *line*. Since, however, balanced and unbalanced circuits exist (as, for instance, the circuits of amplifiers and radio transmitters) which do not consist of one or two wires, in the prèsent discussion the term " circuit " is retained.

1. Unbalanced Circuits.

Early telegraph circuits consisted of a single wire running from one place to another, the return circuit being through earth as indicated in Fig. 1 (*a*) where a generator of internal impedance R_0

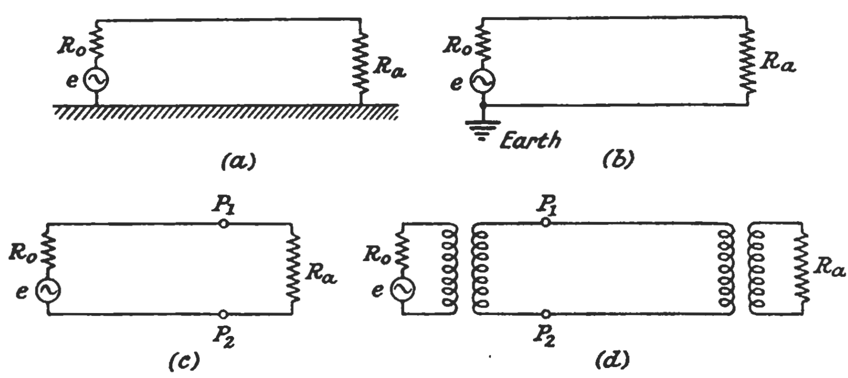

FIG. 1/XVII:1.—Unbalanced and Balanced Circuits.

and internal e.m.f. e is connected through an unbalanced line to a distant piece of apparatus of impedance R_a. This constitutes a type of circuit which has gone out of fashion for long circuits because, if the wires of two such circuits run close together, *interference* or *cross-talk* occurs from one circuit into the other due to two effects : electromagnetic and electrostatic induction.

2. Electromagnetic Induction.

This is an effect, not previously discussed, by which a varying current in a wire induces an e.m.f. in a parallel wire, proportional to the magnitude and rate of variation of the current. It follows that electromagnetic induction is of greater magnitude at high frequencies than at low. The effect is due to the magnetic field which surrounds any conductor carrying a current.

3. Electrostatic Induction.

This is caused by current which flows from one wire to the other through the capacity between the wires, due to any potential difference between the wires. The electrostatic induction is also of greater magnitude at high frequencies than at low.

Electrostatic induction can be substantially reduced by surrounding the wire with an earthed screen.

A second type of unbalanced circuit is shown in Fig. 1 (*b*) in which an earth return circuit is not used, but in which one leg of the circuit is earthed at one or more points. The difficulties introduced by multiple earths will be discussed later.

4. Balanced Circuits.

The difficulties experienced in the case of unbalanced circuits have led to the use of balanced circuits, examples of which are shown in Fig. 1 at (*c*) and (*d*).

The requirements of a balanced circuit are that the impedances to ground measured from corresponding points such as P_1 and P_2 on each leg of the circuit shall be equal. The circuit is then said to be balanced to ground.

Since it is seldom the case that a piece of apparatus is constructed with its two output legs inherently balanced to ground, the circuit of Fig. 1 (*c*) has no practical interest. The practical circuit is shown in Fig. 1 (*d*) where transformers have been inserted between the pieces of terminal apparatus and the line. The transformers may of course be (and generally are) incorporated in the terminal apparatus, in which case the terminal apparatus has its input or output balanced to ground.

The reason for inserting the transformers is that it is possible to construct transformers with windings balanced for impedance to ground. Before discussing this further it is necessary to consider how a balanced circuit reduces interference.

5. Reduction of Electromagnetic Induction.

In Fig. 1 (*a*) are shown the conductors of two pairs of wires, *ab* and *cd*, each pair constituting the conductors of a balanced circuit. The crosses indicate current " entering the paper " and the dots indicate current " leaving the paper ". If *a* and *b* are the conductors of one pair and *c* and *d* are the conductors of a second pair, it is evident that the currents induced in leg *c* by the current in *a* are in opposite sense from the currents induced by the current in *b*; they therefore subtract. If the distance from *a* to *c* is made equal to the distance from *b* to *c* the induced currents cancel. Similar remarks apply to currents induced in leg *d*.

Two methods are employed for ensuring that each leg of one pair is always equidistant from each leg of another pair, or from any disturbing source.

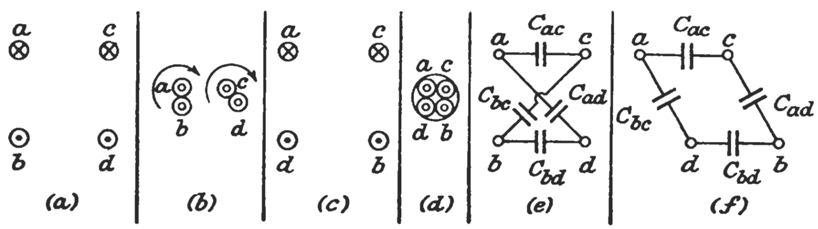

FIG. 1/XVII:5.—Arrangement of Conductors in Balanced Circuits.

In the first method each pair is twisted as indicated by the arrows in Fig. 1 (*b*). The sense of twist is unimportant.

In the second method the conductors of a pair, such as *ab* or *cd*, constitute the diagonally opposite corners of a group of four wires. This method is shown at (*c*), which represents overhead line construction, and at (*d*), which represents underground cable construction. When in a cable, the four wires are called a *quad*, and to prevent interference between the wires in one quad and neighbouring quads, each quad is twisted as a whole. In overhead construction the wires are rotated as they go from pole to pole, e.g. in four successive poles each conductor occupies each of the four positions in turn.

6. Reduction of Electrostatic Induction.

Regardless of the position of the wires in space, Fig. 1(*e*)/XVII:5 illustrates the capacities between the legs of two pairs, *ab* and *cd*. This arrangement is redrawn diagrammatically in Fig. 1 (*f*), from which it is evident that in order that no voltage shall appear across *cd* due to a voltage across *ab* the condition must hold that

$$\frac{C_{ac}}{C_{bc}} = \frac{C_{ad}}{C_{bd}} \qquad \text{see XX:8}$$

Twisting or rotating the pairs as in Fig. 1 (*b*) has the effect of making $C_{ac} = C_{bc} = C_{ad} = C_{bd}$ approximately, in which case the condition for no electrostatic inductance is realized approximately. Owing to random effects in the make-up of cables or the construction of overhead lines, exact equality of these capacities is never quite realized, but for normal telephone circuits cross-talk can be reduced to tolerable levels.

Where a high degree of freedom from interference between circuits is required (e.g. in broadcast circuits) it is therefore necessary to add a further precaution which consists of surrounding each pair with an individual screen. In British practice this screen is usually left floating; but if earthed, it should be earthed at one end only, except in very special circumstances.

Any number of pairs or quads, up to several hundreds, may be twisted together and surrounded by an outer screen, which is usually a lead sheath, to constitute a cable.

From the point of view of the radio maintenance engineer, however, the most important circuits to be considered are individual circuits connecting pieces of apparatus.

7. Concentric Cables.

The concentric cable is of particular interest because it is the only form of circuit which gives a large measure of protection against both electromagnetic and electrostatic induction. The protection against electromagnetic induction is almost perfect, while the degree

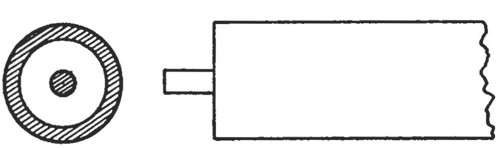

FIG. 1/XVII:7.—Concentric Cable.

of protection against electrostatic induction depends on the longitudinal impedance of the outer conductor, which should be low.

A concentric cable consists of a single conductor supported on insulators in the centre of a circular cylindrical sheath or outer conductor, as shown in Fig. 1. The inner conductor constitutes one leg of the circuit and the outer conductor the other leg of the circuit.

It evidently constitutes an unbalanced circuit.

At zero frequency (direct current) and at low frequencies where skin effect is negligible, the voltage drop along the outer conductor per unit length is equal to the product of the resistance of the outer conductor per unit length and the total current flowing through the outer conductor. *If the outer conductor is insulated from ground* this will evidently introduce only a comparatively small potential gradient along the cable because in general the resistance of the outer conductor is low. A neighbouring insulated circuit which was unbalanced to ground would therefore be subject to interference of a small amount due to electrostatic induction. As the frequency is increased this interference would increase approximately proportional to frequency, until skin effect begins to become appreciable, when with further increase in frequency the interference begins to fall off. By the principle of reciprocity interference into the insulated concentric cable would behave in the same way.

When skin effect appears, as the frequency increases, the current in the outer conductor distributes itself more and more towards the inner surface of the conductor, until, at a very high frequency where the depth of penetration (see II:13.1) is very small compared with the thickness of the outer conductor, the current density falls off exponentially from the inner to outer surface of the outer conductor (a slight departure from the exponential law occurring near the outer surface), so that the current density at the outer surface is only a small fraction of the current density at the inner surface. Practical conditions can be realized in which the current density at the outer surface is entirely negligible, and under these conditions the cable has no external field and can cause substantially no interference into neighbouring circuits. Conversely, it can pick up substantially no interference. With many high-frequency cables in common use such a condition can be reached below a frequency of one megacycle per second.

When the cable is buried in the earth the potential gradient along the length of the cable may give rise to earth currents at low frequencies, and, reciprocally, currents in the earth may cause interference into the cable. At frequencies where the current is substantially confined to the inner skin of the outer conductor, substantially no current flows through the earth, and, reciprocally, earth currents cause substantially no interference into the cable.

In the heavy feeder tubes used for carrying high powers to aerials, and even in the much smaller lead-sheathed concentric cable used in connection with low-power drive circuits and receiving-aerial circuits, the external field is so small as to be nearly negligible at medium and short waves. The braided copper screens which surround single and twin conductors used for interconnections between apparatus have, however, not such a complete screening effect, and trouble is liable to be experienced even at audio frequencies when very low-level unbalanced circuits exposed to high-level disturbance fields are run in this type of conductor. When used in balanced circuits the screening afforded by this type of shield is usually adequate.

8. Comparative Magnitudes of Electrostatic and Electromagnetic Interference.

Since in high-impedance circuits the voltages are high, and in low-impedance circuits the currents are high, there is a tendency for electrostatic induction to be more serious in high-impedance circuits and electromagnetic interference to be more serious in low-impedance circuits.

In general, the impedance level in programme and radio circuits is such that the most serious interference between conductors is caused by electrostatic effects. Electromagnetic effects do, however, play a serious part in interference from A.C. mains supplies, particularly in the leads to A.C. heated filaments or heaters in amplifiers handling low programme levels. For this reason A.C. filament leads should always be twisted right up to the filament terminals and the connection to the filament made so as to introduce as small a loop of current as possible. This is particularly important because the flux of such a loop may link the core of an audio-frequency transformer in the amplifiers and so give rise to interference. Electromagnitude induction due to coupling between coils in different parts of radio-frequency circuits occurs but is not considered here.

9. Effect of Unbalance in Balanced Circuits.

The effects consequent on the presence of unbalance in a circuit depend on

(1) Any unbalances present elsewhere in the system.
(2) The presence of disturbing fields *or* fields due to unbalances elsewhere in the system, or due to another system.
(3) The impedance to ground of the earth point in use.
(4) Whether any currents are flowing to ground through the earth point chosen : due either to other sources of current of frequency likely to cause interference, or to unbalances elsewhere in the circuit.

(5) The type of apparatus involved.
(6) The degree of screening of the circuit, and the resultant amount of electrostatic coupling between different parts of the circuit.
(7) The degree of balance and screening of transformers at different points of the circuit.
(8) The circuit impedance.

Broadly, the effects consist either of interference from one programme circuit to another, from power circuits into a programme circuit, or interference between different parts of the same circuit. In the last case equalizers and attenuators do not introduce their correct value of attenuation at each frequency, but too low a value; while amplifiers do not give their proper gain at each frequency: the gain may be higher or lower than the correct value, the response characteristic may be degraded, and in an extreme case an amplifier may sing due to the presence of unbalance.

While all cases of unbalance exhibit the feature that the series impedances in each leg and/or the impedances to ground of each leg of the circuit are not equal, it is convenient to consider this as being the main feature of a disturbed circuit, while the main feature of an unbalanced disturbing circuit is that the voltages to ground of each leg of the circuit are not equal: these voltages are of course opposite in sign.

9.1. Way in which Unbalance of Voltages to Ground Occur in a Disturbing Circuit. The general case is indicated in Fig. 1. A generator of internal e.m.f. e and internal impedance R_0 is connected to the legs a and b of a circuit having impedances to ground which are respectively Z_a and Z_b. It is evident that the voltages to ground are

$$V_a = \frac{Z_a}{Z_a+Z_b+R_0}e$$

$$V_b = \frac{Z_b}{Z_a+Z_b+R_0}e$$

Hence

$$\frac{V_a}{V_b} = \frac{Z_a}{Z_b}$$

The presence of a balanced load across terminals 2,2 does not change the ratio $\frac{V_a}{V_b}$.

Considering voltages with regard to ground, V_b is evidently negative with regard to V_a and it is permissible to consider V_a and

V_b (V_b negative) as being each made up of two voltages with regard to ground. Algebraically :

$$V_a = \frac{V_a+V_b}{2} + \frac{V_a - V_b}{2} = v_1 + v_2$$

$$V_b = \frac{V_a+V_b}{2} - \frac{V_a - V_b}{2} = v_1 - v_2$$

The two voltages v_2 and $-v_2$ constitute a balanced voltage to ground, while the voltages v_1 constitute an unbalanced voltage to ground. Algebraically $v_1 = \frac{1}{2}(V_a+V_b)$ = half the difference of the numerical magnitudes of V_a and V_b.

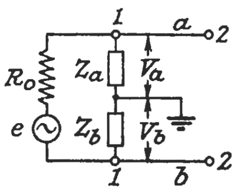

FIG. 1/XVII:9.—Unbalanced Voltages.

The balanced voltages to ground give rise to balanced currents in the two legs, equal in magnitude and opposite in sense, while the voltage v_1 gives rise to currents in the two legs of the circuit which are equal in magnitude and of the *same sense*. These currents are called *longitudinal* currents.

9.11. Case 1. Unscreened Transformers : Screened Conductors : Ideal Earths. A particular case which is really the general practical case is shown in Fig. 2, where a transformer T_1

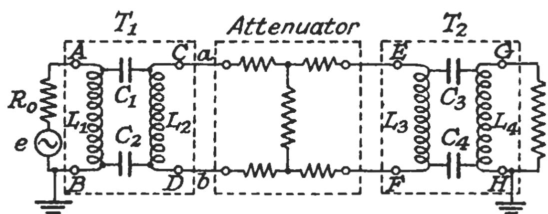

FIG. 2/XVII:9.—Practical Case of Unbalance.

which might, for instance, be the output transformer of an amplifier, is driven by a generator with one side earthed, which might for instance be constituted by the anode circuit of a thermionic valve. The transformer secondary is connected to the legs *a* and *b* of a balanced pair of wires connected through an attenuator to a transformer T_2 which drives a load having one side earthed, which might be constituted by the input circuit of a valve. C_1, C_2 and C_3, C_4 represent respectively the lumped interwinding capacities of T_1 and T_2.

Owing to the presence of the earth at *B* and capacities C_1 and C_2, the impedance to ground of points C and D are not the same (neglecting the effect of T_2). Neglecting the effect of T_1 by a similar argument, the impedances to ground at E and F are unbalanced. There is, therefore, unbalance at both ends of the circuit.

The unbalance of voltages in the output of T_1 can be analysed in detail as follows. The e.m.f. induced in L_2 by its mutual coupling with L_1 is unbalanced by the presence of unequal impedances to ground at C and D. Further, owing to the presence of capacities C_1 and C_2 a circulating current flows from the generator through C_1, L_2 and C_2, and so introduces a further unbalance which may add or subtract from the first unbalance. The net result in practice is always an unbalance over the whole frequency range, except for some chance arrangement of values when balance may occur at one frequency only. The result is the production of longitudinal currents which suffer reduction of intensity owing to the series elements of the attenuator, but are unaffected by the shunt element. The longitudinal current in b flows harmlessly to ground through C_4, while the longitudinal current through a flows via C_3 through L_4 to ground, and through L_3 and C_4 to ground, thereby setting up voltages across the secondary of T_2. As these longitudinal currents have not suffered the full attenuation of the attenuator they may, and do in certain cases, give rise to voltages at the output of T_2 greater than those produced by the balanced voltages and currents. It is to be particularly noticed that, if in this case the attenuator were fitted with perfectly screened and balanced transformers, there would be no path for the longitudinal currents through the attenuator and no anomalous effects would result.

9.12. Case 1a. Screened and Balanced Transformers. Nearly all cases of unbalance are reduced by the use of balanced and screened transformers: they are not completely removed because no transformer has a perfect balance or a perfect screen.

A *screened transformer* is one in which an electrostatic screen is provided between windings by means of a piece of copper foil wrapped completely round the inner winding and insulated so as not to form a short-circuited turn. This is brought out to a terminal.

A balanced transformer is one in which each winding is balanced for impedance to earth and to the screen, also each half of the winding has the same inductance. (As this balance is usually effected by constituting each half of the winding from one leg of a bifilar (twin) winding, and as both ends of each half of the winding are brought out to terminals, a balanced transformer affords a source of two equal e.m.f.s, which is sometimes useful.)

Fig. 3 shows a circuit identical with that of Fig. 2 except that T_1 and T_2 have been replaced by T_3 and T_4 which are screened and balanced transformers. In T_3 the secondary is balanced and in T_4 the primary is balanced so that $C_1 = C_2$ and $C_3 = C_4$. Further, the inductances of each half of any balanced winding are equal.

It follows that the impedances to ground of points C and D are the same and E and F are also balanced to ground. Further, there is no reactive path (as through C_1 and C_2 in Fig. 2) by which the generator can supply longitudinal currents to the line, since all longitudinal currents are led to earth immediately by the screen of T_3.

Finally, if by any chance longitudinal currents do flow, they go to earth through C_3 and C_4 and the screen of T_4. In practice, of course, C_3 and C_4 are distributed capacities so that the longitudinal currents flow to the screen partly through the two halves of the

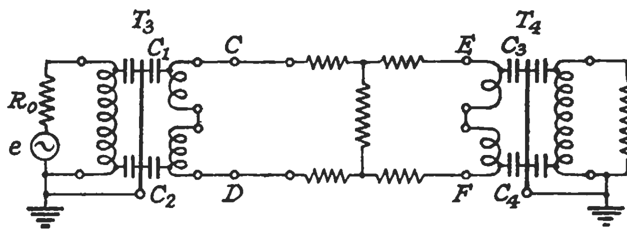

FIG. 3/XVII:9.—Use of Screened and Balanced Transformers.

primary winding of T_4; but as long as everything is balanced the sense of flow of the longitudinal currents is such that the resultant induced e.m.f.s in the secondary winding cancel.

In amplifiers the screens of input and output transformers should normally be connected to the cathodes of the valves to which they are respectively connected.

9.13. Case 2. Unscreened Transformers : Unscreened Conductors : Ideal Earths. Fig. 4 shows the circuit of an amplifier with unscreened input and output transformers. In this case feedback occurs due to coupling from the output circuit to the input circuit, contributed by the unbalance at output and input circuits in conjunction with the stray capacities C_s between input and output conductors.

The output unbalance gives rise to longitudinal voltages and longitudinal currents which flow through the two stray capacities C_s, one going harmlessly to earth through C_2; the other, via C_1, gives rise to an input voltage across the grid circuit. In general, the feedback arising in this way varies widely with frequency.

If the gain of the amplifier is greater than the attenuation in the feedback circuit the amplifier will sing (see XXIII for limitations on singing imposed by phase relations). In any case, the presence of feedback, varying with frequency, modifies the frequency response characteristic of the amplifier, and if of such sense as to increase the gain, also increases the non-linearity of the amplifier.

The introduction of screened and balanced input and output transformers reduces the unbalance and so the feedback. In practice, in spite of the use of screened and balanced transformers it is customary to screen all programme circuits, both from the point of view of cross-talk and from the point of view of feedback, although feedback due to unbalance and lack of screening is usually com-

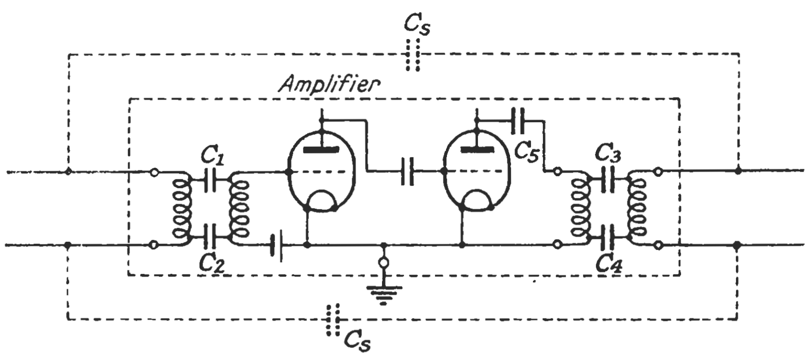

FIG. 4/XVII:9.—Effect of Unbalanced Transformers on Stability of Amplifier.

paratively unimportant at audio frequencies (even with unscreened circuits) with gains of 40 db. and under. Singing may, however, occur at supersonic frequencies, and this must be avoided.

9.14. Case 3. Unscreened Transformers : Screened Conductors : Common Impedance in Earth Circuit. Fig. 5 shows two amplifiers in tandem, each of which may be regarded as being similar to the amplifier in Fig. 4. Both amplifiers share a common earth lead which ultimately goes to a true earth, but the impedance from *P*, the common junction point of the earth leads, to true earth, is Z_e. The output circuit of amplifier 2 generates a longitudinal current which flows across the capacities of the output transformer, along the output circuit and through the capacity to the screen, the screen capacity to ground and the apparatus at the far end, to earth, returning to the filament circuit of the output valve via Z_e. A voltage is therefore established across Z_e and the point *P* is at some voltage above ground. The whole of the earth circuit of amplifier 1 is therefore at a potential to ground, and a longitudinal current flows out along the input circuit of amplifier 1 and in so doing applies a voltage between grid and cathode of the first valve of amplifier 1. This can be seen from the fact that the longitudinal current from the filament flows partly through C_2 and L_1 and partly through L_2 and C_1.

In the circuit shown, this gives rise to feedback, but if the two amplifiers were not in tandem as shown, but were in different programme circuits, the effect would be to introduce cross-talk from one amplifier to the other. A secondary feedback circuit may also be traced from output to input of amplifier 2 via the output circuit of amplifier 1. While the introduction of screened and balanced input and output transformers for both amplifiers reduces this source of interference, it is also essential to ensure that the common impedance in the earth circuit is as low as possible. For this reason

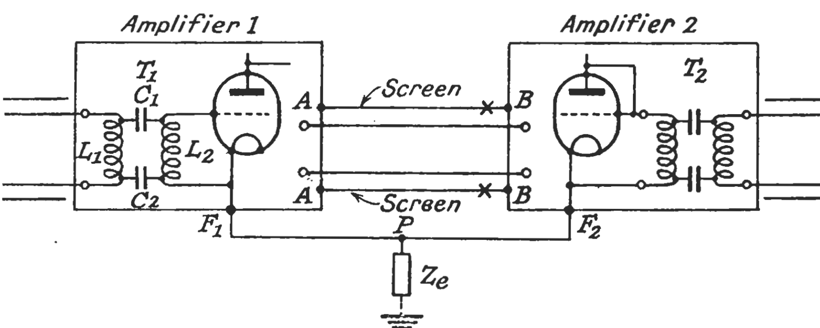

FIG. 5/XVII:9.—Effect of Common Earth Impedance.

it is sometimes desirable to run all earth leads as far as is necessary to reach a good earth, if necessary insulating them from one another, so that no common point is established until a low impedance earth is reached.

Need for Break in Screen. With very high gain amplifiers a second point arises in connection with Fig. 5 even when screened and balanced transformers are used. This relates to the screen round the conductors connecting the two amplifiers. The longitudinal current flowing to ground from the output of amplifier 2, in flowing through its individual earth connection F_2P, induces an e.m.f. in it. This e.m.f. is thus introduced into the loop circuit $F_2PF_1ABF_2$, where F_1 and F_2 are the screens of the amplifiers and A and B are the points at which the screen of the interconnecting wiring is bonded to F_1 and F_2. A current, therefore, circulates round this circuit, and by electromagnetic induction induces a longitudinal current in the inter-amplifier circuit. If the balance of this circuit is perfect no voltages are applied to the input of amplifier 2, but in practice the balance never is perfect and feedback results. The screen must, therefore, be broken at some point along its length such as *XX*. The usual practice is to earth the screen at one end only. This means also that the screen should be provided with an outer insulating covering to prevent contact with casual earths.

It should be noticed that the unwanted coupling introduced when the screen is not broken is increased rather than reduced by the provision of an earth with a low value of Z_e. This is no argument for a high-impedance earth, but is brought out to emphasize the fact that a low-impedance earth does not remove the trouble.

9.15. Case 4. Use of Earth to Reduce Effect of Earth Currents. Regardless of the source of interference or the method of induction, the effect of longitudinal currents can sometimes be

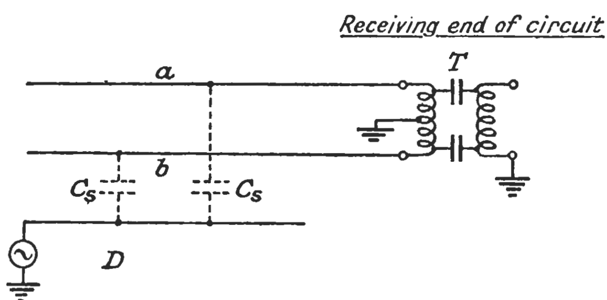

FIG. 6/XVII:9.—Use of Centre Point Earth.

reduced by earthing the midpoint of a suitable transformer. *For the reasons given below, the use of such earths is dangerous, and they should be applied with caution.*

Fig. 6 shows a circuit *ab* subject to disturbance from a neighbouring circuit *D*, which is maintained at an A.C. potential above ground : longitudinal currents are induced through the stray capacities C_s and flow into the balanced primary winding of transformer *T*. If the centre point of the primary winding of this transformer is earthed, the two halves of the primary operate in parallel opposing and so present to the longitudinal currents only a small impedance corresponding to the leakage reactance. The consequence is that the longitudinal currents flow to earth through the primary winding rather than through the stray capacities, and no e.m.f. is induced in the secondary circuit due to the longitudinal currents.

Earths can only be used in this way if experiment shows them to be successful. In general an earth should not be necessary when a balanced and screened input transformer is used. *The use of an earth in this way may be found to increase the interference if the balance of the transformer is poor, or if the series impedances of the legs of the circuit are unequal.*

It should be noted that this earth is located at the receiving end of the circuit, and is for the purpose of improving the circuit considered as a disturbed circuit.

In the same way, considering a circuit as a disturbing circuit, an earth *may* be placed on the centre point of the output transformer at the sending end. This will balance the sending-end voltages and reduce the disturbance caused by the circuit into neighbouring circuits. It has the potential disadvantage that, while it reduces the disturbance into other circuits, it may increase the disturbance *from* other circuits as indicated above. Further, in general, it prohibits the use of a centre-point earth at the receiving end since this introduces an earth loop ; see section 10 below.

10. Avoidance of Earth Loops.

Up to now it has been assumed that, apart from the question of impedance, the provision of an earth is a simple matter. This is far from fact. In practice, earths have to be found in buildings which carry a multiplicity of programme circuits and power circuits, and in which some of the power circuits make use of earth connections at one or more points. Currents of all kinds therefore flow through the earth and through the iron or steel framework of the building. Different points of the framework therefore have a potential difference between them corresponding to the variety of currents spreading out in the building. The same remark also applies to different points in the real ground when earths are realized by means of buried earth plates.

It follows therefore that, if more than one earth is used, an earth e.m.f. may be introduced into the system with the consequent introduction of longitudinal currents generated by the earth e.m.f.

Fig. 1 shows two pieces of apparatus, A_1 and A_2, in tandem with two separate earths, E_1 and E_2, in the neighbourhood of a power earth. The dotted lines indicate the earth currents induced by the power circuit. It is evident that a potential difference V_e exists between E_1 and E_2 and that this causes a longitudinal current of the power-circuit frequency to flow round the loop constituted by the two earth leads to E_1 and E_2 and the interconnecting circuit between the two pieces of apparatus.

The correct method of earthing to avoid the earth loop is shown in Fig. 2. Note that the length of common lead *CE* is kept as short as is economically possible, also it is made of heavy copper strip, e.g. with an area of cross-section of $\frac{1}{8}$ in. to $\frac{1}{4}$ in. or even larger.

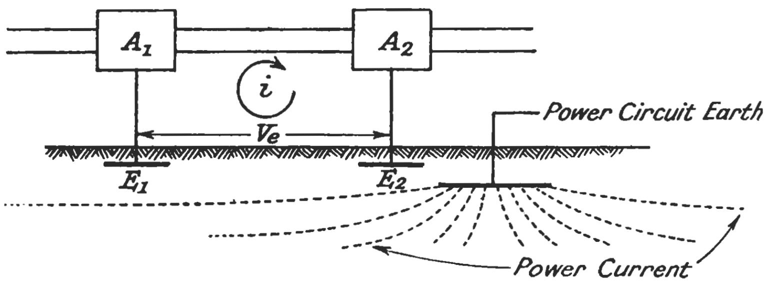

FIG. 1/XVII:10.—Effect of Earth Loop.

In the case of apparatus for use at ultra-high frequencies, i.e. from 30 Mc/s upwards, the overall screen and/or metal housing must be treated as a distributed earth. That is to say, no loops must be formed involving the passage of current through the screen and/or housing. This means that all points on the circuit which

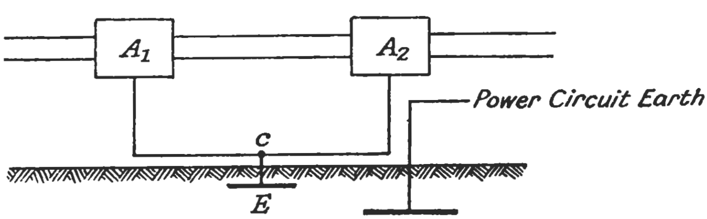

FIG. 2/XVII:10.—Correct Method of Earthing.

are required to be at earth potential must be led by separate insulated conductors to a point on the screen and/or housing as close as possible to the point at which the external earth is connected.

The precautions advocated in this section (XVII:10) evidently cannot be applied in the case of long-distance repeatered cable systems where the amplifier stations may be many miles apart. They can, however, be applied in the case of any one repeater station.

11. Separation of Power Earth Leads and Apparatus Earth Leads.

It is to be noted that if the power earth lead runs parallel to one of the apparatus earth leads as shown, induction from the power earth lead into the apparatus earth may occur. For this reason apparatus earth leads should be kept as far away as possible from power earth leads.

12. Insulation of Earth Leads.

It follows that, in general, earth leads should be insulated to ensure that they make contact only with earth and avoid contact with the framework of the building and any cable sheaths or other conductors capable of providing spurious earths.

CHAPTER XVIII

INTERFERENCE AND NOISE

1. Definitions.

Interference is any form of unwanted disturbance resulting from a source external to a circuit.

Noise is any form of unwanted disturbance resulting from a source internal to a circuit. In the case of an amplifier, transmitter or receiver, having an associated power supply circuit, this will be considered to be an external circuit. Noise does not include distortion products due, for instance, to non-linearity, but does include crackles due to bad connections.

In the case of radio links this classification is inadequate because much of the disturbance which is picked up on a receiving aerial is a definite characteristic of the medium of communication being used : e.g. all atmospherics due to natural causes. The first obvious division therefore is to class all man-made disturbance, whether due to other transmitters or incidental radiation from any kind of electrical apparatus, as Interference, and other disturbances as Noise. Certain man-made interference such as that produced by electric railways has, however, all the characteristics of atmospherics : it produces a number of e.m.f.s in random phase and random directional distribution, and the methods of reducing its harmful effects are identical with those for minimizing the effects of atmospherics.

The most useful classification to use for Interference and Noise incoming to a receiving aerial is therefore as follows :

Interference is any form of man-made disturbance which is identifiable as such.

Noise is any form of disturbance which persists after all specified means for the reduction of interference, which are possible and economical, have been applied. (Directional aerials are not specified for the reduction of interference but only for the reduction of noise.)

This means that all forms of interference may rank as noise, and on a statistical basis—averaging the results on one channel over a period of time—may be reduced by the methods applied to reduce noise.

If any other classification than the above is used an inevitable conflict of requirements for the reduction of interference and noise incoming to an aerial must result. For instance, interference from a transmitter on a certain bearing from the receiving-point may require a directional aerial with a minimum on a certain bearing, while the noise requirements (if any other classification than the above is used) may require an aerial with a maximum on another bearing, of a type which cannot provide a minimum in the required direction. The above classification ensures that both requirements will be considered together and the optimum compromise aerial adopted.

For all interference and noise in a radio receiver which is not incoming to the aerial the original classification holds.

2. Interference.

The more important causes of interference and the means of reducing their effect are discussed below under the heading of the apparatus affected. The reduction of interference in long-distance land lines is a specialized technique outside the scope of this discussion. Interference on the programme lines incoming to a radio transmitter are dealt with by the appropriate government organization.

It is assumed in the following that all the appropriate precautions outlined in XVII have been taken. Evidently the presence of unbalances in conjunction with earth impedances or earth loops is a fruitful source of interference.

2.1. Cross-talk. This is the type of interference in which speech or music from one line, which may be a music or a control line, appears in another music or control line. The true figure of merit, which gives the disturbing effect of the cross-talk, is the difference in level between the programme or speech proper to the line and the disturbing programme or speech. This will be called the *relative cross-talk level* and evidently varies with the levels of the subject matter in disturbing and disturbed circuits. It is normally measured as the difference between the peak levels in disturbing and disturbed circuits.

In the case of lines the *cross-talk attenuation* between unrepeated lengths of pairs of conductors *traversing the same route* is the difference in level between the sending-end level, of the disturbing programme or speech, in the disturbing circuit, and the receiving-end level, in the disturbed circuit of the disturbing programme or speech. When disturbing and disturbed circuits are carrying programme or speech in the same direction, the receiving end of the disturbed circuit is at the opposite end of the circuit from the sending end of the disturbing circuit and the cross-talk attenuation is spoken of as *far-end cross-talk.* When the disturbing and disturbed circuits are carrying programme or speech in opposite directions, the sending end of the disturbing circuit is at the same end as the receiving end of the disturbed circuit, and the cross-talk attenuation is called *near-end cross-talk.*

If the levels in the disturbing and disturbed circuits are known, as well as the cross-talk (attenuation) and the attenuation of the disturbed circuit between sending end and receiving end, the cross-talk level can be calculated very simply as follows:

Conventions.

L_{ing} = level of disturbing programme or speech at sending end of disturbing circuit.

L_{ed} = level of disturbed programme or speech at sending end of disturbed circuit.

B = attenuation of disturbed circuit in decibels.

C_n = near-end cross-talk in decibels.

C_f = far-end cross-talk in decibels.

In the case of transmission through both circuits in the same direction the relative cross-talk level is then equal to

$$L_{ed} - B - (L_{ing} - C_f).$$

In the case of transmission in opposite directions through the two circuits the relative cross-talk level is equal to $L_{ed} - (L_{ing} - C_n)$.

As an example, consider the case where two broadcast circuits run in a cable, where the attenuation of each circuit is 30 db., the near-end cross-talk is 80 db., the far-end cross-talk is 100 db. and the sending-end levels, L, and level range of each programme are the same. For transmission in the same direction the cross-talk level is given by the far-end cross-talk and is $L - 30 - (L - 100) = 70$ db. For transmission in opposite directions the relative cross-talk level is given by the near-end cross-talk, and is $L - (L - 80) = 80$ db.

In the case of transmission in the same direction, peak programme level in either circuit will give rise to a cross-talk level in the other circuit which is 70 db. below the wanted peak programme level in that circuit. If the control operator does his job and maintains the level range of the wanted programme at 20 db. the maximum cross-talk level is 50 db. below minimum wanted programme level. If, as is most likely, he is not uniformly successful, but occasionally allows the programme to fall lower than 20 db. below peak programme level, the margin between the cross-talk level and minimum programme level is correspondingly less. See XXII:1.

The C.C.I.T. defines the far-end cross-talk as the difference in level between the disturbing programme level entering the receiving end of the disturbing circuit and the level of the disturbing programme level entering the receiving end of the disturbed circuit. This used to be known as uncorrected cross-talk, while the definition above defines corrected cross-talk. Evidently the corrected cross-talk of a circuit is greater than the uncorrected cross-talk by the amount of attenuation in the disturbing circuit proper between sending and receiving end. The C.C.I.T. definition of near-end cross-talk conforms to the definition of near-end cross-talk given above.

In terms of its own definitions the C.C.I.T. recommends that the near-end and far-end cross-talk attenuation shall be at least 78 db. for cable circuits and at least 61 db. for open-wire lines, when either is used for broadcast programmes.

In the B.B.C. it is customary to measure relative cross-talk levels directly using a peak programme meter with an amplifier of known gain to bring the cross-talk level up to a value which will give a readable deflection on the programme meter. This very much simplifies matters.

In the case of cross-talk between radio-frequency circuits, it is necessary to make a clear distinction between relative levels of wanted and unwanted carrier frequency, and between levels of wanted and unwanted audio frequency. It may happen, for instance, that, owing to coupling between two adjacent aerials, a modulated transmission on one wavelength on one aerial at a transmitting site may be fed into the output of the transmitter driving the other aerial on a different wavelength. The resultant troubles are various and are dealt with under their various sections. Among these are audio-frequency cross-talk from one programme into the other, consequent on intermodulation in the output valves of the transmitter. This is better expressed in terms of relative cross-talk level than in terms of cross-talk attenuation. The relative cross-talk level is evidently given by the number of decibels corresponding to the ratio of the wanted to unwanted level of audio frequency resulting from detecting the radiated transmission.

In such cases the ratios between the wanted and unwanted radio-frequency powers or voltages effective in the aerial feeder circuits and transmitter output circuits are best expressed as simple ratios and should not be referred to as cross-talk.

3. Interference in Audio-Frequency Circuits.

3.1. Induction from A.C. or D.C. Power Circuits into Audio-Frequency Lines. This appears as a hum or a musical note and is minimized by the use of screened and balanced circuits and by separating the runs of power circuits and audio-frequency circuits.

Electrostatic induction from power circuits into amplifiers is minimized by screening the amplifier electrostatically as a whole. Electromagnetic induction is reduced by screening the transformers of low-level circuits with screens made of permalloy, mumetal or some other high-permeability alloy which provides electromagnetic screening. Also all audio-frequency transformers should be located as far as possible from power transformers.

Electrostatic screening reduces interference radiated by commutators and slip-rings of machines when sparking, by mercury-arc rectifiers, relay circuits, interruptions of D.C. circuits and any other source of a radiated field, including any local radio transmitter.

3.2. Interference from Supply Sources. Where D.C. machines are used for supplying H.T. or the grid bias or filament supply, a hum may be introduced into the circuit by the commutator ripple. This is removed by filtering. In the case of H.T., and grid-bias circuits which are of abnormally low impedance, inductance capacity filters may be used; for grid-bias circuits where little current flows resistance capacity filters are used where possible. For filament circuits taking high currents, e.g. hundreds of amperes, it is seldom possible to introduce any effective form of filter, but in certain cases the use of electrolytic condensers with a capacity of a few thousand microfarads may be beneficial.

Where power supplies are derived from the A.C. mains the H.T. and grid-bias supplies are provided by rectifiers followed by smoothing filters, and it is of course essential to provide adequate filtering. Raw A.C. is normally applied to the heaters of separately heated cathode valves, even in amplifiers handling quite low-power levels, and no serious hum is normally experienced, provided the heater leads are twisted along the whole of their length, and care is taken that no current loops are introduced which might give rise to an A.C. flux linking the core of an audio-frequency transformer.

The filaments of small amplifiers are sometimes heated by rectified A.C., even when directly heated filaments are used. Again, it is evidently necessary to provide adequate smoothing.

In high-power amplifiers raw A.C. is sometimes applied to directly heated filaments. In this case, when the filament current is of the order of 100 amps., hum is sometimes introduced owing to interaction between the magnetic field of the filament and the electron stream from filament to anode. When this occurs there is normally no remedy without the introduction of special measures such as feedback (see XXIII) and hum-bucking. Hum-bucking consists in generating from the power supply all the harmonic interfering frequencies of importance, and feeding them into a low-level part of the circuit, in such amplitude and phase that they cancel the hum frequencies generated in the A.C. heated stage in question. By this means a reduction in hum frequencies between 10 and 20 db. may be obtained.

Where raw A.C. is used directly on the filaments of small valves the noise is sometimes minimized by leading away the earth connection of the filament from a tapping on a potentiometer bridged across the filament. When this potentiometer is variable it is called a hum-bucker or hum-dinger.

A.C. on separately heated cathode valves does not usually lead to trouble except in valves handling very low levels, where induction may take place in the *pinch* of the valve between the heater leads and the leads to the other electrodes.

In amplifiers working from A.C. supplies the following principles are generally applicable.

1. A.C. power circuits should be screened and should be in separate cable forms and ducts well away from programme circuits. A.C. power circuits which are integral parts of amplifiers should be screened from the amplifier circuits.
2. Adequate smoothing should be provided following all rectifier circuits.
3. In low-power amplifiers all A.C. leads must be twisted and no A.C. current loops must be formed. Twisting is not practicable and is unnecessary in the case of heavy conductors leading current to the high-power stages of a transmitter.
4. All grid circuits must be decoupled from their bias circuits and connected by means of a condenser of adequate capacity to the cathode of the valve in question. Cathode feedback circuits constitute an exception.
5. In the case of separately heated cathode valves the A.C. heater circuits must be completely isolated from the remainder of the circuit. Alternatively, one end or the centre point of the heater transformer may be earthed.
6. In low-power amplifiers the A.C. power transformer should have an earthed screen between primary and secondary windings.
7. No coupling must exist between the magnetic circuits of power transformers or chokes and the magnetic circuits of audio-

frequency transformers. This is particularly important in low-level circuits. Coupling may be reduced by orientation of the transformers in space, by the use of permalloy or mumetal screens around A.F. transformers, and, when steel panels are used, by raising A.F. transformers about half an inch away from the panel on non-magnetic supports.

8. Raw A.C. should not be used on the filaments of directly heated valves except in high-level circuits, and then only after experiment has established that the noise level is tolerable.
9. All screens should be well bonded together and to earth : see, however, XVII:9.14, " Need for break in screen ".

3.3. Interference from Charging Generator. It is common practice to double up every battery with a spare so that the spare can be on charge while the main battery is in use. Owing to the high charge currents involved in even medium-sized installations, if any inductive coupling exists between the charge circuit of one battery and the discharge circuit of the other battery inductive interference in the form of a musical note may be experienced owing to induction from the charging circuit to the discharge circuit which is connected to amplifiers in use. Care must, therefore, be taken in designing the layout of the conductors in the charge and discharge circuits to see that all current loops are kept as small as possible and that no inductive coupling exists between the charge and discharge circuits. Serious inductive interference may be experienced, even if such loops are separated by the width of the battery room, if they are large enough.

3.4. Interference from High-Frequency Field of Transmitter. In certain cases electrostatic screening alone is found to be inadequate, and it is then necessary to insert a radio-frequency low-pass filter in the input of the apparatus affected. It is economical to give this filter as high a cut-off as possible, and good results are usually obtained with a single mid-shunt terminated section of prototype low-pass filter with a cut-off at about a quarter of the carrier frequency. If this is found inadequate two sections may be used. The presence of radio-frequency pick-up may be observed variously as an increase in plate current of certain valves, non-linear distortion, loss of gain in the amplifier accompanied by distortion, or in extreme cases, where the pick-up provides feedback, as singing. Where the carrier picked up is modulated with a different programme from that passing through the amplifier, singing cannot occur but cross-talk may be observed.

3.5. Fields Radiated by the Commutators of Machines may be reduced by connecting a condenser between the brushes and the frame of the machine. For medium-voltage (e.g. below 2,000 volts) low-current machines the value of condenser required is one or two microfarads. For low-voltage (e.g. 24 volts) high-current (e.g. 1,000 amps.) machines, such as the filament machines of large transmitters, an electrolytic condenser of about 50 μF is required.

3.6. Interference from Relay Circuits, which appears as clicks when the relays are operated, is reduced by the following precautions:

1. The relays are operated from a separate battery *with a separate earth.*
2. The lines connecting relays are separated from programme circuits.
3. In cases where the above precautions are inadequate the make-and-break contacts which interrupt all inductive circuits are shunted by a *spark quench* consisting of a 2-μF condenser in series with 200 ohms. These values are evidently not critical.

3.7. Cross-Talk between Programme Circuits. Assuming the use of balanced and screened circuits, this may occur owing to a variety of causes such as:

1. The impedance of the common H.T. or L.T. supply source. This is usually reduced in the design of the supply circuit by providing individual filtering for each circuit or each valve circuit, constituted in certain cases by series chokes and shunt paper or electrolytic condensers; in others by series resistances and paper or mica condensers in shunt to ground.

   With the tendency towards the use of mains-operated amplifiers, using individual power packs for supplying the H.T., this source of trouble will become less frequent.
2. Magnetic coupling between the cores of transformers in amplifiers on the same rack. This coupling usually occurs in the case of steel panels mounted on iron racks by the passage of linkage flux through rack and panels. It can be reduced by mounting the transformer half an inch or an inch above the panel. Rotation of the transformer about an appropriate axis sometimes clears the trouble.
3. Capacity between neighbouring jacks and neighbouring keys. This can very often be cleared by inserting small screens between the offending jacks or keys.
4. High-level circuits run in the same ducts as low-level circuits.

5. A fault condition in any of the anti-cross-talk arrangements, e.g. a shorted choke, a disconnected condenser, a high resistance in an earth circuit, etc.

4. Interference in Transmitter Circuits.

4.1. Interference from the High-Power Stages of One Transmitter into the Low-Power Stages of another Transmitter at the Same Site. This appears, in the A.F. output of a receiver tuned to one transmitter, as cross-talk constituted by the programme supplied to the other transmitter. If in one stage only, it can be located to that stage by successive disconnection or disabling of each stage. The cause of interference is often extremely difficult to locate, since, owing to the size of a transmitter, it is more than probable that the principles relating to earth connections established in XVII:9 to 12 have not been rigidly observed. The interference may, therefore, be introduced in the earth circuits, or by electromagnetic pick-up in the coils of the disturbed unit, or by electrostatic pick-up in that unit.

The run of the filament circuits and earth circuits should be examined and an attempt made to clear the fault by trying one or two rearrangements. Large dry electrolytic condensers, e.g. 1,000 μF, may be used to connect together various points at low p.d. to see if any improvement results. If unsuccessful, with the stage isolated and all supplies removed, attempts may be made to measure the currents induced in the tuned circuits of the stage, tuning them to the interfering frequency for this purpose. Once a means of observing the disturbing current has been established it is a comparatively simple matter to observe whether the interference is introduced from the earth circuits or by direct induction. According to the final diagnosis, the remedy is either a rearrangement of the earth system, improved screening, or possibly re-orientation of the tuning-coils.

In the case of apparent interference into a low-power-drive stage, or even the crystal drive itself, care should be taken to make sure that the R.F. feeder coupling the crystal drive to the first drive stage of the transmitter is properly bonded and that its sheath does not make contact with any possible sources of disturbing e.m.f., which may even be constituted by earth connections or the frame of the building.

4.2. Interference between Aerials of Two Transmitters on the Same Site. This is most serious when the difference between wavelengths is small, and in the case of short-wave stations the effect is accentuated when one beam array fires through another.

The effect observed is that with one transmitter alone radiating, volts are built up in the anode output circuit of the other, due to the *throw-in* or pick-up from the radiating transmitter, via the two aerials and feeder circuits. The effect is reciprocal : each transmitter builds up volts on the anode output circuit of the other transmitter. Owing to the non-linearity of the anode impedance, when both transmitters are radiating together, intermodulation occurs in the anode output circuits of each transmitter. The result is that, f_1 and f_2 being the respective carrier frequencies of each transmitter, in addition to the main carrier frequencies, each modulated with its own programme, there are also radiated carrier frequencies, $2f_1$, $2f_2$, $f_1 \pm f_2$, $f_1 \pm 2f_2$, $f_2 \pm 2f_1$, etc., each modulated with *both* programmes. The disadvantages of this are many : each transmitter loses power which is dissipated in the anode circuit of the other transmitter, the volts and currents in the circuit are increased and may bring components over their ratings, and power is radiated in unwanted intermodulation frequencies which jam other channels. Depending on the extent to which third-order non-linearity is present (see XVIII:5.226), the programme of each transmitter will appear as cross-talk modulation of the other transmitter's carrier.

The remedy is to insert some form of selective circuit in each transmitter output circuit, feeder, or aerial coupling circuit, which substantially passes only the frequency of the transmitter concerned and rejects the frequency of the other transmitter. The particular arrangement used varies with circumstances and with wave-lengths. See XVI:5 and 9.

5. Interference in Radio Receivers.

5.1. Interference Not Incoming to Aerial. It is evident that since a receiver contains an audio-frequency amplifier it is subject to the forms of interference described with reference to audio-frequency low-power circuits ; see XVIII:3.

In addition, the receiver is subject to direct pick-up of radio frequency : this can be reduced by enclosing the whole receiver in a screen. Radio frequencies are generated by mercury-arc rectifiers, commutators of rotating machines, especially when sparking, by electric railways, magnetos in cars, and by a variety of other local sources.

At radio frequencies a measure of electromagnetic screening is obtained by the use of a screen, in addition to electrostatic screening. This is due to the induction of eddy currents in the screen. Practically any thickness of screening which is mechanically strong enough is suitable ; the screens should be made of copper making continuous close contact.

The treatment of commutators with condensers, as described previously in XVIII:3.4, is particularly effective in reducing interference at radio frequencies.

Little or no advantage will be obtained by screening a receiver if the down lead from the aerial is not completely screened. The screen of the down lead must run right up to the screen of the receiver so that not a fraction of an inch of down lead is exposed, and then be bonded to the receiver. See XVIII:5.211. R.F. interference incoming on the A.C. mains supply leads to the receiver may be reduced by using a power transformer with an earthed screen between the windings. In addition an R.F. filter may be inserted in the mains leads. This filter should be balanced and the shunt elements should each consist of two condensers in series, the junction of the condensers being connected to the receiver earth.

R.F. interference sometimes enters a receiver through the audio-frequency output leads. This again can be removed by the use of a balanced R.F. filter with split elements connected from each leg of the circuit to the receiver earth.

5.2. Interference Incoming to Aerial. Means to reduce interference incoming to the aerial fall naturally into three categories according to the source of interference, which are as follows :

1. Nearby localized sources, within a distance comparable with the height of the aerial.
2. Other radio-frequency transmissions.
3. Other distant man-made sources.

5.21. Nearby Localized Sources. These are dealt with in two ways : by reducing the interference at its source and by screening the receiver and down lead.

Means for reducing the interference from commutators of machines and relays are described in XVIII:3.5 and 3.6.

At transmitting stations small discharges occur across aerial and stay insulators in wet weather and heavier discharges occur in thunder weather. In addition, if any loose metal contacts exist in any of the stay wire supports or between any pieces of metal in light contact which are modified when the wind blows, these provide a pernicious form of interference which is hard to locate. The disturbance is produced by the modification of the radiated field consequent on the redistribution of currents and voltages resulting from the variation of contact resistance. The only remedy is to locate the loose contact and bond the two surfaces together. A portable receiver is of some assistance in locating these sources, but not much.

Both these sources give rise to irregular sizzles and crackles in a receiver located on the site, and the only satisfactory method of eliminating them is to remove the receiver to some distance from the transmitting aerial. A palliative may be provided by locating the receiving aerial at a point (found with a portable receiver or, better still, a field strength measuring set) where the disturbances are least, and connecting it to the receiver through a length of concentric cable.

A department exists in the G.P.O. for dealing with interference from local sources. This department is not only prepared to give advice on matters relating to interference but also has the authority to deal with any local undertaking which may be operating equipment which is causing interference. Complaints from listeners which reach the B.B.C. are ultimately routed to this department.

5.211. Screening of Downlead. Interference sources at ground level, within a distance comparable with the height of the aerial, can be dealt with to some extent by screening the downlead.

The downlead of an aerial usually contributes very largely to the received signal strength, and if the downlead is screened, in so far as it diminishes the strength of any radiated field traversing the region of the aerial, it diminishes equally the strength of all radiated fields on any given wavelength, whether due to a wanted transmission, an unwanted transmission, or a distant disturbing source including noise.

In cases where the interfering source is nearer to the downlead than to the aerial, the effect of screening the downlead is effectively to move the aerial farther from the noise source. In such cases an appreciable improvement in signal-to-noise ratio can be obtained by screening the downlead.

The simplest possible form of screened downlead is shown in Fig. 1 (*a*), where S is the disturbing source and C_1 the original capacity through which disturbing currents would flow to the part of the downlead indicated if the screen were removed. It is to be noted that disturbing currents still flow through capacity C_2 to the unscreened part of the aerial.

Circuits have been devised to run a balanced pair in a screen through the region where disturbance is experienced. Two of these, one for vertically polarized waves and one for horizontally polarized waves, are shown at (*b*) and (*c*) in Fig. 1, the aerial, which is nominally balanced, being connected to the balanced feeder through a transformer with both windings balanced. The arrangement at (*b*) has the disadvantage that the lower end of the aerial projects down towards the disturbing field, and further, the aerial is not balanced. In the case of 1 (*c*) the transformer is only necessary to match the aerial to the feeder in the case where the overall length of the hori-

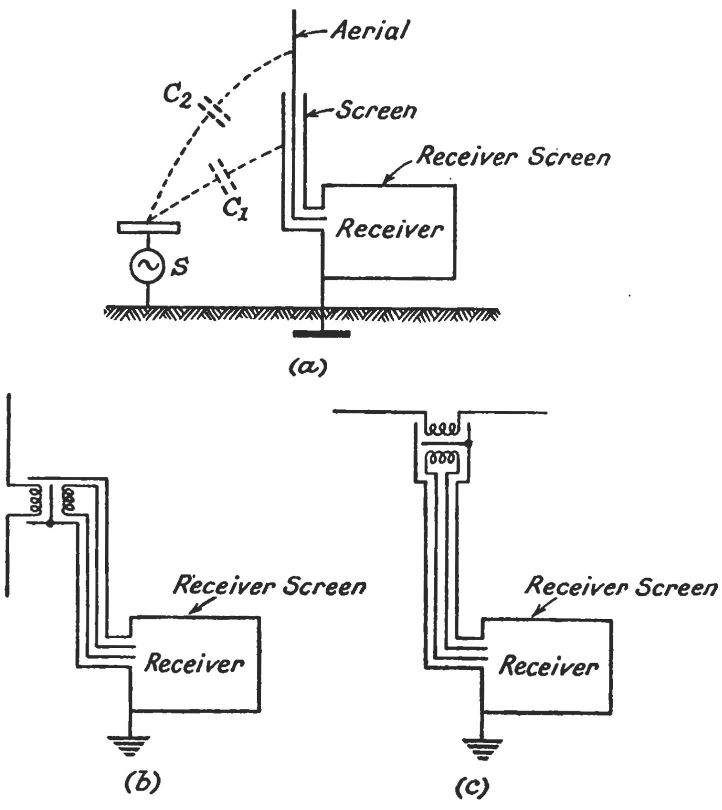

FIG. 1/XVIII:5.—Screened Downlead Arrangements.

zontal part of the aerial is other than half a wavelength, a condition which arises on medium and long waves. Since most medium-wave transmissions are vertically polarized, the arrangement in Fig. 1 (*c*), when used for medium waves, depends for successful transmission entirely on the ray reflected from the ionosphere. For this reason it is suspect. When used for horizontally polarized short waves, the overall length is made equal to half a wavelength (the aerial then being called a dipole), the transformer is omitted and the arrangement is ideal. Since the downlead constitutes a balanced circuit the screen could theoretically be omitted ; but as the receiver input is usually badly balanced an advantage may result from screening the downlead.

5.22. Interference from other Radio-Frequency Transmissions. It is assumed that the reader is familiar with the principles of receiver design outlined in XIX.

5.221. Adjacent Channel Interference. Sideband Overlap. The spacing of transmitter carrier frequencies in the broadcast bands is 9 kc/s, and since it is permitted to modulate broadcast transmitters with an audio-frequency band extending up to 10 kc/s, the sidebands of each transmitter may extend 10 kc/s each side of the carrier frequency. The upper sideband of the lower frequency channel of two adjacent channels may therefore completely overlap the lower sideband of the higher frequency channel. In practice, the interference consequent on this is minimized by allocating adjacent wavelengths to stations remote from one another. In addition, many transmissions do not extend above 6 kc/s in the audio-frequency range, while the average spectrum of speech and music is such that at frequencies above about 2 kc/s there is a fairly rapid falling away in amplitude. The effect of sideband overlap, when it does occur, is an irregular unintelligible sound, resultant on the fact that the sideband, corresponding to each original audio-frequency f (of the disturbing channel), on being intermodulated with the wanted received carrier 9 kc/s from the original frequency, is reproduced with a frequency $= 9{,}000 - f$ c/s. The resulting cacophony has been likened to monkey-chatter.

5.222. Adjacent-Channel Interference : Heterodyne Note. As a corollary of the above, it may be noted that the adjacent carrier frequency itself, intermodulating with the wanted carrier, will give rise to a 9-kc/s note. This is usually referred to as a heterodyne note : since the term heterodyne is quite general, it should really be called an adjacent-channel heterodyne. Heterodyne notes are reduced by the use of I.F. filters of reduced width, and by rejector circuits tuned to 9 kc/s inserted in the A.F. circuits of the receiver.

5.223. Adjacent-Channel Interference : Sideband Splash. In all transmitters a certain amount of non-linearity exists, giving rise to harmonic distortion in the audio-frequency envelope.

Second harmonics of audio frequencies greater than 5 kc/s and third harmonics of frequencies greater than $3\frac{1}{3}$ kc/s give rise to sidebands located more than 10 kc/s away from the carrier frequency. These give rise to frequencies in the adjacent channels which usually are noticeable only on the peaks of modulation and generally appear as intermittent irregular squeaks and squawks.

They can only be eliminated at the transmitter. Assuming the distortion to occur in the modulator and to have been reduced to a minimum, sideband splash can be reduced by inserting an audio-frequency low-pass filter, with a cut-off at or below 10 kc/s, between the modulator and modulated amplifier, except in the case where series modulation is in use. If sideband splash occurs in the modulated amplifier the only remedy is to improve the modulated amplifier : in practice, no transmitter aerial circuits can be made sufficiently selective to have any useful effect in reducing it.

5.224. Adjacent-Channel Interference due to Poor Selectivity. In a straight receiving set, consisting of a tuned radio-frequency (T.R.F.) amplifier followed by a detector and low-frequency stages, the adjacent-channel discrimination is always poor, and such sets can only receive stations of which the field strengths are very much larger than the fields of adjacent-channel transmitters.

In a superheterodyne receiver discrimination against adjacent-channel interference depends on the width of the I.F. filter and its rate of rise of attenuation. The I.F. filter response curve, therefore, determines the performance of the receiver from the point of view of adjacent-channel interference.

Owing to poor selectivity, an unwanted neighbouring signal may reach the detector where, in addition to beats between the wanted carrier and the unwanted signal, beats occur between the unwanted carrier and its sidebands, so that the unwanted programme appears as cross-talk.

5.225. Adjacent-Channel Interference Due to Non-linearity : Cross Modulation. Since the signal frequency circuits of a superheterodyne receiver provide substantially no discrimination against adjacent-channel frequencies, all the signal frequency stages receive and amplify the adjacent-channel frequencies. (A qualified exception may be made in the case of long waves.) If any of these valves possess third-order non-linearity, one of the products of intermodulation of the wanted carrier and an adjacent-channel modulated carrier is the wanted frequency modulated with the adjacent-channel programme (see XVIII:5.226). The adjacent-channel programme, therefore, appears as interference on the wanted programme.

With receivers of poor selectivity this effect, which is called cross-modulation, can take place with channels differing in frequency from the wanted frequency by more than 9 kc/s, i.e. farther away than the adjacent-channel. Cross-modulation in receivers sometimes leads to spurious complaints from listeners of (radiated) cross-talk between programmes, and this is one of the first points to investigate when such complaints are received.

It is not essential that the non-linearity should occur in the receiver ; a poor joint or connection in the aerial circuit may constitute a source of non-linearity capable of giving rise to cross-talk.

In all arts there are certain unestablished theories to explain anomalies, and the occurrence of cross-modulation under circumstances where the receiving circuit was believed to be guiltless has led to the belief that tuned non-linear circuits occasionally occur naturally in buildings or elsewhere, giving rise to cross-modulation observed by receivers in the neighbourhood. Cases where this cause has been suspected may, however, be due to the Luxembourg effect described in XVIII:5.227.

5.226. Analysis of Effect of Third-Order Non-linearity in a S.F. Amplifier. In general, the anode-current grid-voltage characteristic of a valve over its operating range can be expressed by the relation

$$i_a = k_0 + k_1 e + k_2 e^2 + k_3 e^3 + \quad . \quad . \quad . \quad (1)$$

when e is the instantaneous value of the grid volts.

The output will therefore contain contributions corresponding to all these terms.

Consider the output

$$i_3 = k_3 e^3 \quad . \quad . \quad . \quad . \quad (2)$$

and suppose that two modulated waves are applied to the input of the last signal frequency stage (in which the amplitude is largest and therefore the non-linear effects are most pronounced).

Let the wanted wave be $A_1(1+m_1 \sin v_1 t) \sin c_1 t$, corresponding to a carrier frequency f_1, and the unwanted wave be

$$A_2(1+m_2 \sin v_2) \sin c_2 t,$$

corresponding to a carrier frequency f_2. The sum of these two waves may be considered to constitute the input grid voltage e.

If E_g is the grid bias

$$i_3 = k_3[\underset{(a)}{A_1(1+m_1 \sin v_1 t) \sin c_1 t} + \underset{(b)}{A_2(1+m_2 \sin v_2) \sin c_2 t} - \underset{(c)}{E_g}]^3 \quad (3)$$

Calling the terms inside the cube bracket a, b and c respectively,

$$i_3 = k_3(a^3+b^3-c^3+3ab^2+3a^2b$$
$$+3bc^2-3b^2c+3ac^2-3a^2c-6abc) \quad (4)$$

The terms a^3, b^3, c^3, $3b^2c$, $3bc^2$, $3a^2c$, $3ac^2$ evidently contain no intermodulation products between a and b. This leaves only three terms, $3ab^2$, $3a^2b$ and $-6abc$. The term

$$6abc = 6E_gA_1A_2(1+m_1 \sin v_1t+m_2 \sin v_2t$$
$$+m_1m_2 \sin v_1t \sin v_2t) \sin c_1t \sin c_2t$$

This corresponds to two carrier frequencies respectively of frequency f_1+f_2 and f_1-f_2, each modulated with the two programmes and with the sum and difference frequencies of the two programmes. It illustrates the type of intermodulation which occurs also due to square-law non-linearity, and is not of interest to the present discussion.

The term ab^2 is however of interest.

$$ab^2 = A_1A_2^2(\sin c_1t+m_1 \sin c_1t \sin v_1t)$$
$$\times (1+2m_2 \sin v_2t+m_2^2 \sin^2 v_2t) \sin^2 c_2t$$

But $$\sin^2 c_2t = \tfrac{1}{2} - \tfrac{1}{2} \cos 2c_2t$$

From which it follows that among others a term appears:

$$\tfrac{1}{2}A_1A_2^2 \sin c_1t(1+2m_2 \sin v_2t - \tfrac{1}{2}m_2^2 \cos 2v_2t) \quad . \quad (5)$$

This is a carrier of frequency equal to the wanted carrier modulated by the modulation of the unwanted transmission, but to twice the depth of modulation of the original unwanted carrier, and modulated also (to half the depth) by double-frequency components of the unwanted programme. It follows that third-order non-linearity in the signal frequency stage must be avoided. Such interference may occur from a channel other than an adjacent channel, if the signal is sufficiently strong and is not so far away that the signal frequency attenuation suppresses it.

5.227. Luxembourg Effect. When one transmitter gives rise to a high field strength in the ionosphere and another transmitter gives rise to a weak field strength in the same region, the weaker field-strength signal, on reception after reflection at the ionosphere layer, is sometimes found to be partially modulated by the modulation signal carried by the high-strength carrier. This effect is sometimes quite serious. Third-order non-linearity therefore appears to exist under certain conditions of the ionosphere.

5.228. Second-Channel Interference. See also XIX:9. To receive the wanted frequency the oscillator of a superheterodyne receiver is usually set to a frequency f_0, greater than the wanted frequency f_1 by an amount equal to the I.F. frequency f. The difference frequency $f_0 - f_1 = f$ traverses the I.F. circuits. If a carrier frequency f_2 greater than f_0 reaches the aerial and f_2 is such that $f_2 - f_0 = f$, the difference frequency will pass through the I.F. circuits and cause *second-channel interference.* Since the signal frequency circuits are tuned to the wanted frequency f_1, and since $f_2 = f_1 + 2f$, the response of the signal frequency circuits at f_2, the second-channel frequency is much less than at the wanted frequency. The difference between the response of the signal frequency circuits at f_1 and f_2, measured in decibels, determines the second-channel selectivity of the receiver.

Similar arguments to the above apply with equal force in the rare cases where the beat oscillator is set below the wanted frequency. The remedy is a rejector circuit directly in the aerial lead, tuned to the frequency of the unwanted second-channel transmission.

5.229. Third- and Fourth-Channel Interference, etc. It will be evident that if the beat oscillator produces harmonics, any frequency incoming to the aerial which is distant from any harmonic by a frequency interval equal to the I.F. frequency will give rise to a difference frequency which will traverse the I.F. filter and give rise to a signal in the output of the receiver. The only remedy for this, apart from improving the purity of the beat oscillator, which is a major design problem, is to insert a filter between the beat oscillator and the mixer. As the beat oscillator covers a series of bands of frequencies, this also is a measure which is only applicable in desperate cases. The terms " third and fourth channel " are not in general use but appear as good as any other to describe this type of interference.

5.22.10. Double Modulation in the Mixer. If the mixer, after producing the normal difference or beat frequency, then proceeds to double it, any frequency incoming to the aerial which is situated a distance from the beat-oscillator frequency (or any of its harmonics) equal to half the I.F. frequency, will give rise to a frequency which will traverse the I.F. filter. The remedy is to try a new mixer valve. This is one of the classified effects, but it is rather hard to see how the mixer can act in this way.

5.22.11. Direct Reception of I.F. Frequency. If the signal frequency selectivity is not adequate, a frequency incoming from the aerial which lies within the pass range of the I.F. filter will pass through the I.F. filter and give rise to a signal in the output of the receiver. The remedy is a rejector circuit directly in the aerial lead, tuned to the I.F. frequency.

5.22.12. Mixed-Channel Interference. If two carrier frequencies occur with a frequency difference between them equal to the I.F. frequency of a receiver, when the signal frequency circuits are tuned to either carrier, the difference frequency will pass through the I.F. circuits and contribute interference consisting of the modulations on both carriers. It is evident that the interference will rise to a maximum with the signal frequency circuits tuned somewhere between the carriers in question. It is not usual to use a criterion for the selectivity of a receiver against mixed-channel interference, but if one is required, the attenuation of the signal frequency circuits, where the band width is equal to the I.F. frequency, is as good a guide as any other.

Fortunately mixed-channel interference is usually not very serious. A predisposing cause is the proximity of a transmitter.

5.22.13. Interference from Harmonics of Transmitters. If a transmitter radiates harmonics of its carrier frequency these will occupy and jam the channels allocated to those frequencies. The remedy is to suppress the harmonics at the transmitter. This is done by increasing the kVA/kW ratio of transmitter output and aerial coupling circuits (see VII:14 and XVI:8), by the proper choice of coupling circuits, by inserting low-pass filters of conventional type, by using feeders as filters (see XVI:5), and in certain extreme cases by inserting rejector circuits as described in XVI:9 and 10, where their use is described for a different purpose.

As a guide to the degree to which harmonics of the carrier frequency must be suppressed, the requirements laid down in Appendix I of International Radio Communication Regulations, Cairo 1938, are useful : these are as follows :

Frequencies below 3000 *kc/s* (*Wavelengths above* 100 *Metres*) : The harmonic field strength must not exceed 300 microvolts per metre at 5 kilometres from the transmitting station. In the case of directive aerials this appears rather ambiguous ; for aerials designed to have equal radiation in all horizontal directions and to have maximum radiation in a horizontal direction it is however quite clear. The criterion below is evidently designed for directive aerials.

Frequencies above 3000 *kc/s* (*Wavelengths under* 100 *Metres*) : The power of harmonic frequency in the aerial must be 40 db. below the power of the fundamental (carrier) frequency, and in no case must it be above 200 milliwatts.

5.22.14. Use of Rejector and Acceptor Circuits. Assuming that the radiated field of the wanted stations is the highest that can be obtained, the receiver is the best available, and the location of the receiving site is either the optimum or else there is no other choice of site, the pick-up from unwanted transmissions may be reduced by the use of rejector and acceptor circuits.

These are circuits inserted in the aerial lead, introducing a high attenuation at an unwanted frequency. They are only of use when the frequency of the unwanted station differs from the frequency of the wanted station by an adequate frequency interval. The necessary value of this interval depends on the relative field strength of the two transmissions and the type of rejector used. In Fig. 2 (*a*) is shown a simple type of rejector circuit made of the inductance L_1 and capacity C_1. In Fig. 2 (*b*) is shown an alternative type of circuit which is called an acceptor circuit and consists of inductance L_2 and

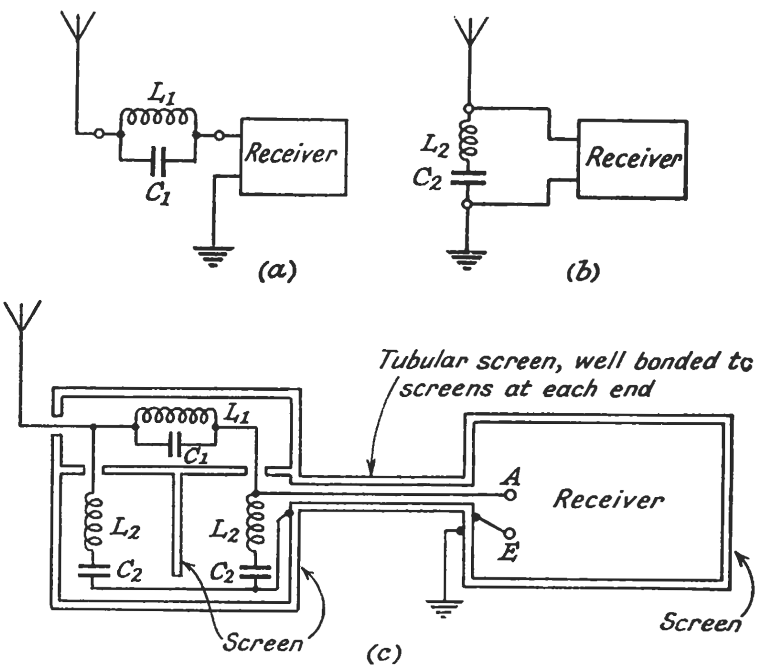

FIG. 2/XVIII:5.—(*a*) Rejector. (*b*) Acceptor. (*c*) Band Elimination Filter.

capacity C_2. Rejector circuits operate by offering a high impedance to the unwanted frequency, while acceptor circuits operate by offering a low shunt impedance to the unwanted frequency and so bypassing the receiver.

In Fig. 2 (*c*) is shown a screened π rejector circuit which gives a much greater attenuation of the unwanted frequency and can be made to discriminate between wanted and unwanted frequencies which are nearer together than in cases where the circuits of Figs. 2 (*a*) and 2 (*b*) are suitable. This consists of a combination of inductances and capacities respectively of value L_1, C_1, L_2 and C_2.

If the band of frequencies to be rejected is from f_1 cycles per second to f_2 cycles per second, the values of L_1 and C_1, L_2 and C_2, in Fig. 2 (*c*) are given by

$$L_1 = \frac{(f_2 - f_1)R}{\pi f_1 f_2} \qquad C_1 = \frac{1}{4\pi(f_2 - f_1)R}$$

$$L_2 = \frac{R}{2\pi(f_2 - f_1)} \qquad C_2 = \frac{f_2 - f_1}{2\pi f_1 f_2 R}$$

The values of inductance are in Henrys and the capacities in Farads.

R is the impedance looking into the receiver, which, *for this purpose*, can usually be assumed to be in the neighbourhood of 200 to 400 ohms.

Strictly, the values of the π section should conform to these values, but if the values of L_2 are difficult to realize it is permissible to use $\frac{1}{2}L_2$ and $2C_2$ in the shunt arms, instead of L_2 and C_2. An artful means of bringing the value of L_2 within easy reach of practical construction is by assuming a different value for R ; this will be found to give good results in practice, provided the assumed value of R is not greater than 5 times or less than one-fifth the receiver impedance. The choice of f_1 and f_2 depends on the separation between wanted and unwanted signals. In general, f_1 and f_2 should not be less than 20 kc/s apart and should be on each side of the unwanted frequency and equidistant from it. If the wanted frequency is 100 kc/s away from the unwanted frequency, $f_2 - f_1$ should be about 40 kc/s.

The most probable optimum values of L_1, C_1, L_2 and C_2 in Fig. 2 (*a*) and (*b*) respectively are also given by the above relations (assuming values for f_1 and f_2 as for Fig. 1 (*c*)), but in any particular case some advantage may be obtained by adjustment of the L to C ratio. In other words, a rejector circuit is made to consist of the series arm only of the π network and an acceptor circuit is made equal to one of the shunt arms of the π network. The choice between rejector and acceptor circuit is usually determined by the elements which are most easily realized in practice. When the real input impedance of the receiver is not known, experiment must decide which gives the best results.

As stated, the π circuit gives a much higher attenuation of the unwanted frequency than the simpler circuits and is sometimes the solution to eliminating interference from a local transmitter in the case of a receiver at a transmitter site. In such cases it is usually necessary to screen both rejector circuit and receiver. Particular notice should be taken of the arrangement of the conductors, screens and earthing in Fig. 2 (*c*). There is no direct earth on the rejector screen which picks up an earth via the *continuous* screen running to the receiver screen.

It should also be noted that the π rejector circuit is capable of rejecting a *band* of frequencies of *any* required width, i.e. extending from f_1 to f_2. For improved π rejector, see XXV:6.

5.23. Distant Man-Made Sources. Since in general a disturbing source such as an electric railway or a mercury-arc rectifier radiates a broad spectrum of frequencies embracing the wanted frequency, rejector circuits are not applicable. The same of course applies to a transmitter which is operating on a frequency so near to the wanted frequency that no practical rejector circuit of adequate discrimination can be built. None of the methods specified for the reduction of interference is therefore applicable and the disturbing fields have to be treated as Noise, see XVIII:7.2. Also see XXV:6 for further discussion on band elimination filters.

6. Noise.

6.1. Noise in Apparatus, including Noise in Radio Receivers, which is not Incoming to the Aerial. A prolific source of noise is bad contacts. These may be due to badly soldered joints, an utterly inexcusable but common source of trouble, a screw terminal without a lock nut which has worked loose owing to vibration, or to faulty apparatus. Noise due to a bad contact is usually associated with D.C. or A.C. circuits, but may occur in low-level programme circuits owing to the presence of contact potentials. The immense trouble which a simple fault may need to locate it justifies very great care in making all connections.

Bad contacts may occur in switches, plugs and sockets, jacks, and between valve pins and their sockets. Faults which resemble bad contacts in their effects as observed at the output of an audio-frequency amplifier are described immediately below: parasitic oscillations at ultra-short waves and battery noises. See also XVIII:6.4 and 6.5.

6.2. Parasitic Oscillations. These are various. If an A.F. amplifier valve oscillates at ultra-short waves the circuit noise may sound like a loose contact. This is a rare occurrence, but has occurred and is mentioned first because it can be confused with a loose contact. The probable remedy is a stopper resistance, but a rearrangement of the run of the wiring may clear the trouble. Audible oscillations in A.F. amplifiers are usually cleared by elimination of longitudinal currents and proper earthing and screening.

Oscillation in transmitters can hardly be considered as noise, since they are observed by meter readings and cleared without regard to the pathology of the radiated signal, although observation of a distorted output may lead to a search for an oscillation. See XI:12 for full discussion of parasitic oscillations.

6.3. Noisy Batteries. Both H.T. and L.T. batteries, when newly charged, or nearly discharged, are liable to be noisy when used on high-gain amplifiers handling very low-level inputs. Dry batteries, at the end of their life, also become noisy. Circuit design sometimes provides for such contingencies by the insertion of smoothing.

6.4. Faulty Valves. Valves which are going soft sometimes give rise to a rushing sound in A.F. amplifiers. Internal deposits on electrodes and internal faults in valves also give rise to noises like a loose contact. Bad contacts in valve sockets are a frequent source of trouble and sockets and pins should be cleaned periodically.

6.5. Faults in Other Components. Transformers, condensers, chokes and resistances develop internal bad contacts, while electrolytic condensers develop noise associated with the deterioration of the active film. Conductors break inside the insulation. Oil-filled condensers lose their insulation due to flashover. Sparking occurs at points where inadequate clearance has been left. The vanes of air condensers get bent and touch or spark across. Contacts in relays and switches get dirty.

6.6. Mechanical Vibration of Electrodes : Ponging. If a valve carrying a low level of programme energy is subject to vibration, the movement of its electrodes modulates the anode current of the valve with the result that, in the A.F. part of the circuit following, a musical, bell-like, ringing noise is heard, which is called ponging. So sensitive are some valves to ponging that it is only necessary to speak near the valve in order to cause ponging. Ponging is, therefore, caused by mechanical or acoustic vibration. Mechanical vibration is prevented by spring mounting the whole amplifier, by spring mounting valve sockets or by mounting valve sockets on rubber. Acoustic vibration is prevented by putting small felt caps on valves, by enclosing the valves in a box and in extreme cases by lining the box with sound-absorbent material such as Celotex.

6.7. Johnson Noise : the E.M.F. of Thermal Agitation. Consonant with the agitation of molecules and atoms which constitutes heat energy, the electrons in a conductor are in a permanent state of movement which results in a system of random e.m.f.s appearing at the terminals of the resistance. This was first discovered by J. B. Johnson. The sound appearing at the output of a high-gain amplifier due to Johnson noise is rather like that due to the pattering of rain on dry ground. The frequency spectrum of Johnson noise is uniform throughout the frequency range : equal frequency band widths contain equal energy.

The noise voltage is therefore proportional to the square root of the frequency range ; it is also proportional to the square root of the resistance and the square root of the absolute temperature.

The noise e.m.f. in a resistance R ohms developed in a frequency range extending from f_1 cycles per second to f_2 cycles per second with the resistor at T degrees K (Absolute) is

$$E_n = 2\sqrt{kRTb} \text{ volts}$$

where k is Boltzman's constant $= 1{\cdot}37 \times 10^{-23}$ joule per degree and $b = f_2 - f_1$

Hence

$$E_n = 7{\cdot}4 \times 10^{-6}\sqrt{RTb} \text{ microvolts.}$$

With the resistance on open circuit the voltage developed across the terminals of the resistance is E_n.

With the resistance shunted by a capacity C the voltage developed across the condenser due to the e.m.f. of thermal agitation in the resistance is

$$V_n = 7{\cdot}4 \times 10^{-6}\sqrt{\frac{T}{2\pi C}(\tan^{-1} 2\pi RCf_2 - \tan^{-1} 2\pi RCf_1)}$$

microvolts.

6.8. Shot or Schrot Effect : Schotky Noise. A noise e.m.f. with a similar spectrum to Johnson noise is produced in the anode circuit of thermionic valves due to the fact that electrons are not emitted from the cathode in a steady stream, but in packets.

In magnitude it is greater than the e.m.f. of thermal agitation in the anode circuit but, at frequencies below about 5 Mc/s, is usually less than the amplified voltages applied by the e.m.f. of thermal agitation in the circuit connected to the grid of the valve. Above 5 Mc/s, owing to the low values of circuit impedance, shot effect is usually greater than Johnson noise.

The magnitude of the Schotky noise in a valve is usually stated by specifying the value of resistance which, when connected to the grid, will (by virtue of the e.m.f. of thermal agitation in it) produce an e.m.f. in the anode circuit of the same magnitude as the Schotky noise.

The magnitude of shot noise of a triode expressed in terms of the equivalent noise on the grid is

$$V_g = 6{\cdot}17 \times 10^{-3}\sqrt{(f_2 - f_1)/g_m} \text{ microvolts}$$

where g_m is the mutual conductance of the valve in mA/volt.

Partition Noise. When the cathode current divides between two collecting electrodes, as for instance it does in a pentode in which the cathode current is divided between cathode and screen, a new current fluctuation is introduced on each of the divided components. This is known as *partition noise.* For all practical purposes it can be considered as introducing a factor by which the shot noise must be multiplied to give the equivalent noise on the grid.

In the case of a pentode the value by which V_g, determined from the equation immediately above, must be multiplied to give the effective value of anode shot noise including partition noise, expressed as equivalent volts on the grid is

$$\sqrt{\frac{I_a}{I_{ca}}\left(1 + 8\frac{I_{sc}}{g_m}\right)} \text{ (see Bibliography, I 8.1)}$$

where I_{ca} = the total cathode current in mA
I_a = the anode current in mA
I_{sc} = the screen current in mA
g_m = the mutual conductance of the pentode from control grid to anode in mA/volt.

6.9. Flicker Effect. J. B. Johnson (*Phys. Rev.*, 1925, vol. 26, p. 71) found that in the frequency range below 1,000 c.p.s. the shot voltage in the absence of space charge (i.e. in a valve which is in filament saturation so that the anode draws all free electrons to the anode and does not permit a space charge to collect) may rise to a thousand times its normal value. This is believed to be due to bursts of emission and positive ions. Although variable leaks between electrodes have been cited as contributive to this effect, it seems more probable that they constitute a separate phenomenon.

In an unsaturated valve, i.e. a normal valve with space charge, the magnitude of the flicker effect expressed as equivalent volts on the grid of the valve is

$$V_g = 0{\cdot}316\sqrt{\log_e(f_2/f_1)} \text{ microvolts.}$$

6.10. Grid Current Noise. In any valve which has either a positive or negative component of grid current, this current will have a fluctuation which will give rise to an effective noise voltage on the grid.

If the grid resistance R ohms is shunted with a condenser C farads and the grid current is I_g mA, the noise voltage on the grid is

$$V_g = 1{\cdot}8 \times 10^{-5}\sqrt{I_g\,(\tan^{-1} 2\pi CRf_2 - \tan^{-1} 2\pi CRf_1)} \text{ microvolts,}$$

where f_1 and f_2 are the limits of the frequency range in cycles per second.

This formula is not universally applicable because valves have both positive and negative components of grid current simultaneously and it is impossible to separate the two components. Fortunately, however, valves are normally used under conditions in which the ionic grid current predominates, so that the above formula is applicable using the measured value of grid current.

The author would like to express his appreciation of the help he has received from Mr. A. B. Gillespie, who provided the original information from which the above values of valve noise voltages are derived.

6.11. Transit Noise. This is noise induced into the grid of a valve by the passage of electrons past the grid and so causing changes in the potential of the grid.

6.12. Signal to Noise Ratio of a Receiver.

6.12.1. Noise Factor of a Receiver or other Fourpole or Transducer. *Definitions.* The *available power* of a generator supplying signal or noise is the power supplied to a load connected to the generator, the load having an impedance which is the conjugate of that of the generator.

The *energy band* of a receiver or other fourpole is defined as

$$b = \int A\,df/A_0,$$

where A_0 is the power amplification factor at the arithmetic mid band and A is the power amplification factor at any frequency.

Then the Noise Factor is

$$N = \frac{\text{Available Noise Power at Receiver Output before Detection.}}{\text{That part of the Available Noise Power in the Receiver Output contributed by the Impedance } R_a \text{ facing the Receiver Input assumed to be at an Absolute Temperature of } 290^\circ.}$$

As is substantially the case, it will be assumed that the spectra of thermal noise, receiver noise and aerial noise are identical. In this case it may be shown that

$$N = \frac{N_r + N_{290}}{N_{290}},$$

where N_r = the receiver output noise power expressed in terms of equivalent available noise power in the impedance R_a, facing the input of the receiver, required to give the actual receiver output available noise power, in the absence of any other thermal or aerial noise in R_a.

N_{290} = the available noise power in impedance R_a within the energy band of the receiver due to thermal noise when R_a is at an Absolute temperature of $290^\circ = 4kTb$.

In the whole of this discussion it is assumed that R_a is equal to the impedance presented towards the input of the receiver by the transferred aerial impedance, assumed to be transformed through a lossless network, and that the receiver input impedance is unknown, and in general unequal to R_a. It is this ignorance which makes the concept of available power useful, since by its introduction the input transition loss cancels when calculating signal to noise ratio. For dealing with the case where the network between aerial and receiver input is not lossless reference should be made to L. A. Moxon, "The Noise Characteristics of Radio Receivers", *I.E.E. Jour.*, vol. 93, part IIIA, no. 6.

It will be evident that the Noise Factor is not a unique characteristic of the receiver, but is a characteristic of the receiver in combination with a certain source impedance R_a.

The value of $N_{290} = 4kTb$ watts,

where k = Boltzman's constant $= 1{\cdot}37 \times 10^{-23}$ joules per cycle

$T = 290^\circ$ Absolute.

b = the energy band in cycles per second calculated as above.

Hence $N_{290} = 1{\cdot}6b \times 10^{-20}$ watts, and this value is entered into the denominator of the expression for N.

The value of the numerator in the expression for N, i.e. the value of $N_r + N_{290}$ is measured as follows.

A saturated diode is arranged to supply noise energy to a resistive impedance R_a, either by shunting a resistance of magnitude R_a across the diode, or by coupling a resistance R_a to the diode through a lossless transforming network of band width not less than b. In general, therefore, the diode is faced with some other impedance than R_a, which will be referred to as rR_a, where r is the impedance ratio between R_a and the diode. The temperature of R_a is adjusted to 290° Abs.

The noise energy supplied by the diode is adjusted by varying the filament current until the required anode current is obtained, and so the required output noise energy. With the diode connected to the input of the receiver through the impedance transforming network and the resistance R_a in shunt across the input of the receiver and the output of the network, the noise power supplied by the diode is adjusted by varying the diode anode current until the output noise power observed at the output of the receiver with the diode connected is twice that observed at the output of the receiver with the whole input circuit left connected and the diode cold.

The value of $N_r + N_{290}$ is then equal to the noise output of the saturated diode $= 2ebI \times 10^{-3}$ watts,

where e is the charge on an electron in coulombs $= 1{\cdot}6 \times 10^{-19}$ coulombs

I is the diode anode current in mA

and b is the energy band in cycles per second.

The noise factor is therefore $\dfrac{2ebI}{4kTb} \times 10^{-3} = 0{\cdot}02I$ since $T = 290°$ Abs.

The author is indebted to Mr. N. Houlding for information on definition and determination of noise factors.

6.12.2. Noise Temperature of an Aerial. Assuming that the aerial is coupled to the input of the receiver through a network which contains reactances only (and so is lossless), it is possible to express the noise that comes in from space by supposing that it obeys the same laws as thermal noise and assigning a temperature to the transferred aerial impedance R_a indicative of the available noise power due to the aerial. If T_a is the aerial temperature so assigned, the value of available aerial noise power is:

$$N_a = kT_a bR_a \text{ watts.}$$

Value of Aerial Noise Temperature. The noise picked up by an aerial depends on the part of the sky towards which its polar diagram is directed, the minimum value being about 25% of the maximum, see L. A. Moxon, "The Noise Characteristics of Radio Receivers", *I.E.E. Jour.*, vol. 93, part IIIA, no. 6. This noise is greatest when the aerial beam is directed in the plane of the galaxy and least when directed normal to the galaxy. Moxon gives observations of noise temperature taken by himself and others, from which it appears that, within the limits of experimental error, neglecting solar noise, the limiting values of the noise temperature of an aerial are as given below.

Maximum Maximum Value Associated with the Centre of the Galaxy.

$$T_a = 2700 \times \left(\frac{100}{f}\right)^{2{\cdot}7} \text{ degrees Absolute.}$$

Minimum Value in Direction Normal to Plane of Galaxy.

$$T_a = 600 \times \left(\frac{100}{f}\right)^{2 \cdot 1} \text{ degrees Absolute,}$$

where f is the midband frequency of the energy band in megacycles per second, e.g. if $f = 100$ Mc/s, then the maximum maximum value of $T_a = 2{,}700$ degrees Abs.

6.12.3. Determination of Received Signal to Noise Ratio. This depends on :

E = received field strength in millivolts per metre

G = aerial gain expressed as a power ratio = power received from signal direction divided by power received from any direction by an isotropic aerial (i.e. with a spherical polar diagram) situated in the same signal field. (See XVIII:8.1.)

λ = wavelength in metres

$N_a = kT_abR_a$ = available noise input power from aerial (this may conveniently be assumed to be the available noise power in R_a)

T_a = aerial temperature in degrees Abs.

The aerial is then situated in a signal field of power density :

$$W_m = 2 \cdot 65 \times 10^{-9}E^2 \text{ watts per square metre.}$$

The equivalent area of the aerial (i.e. the area measured normal to the direction of propagation of the received signal field, which, when multiplied by the power density of the field, gives the power collected from the field by the aerial) is :

$$S = G\lambda^2/4\pi.$$

The available signal power in the aerial is :

$$S_a = SW_m.$$

From the definition of noise factor, the equivalent available noise power in R_a corresponding to the receiver noise is :

$$N_r = (N - 1)N_{290}.$$

The receiver signal to noise (power) ratio is therefore :

$$\frac{S_a}{N_a + (N - 1)N_{290}}.$$

7. Noise Incoming to Receiving Aerials.

Atmospheric or static disturbances in nature are generated by thunderstorms, aurora and other discharges in the ionosphere, and some originate outside the earth's atmosphere and even outside the solar system. Man-made static is produced by electric railways, magnetos of cars and aeroplanes, and in brief by any generator of transient disturbances in the form of a radiated field. Such disturbances are always strong in the neighbourhood of towns and industrial areas. Where the disturbing sources are near to the receiving aerial certain means of reducing their effect has been discussed under the heading of interference.

7.1. Restriction of Frequency Range. By restricting the received audio-frequency range some measure of reduction of noise due to atmospherics can be obtained. The voltages generated by atmospherics are random in phase and are therefore proportional in magnitude to the square root of the received band width. If, therefore, the received audio-frequency range is reduced for instance to half, the mean noise voltage is reduced by a factor $1/\sqrt{2}$. The effective reduction in annoyance value is however less than is to be expected from the ratio, i.e. less than 3 db., because the normal method of restricting the range consists in cutting out the upper part of the audio-frequency range where the ear is not so sensitive and where the masking effect of the noise is less. It is a practical proposition, for instance, to cut down the received frequency range from 30 to 10,000 c/s to 30 to 5,000 c/s. This is done on certain short-wave receivers by changing the width of the I.F. filter.

7.11. Single Sideband. See also XIX:17. In commercial radio telephony the frequency range occupied by the radiated space wave is reduced by transmitting only one sideband and the carrier is also suppressed and restored at the receiving end. This gives a net effective improvement in speech-to-noise ratio equal to 12 db. This is obtained as follows:

Suppressing one sideband *degrades* noise ratio by 3 db. Doubling amplitude of the other sideband *improves* noise ratio by 6 db.

Restricting frequency range of receiver, to receive one sideband instead of two, *improves* speech-to-noise ratio by 3 db.

Suppressing the carrier and doubling the amplitude of the remaining sideband again so that it is now four times its amplitude in the original normally modulated wave, *improves* the speech-to-noise ratio another 6 db.

The restored carrier is added with an amplitude large compared to the received sideband amplitude so that the detected wave is free from distortion. See VIII:4.2 and Figs. 2 (*a*), (*b*) and (*c*)/VIII:4. This complicates the receiver, but since in point-to-point radio links there is only one receiver, of negligible cost and power consumption compared to the transmitter cost and power consumption, a saving in transmitter power of 16 times (power ratio corresponding to 12 db) is obtained: the transmitter is reduced to one sixteenth of the power required in the case of a normally modulated wave, without degrading the speech-to-noise ratio. Alternatively, of course, the transmitter may be built for the same power, in which case an improvement in speech-to-noise ratio of 12 db. is obtained.

In the case of broadcasting there may be 10 transmitters which provide an adequate noise ratio with an average transmitted power of, say, 100 kW. each and a power consumption of 300 kW. each. Even if the introduction of single sideband saved the whole 300 kW of power in each transmitter a total saving of power of only 3000 kW would result.

In Great Britain there are over 9 million licences, which means a still greater number of receiving sets. If one valve is added to each of these sets with a power consumption of 2 watts only (which is much below the average), in order to provide an oscillator for reintroducing the carrier, a total power consumption of 18 million watts = 18,000 kW is involved. This 18,000 kW is evidently not in continuous use, but in broadcasting single sideband is not necessarily an economical means of improving speech-to-noise ratio or of effecting a reduction in the power of transmitters.

From the point of view of relieving congestion in the space/wave-frequency spectrum, single sideband, with or without suppressed carrier, has very large merits; but that is irrelevant to the present discussion. On short waves, use of single sideband reduces selective fading.

7.2. Use of Directional Aerials. An improvement in speech-to-noise ratio can be obtained by the use of directional aerials.

At the receiver the effect of these is threefold:

1. By the use of an aerial which has a higher efficiency in the direction of reception than the most economical type of non-directional aerial a reduction in the effect of set noise can be obtained.
2. By the use of an aerial which has a greater efficiency in the direction of reception than in other directions, a reduction in random noise picked up on the aerial can be obtained.
3. By the use of an aerial which has lower efficiency in the direction of reception of an unwanted station, or source of noise, than in the direction of the wanted station, a

reductoin in the disturbing effect of these sources can be obtained.

Simple types of directional aerial are described in XVIII:8 immediately below.

The effect of directive transmitting aerials requires no explanation.

8. Directional Aerials.

Directional aerials or arrays are combinations of one or more aerial elements so arranged that the efficiency of reception is not the same in all directions, and, in the absence of contrary requirements, is greatest in the direction from which it is required to receive signals, or in which it is required to send signals.

8.1. Polar Diagrams illustrating the two Classes of Receiving Array. The performance of a directional receiving aerial is represented by drawing a vector in each direction proportional to

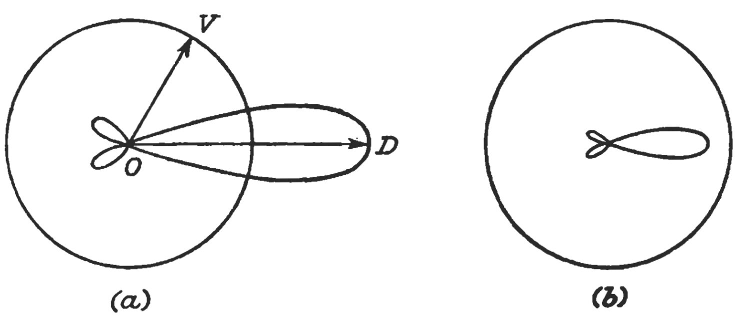

FIG. 1/XVIII:8.—Polar Diagrams.
(*a*) Class 1 Receiving Array. (*b*) Class 2 Receiving Array.

the efficiency of reception from that direction. The diagram resulting from joining the tips of these vectors (by a line in the case of plane polar diagrams, and by a surface in the case of solid polar diagrams) is called a polar diagram. The polar diagrams in most common use for receiving aerials are expressed in terms of the e.m.f. supplied by the aerial for constant field strength propagated from each direction in turn. For polar diagrams of transmitting aerials, see XVI:7.

Two classes of receiving directional aerials may conveniently be distinguished.

In a Class 1 *array* the voltage output from the array, due to a signal from the wanted direction, is greater than that due to a standard aerial used as a basis of reference. The polar diagram of a class 1 array is illustrated in Fig. 1 (*a*), where the circle is the horizontal polar diagram of the reférence aerial and the polar diagram with three lobes is the horizontal diagram of the directional aerial. Such an aerial array is said to have a gain : the magnitude of the gain in any particular case is discussed below.

In a Class 2 array the voltage output from the array, due to a signal from the wanted direction, is equal to or less than that due to a standard aerial used as a basis of reference. The polar diagram of a class 2 array is illustrated in Fig. 1 (*b*).

8.2. Gain of Array. *The power gain of an aerial or array in a given direction is equal to the power density in watts per solid angle radiated in that direction divided by the power density radiated in all directions by an isotropic (spherical polar diagram) aerial radiating the same total power.* This is strictly a power *ratio*, but custom has applied the term gain to it as also to 10 times $\log_{10}$ of this ratio. The statement in XVIII:6.12.3 is true but not any use for calculating gain. The *improvement* of any array may be calculated as the array gain divided by that of the simplest appropriate aerial which is usually one element of the array.

8.3. Noise Ratio of Array. The noise ratio of an array may conveniently be defined as the ratio : e.m.f. induced by unit field propagated from the direction in which the array is most efficient divided by the root of the mean of the squared values of the e.m.f.s induced by unit fields propagated from all directions in turn. This is because the noise fields, being in random phase, add as the square root of the sum of the squares. The point at which the e.m.f. is observed is the point at which the feeder to the receiver makes connection to the array : the receiver is assumed to be impedance matched to the array. This figure is somewhat analogous to, but *not* equal to, the directivity of a transmitting aerial. (The directivity of a transmitting aerial is the energy density in the direction of maximum transmission divided by the mean energy density averaged over all directions.) For both purposes directions in all vertical planes must be taken as well as in the horizontal plane. This is rigidly true in the case of directivity, but useful results can be obtained by determining the noise ratio in the horizontal plane only. It will be assumed that noise ratios are so determined.

Evidently a rough indication of the noise ratio of an array can be obtained by looking at its solid polar diagram. If the array is made up of vertical aerials, for reception of horizontally propagated waves, the horizontal polar diagram gives a rough indication. The sharper the polar diagram, the higher is the noise ratio.

The improvement in signal-to-noise ratio determined by changing from an aerial with a noise ratio of N_1 to an array with a higher noise ratio N_2 is then $20 \log_{10} \frac{N_2}{N_1}$ db. There is evidently no point in using a class 2 array if a class 1 array of equal noise ratio can be built for the same cost, since the higher gain of the class 1 array makes it preferable as it enables the signal to override inherent receiver noise (i.e. not incoming to the aerial).

If, however, a class 2 aerial can be constructed of greater noise ratio than a class 1 aerial of equivalent cost, and if the inherent receiver noise is negligible, there is evidently an advantage to be obtained in using a class 2 array. The criterion which sets a limit to the possibility of using a particular class 2 array is the signal to inherent receiver noise ratio which it affects. If this is better than the required overall signal-to-noise ratio a class 2 array of high noise ratio has an advantage over a class 1 array of low noise ratio.

8.4. Plane of Polarization. The plane of polarization of the received wave determines the type of receiving aerial to be used.

In practice, directive arrays are used more frequently for horizontally polarized waves than they are for vertically polarized waves. This is partly because beam arrays are used chiefly for long-distance transmission at frequencies where the attenuation of horizontally polarized waves is less than that of vertically polarized waves ; but also because a stack of horizontal dipoles lends itself better to a balanced current and voltage distribution and therefore gives more consistent performance.

The relations between direction of propagation and of electric and magnetic vectors are discussed in XVI:7 and the conventions by which horizontally and vertically polarized waves may be defined are enunciated in that section. These will be used here : a vertically polarized wave is a wave in which the magnetic vector is horizontal ; a horizontally polarized wave is a wave in which the electric vector is horizontal.

In the case of a single horizontal aerial the vertical polar diagram taken in a vertical plane making any horizontal angle with the aerial varies with the height of the aerial above ground. For the purpose of determining the improvement of an array radiating or receiving horizontally polarized waves, the most satisfactory practical method is to use as reference a horizontal half-wave dipole situated at a height above ground which gives the optimum reception from the required angle of elevation. Proposals have been made to use as reference a dipole in free space, but this has no practical value.

In theory a horizontal dipole above ground receives (and radiates) zero field in all horizontal directions and generally but not always the direction of reception (and transmission) of interest in the case of horizontally polarized waves is one making an angle with the horizontal.

Provided the above points are remembered, the discussion in terms of a vertical aerial applies equally to horizontal aerials.

Fig. 2 (*a*) shows a horizontally polarized wave ; the vector *E* is horizontal, but if the direction of propagation is not horizontal the vector *M* will remain normal to the direction of propagation and will not be vertical. Similarly Fig. 2 (*b*) shows a vertically polarized wave in which the vector *M* is horizontal and the vector *E* is normal to the direction of propagation. In looking at Fig. 2 the direction

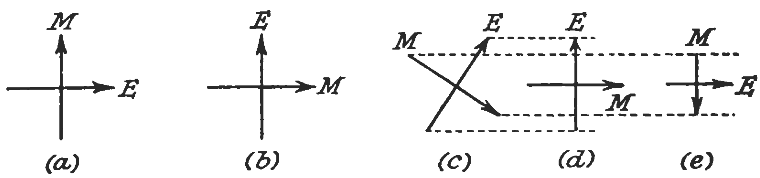

FIG. 2/XVIII:8.—(*a*) Horizontally Polarized Field. (*b*) Vertically Polarized Field. (*c*) Skew Polarized Field resolved into Vertically Polarized and Horizontally Polarized Fields at (*d*) and (*e*).

of propagation should be regarded as being normal to the paper. Fig. 2 (*c*) shows a field in which neither the magnetic vector nor the electric vector is horizontal and which may be resolved into the vertically polarized field at (*d*) and the horizontally polarized field at (*e*). In other words, the field at (*c*) can be reconstituted by adding the magnetic vectors in (*d*) and (*e*) and adding the electric vectors in (*d*) and (*e*). The field at (*c*) may be called a skew polarized field, although this term is not in general use.

8.5. Performance of Loop Aerial in Vertically Polarized and Horizontally Polarized Fields. A vertical loop aerial in a vertically polarized field has a horizontal polar diagram as shown in Fig. 3 by the two small circles, the loop being shown in plan as *LL*.

A vertical loop aerial situated in a horizontally propagated, horizontally polarized field has no e.m.f. induced in it at all.

A vertical loop aerial situated in a horizontally polarized field which is not propagated horizontally has a polar diagram similar to that shown in Fig. 3 but rotated at 90° with regard to the loop direction. In other words, maximum e.m.f. is induced in the loop when it is placed at right angles to the projection of the direction of propagation projected on to the horizontal plane. This will be made clear by reference to Fig. 4, which shows at (*a*) a perspective view and at (*b*) a side elevation of a loop aerial situated in a down-

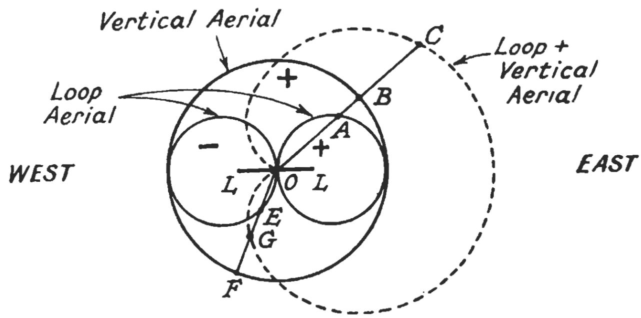

FIG. 3/XVIII:8.—Combination of Loop and Vertical Aerial to give Cardioid Polar Diagram.

coming ray of a horizontally polarized wave. It is evident that maximum linkage with the magnetic vector *M* occurs in the condition specified above for maximum e.m.f. induced in the loop. Speaking colloquially, linkage with the magnetic vector results because the magnetic vector is canted out of the vertical. Loop aerials are not

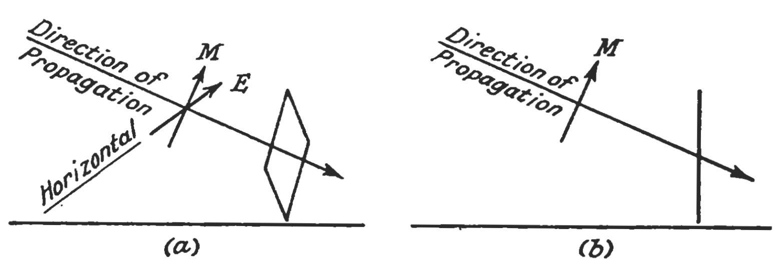

FIG. 4/XVIII:8.—Vertical Loop Aerial in Horizontally Polarized Field which is not Propagated Horizontally.
(*a*) Perspective View. (*b*) Side Elevation.

normally used for reception of horizontally polarized waves, but the behaviour of a loop in the presence of a horizontally polarized wave constitutes an explanation of an aberration known as " night effect ", discussed below.

Owing to reflection from the ground, which introduces a ray with its direction of propagation inclined up with regard to the horizontal, a vertical loop aerial situated in a nominally horizontally propagated horizontally polarized wave also gives maximum received signal with the plane of the loop normal to the direction of propagation.

8.6. Cardioid Polar Diagram. Referring to Fig. 3 which shows a loop aerial in a plane running from east to west, the small circle on the east side constituting part of the loop polar diagram for vertically polarized waves has been marked " plus ", while that on the west side has been marked " minus ". This is to indicate that the sense of the e.m.f.s induced in the loop depends on the direction of propagation. That is, waves arriving from the east induce e.m.f.s in the loop of opposite sense from those due to waves arriving from the west. This characteristic of a loop aerial is used to provide an aerial array in which reception from one direction is cancelled. This is done by adding the e.m.f.s supplied by a loop aerial to the e.m.f. supplied by a vertical aerial located at the same

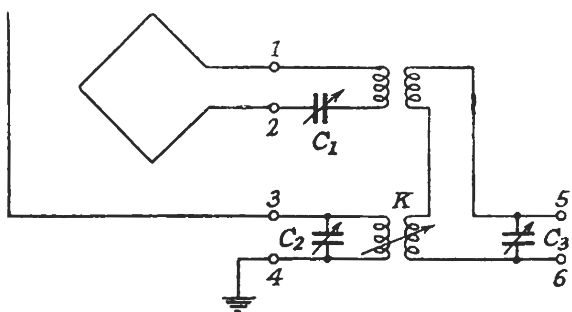

FIG. 5/XVIII:8.—Simple Cardioid System.

place, and adjusting the two e.m.f.s to be of equal magnitude. In this case the polar diagram of the vertical aerial is represented by the large circle and the sense of the e.m.f.s induced in the vertical aerial is independent of the direction of propagation. The consequence is that, if, as is assumed in Fig. 3, the output from the vertical aerial is phased so as to add to the output from the loop for directions of propagation from the east, the two outputs will subtract for directions of propagation from the west. The resultant polar diagram is obtained by adding vectors such as *OA* and *OB* (which give rise to *OC*) and subtracting vectors such as *OE* and *OF* (which give rise to *OG*). The form of the resultant polar diagram is sometimes called a *cardioid*, owing to its rather vague resemblance to a conventional heart shape.

Fig. 5 shows the essentials of a simple arrangement for combining the output of a loop and a vertical aerial. The loop is tuned to a maximum response by means of condenser C_1, while coupling K

and condenser C_2 are adjusted so as to make the e.m.f.s, injected into the secondary circuit by loop and vertical aerial, equal and in phase. Evidently, by commutating either 1 and 2 or 3 and 4, the sense of the cardioid can be reversed, that is to say, the directions of maximum and minimum reception can be interchanged. The secondary circuit is tuned by means of condenser 3, and terminals 5 and 6 are connected to the receiver. The method of setting up a cardioid system to eliminate an unwanted station is fairly obvious. With the vertical aerial disconnected, for instance, at 3, the loop aerial is rotated to give maximum reception of the unwanted station. With the loop aerial left in this position the vertical aerial is connected and commutated in the correct sense, so that by adjustment of C_2 and K the signal from the unwanted station can be balanced out. It is hardly necessary to say that any wanted station can then only be received provided its direction differs adequately from that of the unwanted station.

The method of adjusting a cardioid to give maximum reception of a wanted station is to adjust it first of all to give zero reception of the wanted station and then to commutate either the output of the loop aerial or the vertical aerial.

Simple cardioid systems are used to eliminate unwanted stations, to provide arrays with improved noise ratio, and to give indications for direction finding.

8.7. Bellini-Tosi System. To enable large loops to be used (to provide adequate sensitivity on medium and long waves) which cannot be conveniently rotated, the arrangement shown in Fig. 6 is used. In this each loop is provided with suitable tuning arrangements and fed through one of two fixed coils wound mutually at right angles, as shown in Fig. 6, coils 1 and 2. The axis of each coil is parallel to the axis (and normal to the plane) of the loop to which it is connected. A third coil 3 rotates between these coils as indicated, and provides an output to terminals 1 and 2, which are treated as the output of a single loop and may be regarded as being connected to terminals 1 and 2 in Fig. 5, so that the arrangement in Fig. 6 replaces the loop in Fig. 5. With such an arrangement coil 3 behaves exactly like a direction-finding aerial, and when tuned to give maximum signal strength lies in the direction of propagation of any vertically polarized wave striking the loop assembly. The coil assembly is called a goniometer.

Night Effect. It will have been understood that both the simple cardioid system and the Bellini-Tosi system are employed only for use with vertically polarized waves, such as are constituted by the direct ground ray from an ordinary medium- or long-wave broadcasting station. In practice, however, particularly at night, in addition to the direct ground ray, waves reach the receiving array by reflection from the ionosphere, which means that their direction of propagation is not horizontal. This in itself would not matter, provided that the wave was still vertically polarized, but a further complication is introduced because the plane of polarization is usually rotated about the direction of propagation so that a skew polarized wave results, or, in other words, there is a component field which is horizontally polarized. Loop aerials therefore, have modified polar diagrams resulting from the presence of horizontally polarized waves which are not horizontally propagated. The net result is that the directive properties of the loop aerials, and therefore, of any system employing loop aerials, are spoiled. As this effect is most pronounced at nights, it is sometimes called " night effect ".

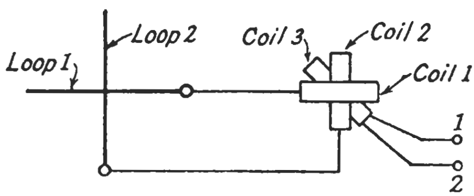

FIG. 6/XVIII:8.—Bellini-Tosi System.

8.8. Marconi-Adcock System. By replacing each loop aerial by two vertical aerials, as shown in plan in Fig. 7, pick-up from the horizontally polarized component of the received wave is eliminated. In Fig. 7, A_1 and A_2 each consist of a vertical aerial connected through a length of concentric cable C (which may be buried) to one side of coil 1, so that coil 1 is in series between the two lengths of concentric cable. Similarly, coil 2 is in series between the two lengths of cable connected to the vertical aerials A_3 and A_4. As before, coil 3 is connected to terminals 1 and 2 and the whole system effectively replaces the loop in Fig. 5. This system, which is known as the Marconi-Adcock system, has improved directional characteristics in the presence of downcoming ray which is not vertically polarized.

In practice, the accuracy of the Marconi-Adcock system for direction-finding purposes is impaired owing to coupling between the vertical aerials and the horizontal sheaths of the cables running from the vertical aerials to the combining coils. The longitudinal e.m.f.s are induced in the cable sheaths by the downcoming ray, and the resultant e.m.f.s induced in the vertical aerials shift the minima of the system, and so reduce the accuracy with which a bearing is indicated. From the point of view of attenuating the energy received from an unwanted station, the effect is to reduce the amount of discrimination obtaining between wanted and unwanted stations on different bearings.

Resort has therefore been made to the system of spaced loops, described immediately below.

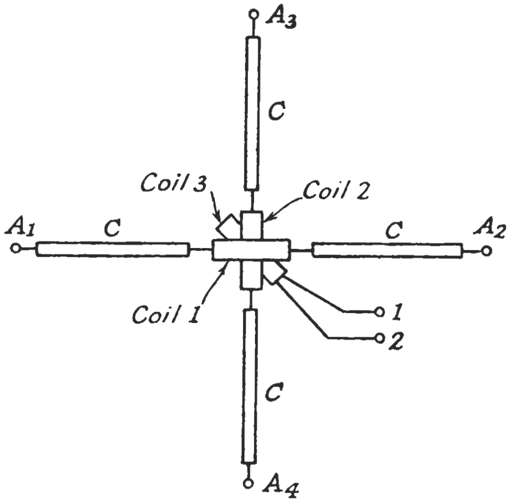

FIG. 7/XVIII:8.—Marconi-Adcock System: Plan View.

8.9. Spaced Loops. In this system two loops are mounted on a common horizontal bar which may be rotatable if the system is used for direction finding, and may be fixed on a given bearing if the system is required to have a receiving polar diagram with a fixed orientation for purposes of reducing interference.

The loops may be either co-planar with one another and with the common supporting bar, as shown in Fig. 8, or may be co-axial and normal to the mounting bar, as shown in Fig. 9.

Fig. 8 shows the co-planar arrangement. The sense of the connection of the loops is indicated at 8 (*a*) which, for this purpose, omits the standard tuning and coupling arrangement associated with each loop, as shown at 8 (*b*).

This system has a horizontal polar diagram for vertically polarized waves, as shown in Fig. 8 (*c*) by the full-line curve. Fig. 8 (*c*) gives a plan view of the spaced loop system. The horizontal polar diagram for horizontally polarized waves is indicated by the dotted curve.

It is therefore evident that, for signals received from any direction in the vertical plane through *NN* (i.e. the normal to the mounting bar), there is a minimum on both diagrams. No errors are therefore introduced on downcoming rays with skew polarization.

Fig. 9 (*a*) gives a perspective of the co-axial spaced loop system with the sense of interconnection between loops indicated as before. Fig. 9 (*b*) shows a plan view of the co-axial loops, with the horizontal polar diagram for vertically polarized waves shown as a full line; the horizontal polar diagram for horizontally polarized waves is shown dotted. It is again evident that there is a minimum for

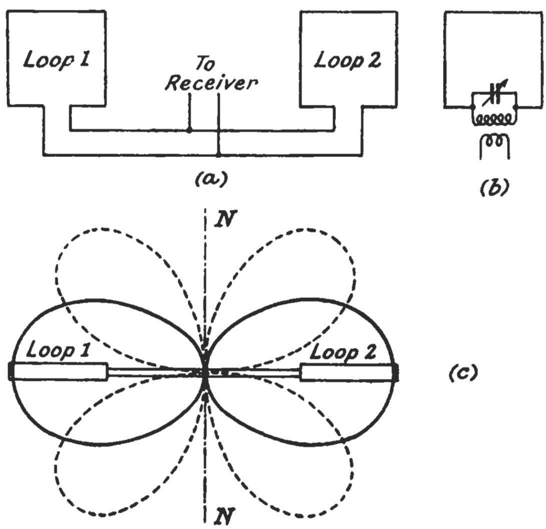

FIG. 8/XVIII:8.—Co-planer spaced Loops and Corresponding Polar Diagrams. Full line = Vertical Polarization. Dotted line = Horizontal Polarization.

signals with any kind of polarization arriving from any direction lying in the vertical plane through *NN*. Of these two systems, the co-axial is to be preferred because:

1. No coupling occurs between the loops and the longitudinal path through the interconnecting wires. (This might cause inaccuracy in the presence of any component of horizontal polarization.)
2. There is no coupling between the loops and any vertical metal supports or calibrating aerials at the centre of the system. (This might cause inaccuracy in the presence of any component of vertical polarization.)
3. The minimum on vertically polarized waves is sharper than in the case of the co-planar arrangement.

It is also true that the co-planar arrangement gives the sharpest minimum for horizontally polarized rays, and, provided the inaccuracy due to 1 above is tolerable, this arrangement can be used for direction finding or discriminating purposes on horizontally polarized waves. *Its sensitivity, when used for these purposes, is, however, low.*

In all cases when loops are used, they should be screened by a tube or housing completely enclosing the conductors of the loop, but not constituting a short-circuited turn. It is important that the break in the loop should be exactly in the centre of the top member

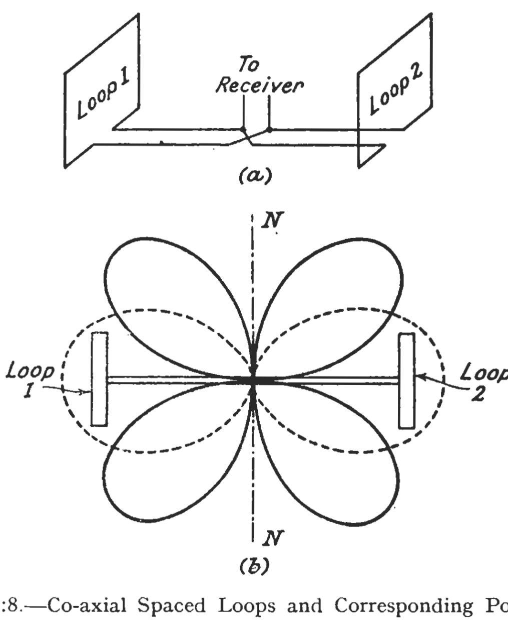

FIG. 9/XVIII:8.—Co-axial Spaced Loops and Corresponding Polar Diagrams. Full line = Vertical Polarization. Dotted line = Horizontal Polarization.

of the loop, otherwise the currents induced in the two halves of the screen may give rise to an e.m.f. in the loop.

The coupling circuit, associated immediately with the loop as shown at (*b*), must be balanced for admittance to ground and admittance to the centre of the output coil. This balance may be assisted by a screen, between the windings, connected to the centre output coil.

The connection to the receiver should be made with a transformer having a winding, on the loop side, which is balanced for admittance to ground. This transformer should be screened and this screen should be connected to the receiver earth.

The screen on the loop should be continuous with the screen on the conductors going to the receiver, and should enclose the coupling coils, tuning condenser and balanced transformer supplying the receiver. If the connection to the receiver is more than a few inches long, the balanced transformer should be connected at the receiver end of this connection.

8.10. Horizontal Polar Diagram of a Horizontal Dipole. The horizontal polar diagram of a horizontal dipole in a horizontally polarized horizontally propagated field is of the same form as that shown for the vertical loop in Fig. 3, with the circles replaced by ellipses of low eccentricity, the dipole being assumed to lie in a plane at right angles to the loop. For the use of horizontal dipoles for the reception of short waves at distances of two or three hundred miles where the received wave is all downcoming ray at a fairly steep angle, this polar diagram is almost meaningless and little difference in reception will be observed as the dipole is rotated in the horizontal plane. For reception on ultra-short waves at distances of twenty or thirty miles appreciable directional effect will be observed. Also for reception over distances of thousands of miles, where the angle of the downcoming ray with the horizontal is small, appreciable directive effect can be obtained.

In the case of interference from a nearby transmitter where the direct ray is much stronger than the downcoming ray, an appreciable improvement may be obtained by orientating a horizontal dipole so that it points at the interfering station.

Many other types of directional aerials and directional arrays exist, but space does not permit their description here.

CHAPTER XIX

RADIO RECEIVERS

A RADIO RECEIVER is a device which converts the modulated wave induced, in a receiving aerial by a modulated space wave to the original band of modulating frequencies, at a suitable level and as free as possible from all forms of extraneous disturbance. The actual process of converting the modulated wave to the original modulating frequency is called *detection*. In addition to the *detector* a receiver may have means for amplifying the incoming waves, before and/or after detection, means for selecting a transmission on any wanted frequency to the exclusion of other transmissions, and means for making the output level independent of variations in the field strength of the received signal.

In this chapter the principles of operation of radio receivers are described, largely from the viewpoint of the user. For information on design of receivers reference should be made to *Radio Receiver Design*, by K. R. Sturley.

1. Diode and Crystal Detectors.

The simplest possible form of a receiver consists of a rectifying device such as a diode, a crystal or a copper oxide rectifier, in association with a coupling circuit to an aerial and a pair of head-

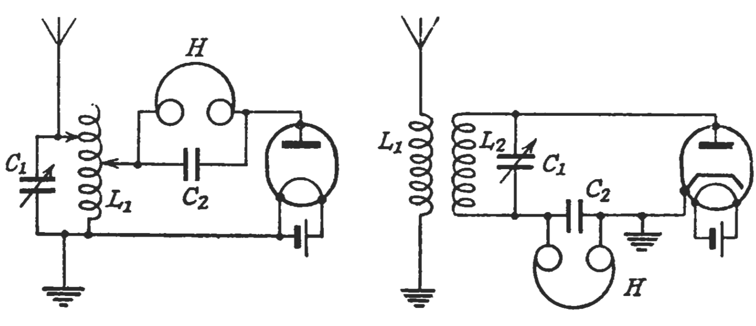

FIG. 1/XIX:1.—Simple Receivers using Rectifiers.

phones. Fig. 1 shows two such receivers. In type (*a*) the double-tapped inductance in conjunction with the tuning condenser *C*, provide coupling and impedance matching between the aerial and the detector circuits. The double-tapped inductance is useful when it is important to secure the maximum efficiency of coupling. The type at (*b*) has a separate coupled tuned circuit to secure greater selectivity and locates the headphones *H* in the cathode lead so that one side of the headphones goes to earth. This is a preferred arrangement to that at (*a*) but cannot be used at (*a*) because the capacity to ground of the filament battery would be located across the headphones. The value of C_2 in each case is about 0·0005 μF.

The principle of operation of these circuits is illustrated in Fig. 2. Fig. 2 (*a*) shows a carrier wave with a sinusoidal envelope produced by modulating a carrier with an audio-frequency wave as shown at 2 (*d*). The envelope of the wave is of the same form as the

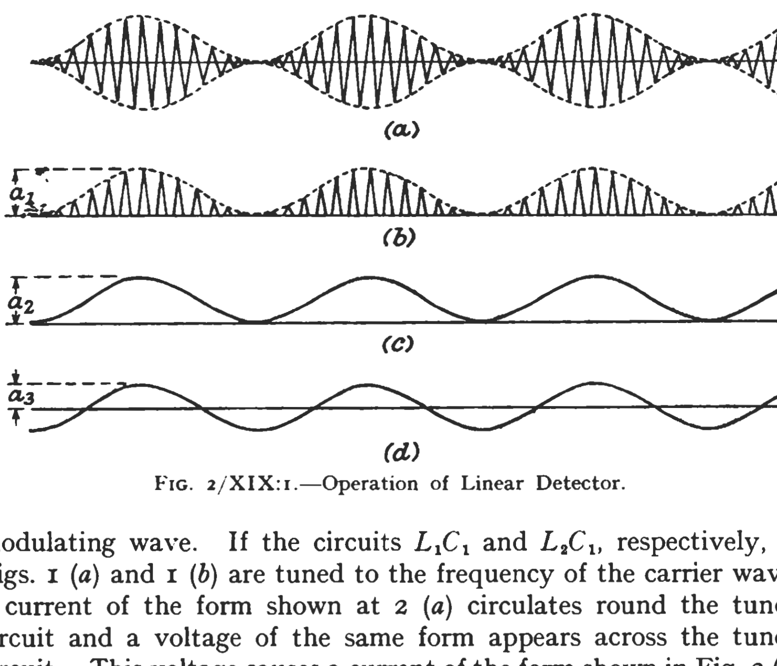

FIG. 2/XIX:1.—Operation of Linear Detector.

modulating wave. If the circuits L_1C_1 and L_2C_1, respectively, in Figs. 1 (*a*) and 1 (*b*) are tuned to the frequency of the carrier wave, a current of the form shown at 2 (*a*) circulates round the tuned circuit and a voltage of the same form appears across the tuned circuit. This voltage causes a current of the form shown in Fig. 2 (*b*) to flow through the diode and through the combination of *H* and C_2 in parallel. The high-frequency components of the current flow through C_2 which presents a comparatively low impedance to high-frequency currents as compared with *H* ; and the audio-frequency component and D.C. component, shown together in Fig. 2 (*c*), flow through the headphones *H*, which present a comparatively low impedance to audio-frequency currents (as compared with C_2) and a still lower impedance to D.C.

The audio-frequency component consists of a sine wave of the form shown at (*d*), which is of the same form as the original modulating frequency. The direct current produces a steady deflection of the headphone diaphragm while the alternating component makes the diaphragm reproduce the original modulation frequency as sound. The process is exactly the same when the modulating wave corresponds to speech or music.

The scales in Fig. 2 are arbitrary. $a_3 = \frac{1}{2}a_2$ and approximately $a_2 = a_1/\pi$.

In either of the circuits of Fig. 1 the diode may be replaced by any low-current and low-voltage rectifier sufficiently free from self-capacity and having a suitable voltage-current characteristic. It is evidently no use using a rectifier in which a very large current or a very large voltage is required to reach the bend of the characteristic.

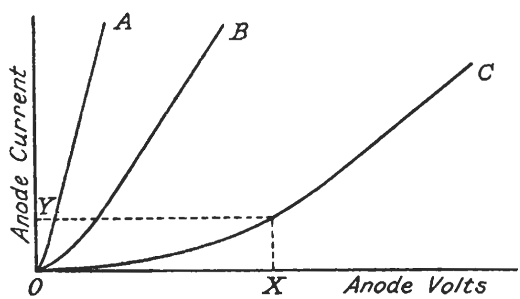

FIG. 3/XIX:1.—Rectifier Characteristics.

This will be made clear by reference to Fig. 3. Evidently curve *C* is unsuitable unless a voltage greater than *OX* and a current greater than *OY* can be obtained. Further, it should be noticed that a rectifier having a characteristic as at *B* requires to work from a circuit of higher impedance than does a rectifier having a characteristic as at *A*. This means not only that the radio circuit impedance must be higher, but also the audio-frequency circuit. The alternative types of rectifier which are available are special copper oxide rectifiers, e.g. the Westinghouse type W6, Westector, crystal detectors and diodes. A crystal detector consists of one or two pieces of certain minerals in contact either with one another or with a metal point, the combination behaving as a rectifier. The impedance of diodes and copper oxide rectifiers is much lower than that of crystals, so that direct replacement of diodes and copper oxide rectifiers by crystals cannot be made. Copper oxide rectifiers require much more power than diodes and crystals. Copper oxide rectifiers possess considerable inherent capacity, and although they may be used in medium- and long-wave circuits, where adequate power is available, the general-purpose rectifier which is used as a *detector* is the diode. Crystals are used at centimetre wavelengths.

Like diodes, crystals contribute to the noise of a circuit in the form of a fluctuation noise greater than the Johnson noise to be expected from their impedance characteristics.

The *Noise Temperature* of a crystal is the ratio : noise power output of the crystal divided by the Johnson noise power output of a passive impedance of the same magnitude and angle.

2. T.R.F. Receiver.

After the simple crystal or diode detector the next stage in development was to add one or more high-frequency amplifiers

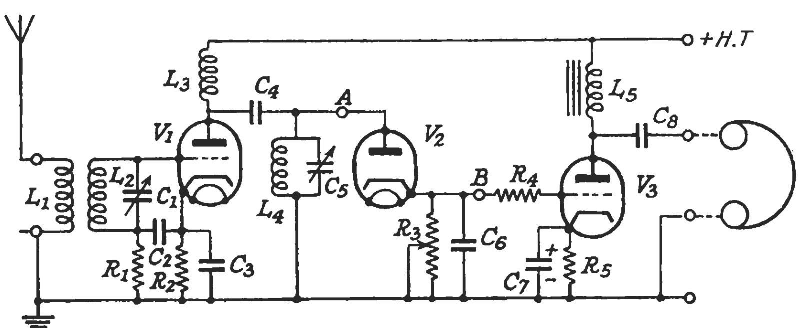

FIG. 1/XIX:2.—Straight or T.R.F. Receiver with Diode Detector.

before the diode, and one or more audio-frequency amplifiers after the diode, resulting in the form of circuit shown in Fig. 1. This is a straight circuit or a T.R.F. (tuned radio-frequency) circuit.

The circuit is shown in terms of separately heated cathode valves, which had not been developed when this type of circuit was first used, since these are in common use.

L_1, L_2 and C_1 constitute the aerial coupling circuit feeding into the grid of valve V_1, which is a class A amplifier biased as a class A amplifier by means of the cathode resistance R_2, which is shunted by C_3, an 0·01-μF mica condenser. The grid circuit is decoupled from earth and coupled to the cathode of V_1 by means of the one-megohm resistance R_1 and the 0·01-μF condenser C_2. L_3 is a high-frequency choke as large as is consistent with non-resonance in the band of frequencies to be received, and C_4 is an 0·01-μF condenser coupling the anode circuit of V_1 to the tuned anode circuit L_4, C_5 and the diode V_2. The diode load R_3 is usually between 100,000 and 500,000 ohms and may be made variable, as shown, to provide a volume control. The condenser C_6 across the diode load is usually about 50–100 $\mu\mu$F and permits positive peaks of voltage to drive a current through the diode so as to charge up C_6 to a mean voltage (averaged over a cycle of the R.F.) which follows the form of the envelope of the incoming wave. The resultant audio-frequency is applied to the grid of V_3, which is an audio-frequency amplifying valve, via R_4, which is inserted so as to constitute, in conjunction with the grid input capacity of V_3, a radio-frequency filter which prevents the R.F. from overloading V_3. V_3 is biased by means of R_5, which is shunted with the electrolytic condenser C_7, which has a capacity of about 20 μF in order to prevent audio-frequency feed-back from anode to cathode circuit of V_3. L_5 is an audio-frequency choke and C_8 is a 1 μF condenser. It is to be noted that, with the diode in the sense shown, the incoming R.F. (owing to the rectifying action of V_2) puts positive bias on V_3 and so reduces the effective grid bias. This is not likely to cause difficulty, but if it does it can be reversed in effect by reversing the diode connection, or, better still, eliminated altogether by inserting an 0·5-μF condenser in series with R_4 at B and connecting a grid leak of about 0·5 megohm from the grid side of the condenser to ground.

3. Anode-Bend Detector.

This is a type of detector which had a short vogue, particularly in America, but was discarded on account of the distortion introduced. Fig. 1 shows a valve arranged as an anode-bend detector, which might be used to replace the diode detector in Fig. 1/XIX:2, by inserting the circuit of Fig. 1 between A and B in Fig. 1/XIX:2.

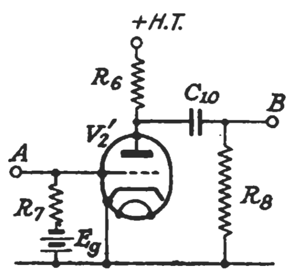

FIG. 1/XIX:3.—Anode-Bend Detector.

In Fig. 1, R_6 is a resistance providing audio-frequency coupling to the next stage in conjunction with condenser C_{10}, which is about 0·5 μF. R_8 in Fig. 1 maintains the mean potential of the grid of the following valve, e.g. V_3 in Fig. 1/XIX:2 at ground potential. V_2' is a valve with a parabolic anode-current grid-voltage characteristic, biased by means of resistance R_7 and battery B to a point on the curved part of the anode-current/grid-voltage characteristic as indicated in Fig. 2/XIX:3, where E_g is the applied grid bias and E_c is the grid bias required to take the valve to cut-off.

This anode-current/grid-voltage characteristic is of the form

$$i_a = a(E_c+e)^2 \quad . \quad . \quad . \quad . \quad (1)$$

where i_a is the anode current

a is a parameter of the valve

e is the value of instantaneous grid potential with regard to the cathode.

Consider the case where the incoming voltage wave applied to the grid of V_2' consists of a carrier wave of amplitude A and angular frequency c modulated to a depth m by a sinusoidal audio-frequency wave of angular frequency v.

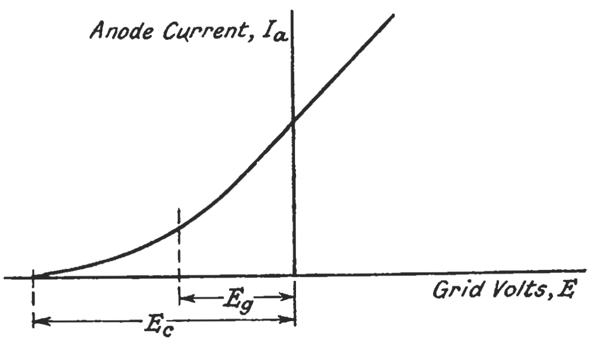

FIG. 2/XIX:3.—Characteristic of Anode-Bend Detector.

The instantaneous voltage of such a wave is expressed analytically by:

$$E_m = A \sin ct + mA \sin ct \sin vt \quad . \quad . \quad (2)$$

$$= A(1+m \sin vt) \sin ct \quad . \quad . \quad . \quad (2a)$$

Suppose that a voltage of this form is applied to the grid of valve V_2' by the output from valve V_1 of Fig. 1/XIX:2. The instantaneous value of grid voltage on V_2' is given by $e = E_m - E_g$, where E_g is the steady value of grid bias. Substituting this value of e in equation (1):

$$i_a = a[E_c+A(1+m \sin vt) \sin ct - E_g]^2 \quad . \quad . \quad . \quad (3)$$

$$= a[\underbrace{(E_c - E_g)^2}_{\text{D.C.}} + \underbrace{A^2 \sin^2 ct}_{\text{Twice carrier frequency + D.C.}} + 2mA^2 \sin vt \sin^2 ct + \underbrace{2A(E_c - E_g) \sin ct}_{\text{carrier frequency}}$$

$$+ \underbrace{2mA(E_c - E_g) \sin vt \sin ct}_{\text{sideband frequencies}} + A^2m^2 \sin^2 vt \sin^2 ct]$$

Neglecting the D.C., carrier frequency, double-frequency carrier, and sideband frequency terms

$$i_a = a[2mA^2 \sin vt(\tfrac{1}{2} - \tfrac{1}{2}\cos 2ct) + m^2A^2(\tfrac{1}{2} - \tfrac{1}{2}\cos 2vt)(\tfrac{1}{2} - \tfrac{1}{2}\cos 2ct)]$$

$$= a[\underbrace{mA^2 \sin vt}_{\substack{\textit{Wanted}\\ \textit{original}\\ \textit{audio}\\ \textit{frequency}}} - \underbrace{mA^2 \sin vt \cos 2ct}_{2c+v \text{ and } 2c-v} + \underbrace{\tfrac{1}{4}m^2A^2}_{\text{D.C.}} + \underbrace{\tfrac{1}{4}m^2A^2 \cos 2ct \cos 2vt}_{2c+2v \text{ and } 2c-2v}$$

$$- \underbrace{\tfrac{1}{4}m^2A^2 \cos 2ct}_{\substack{\text{Twice carrier}\\ \text{frequency}}} - \underbrace{\tfrac{1}{4}m^2A^2 \cos 2vt}_{\substack{\textit{Twice audio}\\ \textit{frequency}}}] \quad . \quad . \quad (4)$$

The only audio-frequency terms in this expression are the first and the last. The first term, which is of amplitude amA^2, constitutes the wanted audio frequency, while the last term, which is of amplitude $am^2A^2/4$, constitutes currents of twice the wanted audio frequency, which therefore appear as distortion. All radio-frequency terms are suppressed in the following circuits.

Expressed as a percentage of the fundamental (original modulating audio frequency), the second harmonic (twice audio-frequency current) amplitude is :

$$\frac{\tfrac{1}{4}am^2A^2}{amA^2} = \frac{m}{4} \quad . \quad . \quad . \quad . \quad (5)$$

Hence at 100% modulation the second-harmonic distortion is 25%; at 50% mod., 12·5%, and so on. This is quite intolerable. As a matter of interest the second-harmonic term corresponds to direct intermodulation between the two sidebands on each side of the carrier. Similarly, it may be shown that if two audio frequencies, f_1 and f_2, are present in the original modulation giving rise to sidebands at each frequency corresponding to percentage modulations m_1 and m_2, in addition to the intermodulation between corresponding sidebands on each side of the carrier, there is cross-modulation between the sidebands corresponding to f_1 and f_2, giving rise to distortion products $f_2 - f_1$ and $f_2 + f_1$ of amplitude $\tfrac{1}{2}am_1m_2A$.

4. Leaky-Grid Detector.

This type of detector is shown in Fig. 1/XIX:4. C_{11} is about 0·0003 μF, R_9 is about 1 MΩ, while R_{10}, C_{12} and R_{11} have the same values and the same object respectively as R_6, C_{10} and R_8 in Fig. 1/XIX:3. Since the valve V_2'' is operated with no grid bias, when R.F. drive is applied between A and ground, the grid is swung alternatively positive and negative and grid current flows during the positive pulses. The circuit between grid and cathode therefore behaves like a diode and it is immediately evident that the detector circuit of Fig. 1/XIX:2 has been reproduced with the position of diode and diode load plus shunt condenser interchanged. A voltage corresponding to the envelope of the wave is therefore built up across the condenser C_{11}, the grid being given a mean negative bias averaged over one cycle, which corresponds to the envelope of the incoming voltage wave. A leaky-grid detector is more sensitive than an anode-bend detector and more sensitive than a diode alone, by the amount of amplification in the valve. Since, however, diode-triode valves are available for substantially the same cost as a triode, and since diodes have a more consistent performance than grid input circuits of valves, the leaky-grid detector has gone out of use.

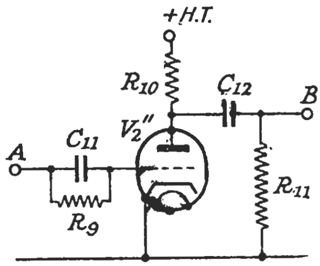

FIG. 1/XIX:4.—Leaky-Grid Detector.

5. Distortion in Diode Circuit and Leaky-Grid Detector.

Distortion occurs in all diode detector circuits and in the leaky-grid detector owing to the time taken to charge and discharge the condenser across the diode load: C_2 in Fig. 1/XIX:1, C_6 in Fig. 1/XIX:2, and C_{11} in Fig. 1/XIX:4. This distortion is illustrated in Fig. 1/XIX:5, where the full-line curve represents the ideal

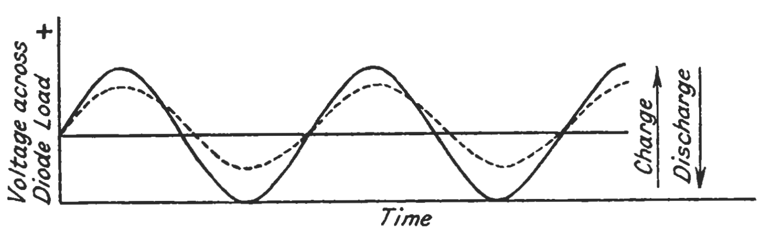

FIG. 1/XIX:5.—Distortion in Leaky-Grid Detector and Simple Diode Circuit.

voltage developed across the diode load and the dotted curve shows how the actual voltage lags behind the ideal voltage by different amounts at different parts of the cycle and so distorts the wave form. In the particular case chosen the time of charge is different from the time of discharge, resulting in further distortion constituted by inequality of the voltage swings each side of the true mean line.

In practice this distortion is kept within tolerable limits by proper proportioning of the diode load and condenser. Typical values taken from a number of good commercial superheterodyne sets at random are tabulated below. In this case, as is explained later, the carrier frequency reaching the detector is substantially constant in frequency, being equal to the intermediate frequency in the table below.

Intermediate Frequency kc/s	*R* Ohms	*C* $\mu\mu F$	*Time constant* = *RC* *Microseconds*
445	200,000	100	20
455	600,000	50	30
465	100,000	50	5

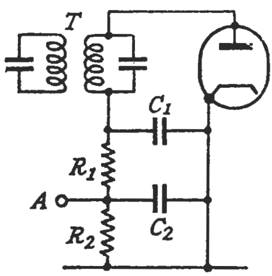

FIG. 2/XIX:5.—Diode Circuit with R.F. Filter.

A variation of the diode load circuit is shown in Fig. 2, in which the diode load consists of two resistances, R_1 and R_2, with two capacities, C_1 and C_2, shunted to the cathode. The transformer *T* is an I.F. or R.F. transformer supplying the signal to the diode circuit and the audio-frequency output is taken to the following valve from point *A*. Typical values of R_1, R_2, C_1 and C_2 taken from commercial receivers are :

	Case 1	*Case* 2	*Case* 3
R_1	100,000 Ω	47,000 Ω	50,000 Ω
R_2	180,000 Ω	1·5 MΩ	600,000 Ω
C_1	100 $\mu\mu$F	50 $\mu\mu$F	100 $\mu\mu$F
C_2	100 $\mu\mu$F	50 $\mu\mu$F	50 $\mu\mu$F

It will be seen that the tendency is to keep R_1 low in value compared to R_2 in order to transfer as much voltage as possible to the following stage. R_1 and C_1 constitute an R.F. filter preventing R.F. from being applied to the grid of the following valve, while R_2 and C_2 constitute the diode load and condenser proper.

6. Distortionless Diode Detector.

For measuring purposes, and other purposes where really distortionless detection is required, arrangements must be made so that the diode load is a pure resistance. Theoretically this is very simple : it is only necessary to use the circuit of Fig. 1, which shows also the following audio-frequency valve. If $R_1 = R_2 = R$ and $L = CR^2$, the diode load impedance presented between cathode and ground is a pure resistance of magnitude *R*. The difficulty in practice is that to preserve linearity the diode load has to be high, while, in order to use a value of *L* which has negligible self-capacity, the value of

R must be low. The probable compromise value of R is about 10,000 Ω, which is very low compared to the usual values of 100,000 to 500,000 ohms. If, however, a low-impedance diode is used, constituted for instance by a low-impedance triode with grid and anode strapped, quite good linearity can be obtained even with a diode load as low as 10,000 ohms.

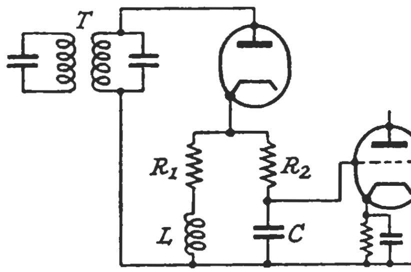

FIG. 1/XIX:6.—Constant Resistance Diode Load.

7. Linearity of Diode.

A second source of distortion occurs owing to the inherent non-linearity of the diode, particularly for low values of voltage and current. This is reduced, as indicated elsewhere, by using a high value of diode load, but, in addition, care is taken that the diode is operated at as high a level as possible. This increases the effective linearity of the diode, since for high input levels the part of the diode characteristic with greatest curvature (which occurs at low levels) constitutes a comparatively small percentage of the range of the characteristic which is used.

8. Superheterodyne Receiver.

The most common type of radio receiver in general use is the superheterodyne. This consists of the following :

1. A tuned coupling between the aerial and the grid of the first valve.
2. A series of tuned signal-frequency amplifiers amplifying the incoming frequency ; these are called R.F. or signal-frequency stages.
3. An oscillator and a mixer (see IX:13 and XIII:8) which change the incoming signal frequency to a (usually) lower or intermediate frequency (usually about 450 kc) at which greater amplification per stage is possible than at the (usually higher) incoming signal frequencies.

A second advantage of greater importance is that selective circuits may be introduced at the intermediate frequency which are designed to accept a single fixed band of frequencies. As a result it is possible to obtain not only a flatter response in the pass band but also a greater attenuation outside the pass band.

When receiving long waves the intermediate frequency is usually higher than the incoming signal frequency ; but when receiving short waves the intermediate frequency is much lower than the signal frequency and the improvement in selectivity over a T.R.F. receiver is very large. This is because the attenuation in the non-pass region of band-pass structures is a function of the amount by which the unwanted frequency differs from the mid-pass-band frequency expressed as a percentage of the mid-pass-band frequency. For instance, it would be much more difficult to design a structure to give a large discrimination between 10 and 10·5 Mc/s, than to design a structure to provide an equal discrimination between 0·45 and 0·95 Mc/s.

4. An intermediate-frequency filter, usually but not always constituted by the couplings between the stages of the intermediate-frequency amplifier.
5. An intermediate-frequency amplifier supplying the amplified I.F. to the detector.
6. A detector which suppresses the I.F. frequency and supplies, to the low-frequency amplifier following, an input voltage corresponding to the envelope of the I.F.
7. A low-frequency amplifier feeding a loud-speaker
8. A loudspeaker.
9. An automatic volume control (A.V.C.) circuit which, by amplification or by direct coupling, derives a negative D.C. bias from the detector circuit, proportional to the amplitude of the signal reaching the detector. This negative bias is applied to the R.F. and I.F. amplifying valves in such wise as to make the signal reaching the diode to a large extent independent of variations of incoming signal strength.

Method of Operation of a Superheterodyne Receiver. The general arrangement is shown in Fig. 1 and the relative locations in the frequency spectrum of wanted and second-channel frequencies (defined below), beat oscillator and intermediate frequency are shown in Fig. 2. The operation of the receiver is as follows :

If f_w in Fig. 2 is the carrier frequency of the station to be received (i.e. the signal frequency), the signal-frequency circuits of the receiver are tuned so that the minimum attenuation of these circuits occurs at f_w, as indicated by the position of the dotted curve marked " attenuation of SF circuits " : this is the normal process of tuning

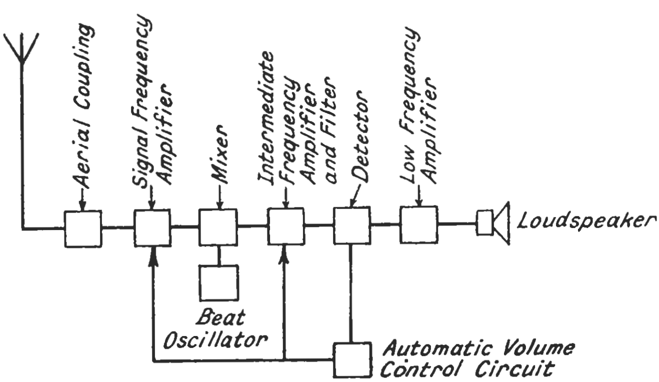

FIG. 1/XIX:8.—Essential Elements of a Superheterodyne Receiver.

in to a signal. Simultaneously, by means of interconnection of the controls, known as ganging, the beat oscillator is adjusted to a frequency f_0 (greater than f_w) such that $f_0 - f_w =$ I.F., where I.F. is the mid-band frequency of the I.F. filter. The mixer therefore has

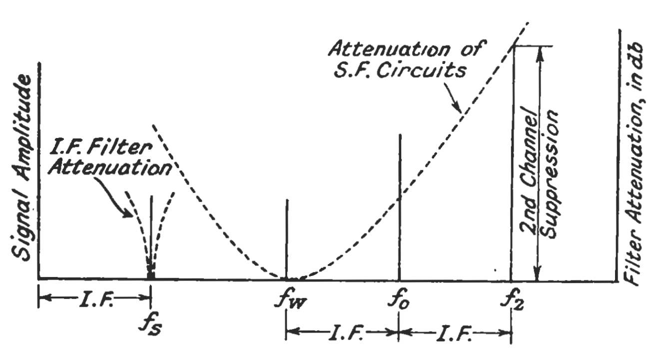

FIG. 2/XIX:8.—Relative Locations in the Frequency Spectrum of Wanted (Signal) Frequency, Second-Channel Frequency, Beat Oscillator and I.F. Frequency.

applied to it the frequency f_w and its sidebands plus the beat oscillator frequency f_0. The mixer is *in effect* a non-linear device so that there appears at its output f_0+f_w and $f_0 - f_w$ plus sum and difference frequencies between f_0 and the sidebands of the wanted carrier. (See XI:9 for behaviour of non-linear device and XIII:8 for more exact description of working of a mixer.) Neglecting the sum frequency, the difference in frequency due to f_0 and f_w is $f_0 - f_w$ = I.F., a shifted carrier frequency appearing in the middle of the I.F. filter pass range at f_s = I.F. Similarly, shifted sideband frequencies appear each side of f_s so that an amplitude modulated wave of frequency f_s is applied to the input of the I.F. filter and I.F. amplifier. Just as the sideband frequencies are transferred while preserving their proper frequency interval from the carrier frequency, so are any other frequencies in the neighbourhood of f_w which may, for instance, be due to a transmitter located on an adjacent channel. The degree to which these are suppressed in traversing the I.F. filter depends on the attenuation characteristics of the I.F. filter. After traversing the I.F. filter the shifted wanted signal frequencies plus any residual adjacent channel frequencies, at the level they have reached after attenuation by the I.F. filter, are applied to the detector, which is usually a diode circuit of the form shown in Fig. 1/XIX:2 or Fig. 2/XIX:5. The shifted carrier interacts with its sidebands to give rise to the wanted audio frequency and interacts with adjacent-channel frequencies to give rise to higher audio frequencies. The detected audio frequency is then amplified in the audio-frequency amplifier and applied to the loudspeaker.

The various types of interference and noise observed in a radio receiver, together with such means as are available for their reduction, are described in XVIII:5 to 8 inclusive. Second-channel interference is rather a special case and is also discussed immediately below.

9. Second-Channel Interference.

Referring to Fig. 2/XIX:8, it will be seen that if an unwanted frequency happens to be located at f_2 such that $f_2 - f_0$ = I.F., the difference frequency at the output of the mixer is equal to the I.F. frequency and will traverse the I.F. filter and be detected just as is the wanted frequency, with one difference. This difference is that the frequency is subject to the amount of attenuation (introduced by the S.F. circuits) marked "2nd channel suppression" in Fig. 2/XIX:8.

Interference due to a frequency located at f_2 is called *second-channel interference.*

It is to be noted that the suppression of second-channel pick-up requires a high S.F. attenuation at a frequency higher than the wanted frequency by twice the I.F. frequency.

A useful point to note in connection with the observed effect of second-channel interference is that, as the beat oscillator is increased in frequency, it can be seen from Fig. 2/XIX:8 that $f_s = f_0 - f_w$

also increases in frequency, while the shifted value of f_2, which is equal to $f_2 - f_0$ decreases in frequency. The wanted frequency and unwanted frequency therefore travel through pass range of the I.F. filter in opposite directions and the audio-frequency beat between them, which occurs at the diode, therefore changes note as the beat oscillator is changed in frequency; in other words, as the tuning of the receiver is changed the beat note appears first at a high frequency, getting lower in pitch, disappears when f_s = the shifted value of f_2 and reappears as a note of increasing pitch as the tuning is moved further.

Heterodyne notes and mixed-channel whistles (see XVIII:5.22 and 5.22.12), on the other hand, do not change pitch as the tuning is changed.

10. Mixers.

The types of valves usually employed as mixers are heptodes, hexodes and octodes, and are shown in Fig. 1/IX:13, and Fig. 1/XIX:10.

In the hexode and heptode the incoming signal is applied to grid 1 and the beat oscillator frequency to grid 2.

The octode has two grids which operate respectively as the grid and anode of a triode connected to an external oscillator circuit, as indicated in Fig. 1 (*c*)/IX:13. The incoming signal is then applied to grid 4.

The operation of a mixer is closely analogous to that of a suppressor grid modulator; see XIII:8. The voltage amplification of the mixer, measured from the grid to which the signal frequency is applied, to the anode, is directly proportional to the potential applied to the other control grid: grid 2 in the case of hexode and heptode, and grid 1 in the case of the octode.

In other words, if e_2 is the voltage applied to the control grid influenced directly by the beat oscillator, the effective voltage amplification, from signal grid to anode, is:

$$\mu_e = ke_2 . \qquad (1)$$

where k is a constant.

If e_1 is the instantaneous voltage applied to the signal grid, the anode voltage swing

$$e_a = \mu_e e_1 = ke_1 e_2 \qquad (2)$$

hence the output voltage is proportional to the product of the input and output voltages. This is the condition for the production of sum and difference frequencies, as can be seen from equations (3)

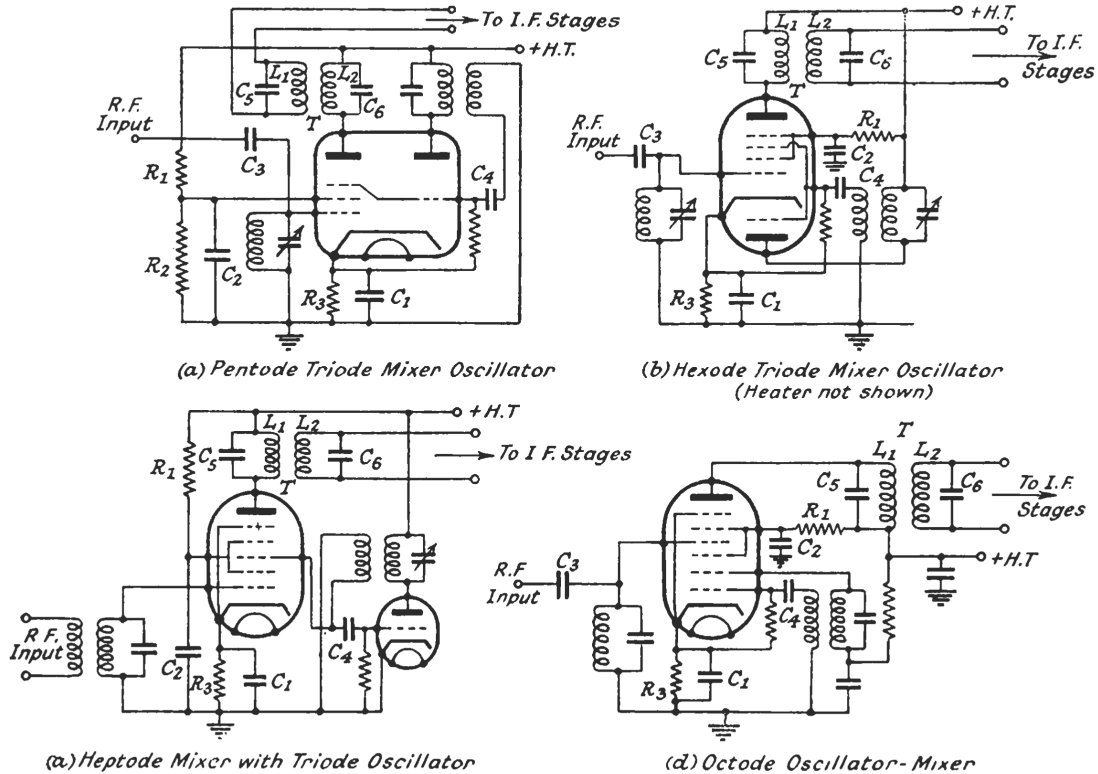

FIG. 1/XIX:10.—Oscillator and Mixer Circuits.

and (4)/XI:9, where the term corresponding to the product of two sinusoidal voltages gives rise to sum and difference frequencies. More explicitly, if a frequency f_w is applied to the signal grid and a frequency f_0 is applied to the oscillator grid, there will appear in the anode circuits, in addition to frequencies f_w and f_0, also the frequencies f_0+f_w and f_0-f_w, the last of which is the frequency required for supplying to the I.F. filter and amplifier.

Typical mixer circuits are shown in Fig. 1 with their associated oscillator circuits, which have all been shown as the tuned anode type to make identification of circuit components simple. In all circuits R_1 is of such a value that the screen-grid current drops the screen grid to the correct value. In Fig. 1 (*a*) the screen-grid current is reinforced by the current through R_2. R_3 is the bias resistor in each case. The decoupling condensers C_1 and C_2 may be mica condensers of about 0·001 μF ; on long-wave sets they should be shunted with a paper condenser of about 0·1 μF. The coupling condensers C_3 and C_4 should be mica condensers about 0·01 μF. The coupling transformer T constituted by L_1, L_2, C_5 and C_6 is really one of the I.F. filter stages and may be designed directly from the chart on Fig. 1/VII:14 for the band width required for the I.F. filter.

The ratio between the input power to a mixer at signal frequency and the output power from the mixer at I.F. frequency is called the *Conversion Gain* (*or Loss*) :

$$\text{Conversion Gain in. db.} = 10 \log_{10} \frac{\text{Output power at I.F.}}{\text{Input power at signal frequency}}.$$

When crystals are used as mixers, as is usual at centimetre wavelengths, the conversion gain is negative, i.e. it is a loss. With a good crystal mixer this loss will be about 10 db. at $\lambda = 10$ cm.

11. I.F. Filter and I.F. Amplifier.

The I.F. filter and I.F. amplifier are usually combined, and are constituted by a number of class A amplifying stages coupled together by transformer or coupling circuits of the type shown at T in Fig. 1/XIX:10. The primary impedance for which this coupling is designed is determined by the load requirements of the preceding valves, and the secondary impedance by the input capacity of the following valve and the percentage band width required in the I.F. transformer.

The determination of the secondary impedance may be illustrated by an example in which the total effective grid input capacity of the following valve is 10 $\mu\mu$F, wiring stray capacity 20 $\mu\mu$F, self-capacity of coil estimated at 5 $\mu\mu$F, total estimated capacity 35 $\mu\mu$F. A trimmer condenser is added with a capacity range from 5 to 25 $\mu\mu$F, so the estimated total capacity range is from 40 to 60 $\mu\mu$F. Hence design for 50 $\mu\mu$F. Assume the I.F. frequency is 450 kc/s and that design is carried out for a band width of 15 kc/s: the percentage band width is then $\frac{15}{450} = 3{\cdot}33\%$. From Fig. 1/VII:14 the value of capacity for 10,000 Ω impedance with a mid-band frequency of 1 Mc/s and 3·33% band width is 475 $\mu\mu$F, and at 450 kc this corresponds to $475 \times \frac{1{,}000}{450} = 1{,}055$ $\mu\mu$F. Hence the secondary impedance is $\frac{1{,}055}{50} \times 10{,}000 = 200{,}000$ Ω approximately.

The remainder of the design is straightforward.

In special cases where high selectivity is required, the coupling stages may be supplemented by a straight band-pass filter inserted at any convenient point in the I.F. chain. If it is inserted between two I.F. stages, the coupling units of the type shown at T may be used at each end of the filter to match it to the valves at each end.

Recent fashion has favoured the use of a number of straight anode tuned stages with their resonant frequencies staggered to provide an approximation to a band-pass characteristic. The advantages are simplicity of design and ease of construction.

12. Signal Frequency ("Radio-Frequency") Circuits.

These are the circuits which amplify and/or select the incoming signal frequency. They are sometimes called radio-frequency circuits, but "signal frequency" is the modern designation.

The most common aerial coupling circuit in use is shown in Fig. 1 (*a*), and this circuit is often used as an interstage coupling between valves.

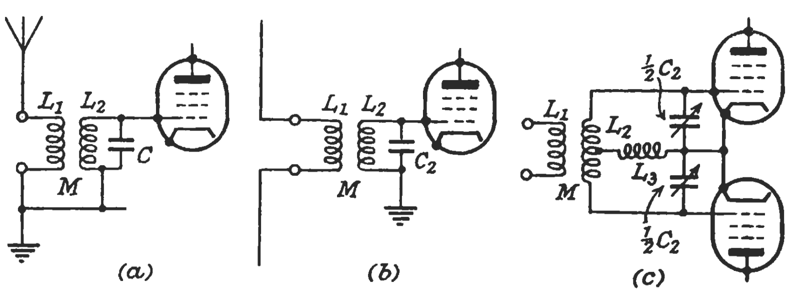

FIG. 1/XIX:12.—Aerial Coupling Circuit.

At (*b*) are shown two of these circuits in use with balanced short-wave aerials, and at (*c*) is shown an improved version for balanced aerials designed to reduce unbalances transferred from the secondary circuit to the primary circuit via stray capacity. The circuit at (*b*) may be improved by the use of an earthed screen between primary and secondary windings.

For interstage coupling, the circuits in Fig. 2 have been used. The coupling at (*a*) is of the same form as in 1 (*a*) constituted by L_1, L_2 and C_2, while the coupling at (*b*) is constituted by L_1, L_2, C_2 and C_3 and has improved band-pass characteristics as compared with the circuit at (*a*). In each case resistance R_1 and condenser C_1 provide H.T. decoupling and do not constitute part of the R.F. coupling.

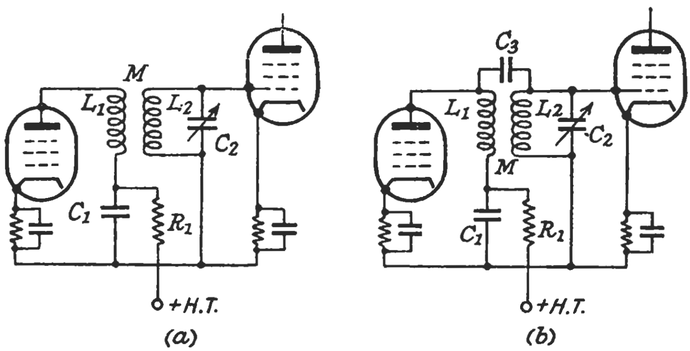

FIG. 2/XIX:12.—Interstage Coupling Circuits.

The present tendency is towards the use of a simple tuned circuit for interstage coupling or a band-pass circuit similar to that used in I.F. circuits.

12.1. Design of Signal Frequency Circuits. In the design of these circuits art plays a large part on account of:

(*a*) Variations in the types of aerials which will be used with any given receiver.

(*b*) The need for providing adequate band width in a circuit in which the mid-band frequency is variable.

(*c*) The necessity for maintaining circuits in track: all amplifier circuits must always be tuned to the same frequency and the oscillator must always be tuned to a frequency higher than the amplifier tuning by a frequency interval equal to the I.F.

Any design indications based on simple assumptions are therefore liable to be misleading. The simple design methods below have, however, been included to give an idea of the approximate dimensions of elements, and because they are sometimes useful in isolated cases, and not because they illustrate the design technique in use by receiver manufacturers.

Aerial Coupling Circuits of Fig. 1. There are various schools of thought relating to the design of these circuits. The first considers the relation of the relative resonant frequencies of aerial, or primary circuit, and secondary circuit. If the secondary circuit is arranged to tune over a given band of signal frequencies the primary circuit may be adjusted to resonate :

(*a*) Below the band of signal frequencies to be received, in which case the value of L_1 is comparatively large, and the circuit is said to have a high impedance primary (i.e. L_1 is large).

(*b*) In the band of frequencies.

(*c*) Above the band of frequencies : the circuit is then said to have a low impedance primary (i.e. L_1 is small).

(*a*) affords relatively high selectivity and relatively low sensitivity, (*c*) affords relatively high sensitivity and relatively low selectivity. A full discussion is given in *L'Onde Electrique*, Nov. 1938, p. 521, "Étude sur la Normalisation des Bobinages, Condensateurs variables, et Cadrans."

The high impedance primary method is the most usual for medium and long-wave commercial receivers which have to receive signals in a part of the frequency spectrum where atmospheric disturbances limit the minimum field strength receivable to a value between 100 and 2,000 microvolts, so that valve noise is relatively unimportant and high sensitivity in the input circuit is not of paramount importance in view of the ease of securing gain with modern valves.

Method (*b*) has application only to special cases of receivers designed to receive a particular frequency, but is described immediately below because it is the most simple method to apply and the one which will probably be of most use to engineers other than designers of receiving sets.

12.11. Application of Method (*b*) to the Circuit of Fig. 1 (*a*). In British practice, at medium-wave frequencies the average aerial is assumed to look like a 200-$\mu\mu$F (U.S.A. 150-$\mu\mu$F) condenser in series with 25 Ω. In the aerial coupling circuits of Fig. 1 the value of L_1 for a given band of frequencies is the value which resonates with the aerial capacity at geometric mid-band. (The average variable condenser covers a range of about 10 to 1 in capacity, so that a single inductance will cover a range of wavelength of about 3 to 1.) As an example, suppose the range to be covered is from 200 to 600 metres, the geometric mean of 200 and 600 is $\sqrt{200 \times 600} = 347$ metres which corresponds to 865 kc/s; and at 865 kc/s the value of inductance resonant with 200 $\mu\mu$F is

$$L_1 = \frac{10^6 \times 10^{12}}{200 \times (2\pi \times 865{,}000)^2} = 170\ \mu\text{H}.$$

Let
$$Q_1 = \frac{L_1\omega}{R_1+R_a}$$
Q_2 = the Q of L_2
R_a = the aerial resistance
R_1 = the resistance of L_1.

Then at secondary resonance the secondary series impedance is $\frac{L_2\omega}{Q_2}$ and the mutual coupling must be adjusted so that at the geometric mid-band frequency the impedance transferred to the primary circuit is equal to the total primary impedance $= R_1+R_a = \frac{L_1\omega}{Q_1}$ where $\omega = 2\pi f$ and f is the geometric mid-band frequency. That is

$$\frac{M^2\omega^2Q_2}{L_2\omega} = \frac{L_1\omega}{Q_1} \quad \text{where } M = k\sqrt{L_1L_2}$$

$$\therefore\ k^2Q_2L_1\omega = \frac{L_1\omega}{Q_1}$$

$$\therefore\ k = \frac{1}{\sqrt{Q_1Q_2}} \quad . \quad . \quad . \quad . \quad (1)$$

The value of L_2 is determined by the input capacity of the valve including stray capacity, and by the frequency range to be covered by variation of C_2. If the frequency range to be covered is 3 to 1, then the maximum value of C_2 must be equal to 9 times the total circuit capacity with C_2 at minimum.

Let C_0 = the minimum condenser capacity
C_c = the circuit capacity including wiring and valve input capacity
C_m = the maximum value of the condenser.

Then
$$\frac{C_m+C_c}{C_0+C_c} = 9$$

$$\therefore\ C_m = 9C_0+8C_c. \quad . \quad . \quad . \quad (2)$$

and
$$L_2 = \frac{1}{C_m\omega_1^2}$$

where $\omega_1 = 2\pi f_1$ and f_1 is the lower limit of the band of frequencies to be covered.

Evidently such a circuit will give an ideal performance only at mid-band, falling off in efficiency towards each end of the band, and while at mid-band this efficiency is much higher than that obtained by the methods of design (*a*) and (*c*), the latter are preferred because they give a more uniform efficiency over the band, and greater selectivity.

12.12. Interstage Coupling Circuits : Circuit of Fig. 2 (*a*). Since the advent of pentodes, which require a load impedance which is only between 1 and 5% of their anode impedance, the practical condition is that the current flowing through L_1 is independent of the impedance presented by L_1 towards the valve anode and independent therefore of secondary tuning. Maximum output voltage is always obtained therefore at secondary resonance. In this condition the impedance transferred to the primary circuit is a maximum and is of magnitude $\dfrac{M^2\omega^2 Q_2}{L_2\omega} = k^2 Q_2 L_1\omega$, retaining the conventions of XIX:12.11. The input impedance looking into L_1 is then $jL_1\omega + k^2Q_2L_1\omega = L_1\omega(k^2Q_2+j)$, neglecting the resistance $Q_1L_1\omega$ constituted by the losses in L_1. Since it is a comparatively easy matter to obtain a coupling factor of 0·5, and since the value of Q_2 is generally of the order of 100 or greater, the value of the impedance looking into L_1 is nearly a pure resistance of magnitude $k^2Q_2L_1\omega$. The method of design is therefore extremely simple : the coupling factor is made as high as possible and L_1 is determined by the equation

$$k^2Q_2L_1\omega = Z_a,$$

where Z_a is the required load impedance for the preceding value,

or
$$L_1 = \frac{Z_a}{k^2Q_2\omega} \quad . \quad . \quad . \quad . \quad (3)$$

This method may also be used for designing aerial coupling circuits instead of the methods described above, in which case Z_a in equation (3) represents the input impedance of the receiver and is usually made equal to about 200 to 400 ohms.

12.2. Ganging and Tracking. *Ganging* is the process by which a number of circuits tuned by condensers on a single spindle are adjusted to be simultaneously in tune for every angular position of the common spindle. This is achieved by using identical variable condensers for tuning each circuit and building up the stray capacities in each circuit to a common value by means of small *trimmer* condensers which are usually made adjustable by means of a screwdriver.

Tracking is the process in the adjustment of a superheterodyne receiver by which the beating oscillator frequency is maintained at its required distance away from the received signal (to which the signal-frequency or radio-frequency circuits of the receiver are tuned by means of a common spindle) for all positions of the common spindle. It is to be understood that the common spindle controls the frequency of the beat oscillator as well as the signal-frequency circuits.

In practice this is achieved by using identical condensers for tuning both signal-frequency circuits and beat oscillator and inserting in series with the beat oscillator tuning condenser a *padding* condenser. A trimmer condenser is also connected either across the oscillator tuning condenser or across the oscillator coil. The oscillator tuning coil is made of smaller value than the tuning coils in the signal-frequency circuits so that the oscillator frequency is higher than the signal frequency due to both smaller inductance and smaller capacity (due to series padding condenser).

By suitable choice of oscillator tuning inductance, padding condenser and trimmer, it is possible to obtain the exact value of required frequency difference at three points in each wavelength band—the band covered by each set of tuning coils.

Ganging is sometimes called alignment, while tracking is sometimes called ganging. The terms ganging and alignment are each used also to indicate the complete process of lining up a receiver, that is, they embrace the complete process of ganging and tracking as defined above, as well as the adjustment of the I.F. filter. The definitions above are, however, preferred.

12.21. Lining up a Receiver. The exact order in which this is done varies with different designs. The usual procedure is to adjust the I.F. filter first. If this is a proper band-pass filter it must be adjusted as described in XXV:8. If, as is more usual, it consists of a series of tuned transformers as shown in Fig. 1/XIX:18, I.F.T.1, I.F.T.2, etc., then it is adjusted by applying the I.F. frequency to the grid of the mixer valve and tuning each transformer in turn for maximum output. Since both primary and secondary of each transformer are tuned, and since classical methods of tuning coupled circuits (see VII:14.4) cannot be used, it is necessary in effect to try all possible positions of each trimmer against all possible positions of the other trimmer on the same transformer, finally choosing the two positions which give the maximum output. The output may be measured as diode current, or, if a modulated oscillator is used to supply the input, any form of audio-frequency meter may be used.

The processes of ganging and tracking may be carried out, using an output meter and adjusting to maximum output, in cases where the I.F. filter consists of tuned transformers and has not proper band-pass characteristics. (There is no reason why such a filter should not have band-pass properties ; see VII:14.3.) Where the I.F. filter has a characteristic which approximates to band pass, then a ganging oscillator and oscilloscope must be used for ganging and tracking (see XX:15).

The best procedure after adjusting the I.F. filter is to adjust the tracking. This is done at two frequencies in each wave band near the ends of the wave band. The receiver dial is set to each frequency in turn, the frequency is applied to the signal grid of the mixer and the oscillator frequency is adjusted until maximum output is observed, or, if a ganging oscillator is used, until a symmetrical response characteristic is obtained. The oscillator frequency is adjusted by means of the oscillator padding condenser at the lower frequency and by means of the oscillator trimmer condenser at the higher frequency, a series of successive adjustments being made at each of the frequencies in turn until satisfactory tracking results at both frequencies.

Finally, the receiver is ganged by aligning the signal frequency or R.F. circuits. This is done at the high-frequency end of each wave band. The receiver dial is set to this frequency, the frequency is applied at the aerial and earth terminals of the receiver and the trimmer condensers in each stage are adjusted in turn for maximum output.

12.3. Neutralization of Anode-grid Capacity. With the advent of pentodes the screening between grid and anode is such that the residual anode-grid capacity is very low, for instance, 0·005 $\mu\mu$F is an average figure. Even at 100 Mc/s the reactance of this is over 100,000 ohms, so that neutralization is usually unnecessary in receivers, except in special cases.

13. Automatic Volume Control.

Automatic volume control operates by deriving a D.C. current, proportional to the signal (or I.F. voltage) arriving at a convenient point in the circuit, which is fed through a network of resistances with filter condensers shunted to ground at essential points, so contrived that the grid bias on any or all of the amplifying stages (signal frequency, I.F. or mixer) increases as the amplitude of the signal increases. The valves used in these stages are of the variable μ type in which the amplification factor drops as the bias increases: see IX:14.

For this purpose the cathodes of all valves to which A.V.C. is to be applied may be connected directly to earth: individual automatic bias is not used. A steady bias is applied to the grid circuits of all valves by any convenient means, and a diode rectifier is used to augment this bias as the incoming signal increases.

13.1. Simple Practical Form of A.V.C. One way of applying the steady bias and A.V.C. voltage which has been used in practice is illustrated in Fig. 1. This shows in schematic form a number of valves, V_1, V_2, V_3 and V_4, constituting the R.F., mixer and I.F. valves of a receiver: for instance, V_1 may be an R.F. amplifier, V_2 the mixer,

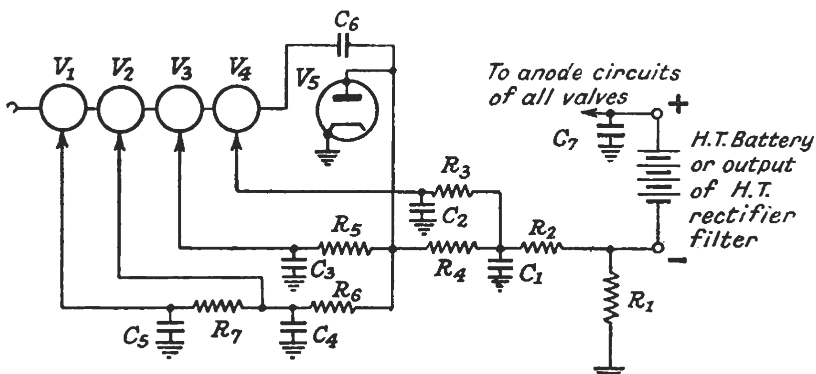

FIG. 1/XIX:13.—Principle of Automatic Volume Control.

and V_3 and V_4 the first and second I.F. stages respectively. The output of V_4 is fed to a detector circuit not shown, and to provide A.V.C. is also fed to the diode V_5 through the coupling condenser C_6. Normal bias is applied to valves V_1 to V_4 by virtue of the voltage drop across the common resistance R_1 in the H.T. circuit: the anode and screen currents of all valves flow through this resistance and cause the non-earthy end of this resistance to assume a negative potential with regard to ground. This negative potential is fed to the bottom ends of the grid circuits of all valves through resistances R_1 to R_7 as indicated. It will be noted that the amount of filtering or decoupling provided by condensers C_1 to C_5 is greater for early stages than for later stages. As the result of the application of the output signal from V_4 to the anode of V_5 the anode assumes a negative potential since all positive charges flow to earth through the diode. This negative potential causes a current to flow through R_4, R_2 and R_1 to ground and so augments the negative bias produced by the anode current flowing through R_1. In the particular case shown the A.V.C. applied to V_4 is less than that applied to the earlier stages by the amount of the voltage drop in R_4.

An important characteristic of A.V.C. circuits, particularly when the receiver is used for circuits subject to rapid fading, is the time constant of the circuit. The simplest measure of the performance of the receiver from this point of view is to take the time for the receiver to restore to full gain after the removal of a steady signal of amplitude sufficient to bring the gain to the knee of the A.V.C. curve. See Fig. 2 below. Experience has shown that the optimum performance is obtained when this time lies in the neighbourhood of a second.

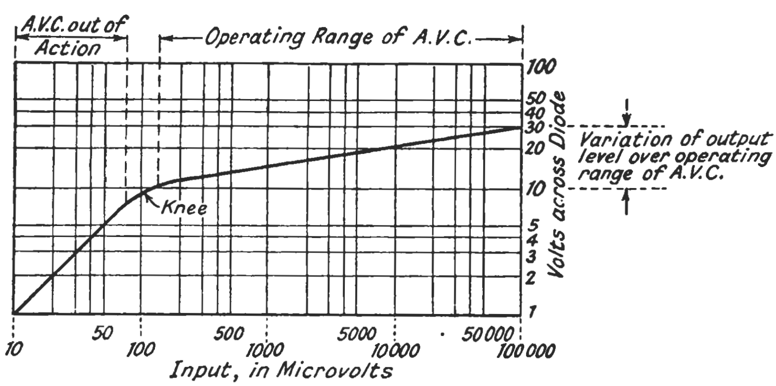

FIG. 2/XIX:13.—A.V.C. Curve.

The performance of a receiver fitted with A.V.C. is defined by a curve plotted between the signal frequency input voltage (applied between aerial and earth) in microvolts, and the output volts across the detecting diode. In an average receiver the curve is of the form shown in Fig. 2. Logarithmic scales are always employed for both input and output voltages. At very low levels the output is insufficient to operate the A.V.C., so that the output is proportional to the input (i.e. the gain stays constant), and when the A.V.C. comes into operation the output rises less rapidly with input. In the curve shown the A.V.C. holds the output constant within ± 5 db. over its operating range. A good A.V.C. will hold the output constant within ± 2 db. over its operating range.

In the type of A.V.C. described above the knee is not so pronounced as is indicated in Fig. 2, since the A.V.C. is operating to some extent even with the lowest voltages. An advantage in A.V.C. operation can be obtained by delaying the action of the A.V.C. until the volts across the diode have built up to a value corresponding to the level desirable for good detection. This can be done by inserting a delay voltage in series with the diode of Fig. 1, such as would, for instance, be provided by a battery between cathode and earth with its negative terminal to ground. The provision of such a battery is evidently impracticable in commercial receivers and other means must be used.

13.2. Amplified and Delayed A.V.C. using Diodes and D.C. Amplifying Valve. One circuit for realizing this is shown in Fig. 3. The main anode current to all valves flowing through resistance R_6 provides bias to all grid circuits via R_4, the cathodes of all valves

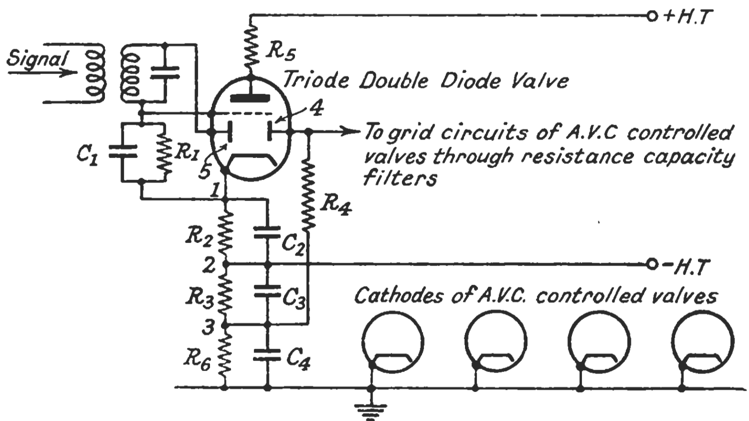

FIG. 3/XIX:13.—Amplified and Delayed A.V.C. using Diodes and D.C. Amplifying Valve.

to which A.V.C. is applied being connected to the receiver screen and earth. (Note the position of H.T. minus.) The main anode current flowing through R_3 makes point 2 negative with regard to point 3. Let E_3 be the voltage across R_3. The anode current of the triode double-diode valve flowing through resistance R_2 makes point 1 and the cathode more positive than point 2 by a voltage E_2.

The potential of the cathode with regard to point 3 is therefore $E_c = E_2 - E_3$ and is positive when $E_2 > E_3$, which is the practical condition in the absence of received signal. The value of $E_2 - E_3$ in the absence of signal is the voltage delay since A.V.C. is applied only when the cathode goes negative with regard to diode-anode 4. Initially, diode-anode 4 is substantially at the potential of point 3, differing from that potential only by an amount corresponding to any small leakage current supplied to the grid and grid filter circuits.

When the cathode goes negative with regard to diode-anode 4 a current flows round the loop constituted by diode anode 4, cathode, R_2 and R_4, applying negative A.V.C. bias to the grid circuits.

When a signal is received, rectification occurs in diode 5, with the result that the grid becomes negative with regard to the cathode, the anode current, and so E_2 is reduced, and finally when E_2 becomes less than E_3 the cathode becomes negative with regard to diode-anode 4 and A.V.C. operation starts.

Such a circuit gives a more pronounced knee because, firstly, the A.V.C. operation does not start until the signal reaches a definite level, and secondly, when it does start the A.V.C. action is greater and the variation of output over the range of operation of the A.V.C.

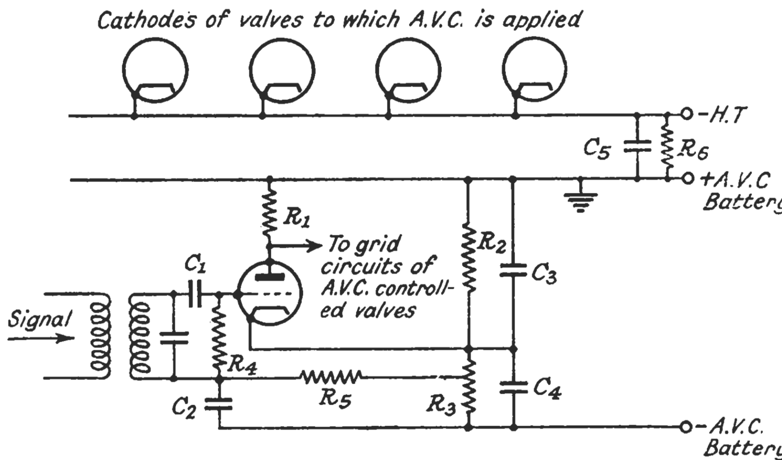

FIG. 4/XIX:13.—Amplified and Delayed A.V.C. using Triode Detector.

is reduced. The adjustment of the voltage across R_2 and R_3 is however critical and is upset by changes in valve parameters with life, and by voltage variation, so that delay is not constant and cases have even occurred where the delay voltage has changed sign.

13.3. Amplified and Delayed A.V.C. using Triode Detector. This form is used in some of the more expensive communication receivers and uses a separate voltage supply source of the order of 100 volts for driving the A.V.C. triode detector valves.

The circuit is shown in Fig. 4. The A.V.C. voltage is developed across the resistance R_1 in the anode of the triode detector. Resistances R_2 and R_3 form a potentiometer across the A.V.C. battery arranged to provide a convenient bias control (for adjusting the delay) to the grid of the triode, this bias being fed through resistances

R_4 and R_5. C_1 is a stopping condenser (it is not quite clear why C_1 and R_4 cannot both be eliminated), while C_2, C_3 and C_4 are decoupling condensers. The triode valve operates as an anode-bend detector, so that when a signal is received, of amplitude capable of overriding the bias, rectification takes place, causing a rise of anode current, and the A.V.C. starts to operate. Normal bias, in the absence of signal, is supplied by the flow of the total anode and screen current to all valves through resistance R_6; the A.V.C. bias is then additive to the normal bias.

This system has the apparent advantages of the last system without the disadvantages of critical adjustment. It has the added advantage that the rectified current rises more rapidly than the incoming signal, with the result that the variation of output level over the operative range tends to be further reduced. In practice, however, it has the disadvantage (which does not occur in the system previously described) that the A.V.C. voltage varies with depth of modulation, and to reduce this effect it is sometimes the custom to operate the triode with a small initial standing feed so that the delay obtained is nil. The absence of delay may cause the speech diode, which is usually (but of course need not be) located at the point of the circuit from which the A.V.C. signal voltage is taken, to receive a low signal voltage giving rise to non-linear detection. If, to avoid low volts across the speech diode, the A.V.C. triode is tapped off across an earlier stage, the gain round the A.V.C. loop is reduced, and the variation of output over the A.V.C. operating range is increased. Further, if the knee of the A.V.C. curve is not to be moved to the right (i.e. if the operating range of the A.V.C. is not to be shifted to higher levels), a lower delay voltage is necessary, which introduces further undesirable reactions.

It should be noted that the A.V.C. is often taken off the I.F. circuits at a point earlier than the speech to avoid rapid removal of the A.V.C. when tuning out of a station. Such rapid removal gives rise to objectionable sideband chatter during the process of tuning.

13.4. Amplified and Delayed A.V.C. using A.F. Balanced Detectors and Triode D.C. Amplifier (B.B.C. Patent No. 533,275). The circuit, which is due to Mr. C. J. W. Hill and Mr. J. Wardley-Smith, is in use at the B.B.C. receiving station on short-wave receivers installed there. It was designed to overcome the disadvantages resulting from the last described circuit, and is shown in Fig. 5.

The incoming signal from the receiver is fed through the condensers C_1 and C_2 to the simple shunt diode rectifier circuits consisting of the diodes V_1 and V_2, and their loads R_1 and R_2 respectively. The output from these diodes is commoned through high resistances R_3 and R_4 and a large isolating condenser, C_3, the centre point of this commoning being taken through an R.F. filter (C_4, R_5, C_5, R_6) to the grid of the triode D.C. amplifier. The other end of the diode circuit is returned to a variable or fixed bias point on R_9. The anode load R_7 of the triode V_3 is made high, and the A.V.C. line taken direct from the anode through the usual filtering circuits to the control grids of the controlled valves. Resistances R_8 and R_9 form a

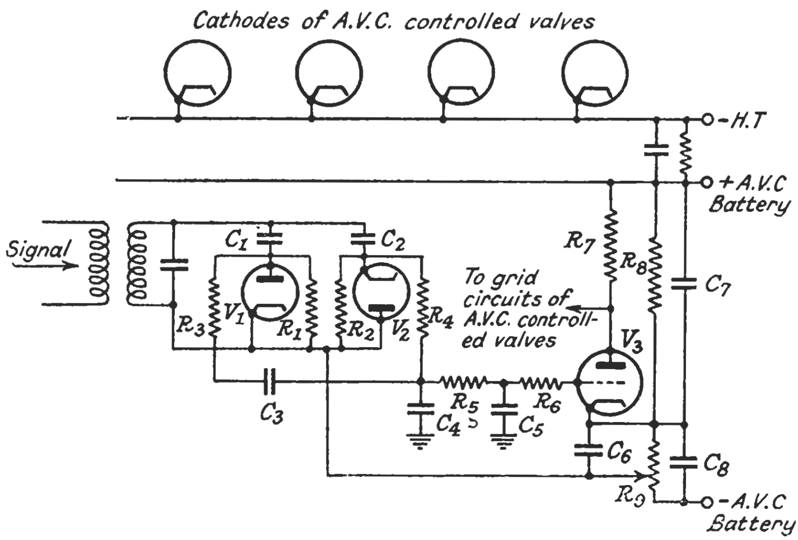

FIG. 5/XIX:13.—Amplified and Delayed A.V.C. using Balanced Detectors and Triode D.C. Amplifier.

potentiometer across the A.V.C. battery (usually about 100 V), thus giving a bias for the triode across R_9. Condensers C_6, C_7 and C_8 are decoupling condensers.

The operation of this system is as follows. The full signal voltage is applied without delay to the diode circuits D_1 and D_2. As the two diodes are connected in reverse sense, they rectify from opposite halves of the modulated wave. Hence at the combination point of the two outputs, the audio frequencies are in opposite phase and of equal amplitude, so cancelling, and giving no resultant audio-frequency output. The D.C. output from the diodes, however, is not commoned, and hence the D.C. output of one diode only is passed to the triode grid. It should be noted here that this D.C. voltage is proportional to the peak value of the mean carrier level, and is not affected by modulation depths up to 100%.

A negative bias for the triode is obtained from R_9, and it will be seen that the total D.C. voltage applied to the grid of the triode is the sum of the bias from R_9 and the voltage from the diode D_2. As these two voltages, the one static, and the other derived from the signal, are of opposite sign, the actual voltage applied to the grid of the triode is the difference between these two. The voltage from R_9 can therefore be regarded as a delay voltage, and when the difference between the two voltages is less than the grid base of the triode, current will flow in the anode load R_7, thus creating a voltage across R_7 available for use as A.V.C.

While this system shares with the previous system the disadvantage of a separate battery or source of supply for the A.V.C., it gives an unusually good performance.

It provides effective delay and so full gain on low signals, the speech diode is worked at high level, no audio-frequency voltages reach the A.V.C. amplifier, and the A.V.C. voltage is independent of modulation. The knee of the A.V.C. curve is sharp and can be moved along the scale of input level (see Fig. 2) without appreciably changing its shape. The variation of output level over the range of operation of the A.V.C. is very small, and the A.V.C. performance is said to be substantially independent of receiver gain, so that, by manual control of the receiver sensitivity, a signal can be placed at such a point on the A.V.C. curve that the gain during a fade can be increased by a predetermined amount : by such an adjustment the receiver can be set so that even on a bad fade the receiver and other noise does not rise to objectionable levels. Finally, the system is not critical to adjust.

14. Triple Detection Receiver.

This is sometimes called a double superheterodyne receiver ; it is used for short-wave reception where high selectivity is required. It is possible to regard the circuit of Fig. 1/XIX:8 as one in which a simple receiver constituted by the I.F. amplifier and filter, the detector and L.F. amplifier, is preceded by a signal-frequency filter and a frequency changer. The I.F. frequency is fixed and such a simple receiver can only receive one frequency, the I.F. frequency, but the addition of the frequency changer extends the frequency range of the receiver. The present point is that by introducing a frequency changer in front of any receiver which is capable of receiving only a limited range of frequencies (e.g. medium waves), the frequency range of that receiver can be extended (e.g. to receive short waves). If the receiver is a superheterodyne receiver, then the combination of frequency changer and receiver contains two beat

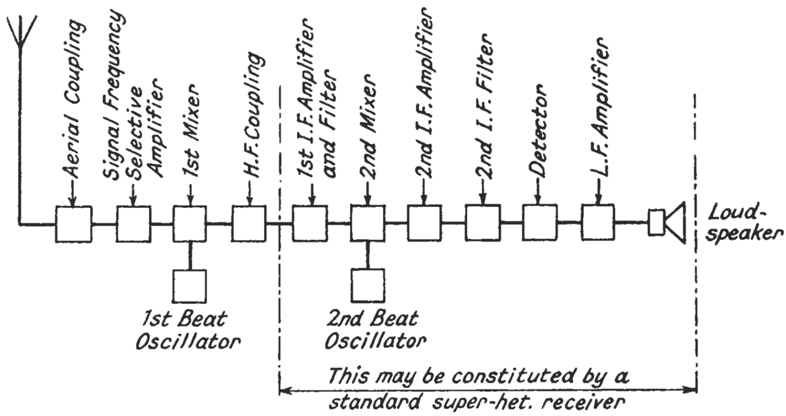

FIG. 1/XIX:14.—Triple Detection Receiver.

oscillators, two mixers, and one diode detector. If it is permissible to call the beating process taking place in the mixer, detection (although little justification appears to exist for this), then the composite receiver may be called a triple detection receiver.

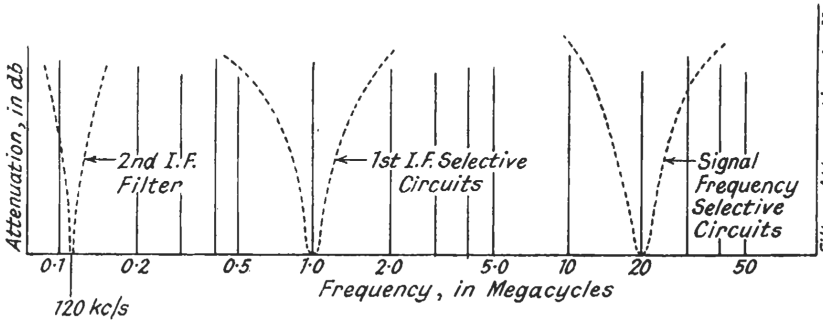

FIG. 2/XIX:14.—Triple Detection Receiver: Location of Selective Circuit Attenuation Characteristics in Frequency Spectrum for One Adjustment of Receiver.

It is evident that a triple detection receiver may be built as a single unit, but the rare occasions on which such receivers are required usually make it more economical to add a frequency changer to an existing receiver. The block schematic of a triple detection receiver is shown in Fig. 1. In Fig. 1 the part of the receiver to the right of the chain-dotted line may be constituted by an ordinary superheterodyne receiver. In this case the second I.F. filter and second I.F. amplifier have been shown separately, and while it is not always essential that they should be separate, in cases where triple detection is justified, it is sometimes justifiable to go to the expense of a separate I.F. filter to obtain sharper attenuation characteristics. If the assembly is constituted by a normal receiver plus a frequency changer, the normal signal frequency amplifying stages of the receiver may be regarded as the first intermediate frequency amplifier and filter of the whole receiver.

Fig. 2 shows the form of the attenuation frequency characteristics of the signal frequency tuned circuits (in aerial coupling and signal frequency selective amplifier in Fig. 1), and the two I.F. circuits, of a typical triple detection receiver constituted by a normal receiver plus frequency changer when tuned to receive 20 Mc/s, the normal receiver being tuned to receive 1 Mc/s. The advantage of using a normal receiver to provide the first I.F. selectivity is that the first intermediate frequency can be changed at will merely by changing the tuning of the receiver. This enables (first) second-channel effects, due to inadequate selectivity of the signal frequency selective circuits, to be eliminated by changing the first intermediate frequency to such a frequency that the (first) second-channel lies in a silent part of the frequency spectrum. Second second-channel effects occur in the normal way due to inadequate selectivity of the first I.F. selective circuits, which are of course the radio frequency circuits of the normal receiver.

The beat oscillator frequencies are not shown in Fig. 2, but evidently the first beat oscillator must oscillate at 21 Mc/s and the second beat oscillator at 1·12 Mc/s (1,120 kc/s).

Such a triple detection receiver might conveniently be arranged to cover a band of received signal frequencies from 6 to 30 Mc/s, in which case the initial signal frequency circuits must be capable of being tuned over that range. If a receiver covering the medium-wave band is used to provide the I.F. circuits, these can be tuned to any frequency in the range 0·5 to 1·5 Mc/s. The first beat oscillator must therefore be capable of being tuned to any frequency in the range 6·5 to 31·5 Mc/s. Evidently, since the first I.F. frequency may be varied, the first beat oscillator must be capable of being tuned independently of the signal frequency circuits.

The above description illustrates the type of triple detection receivers in use at the B.B.C. receiving station.

15. Use of Reaction or Regeneration = Positive Feedback.

Feedback, or reaction, from anode to grid, due to grid-anode capacity, increases the effective input capacity of valves and often leads to instability. It is for this reason that valves have been developed with screens between anode and grid. Also, in short-wave receivers particularly, screening is usually introduced to prevent stray capacity between anode and grid circuits.

In low-price receivers where only a few valves are used, positive feedback is however introduced by deliberate coupling from anode to grid in such phase that the gain is increased. This feedback is adjustable and is generally effected by coupling a coil in the anode circuit, often the tuned anode coil, to a coil in the previous grid circuit.

It is shown in XXIII that if positive feedback exists round an amplifier with a voltage amplification μ, such that the voltage amplification round the feedback loop is $+\mu\beta$, the effective voltage amplification of the stage is $\frac{\mu}{1 - \mu\beta}$.

If, for instance, $\mu\beta = 0{\cdot}5$, the amplification is doubled, and the stage has a stability margin or singing margin of 6 db., since, if the loop amplification is doubled, the stage will oscillate. Hence there is a limit to the increase of amplification which can be obtained with an adequate singing margin. Further, the effect of positive reaction (reaction in such sense as to increase the gain) is to multiply all distortion products of non-linearity (in the valve round which reaction is applied) by the factor $\frac{1}{1 - \mu\beta}$. In other words, the distortion products are increased in the same ratio that the voltage amplification is increased. As indicated already, this is the reason that positive reaction is not used in normal receivers.

16. The Super-Regenerative Receiver.

This receiver is an attempt to develop the principle of positive feedback or regeneration to its logical limit. A receiver with reaction on one stage has the reaction and tuning adjusted to the point where, in the absence of incoming signal, the stage oscillates as near as possible to the frequency of the signal to be received. By means of a separate oscillator of frequency very much lower than the signal, but of supersonic frequency high enough to be easily suppressed in any following audio-frequency circuits, a quenching wave is applied to the oscillating stage of such amplitude that the signal-frequency oscillation is completely quenched during each negative cycle of the quenching frequency. During each positive cycle of the quenching frequency the oscillation builds up.

When the signal frequency is received the gain at the signal frequency is very large, or from another point of view the build-up during each half-cycle of the quenching signal is constituted by build-up of the signal frequency which takes charge of the circuit. The resultant signal is of course modulated by the quenching frequency, but after detection this modulation is removed from the audio-frequency circuit by a filter.

Such receivers are inclined to high background noise, to distortion and instability, and are therefore out of fashion, but revivals occasionally occur owing to the high sensitivity obtainable with only a few valves. Radar use has been fairly extensive.

17. Single Sideband Receivers. See also XVIII:7.11.

These are not likely to come into general use for broadcasting until the problem of distortionless detection of single sideband transmission has been satisfactorily solved. It will be noted that in a normally modulated wave the intimation as to what audio frequencies are present in the envelope is given twice : by the difference between the carrier frequency and the lower sideband frequencies, and by the difference between the upper sideband frequencies and the carrier. It is therefore possible to obtain satisfactory reception of intelligence by transmitting only one sideband, and this has important results in the suppression of noise. If one sideband in the transmitter uses the same voltage excursion that was previously utilized by one, the sideband power is doubled and the frequency band width is halved ; if then the carrier is transmitted at normal amplitude, an improvement of signal-to-noise ratio of 6 db. results since the noise voltage is proportional to the square root of the band width. If the carrier also is suppressed at the transmitter and restored at the receiver, the same total peak power being radiated, a further improvement of 6 db. in speech-to-noise ratio results because the normal peak carrier voltage in the transmitter is equal to the peak voltage in the sidebands, so that suppression of the carrier enables the sideband voltages to be doubled, and the sideband power to be multiplied by 4.

If a square-law detector is used, in the case of transmission of a carrier and one sideband of equal amplitude, distortion products occur owing to beating between sideband frequencies. Where only one sideband frequency exists, in other words, when the modulating frequency consists of a sine wave, no distortion occurs, but this is such a rare case in practice as to be of little importance. The amplitude of the distortion products is proportional to the sideband amplitudes, so that for lower sideband amplitudes less distortion occurs.

To give some idea of the magnitude of these distortion products, assume a carrier amplitude C, and any number of sideband frequencies of amplitudes, respectively, m_1C, m_2C, m_3C, etc., corresponding to original modulating audio frequencies. Assume also that after detection these sideband frequencies appear as the original audio frequencies of amplitudes am_1C, am_2C, am_3C, etc., where a is a constant. The amplitudes of the corresponding intermodulation frequencies are then :

am_1m_2C	am_1m_3C	am_1m_4C	etc.
am_2m_3C	am_2m_4C	am_2m_5C	etc.
am_3m_4C	am_3m_5C	am_3m_6C	etc., and so on.

In the case of a diode detector distortion occurs with a single sideband frequency, the magnitude of the distortion being given in VIII:4.2. The magnitudes of the intermodulation products, where more than one sideband is present, have not been evaluated. In both cases the distortion products are reduced in relation to the wanted frequencies as the carrier amplitude is increased in relation to the peak amplitude of the sum of the sideband frequencies ; but the distortion in both cases is intolerable with economic relative levels of carrier and sidebands.

Since, as the carrier amplitude is increased in relation to the sidebands, the distortion drops, one proposed remedy which is sometimes used in point-to-point communication links is to restore a large amplitude carrier at the receiving end. In this case it is only necessary to transmit a small amplitude pilot carrier which is used as an indication to enable the restored carrier to have the correct frequency. Another remedy which has been proposed by Nyquist, Eckersley and Koomans is to transmit single sideband and normal carrier plus the part of the other sideband corresponding to audio frequencies below 1 or 2 kc. Owing to the form of the spectrum of speech and music, higher audio frequencies are usually of such amplitude that serious distortion does not arise when only one sideband is transmitted.

18. Examples of Commercial Receivers.

In Table I, pp. 96 and 97, typical characteristics of six commercial broadcasting receivers are shown.

Fig. 1, facing p. 100, by courtesy of the G.E.C., shows the circuit of a very successful type of superheterodyne receiver ; the G.E.C. Overseas 10.

The values of the components of this receiver are shown in Table II, the operating conditions are shown in Table III, while the wave bands are given in Table IV.

The operation of this receiver can be very easily followed from the circuit diagram. The aerial is earthed through the 9,900-ohm resistance R_1 and is also connected to switch " A ", which selects the signal-frequency circuits for each wave band, which are applied to the grid of the signal-frequency amplifier valve V_1 by means of switch " B ".

The anode of V_1 is connected to selector switch " C ", which again chooses the circuits appropriate to the wave band, and the output of the circuit chosen is fed through switch " D ", another selector switch, through blocking condenser C_{13} and stopper resistance R_{11} to the grid of V_2, which is a hexode-triode valve containing respectively the mixer and a triode valve serving as an oscillator.

The appropriate oscillatory circuit is chosen by means of switches " E " and " F " and the grid of the triode is directly coupled to one of the control grids of the hexode.

The anode of the mixer is connected to the band-pass coupled circuit labelled $I.F.T_1$, which drives the grid of V_3, the first I.F. amplifier valve. This is coupled through transformer $I.F.T_2$ to the grid of V_4, the second I.F. amplifier valve, which is connected to two diodes of the valve V_5, which is a double diode triode. The left-hand diode of V_5 constitutes the normal detector driving the diode load R_{24} through R_{23}, which, in conjunction with C_{28}, constitutes a filter to remove I.F. frequency. R_{25} (1 MΩ) constitutes the output potentiometer, and so the effective diode load impedance is constituted by R_{24} and R_{25} in parallel and so is about 90,000 ohms.

R_{27} provides cathode bias for the triode of V_5 and is shunted by electrolytic condenser C_{27}.

The output from R_{25} is fed from the moving contact on R_{25} to the grid of the triode valve. The output of the triode valve is resistance coupled to the grid of valve V_6, which is a double diode-triode valve used as a triode, the diodes being shorted to the cathode.

The anode circuit of V_6 is resistance capacity coupled to the balanced choke or auto-transformer, shown but not designated, which drives the grid of V_7 and V_8 in push-pull. The anodes of V_7 and V_8 are connected through stopper resistances R_{45} and R_{47} to the push-pull output transformer which feeds the loudspeakers.

TABLE I

Characteristics of Typical Commercial Broadcasting Receivers of the Superheterodyne Type

	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
Controls.	R.F. gain. Band switch. A.F. gain. A.V.C. on-off switch. Main tuning. B.F.O. on-off switch. Anti-noise level on-off switch. Band spread tuning. Combined mains on-off switch, and tone control. Pitch control for B.F.O. Send-receive switch.	R.F. gain. Band switch. Mains on-off and tone control. A.V.C.-B.F.O. switch. Variable selectivity A.F. gain. B.F.O. pitch control.	Volume control. Bass tone control. Treble tone control and main switch. Sensitivity and gram switch. Band switch. Tuning control.	Treble tone control and high fidelity switch. Volume control. Tuning. Band switch. Brass control and on-off switch.	On-off and tone control. Volume control. Tuning. Push buttons for each band.	On-off switch. Volume control. Band switch. Tuning. Tone control.
Frequencies covered.	540 kc/s–44 Mc/s in 4 bands.	25 Mc/s–66 Mc/s in 2 bands.	143 kc/s–333 kc/s 545 ,, 26·3 Mc/s in 5 bands.	150 kc/s–422·5 kc/s 526 ,, –1,550 ,, 2·8 Mc/s–61·6 Mc/s in 5 bands.	150–300 kc/s 546–1,500 ,, 6–7·15 Mc/s 9·47–9·87 ,, 11·67–12·2 ,, 15·08–15·38 ,, 17·75–17·85 ,, 21·42–21·6 ,,	140–360 kc/s 510–1,580 ,, 5·85–18·4 Mc/s in 3 bands.
Number of valves.	9	9	10	11	6	5
Power supply.	110–250 volts 25–60 c.p.s.	110–250 volts 25–60 c.p.s.	100–250 volts 40–100 c.p.s.	195–255 volts 50–100 c.p.s.	200–250 volts 50–100 c.p.s.	200–250 volts 50–100 c.p.s.
Intermediate frequencies.	455 kc/s	1,600 kc/s	445 kc/s	465 kc/s	465 kc/s	465 kc/s
Output impedance.	Speech coil 3·5 Ω	Speech coil 3·5 Ω	2 speech coils 2 Ω	Speech coil 4 Ω	Speech coil 3 Ω	Speech coil 2·25 Ω

Second-channel ratio. (Attenuation of S.F. circuits at second-channel frequency expressed as voltage ratio.)	1 Mc/s . . 3,700 3 „ . . 1,200 10 „ . . 73 20 „ . . 32	32 Mc/s . . 35 50 „ . . 22	260 kc/s . 10^6 1,000 „ . 1,000 3 Mc/s . 700 8 „ . 90 20 „ . 15	600 kc/s 47,000 857 „ . 4,000 7·5 Mc/s . 90 10 „ . 71 15 „ . 21	200 kc/s . 2,300 1 Mc/s . 20 6.67 „ . 87 12 „ . 250 21·4 „ . 195	260 kc/s . 2,000 1,000 „ . 175 6 Mc/s . 11·5 10 „ . 6 18 „ . 1·3
Signal-noise Ratio: Microvolts in at 30% mod. by 400 cycles for 20 db. output ratio "mod. on to mod. off".	1 Mc/s . . 35 µV 3 „ . . 22 „ 9 „ . . 28 „ 25 „ . . 20 „	27 Mc/s . 40 µV 32 „ . 25 „ 50 „ . 32 „	240 kc/s . 130 µV 920 „ . 48 „ 2·5 Mc/s . 22 „ 7·2 „ . 32 „ 18 „ . 30 „	212 kc/s . 20 µV 860 „ . 7 „ 5 Mc/s . 22 „ 15 „ . 30 „ 33·4 „ . 300 „	225 kc/s . 40 µV 1,000 „ . 25 „ 7 Mc/s . 7 „ 10 „ . 5 „ 12 „ . 4 „ 15 „ . 5 „ 18 „ . 8 „ 22 „ . 25 „	240 kc/s . 70 µV 920 „ . 120 „ 12 Mc/s . 120 „
Selectivity Measured at	1,000 kc/s	50 Mc/s 32 Mc/s	250 kc/s 1,000 kc/s	214 kc/s 1,000 kc/s	180 kc/s 850 kc/s	250 kc/s 1,000 kc/s
Band widths for attenuation of 20 db. 40 db. 60 db.	14·5 „ 21 „ 28 „	15 kc/s 24 kc/s 37 „ 46 „ 55 „ 64 „	9 „ 14 „ 16 „ 18 „ 24 „ 31 „	9 „ 7 „ 13 „ 14 „ 18 „ 22 „	11 „ 15 „ 24 „ 30 „ 37 „ 47 „	9 „ 12 „ 10 „ 19 „ 25 „ 29 „
Fidelity Frequencies at which attenuation is 3 db. 6 db. below response at 1,000 cycles.	*cycles*/s 140; 1,600 108; 2,300	*cycles*/s 75; 3,700 44; 5,300	*cycles*/s 20; 3,400 18; 4,000	*cycles*/s 68; 115; 250; 5,200 58; 5,800	*cycles*/s 58; 2,200 47; 3,000	*cycles*/s 64; 2,000 53; 2,700
Automatic volume control threshold	None	6 µV	4 µV	40 µV	70 µV	1,000 µV
Change of output in db. for	40 db.	32 db.	19 db.	26 db.	5 db.	12 db.
change of input in db. above threshold.	100 db.	84 db.	108 db.	88 db.	83 db.	60 db.
Maximum undistorted output (A.F.)	Not measured	Not measured	4·8 watts	4 watts	Ill-defined; approx. 1·3 w.	2·8 watts
Maximum output.	Not measured	Not measured	5·8 watts	9·2 watts	1·8 watts	5·2 watts

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Number of valves.	9	9	10	11	6	5
Power supply.	110–250 volts 25–60 c.p.s.	110–250 volts 25–60 c.p.s.	100–250 volts 40–100 c.p.s.	195–255 volts 50–100 c.p.s.	200–250 volts 50–100 c.p.s.	200–250 volts 50–100 c.p.s.
Intermediate frequencies.	455 kc/s	1,600 kc/s	445 kc/s	465 kc/s	465 kc/s	465 kc/s
Output impedance.	Speech coil 3·5 Ω	Speech coil 3·5 Ω	2 speech coils 2 Ω	Speech coil 4 Ω	Speech coil 3 Ω	Speech coil 2·25 Ω
Second-channel ratio. (Attenuation of S.F. circuits at second-channel frequency expressed as voltage ratio.)	1 Mc/s . . 3,700 3 ,, . . 1,200 10 ,, . . 73 20 ,, . . 32	32 Mc/s . . 35 50 ,, . . 22	260 kc/s . 10^6 1,000 ,, . 1,000 3 Mc/s . 700 8 ,, . 90 20 ,, . 15	600 kc/s 47,000 857 ,, . 4,000 7·5 Mc/s . 90 10 ,, . 71 15 ,, . 21	200 kc/s . 2,300 1 Mc/s . 20 6.67 ,, . 87 12 ,, . 250 21·4 ,, . 195	260 kc/s . 2,000 1,000 ,, . 175 6 Mc/s . 11·5 10 ,, . 6 18 ,, . 1·3
Signal-noise Ratio: Microvolts in at 30% mod. by 400 cycles for 20 db. output ratio "mod. on to mod. off".	1 Mc/s . . 35 μV 3 ,, . . 22 ,, 9 ,, . . 28 ,, 25 ,, . . 20 ,,	27 Mc/s . 40 μV 32 ,, . 25 ,, 50 ,, . 32 ,,	240 kc/s . 130 μV 920 ,, . 48 ,, 2·5 Mc/s . 22 ,, 7·2 ,, . 32 ,, 18 ,, . 30 ,,	212 kc/s . 20 μV 860 ,, . 7 ,, 5 Mc/s . 22 ,, 15 ,, . 30 ,, 33·4 ,, . 300 ,,	225 kc/s . 40 μV 1,000 ,, . 25 ,, 7 Mc/s . 7 ,, 10 ,, . 5 ,, 12 ,, . 4 ,, 15 ,, . 5 ,, 18 ,, . 8 ,, 22 ,, . 25 ,,	240 kc/s . 70 μV 920 ,, . 120 ,, 12 Mc/s . 120 ,,
Selectivity Measured at Band widths for attenuation of 20 db. 40 db. 60 db.	1,000 kc/s 14·5 ,, 21 ,, 28 ,,	50 Mc/s 32 Mc/s 15 kc/s 24 kc/s 37 ,, 46 ,, 55 ,, 64 ,,	250 kc/s 1,000 kc/s 9 ,, 14 ,, 16 ,, 18 ,, 24 ,, 31 ,,	214 kc/s 1,000 kc/s 9 ,, 7 ,, 13 ,, 14 ,, 18 ,, 22 ,,	180 kc/s 850 kc/s 11 ,, 15 ,, 24 ,, 30 ,, 37 ,, 47 ,,	250 kc/s 1,000 kc/s 9 ,, 12 ,, 10 ,, 19 ,, 25 ,, 29 ,,
Fidelity Frequencies at which attenuation is 3 db. 6 db. below response at 1,000 cycles.	*cycles*/s 140 ; 1,600 108 ; 2,300	*cycles*/s 75 ; 3,700 44 ; 5,300	*cycles*/s 20 ; 3,400 18 ; 4,000	*cycles*/s 68 ; 115 ; 250 ; 5,200 58 ; 5,800	*cycles*/s 58 ; 2,200 47 ; 3,000	*cycles*/s 64 ; 2,000 53 ; 2,700
Automatic volume control threshold Change of output in db. for change of input in db. above threshold.	None 40 db. 100 db.	6 μV 32 db. 84 db.	4 μV 19 db. 108 db.	40 μV 26 db. 88 db.	70 μV 5 db. 83 db.	1,000 μV 12 db. 60 db.
Maximum undistorted output (A.F.)	Not measured	Not measured	4·8 watts	4 watts	Ill-defined ; approx. 1·3 w.	2·8 watts
Maximum output.	Not measured	Not measured	5·8 watts	9·2 watts	1·8 watts	5·2 watts

Table II

Values of Components in Fig. 1

CONDENSERS

No.	Capacity	Type	No.	Capacity	Type
C1	0·02 μF	G.E.C. TUB (500 V)	C25	0·05 μF	G.E.C. TUB (500 V)
C2	0·003 μF	LEMCO 3017	C26	0·02 μF	G.E.C. TUB (500 V)
C3	4 μF	EL (350 V) Unit No. 3	C27	30 μF	ELECT (30 V)
C4	0·05 μF	G.E.C. TUB (250 V)	C28	0·0001 μF	DUB. 635
C5	0·0005 μF	DUB. 635	C29	0·1 μF	G.E.C. TUB (500 V)
C6	0·05 μF	G.E.C. TUB (250 V)	C30	0·1 μF	G.E.C. TUB (500 V)
C7	4 μF	EL (350 V) Un. No. 3	C31	0·005 μF	G.E.C. TUB (500 V)
C8	2 μF	EL (500 V) ,, ,, ,,	C32	0·0001 μF	DUB. 635
C9	0·05 μF	G.E.C. TUB (500 V)	C33	0·05 μF	G.E.C. TUB (250 V)
C10	0·02 μF	G.E.C. TUB (500 V)	C34	0·02 μF	G.E.C. TUB (500 V)
C11	0·0005 μF	DUB. 635	C35	0·1 μF	G.E.C. TUB (500 V)
C12	0·0032 μF	DUB. 5815	C36	0·05 μF	G.E.C. TUB (250 V)
C13	0·0005 μF	DUB. 635	C37	30 μF	ELECT (30 V)
C14	0·0005 μF	DUB. 675	C38	1 μF	EL (500 V) Un. No. 2
C15	0·0001 μF	DUB. 635	C39	0·1 μF	G.E.C. TUB (500 V)
C16	0·1 μF	G.E.C. TUB (500 V)	C40	30 μF	ELECT (30 V)
C17	1,750 μμF	LEMCO 3017	C41	0·005 μF	G.E.C. TUB (500 V)
C18	1,500 μμF	LEMCO 3017	C42	0·005 μF	G.E.C. TUB (500 V)
C19	0·0005 μF	LEMCO 3017	C43	16 μF	EL (500 V) Un. No. 2
C20	0·0002 μF	LEMCO 3010	C44	16 μF	EL (500 V) ,, ,, 1
C21	0·02 μF	G.E.C. TUB (500 V)	C45	8 μF	EL (500 V) ,, ,, 3
C22	0·25 μF	G.E.C. TUB (250 V)	C46	0·05 μF	G.E.C. TUB (250 V)
C23	0·05 μF	G.E.C. TUB (250 V)	C47	0·02 μF	G.E.C. TUB (500 V)
C24	1 μF	EL (500 V) Un. No. 2			

RESISTANCES

No.	Resistance	Watts	Type	No.	Resistance	Watts	Type
R1	9,900 Ω	⅓	B.T.	R26	55,000 Ω	⅓	B.T.
R2	22,000 Ω	½	B.T.	R27	3,300 Ω	½	B.T.
R3	55,000 Ω	1	B.T.	R28	2 MΩ	⅓	B.T.
R4	440,000 Ω	⅓	B.T.	R29	220,000 Ω	⅓	B.T.
R5	9,900 Ω	1	B.T.	R30	99,000 Ω	⅓	B.T.
R6	9,900 Ω	1	B.T.	R31	1 MΩ	½	B.T.
R7	44,000 Ω	1	B.T.	R32	66,000 Ω	½	B.T.
R8	22,000 Ω	½	B.T.	R33	33,000 Ω	½	B.T.
R9	2,200 Ω	½	B.T.	R34	150,000 Ω	⅓	B.T.
R10	220,000 Ω	⅓	B.T.	R35	1 MΩ	Tone Cont.	Dubilier
R11	50 Ω	½	3N.	R36	22,000 Ω	⅓	B.T.
R12	440,000 Ω	⅓	B.T.	R37	22,000 Ω	½	B.T.
R13	220,000 Ω	⅓	B.T.	R38	22,000 Ω	½	B.T.
R14	33,000 Ω	1	B.T.	R39	66,000 Ω	½	B.T.
R15	55,000 Ω	⅓	B.T.	R40	3,300 Ω	½	B.T.
R16	77,000 Ω	⅓	B.T.	R41	55,000 Ω	Tone Cont.	Dubilier
R17	2,200 Ω	⅓	B.T.	R42	30 Ω	½	3N.
R18	1,300 Ω	Sensy. Cont.	Stackpole	R43	99,000 Ω	⅓	B.T.
R19	440,000 Ω	⅓	B.T.	R44	90 Ω	1	2
R20	77,000 Ω	⅓	B.T.	R45	100 Ω	¼	4N.
R21	88,000 Ω	1	B.T.	R46	100 Ω	¼	4N.
R22	220,000 Ω	⅓	B.T.	R47	100 Ω	¼	4N.
R23	99,000 Ω	⅓	B.T.	R48	100 Ω	¼	4N.
R24	99,000 Ω	⅓	B.T.	R49	1 MΩ	⅓	B.T.
R25	1 MΩ	Vol. Cont.	Dubilier	R50	2 MΩ	⅓	B.T.

Table III

Operating Conditions

V No.	Function	Type	Heater Volts A.C.	Valve Electrode Potentials and Currents						
				Voltage Relative to Chassis Volts			Currents (mA)			
				Cathode	Anode	Screen	Cathode	Screen	Anode	Osc. Anode
1	R.F. Amplifier	KTW61	6·3	0	(155)	(53)	4·9	1·2	3·7	—
2	Frequency changer	X65	6·3	0	(247)	(55)	5·4	1·0	0·1	4·3
3	1st I.F. Amplifier	KTW61	6·3	0	(250)	(53)	5·3	1·2	4·1	—
4	2nd I.F. Amplifier	KTW61	6·3	0	(250)	(53)	5·3	1·2	4·1	—
5	Det. and first L.F.	DL63	6·3	3·0*	(118)	—	1·0	—	1·0	—
6	Tone corrector	DL63	6·3	3·0*	(110)	—	0·9	—	0·9	—
7	Output	KT61	6·3	5·5†	(259)	(262)	33·5	5·5	28	—
8	Output	KT61	6·3	5·5†	(259)	(262)	33·5	5·5	28	—
9	Tuning indicator	Y63	6·3	0	—	—	4·2	0·2	4·0	—
10	Rectifier	U50	5·0	—	—	—	—	—	—	—

Smoothed H.T. across C43 = (262) volts. Smoothed H.T. across C45 = (250) volts.
Volts across R42 = +2·9. H.T. volts across C44 = (315) volts.
Total H.T. current through choke No. 1 = 99 mA. Current through choke No. 2 = 32 mA.
Figures in brackets denote measurements made with Avometer (1,200-V range).
Receiver tuned to 1 Mc/s and sensitive control set for maximum sensitivity.
* Voltages measured on 0–120-V range of avometer.
† Voltages measured on 0–12-V range of avometer.

Table IV

Wave Bands and Switch Positions

RANGE SWITCH

Position	Range
1	11– 25 metres
2	25– 75 ,,
3	75– 200 ,,
4	200– 550 ,,
5	900–2,100 ,,

SWITCH OPERATIONS

Gram.	S1 S2 Closed
Radio	S1 S2 Open

Valve V_{10} is a "cat's eye" signal-strength indicator, driven by the detected output of transformer $I.F.T_3$.

A.V.C. Circuit. This is constituted by the righthand diode of V_5 which supplies current through resistances R_{29}, R_{30} and R_{42} to ground and so augments the bias normally provided by the total anode current which flows through R_{42}. The paths by which bias is applied to the various grids can easily be traced. R_{27} is the resistance producing A.V.C. delay volts.

For purposes of illustration, the performance of such a receiver may be taken as somewhat similar to receiver No. 3 in Table I.

19. Measurements on Radio Receivers.

The following instructions relating to measurements on radio receivers are intended for practical guidance to engineers who have to make acceptance tests on receivers, and are of interest to other engineers only in so far as they describe the form in which the performance of receivers should be presented. The descriptions of tests are not complete in themselves, reference being made to the "Specification for Testing Radio Receivers" issued by the Radio Manufacturers' Association of Great Britain.

19.1. Equipment Required. *Signal Generator.* A signal generator is a calibrated radio-frequency oscillator with a very low impedance output, e.g. between 3 Ω and 60 Ω, capable of delivering a modulated or unmodulated output into an unbalanced circuit (and preferably also into a balanced circuit). Signal generators should give readings of microvolts output to an accuracy of $\pm 10\%$ up to 10 Mc/s and $\pm 25\%$ above 10 Mc/s. Their frequency calibration should be accurate within 1% or within ± 10 kc/s, whichever is the better, and incremental frequencies should be within $\pm 0{\cdot}5$ kc/s; the modulation should be accurate to within $\pm\frac{1}{10}$ of the nominal percentage.

Dummy Aerial. For medium- and long-wave measurements up to 2,000 kc/s the dummy aerial should comprise three series elements, viz. a resistance of 25 ohms, an inductance of 20 microhenrys and a capacity of 200 micro-microfarads. These elements include the internal impedance of the generator and all connections.

For short-wave measurements above 2 Mc/s the dummy aerial should consist of a 400-ohm series resistance, which shall include the impedance of the generator and all connections.

Shielded Coil. This shall be a cylindrical coil, 5 cms. in radius and 6 cms. deep, each winding wound with 20 turns to an approximate inductance of 40 microhenrys. The whole coil shall be shielded

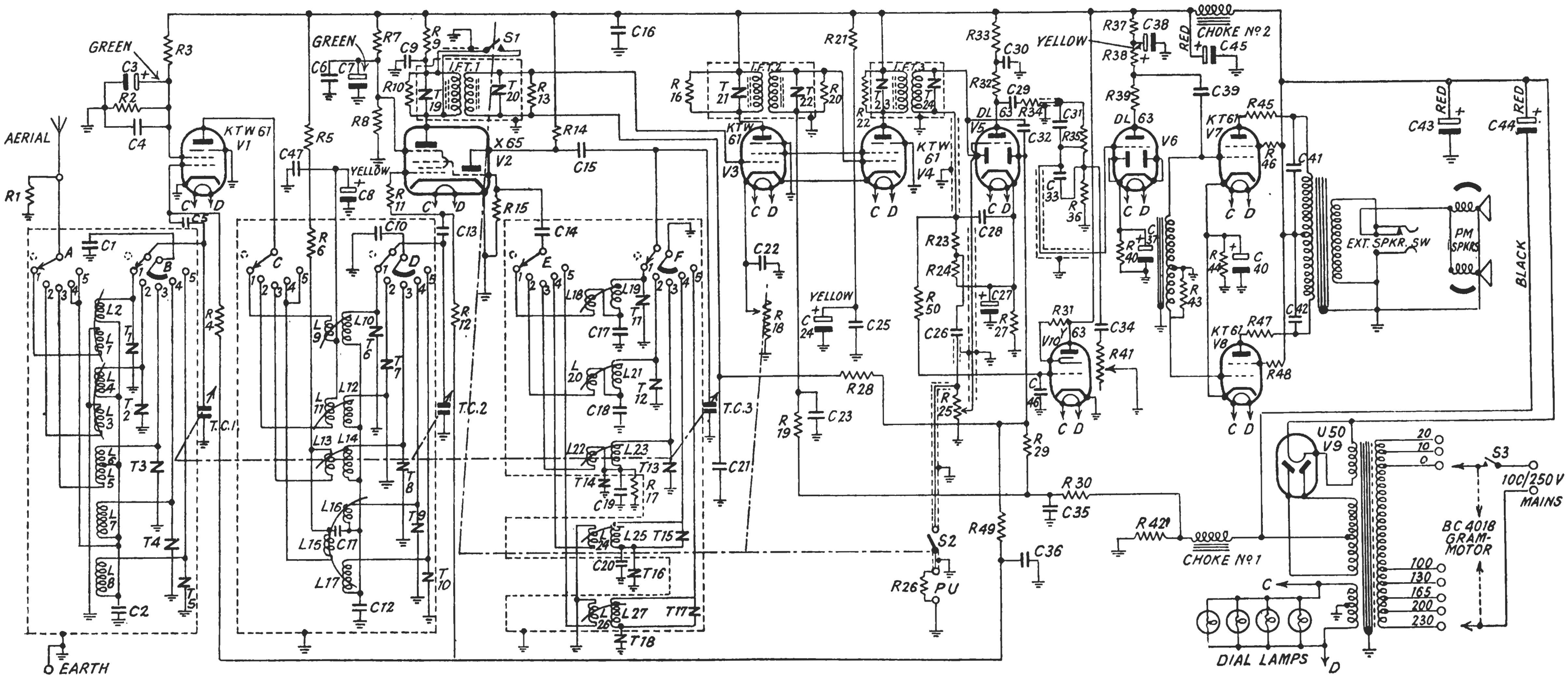


FIG. 1/XIX:18.—Circuit of G.E.C. Overseas 10 Receiver.

(By courtesy of the General Electric Co., Ltd.)

by a wire cage arranged to avoid magnetic screening (i.e. there shall be no completed circuits whose planes are normal to the axis of the coil). The connecting leads shall also be screened. An earthed screen which does not constitute a short-circuited turn shall be provided between the windings.

Valve Voltmeter, Thermocouple or T.M.S. for measuring output level.

Two Microammeters, 0–25 microamps. and 0–250 microamps.

Special Screened Low-Capacity Switch. This is a double-pole double-throw anti-capacity switch with screening between all contacts and connections.

Ganging Oscillator and Oscilloscope.

19.2. Automatic Volume Control. Measurements to be carried out in accordance with the procedure laid down in the Radio Manufacturers' Association Specification for Testing Radio Receivers, Part 1, page 12.

Results to be expressed as a plot of R.M.S. audio-frequency output volts against input microvolts, otherwise as described on page 12 of the above specification.

19.3. Sensitivity. Measurement to be carried out in accordance with the procedure laid down in the R.M.A. Specification for Testing Radio Receivers, Part 1, pages 4 and 8. All gain controls to be set at maximum gain.

The sensitivity to be expressed as the input microvolts needed to give the standard output, and to be plotted as described on page 8 of the R.M.A. Specification, Part 1. The standard output is 50 milliwatts for receivers operating loudspeakers and 1 milliwatt for rebroadcast receivers and check receivers.

Should it be inconvenient to measure a particular receiver at the standard output, this output may be changed and the result reduced to conform to the definition given above.

The output shall be measured in a non-inductive load connected across the output. The value of this load shall be as follows :

Loudspeaker Receivers : Magnitude of load resistance to be equal to the modulus (magnitude) of the loudspeaker voice coil or operating winding.

Check Receivers and Rebroadcast Receivers : Magnitude of load resistance to be equal to nominal impedance of circuit into which the receiver is designed to work.

19.4. Signal-Frequency Response, Second-Channel and Mixed-Channel Selectivity. *The Signal-Frequency Response* *curve* is a plot of the S.F. loss, i.e. the loss in the signal-frequency (radio-frequency) pre-mixer circuits.

The signal-frequency loss for a given setting of the tuning circuits is defined at any given frequency by the decibel ratio between the e.m.f. required to be inserted in series with the dummy aerial supplying the input circuit of the receiver, at the frequency of maximum signal-frequency response, and at the given frequency, in order to produce the same input voltage to the mixer. It is measured over the frequency range situated ± 2 (I.F.) with regard to the frequency to which the receiver is tuned, where (I.F.) is the intermediate frequency.

The Second-Channel Selectivity is the signal-frequency loss at a frequency situated 2 (I.F.) above or below the frequency to which the receiver is tuned according to whether the receiver operates with its local oscillator frequency above or below the signal.

The Mixed-Channel Selectivity is the signal-frequency loss at the points where the width of the response curve on the frequency axis is equal to the intermediate frequency.

Method of Measurement. Shorts and connections in all cases to be made by means of blocking condensers unless direct connection is specified.

Measurement is accomplished by means of a null method.

The position of all variable selectivity controls should be adjusted to maximum or minimum as required and the position noted. The tone control to be at maximum fidelity.

A signal generator is arranged to supply an e.m.f. in series with the dummy aerial and the aerial and earth terminals of the receiver.

An auxiliary receiver is employed as an R.F. valve galvanometer, and has a meter inserted in a suitable position to give indication of the amplitude of the input signal it receives, e.g. a microammeter in the earth side of the detector load resistance.

The receiver under measurement has its mixer anode circuit terminated by direct connection in 1,000 ohms, which constitutes the output load across which the second receiver is connected to obtain its signal. The first I.F. transformer normally connected to the anode of the mixer is shorted to ground through a blocking condenser ; this effectively removes the automatic volume control.

The signal generator is then set on the standard frequency (unmodulated) *which should be one of those specified for the measurement of sensitivity* (*see R.M.A. Specification for Testing Receivers, page* 8). The output of the signal generator is then adjusted to give a convenient value of deflection on the microammeter with the receiver under measurement in tune with the standard frequency, care being taken to see that the auxiliary receiver is not overloaded. This deflection and E_1, the output of the signal generator, is noted.

The signal-generator output is now removed from the receiver under test and its live lead taken through a series resistance of 1,000 ohms to the input of the measuring receiver, which is also disconnected from the receiver under test. The special screened low-capacity switch is used for this change-over. (The effective earth connections of all units, signal generator and the two receivers, etc., are connected permanently to the common earth point.)

The signal generator is now adjusted to give the same deflection on the microammeter, and its output E_2 noted.

Call the difference in level between E_1 and E_2 in decibels, D_0.

The procedure in the previous two paragraphs is now repeated at frequency intervals, determined by the course of the resulting curve and extending over the frequency range (2 I.F.) each side of the frequency used in determining D_0.

Call the decibel difference obtained at any frequency D_f, then the R.F. loss at this frequency is given by $D_0 - D_f$.

During these measurements the tuning of the receiver under test remains unchanged, while it is necessary to retune the auxiliary receiver each time the frequency is changed, care being taken not to touch the auxiliary receiver between the two observations which are subtracted to give the figure D_f.

Method of Expressing Results. The results are to be plotted on graph paper ruled in centimetre and millimetre squares. The loss figures to be plotted vertically to a scale of 1 centimetre = 10 decibels, the zero of the scale being at the bottom of the paper, which is constituted by one of the longer sides.

The frequency scale chosen depends on the value of the intermediate frequency.

Where the I.F. is in the neighbourhood of 125 kc/s, the frequency scale to be used is 1 centimetre = 25 kc/s, and when the I.F. is near 500 kc/s the scale to be used is 1 centimetre = 100 kc/s.

The frequency scale shall be marked in kilocycles above and below the carrier frequency.

19.41. Second-Channel Selectivity for Acceptance Tests on Approved Receivers. This measurement will be an abbreviated version of the measurement just described, in that a single measurement only will be made at 200 kc/s and 1,400 kc/s. This measurement will consist of a comparison of the potential gain of the receiver at the frequency to which it is tuned, and at a frequency equal to

2 (I.F.) above or below this frequency, depending on whether the local oscillator operates above or below the frequency to which the receiver is tuned.

Method of Measurement. The automatic volume control is disconnected in such a way as to leave the loading on the speech diode substantially unaltered. This means disconnecting the D.C. output of the automatic volume control, and care must be taken to reconnect the grid-biasing circuit to a potential point equivalent to that of the automatic volume control in the most sensitive condition of the receiver.

A microammeter is inserted in series with the earth end of the speech diode load.

A signal generator is arranged to drive the receiver through the standard dummy aerial, both the signal generator and the receiver being set for operation on one of the test frequencies above. The output of the signal generator is adjusted so that the microammeter gives a convenient indication, and the output noted.

The frequency of the signal generator only is then moved to 2 (I.F.) above or below the test frequency and the output adjusted until the microammeter gives the same reading as before. The difference between these two successive readings of the signal generator output represents the receiver attenuation to second-channel effects, and will be recorded in decibels against the frequency to which the receiver is tuned.

Method of Expressing Results. The results of this measurement shall be tabulated together with other written matter relating to measurements on the receiver.

19.42. Intermediate Frequency Response. The I.F. response is a plot of the loss in the intermediate frequency stages relative to the frequency of maximum gain.

The I.F. loss is defined at any given frequency by the decibel ratio between the voltage required to be applied to the input of the mixer at the frequency of maximum I.F. gain and at the given frequency, in order to give the same current through the speech diode.

The I.F. loss shall be measured to a frequency 50 kc/s each side of mid-band or to a frequency at which the I.F. loss reaches 80 db., whichever is the less in frequency width.

Method of Measurement. The automatic volume control is disconnected in such a way as to leave the loading on the speech diode substantially unaltered. This means disconnecting the D.C. output of the automatic volume control, and care must be taken to reconnect the grid-biasing circuit to a potential point equivalent to that of the automatic volume control in the most sensitive condition of the receiver. The receiver local oscillator is put out of action.

The microammeter is inserted in series with the earth end of the speech diode load.

A signal generator is arranged to drive the grid of the mixer directly and is adjusted at each test frequency to give constant deflection on the microammeter. The difference between the output of the signal generator at the frequency of maximum I.F. response and at any other given test frequency gives the I.F. loss at the given frequency.

Method of Expressing Results. The results are to be plotted on graph paper ruled in centimetre and millimetre squares. The loss figures to be plotted vertically to a scale of 1 centimetre = 5 decibels, the zero of the scale being at the bottom of the paper, which is constituted by one of the short sides of the paper.

The frequency scale shall be linear and shall be 2 kilocycles per centimetre or 10 kilocycles per centimetre, according to the goodness of the receiver under investigation, and shall be marked in kilocycles to right and left of the mid-band frequency.

19.43. Selectivity. The selectivity curve of a receiver is a plot of the loss in the radio and intermediate frequency stages relative to the frequency of maximum gain.

The loss in the radio and intermediate frequency stages is defined at any given frequency by the decibel ratio between the voltage required to be applied to the input of the receiver at the frequency of maximum radio and intermediate frequency gain and at the given frequency, in order to give the same current through the speech diode.

The loss shall be measured to a frequency 50 kc/s each side of the frequency to which the receiver is tuned, or to frequencies at which the loss reaches 80 db., whichever is the less in frequency width. Curves shall be taken at frequencies laid down in Section 2, Part 1, page 8, of the R.M.A. Specification.

Method of Measurement. The automatic volume control is disconnected in such a way as to leave the loading on the speech diode substantially unaltered. This means disconnecting the D.C. output of the automatic volume control, and care must be taken to reconnect the grid-biasing circuit to a potential point equivalent to that of the automatic volume control in the most sensitive condition of the receiver.

The microammeter is inserted in series with the earth end of the speech diode load. With the receiver tuned to one of the frequencies laid down in Section 2, Part 1, page 8, of the R.M.A. Specification, a signal generator is arranged to drive the input of the receiver through the standard dummy aerial. Without altering the receiver tuning, the signal generator output is adjusted at each test frequency to give constant deflection on the microammeter. The difference between the output of the signal generator at the frequency of maximum receiver response and at any other given test frequency gives the attenuation to adjacent frequencies.

Method of Expressing Results. Results shall be expressed as in the case of I.F. response, a family of smooth curves being plotted, one for each of the frequencies laid down in Section 2, Part 1, page 8, of the R.M.A. Specification.

19.44. Audio-Frequency Response Overall. The overall audio-frequency response is the relation between the envelope of the modulated radio-frequency e.m.f. in a dummy aerial and the audio-frequency voltage appearing across a non-reactive load which terminates the output of the receiver in its nominal impedance.

Method of Measurement. The position of the tone control is adjusted to maximum or minimum and noted.

A modulated signal generator is connected to the input of the receiver through a dummy aerial and the output level adjusted to a value bringing the automatic volume control approximately to the middle of the operating part of the automatic volume-control curve, e.g. on the specimen automatic volume-control curve in the R.M.A. Specification for Testing Receivers, Part 1, this would correspond to an input of between 10^4 and 10^5 microvolts ; the value is in no way critical. The receiver is then tuned to the output of the signal generator, using the tuning indicator device if provided, and if none is provided, is tuned to a position halfway between the two positions of maximum hiss located either side of the true tuning position.

With the signal generator modulated to 30 %, the audio-frequency gain control is adjusted to give an output equal to a quarter of the standard output power. (The standard output is 1 milliwatt for check receivers and rebroadcast receivers, with the output of the receiver terminated in the value of load resistance laid down immediately above.)

The audio frequency modulating the signal generator is then varied over the range 30 to 10,000 c/s, keeping the depth of modulation constant at 30 %, and the output level is measured on a transmission measuring set, thermocouple or valve voltmeter. Observations of response are taken at the R.F. frequencies laid down in the R.M.A. Specification, Part 1, page 8. (The constancy of modulation with audio-frequency variation afforded by any given signal-generator requires to be checked by the use of a diode.)

Method of Expressing Results. The results are plotted on 3-cycle log-ordinary graph paper, the ordinary scale being graduated in centimetres and millimetres.

The results are expressed as a plot in decibels of the output level at each audio frequency referred to the output level at 1,000 cycles = zero.

The scale of decibels used shall be 1 centimetre = 5 db.

19.5. Frequency Stability. The frequency stability is defined by the change of the beat oscillator frequency due to all causes, e.g. variation of temperature and mains voltage.

Method of Measurement. An auxiliary receiver is tuned to a signal generated by a crystal oscillator and the beat oscillator of the receiver under test is tuned approximately 1,000 p : s away from this frequency and coupled into the input of the auxiliary receiver. The frequency of the resulting beat tone is measured by beating it with a standard frequency oscillator. The accuracy of the standard oscillator frequency need not be better than $\pm 5\%$, but must not be worse than this.

The frequency of the beat tone (between the oscillator of the receiver under test and the crystal generated signal) should be observed immediately the receiver under test is switched on and at intervals of one hour for a day. During this test and the subsequent test the controls of the oscillator under observation should not be moved.

At the end of the day the oscillator controls are left in the same position and the test repeated for three hours next day.

Method of Expressing Results. It will probably be found that, during the period of heating up, the receiver oscillator frequency shifts appreciably and that after that the change in frequency is small, so that observations on the second day two hours after switching on will agree closely with observations taken after the same time on the first day.

The frequency stability is therefore to be recorded as two figures :

(1) Change of frequency during heating up. This to be the difference between the oscillator frequency when first switched on and after three hours' running, taken as the mean of the observation on two days.

(2) Day-to-day stability. This to be the difference between the observations of frequency made twenty-four hours apart, after the set has been running two hours.

19.6. Tracking. The object of this measurement is to determine whether at all frequencies the R.F. and I.F. circuits are maintained in correct relative alignment by means of the beating oscillator.

This is indicated by the combined selectivity curves of R.F. and I.F. circuits obtained as a plot of the diode current against input radio frequency. It can be adequately and most conveniently measured by means of a ganging oscillator and oscilloscope such, for instance, as the 3343 ganging oscillator and the 3332 oscilloscope made by Messrs. Cossor, Limited.

Method of Measurement. The receiver is tuned in to one of the standard frequencies given on page 8 of the R.M.A. Specification, Part 1. The automatic volume control is disconnected as described in Section 31. The ganging oscillator is connected to the receiver under test through a dummy aerial and the output of the receiver speech diode is connected through 3 megohms to the input amplifier of the oscilloscope in such a way as to introduce no capacitative loading on the receiver at the point of measurement.

The frequency sweep of the ganging oscillator is adjusted to its standard rate of sweep and the gain of the oscilloscope amplifier is adjusted until a convenient size of picture is obtained. The amplitude of sweep is adjusted to give a scale of 10 kilocycles = 1 inch. The input level to the receiver is reduced until overloading is substantially removed, as is shown by the fact that further reductions in level do not change the wave form observed on the oscilloscope.

In the case of the Cossor 3332 oscilloscope and 3343 ganging oscillator the amplitude of frequency sweep is always 30 kilocycles, but the length of sweep on the oscillograph is adjustable and should be adjusted to 3 inches, which gives the scale calibration required.

Method of Expressing Results. These are traced on squared paper marked in inches and tenths on which is marked the mid-band frequency as judged by eye, and frequencies at intervals of 5 kilocycles either side to a scale of 10 kilocycles = 1 inch. The results should be given with the receiver tuned in to each of the standard frequencies given on page 8 of the R.M.A. Specification, Part 1.

19.61. Alignment of Tuning Devices. This test applies only in the case of receivers containing tuning indicators which derive their indication from separate tuned circuits usually bridged across some portion of the I.F. circuit. The object of the test is to determine whether the frequency to which the indicating circuit is tuned lies effectively in the centre of the I.F. band.

Method of Measurement. The automatic volume control is disconnected as described in Section 31.

A signal generator modulated with 400 p : s is connected to the input of the receiver, and the receiver tuned in to one of the standard frequencies given on page 8 of the R.M.A. Specification. The input level is then adjusted until the receiver gives a quarter of the standard output power (see Section 28) with the A.F. gain control at maximum.

Keeping the modulating frequency constant at 400 p : s the signal generator frequency is now changed in both directions until the audio-frequency output is reduced 6 db.

The amount of frequency change in each direction is recorded; call the magnitudes of these changes a and b. The amount that the tuning indicator is off centre is given by $\frac{1}{2}(a - b)$.

This measurement should be repeated with the signal generator tuned in to each of the standard frequencies given on page 8 of the R.M.A. Specification, Part 1.

Method of Expressing Results. The results are to be expressed as a statement of the number of cycles the tuning circuit is "off centre" when the set is tuned into each of the standard frequencies.

19.7. Hum and Noise Level. The hum-plus-noise level shall be defined as the number of decibels difference in level *between* the output of the receiver, terminated with its nominal impedance at 1,000 cycles, when a signal of specified level at a megacycle modulated 30% with 1,000 cycles is applied between aerial and earth, the audio-frequency gain control being adjusted so that the set delivers its rated output into a non-reactive resistance R equal to its nominal output impedance, *and* the output of a specific weighted network inserted between the receiver and the terminating impedance, with the *modulation* removed. The specified level of the input will be varied in steps of 20 db. from 1 microvolt to 1,000 millivolts. During measurement the R.F. gain control to be at maximum gain, the variable selectivity controls, if any, to be at maximum selectivity, and tone control at maximum fidelity.

19.71. Output Network for Signal-to-Noise Ratio. This shall be constituted by a four-terminal network designed to work between equal terminating resistive impedances R and having the following loss characteristic:

Frequency p : s	*Insertion Loss in Decibels*	
25	35·0	
50	30·0	
100	23·5	
150	18·6	
200	15·0	
250	12·0	
300	10·0	
400	7·2	This curve is
500	5·3	derived from the
600	4·5	ear sensitivity
700	3·9	curve, the rise
800	3·4	at high
1,000	2·6	frequencies
2,000	1·0	being neglected
5,000	0·5	for simplicity.
10,000	0	

The figures used in the right-hand column shall be subject to a tolerance of ± 1 db. The meter used to measure output level shall read R.M.S. values.

A practical method of realizing the above characteristic is by means of an equalizer in the form of a T-network designed to work between 600-ohm terminating impedances.*

The series arms are equal, and each consists of a condenser of capacity 0·4 μF shunted by a resistance of 5,000 ohms. The shunt arm is composed of an inductance of value 0·5 H (approx.) at 1,000 p : s, in series with a resistance of 1,000 ohms (approx.), and may be constituted conveniently by a 50,000-μH Bulgin H.F. choke, in series with a 500-ohm resistance.

Method of Expressing Results. The results in each case to be expressed as the observed difference between the quantities defined in Section 1.10 at each specified level.

19.8. Overall Envelope Harmonics. These measurements are intended to show to what extent the audio-frequency output wave form departs from the envelope of the modulated input signal when this envelope is sinusoidal.

Method of Measurement. A special signal generator is modulated separately with 100 p : s and 1,000 p : s (filtered with a low-pass filter to remove harmonics) to a modulation depth of 80%.

This signal generator is arranged to feed directly into the receiver through the standard dummy aerial and to deliver outputs of 1 millivolt and 100 millivolts.

* Evidently with terminations R, values of circuit elements given below must be modified by multiplying impedance by factor $\frac{R}{600}$.

For each condition as set up, and with the receiver delivering its normal output level into its rated output load impedance, the levels of the fundamental modulating frequency and its second and third harmonics shall be measured.

These measurements shall be made at each of the radio frequencies laid down in Section 2, page 8, of the R.M.A. Specification for Testing Receivers.

Method of Expressing Results. The results shall be expressed as the level of the second and third harmonics of the fundamental audio frequency chosen, expressed as a percentage of the fundamental frequency on a voltage basis.

19.9. Output Impedance. The output impedance shall be measured on an audio-frequency bridge at the following frequencies :

30, 1,000, 5,000 and 10,000 p : s.

19.10. Acceptance Tests on Approved Receivers. The list of measurements given above is intended primarily to determine the performance of a new type of receiver. For accepting approved types of receiver a simpler set of tests can be applied. The following is a list of measurements necessary in order to determine the more important characteristics of a receiver. Every type of receiver must, however, be treated as a special case and possible amendments considered :

Measurement	*Section*
Automatic Volume Control	19·2
Sensitivity	19·3
Second-Channel Selectivity	19·41
Selectivity	19·43
A.F. Response Overall	19·44
Tracking	19·6
Alignment of Tuning Devices	19·61
Hum-and-Noise Level	19·7
Overall Envelope Harmonics	19·8
Output Impedance	19·9

20. Diversity Reception.

This is a means of reducing the effects of fading, interference and noise by the use of two or more receivers, each connected to a separate aerial. The audio-frequency outputs of the two receivers feed into a common channel and the aerials are spaced in the horizontal plane.

One type of fading is due to the shifting interference pattern produced by the interaction of rays, of intensity varying with time, which reach the receiving aerial by different paths. The effect of this is that the procession of nodes and anti-nodes at each point in space is different from that at every other point in space. As a result it is found by experiment that, if two aerials are separated by not less than 10 to 20 wavelengths, fading phenomena on the two aerials are not simultaneous. It makes very little difference whether the line joining the aerials is along the direction of propagation or normal to it. If the audio-frequency outputs of the two receivers are combined it is found that the combined output is less subject to fading than the single output of either receiver alone. The more aerials and associated receivers that are used the greater is the freedom from fading. The improvement obtained by using more than three receivers is, however, small, and in the B.B.C. this is the number normally employed.

With the simple arrangement as described, when the carrier input to one receiver falls, the A.V.C. of the receiver operates to increase the gain, and the noise output of the receiver increases. The simple arrangement is therefore unsatisfactory and a great improvement in noise ratio is obtained by connecting the A.V.C. lines of the receivers in parallel. This parallel connection is usually made through a low-pass filter to prevent interaction between the beat oscillators of the receivers. The A.V.C. line of a receiver is the common connection which carries the A.V.C. bias voltage back to the grid circuits of the A.V.C. controlled valves. See Figs. 4 and 5/XIX:13.

When a number of identical receivers have their A.V.C.s connected in this way the receiver connected to the aerial in the strongest field determines the A.V.C. voltage supplied to all receivers. The receivers with the lowest signals are then prevented from contributing excess noise by increasing their gain. As the signals reaching the receivers vary in intensity, each receiver takes charge in turn, maintaining the output with a reduced variation of level.

In practice, since the process of mixing the signals arriving by two separate paths (to separate aerials) produces a certain amount of distortion, particularly noticeable in music, with slow fading of moderate depth, it is usual to make the sensitivity of one receiver greater than the rest so as to reduce the number of times that change-over occurs. If there is any difference in the inherent noise level in the receivers (which may be due to one aerial being more efficient than the others) the receiver with least noise is naturally the one chosen to be made most sensitive. Deep slow fading requires a more uniform setting of the maximum gain of the receivers, while fading of the slow fall and quick return type is best dealt with by a low value of maximum gain and slow-acting A.V.C. That is, the receiver is operated nearer the knee of its A.V.C. characteristic. It is usual to provide diversity receivers with variable time constants in their A.V.C. bias circuit.

In addition to using spaced aerials, diversity reception may be effected, using aerials with different vertical polar diagrams, receiving different angles of downcoming ray, or with differently orientated horizontal polar diagrams receiving from different angles in the horizontal plane. At times the direction of the incoming ray in the horizontal plane varies within wide limits. The combination of reception of vertical polarization with the reception of horizontal polarization may also be used. So far the use, if any, of such variations has only been experimental.

Finally, when transmissions of the same programme are carried out on more than one wavelength, the reception of the several transmissions may be combined just as in spaced-aerial diversity.

21. Notes on Fault-Finding in Radio Receivers.

These notes are not intended to provide complete information for the servicing of receivers. They are only intended to serve as a guide to beginners and to indicate the general method of approach to be adopted in clearing the most simple kinds of fault. *The manufacturer's service manual is an almost indispensable aid to fault-finding.*

Fault-finding in a modern radio receiver is really a specialist's job and needs proper equipment. There are very many kinds of fault, however, which can be located by means of comparatively simple systematic tests.

Examples of Faults which might cause a Receiver to give no Output from the Loudspeaker. Broadly, this may be due to a disconnection or a short circuit anywhere in the circuit, or a faulty valve or component. In greater detail, it may be due to:

(1) Aerial disconnected. Earth disconnected may make a difference but probably not so much. In each case disconnection may be internal or external to the receiver.

(2) Valve failed; filament burnt out or suffering from low emission, electrode disconnected internally, two electrodes touching, or valve gone soft.

(3) Tuning-coil failed; disconnected, shorted or partly shorted.

(4) Condenser disconnected or shorted with a bit of solder or dirt or a loose screw which has fallen into it. Vanes of condenser may be bent or touching.

(5) Dirty contacts on valve pins on any switch.

(6) Disconnection in H.T. or L.T. circuit.

(7) Smoothing condenser short circuited ; this may affect H.T. or bias to valves.

(8) Rectifier valve failed.

(9) Disconnection in A.C. mains input to receiver either internal or external. See that built-in fuses are not burnt out.

(10) Disconnection or short in R.F., I.F. or A.F. circuits.

(11) Failure of loudspeaker due to disconnection or short in field coil or speech winding (this might result in distorted and/or low output accompanied by excessive hum).

(12) Disconnection in bias circuit to any valve so that grid is free to assume an arbitrary potential.

(13) Internal disconnection in R.F., I.F. or A.F. chokes, or in smoothing chokes.

With appropriate qualifications the above faults may occur in :

A. The S.F. circuits.

B. The I.F. circuits.

C. The A.F. circuits.

D. The oscillator circuits.

E. The power pack. This comprises the A.C. input, the H.T. circuit, including H.T. transformer secondary, rectifiers and smoothing, and the filament circuits.

Fault Location.

No output from loudspeaker.

1. Turn main tuning-dial round to check that set is not tuned into a vacant channel.

2. Try on other wave-bands to see if set is out of action on all bands. If out of action on one band only, fault is connected with the parts of the circuit which are brought into action by operating the wave-band switch.

3. If fault is on all wave-bands, check that faults 1 and 9 are absent.

(4, 5 *and* 6, *Check supplies to Receiver.*)

4. Look at all valves to see if filaments are alight. If filaments are invisible leave set on for five minutes and feel valves to see if they are all warm. (Ineffective in a battery set where volts on filaments and continuity of filaments must be checked by a voltmeter.) Replace any valves which are cold and if these valves still remain cold look for a disconnection in the heater circuit. If all valves become warm, but a spare set of valves is available, replace all valves and if set works replace all valves one by one to find faulty valve.

5. By measuring between anode and ground check that H.T. reaches all valves. If voltage on anode of one valve is absent check back along path to anode (from H.T. source) until volts are found. This gives the exact location of the fault.

6. If H.T. is missing from all valves, check H.T. voltage at output of power pack and if absent check back along smoothing circuit until voltage appears. This may reveal a disconnection in a choke or a faulty rectifier valve.

7. *Check L.F.* With set switched on, put finger on first L.F. grid. This should cause a click in the speaker and, if A.C. mains are in the vicinity, a hum. If nothing heard, tap along L.F. circuit with a pair of headphones, starting from output of detector with set tuned in to station. Point at which signal appears locates fault. If no output from detector when tuning is varied there is a fault earlier in the circuit. A condenser of about 0·5 μF capacity should be connected in series with headphones to hold off H.T.

8. *Check Oscillator.* If no output from detector, put finger on oscillator grid. If oscillator is working this should produce a plop.

9. *Check H.F.* Put aerial directly on grid of mixer. If output is obtained fault is in H.F. Tap aerial back along H.F. circuit until output disappears. This locates fault.

10. When neighbourhood of fault has been found the circuit in that part should be examined mechanically and a number of tests made to localize the fault more closely. If a valve is suspected and it is receiving supplies, check that valve is taking current (assuming that replacing valve does not remedy matters). If not, check grid bias ; this entails finding a low-impedance point of grid-bias supply : even if this bias is correct the grid decoupling may be disconnected or the decoupling condenser may be shorted.

Note 1. All suspected parts of the circuit should be tested for continuity and all condensers should be tested for shorts.

Note 2. If fault is on one wave-band only it is almost certainly in a switch contact or a coil, so that when neighbourhood of fault has been found these are the first things to look for.

Excessive Hum in Output of Loudspeaker.

First see that hum is not incoming to aerial.

1. Disconnect speech-coil of loudspeaker. If hum persists it is due to field-current of loudspeaker. (The field winding of loudspeaker is sometimes used as a choke in the H.T. smoothing circuit.) Extra smoothing must be added before the field-coil in the H.T. smoothing circuit. This can probably be provided by

an 8-μF condenser of a high-voltage (e.g. 400 volts) electrolytic type.

2. Short input to the last stage. If hum persists it is in this stage and may be due to valve, H.T. grid bias, or filament supplies.

(*a*) Replace valve by resistance connected between anode contact and minus H.T. If hum persists it is due to H.T. smoothing being inadequate.

(*b*) If (*a*) gave no hum, replace valve and try effect of extra grid decoupling. If this is ineffective, try replacing centre tap on mains transformer winding supplying heater, by hum-bucking potentiometer. If L.F. has a high gain it is possible for hum to be introduced due to coupling between heater leads and the core of any audio-frequency transformer at a low-level point in the circuit. The remedy is to twist the heater leads along their whole length up to a point as close as possible to the heater.

(*c*) If the valve is a pentode the smoothing of the screen grid should be checked.

Note. In checking the smoothing, check that the smoothing condensers are not taking excessive current. An L.T. electrolytic condenser should not take more than about 100 μA, while an H.T. electrolytic condenser should not take more than about 5 milliamps. Paper condensers should take no current.

3. Hum may be due to coupling between the core of the input transformer to the stage and the mains transformer. Disconnect primary of transformer and connect across its input a resistance equal to anode impedance of previous valve within plus or minus 50%. If pick-up is due to coupling with mains transformer the transformer must be reorientated in space.

4. Hum may be due to lack of screening. To check for electrostatic interference replace secondary of transformer by 100,000 ohms connected between grid bias and grid. If hum persists it must be removed by an earthed screen. It may be possible to remove an electrostatic pick-up by screening leads only, or it may be necessary to screen the whole stage.

5. If hum is observed at output of detector it may be due to inadequate H.T. smoothing on any of the R.F. stages. Check as before by replacing each valve by a resistance.

6. In any set equipped with gramophone pick-up terminals, whether equipped with a turntable or not, turn switch to " Gram " and see if hum persists. If it does, then it is in L.F. circuit ; if not, then it is earlier in the circuit.

Set Equipped with Gramophone Turntable.

7. By disconnecting it, check that gramophone pick-up is not a source of hum. Check also that leads from pick-up are not picking up hum.

8. See that motor frame is earthed and that the insulation of the windings from the frame is satisfactory.

Set near a Powerful Local Station.

9. A powerful local transmitter may induce R.F. in the power mains. This R.F. may traverse the mains transformer and become modulated by the mains frequency and its harmonics. The resultant modulated wave passes into the R.F. stages by a variety of routes and causes hum in the output. Normally this is prevented by the use of a screen between the primary and secondary winding of the mains transformer. This screen should be earthed, and if therefore this trouble is suspected, the first step is to see that the transformer screen, if any, is earthed. Whether or not a screen exists, if hum persists, the next step is to insert an R.F. filter in the mains. This may simply consist of two 0·01-μF condensers, one from each leg of the mains circuit to ground, but if such is not adequate a more elaborate filter must be installed. The condensers should be rated for 250 A.C. volts working, but it is almost safe to say that all condensers of this size are so rated or are rated higher.

10. Check that hum is not induced into earth lead or directly into receiver. This may entail moving the receiver to a new location. If screening the earth lead does not remove the hum it may be necessary either to completely screen the receiver or else to operate it permanently in new location.

Output from Loudspeaker Distorted.

1. If output is high-pitched, check that receiver is properly tuned in to the station being received, and see that the loudspeaker coil is free to move. Then check back along circuit with a pair of headphones until point is found at which change of quality appears. This locates the fault which is certainly in the L.F. circuit and may be due to a disconnected or faulty stopping condenser or a partially shorted choke or transformer.

2. Output from loudspeaker non-linear, e.g. the output sounds as if the set were overloaded even when operating at normal volume.

(*a*) Check supplies.

(*b*) Tap along L.F. circuit with headphones until neighbourhood of fault is located. Replace any suspected valve, and if fault persists check bias supplies and decoupling resistance and condenser in grid circuit.

(c) Fault may be due to an incipient or supersonic oscillation round any part of the circuit or due to leakage of R.F. past the detector. The latter may be eliminated by shunting a 0·0003-μF condenser across the grid of the first L.F. valve, or probably better by inserting 100,000-ohm resistance in series with the first L.F. grid. Supersonic oscillation may sometimes be removed by a similar method applied either to the first L.F. grid or later in the circuit. Incipient oscillation may be due to inadequate anode and/or grid decoupling. It is unlikely that supersonic oscillation is due to this cause.

Whistles.

Whistles are broadly of three kinds:

1. Those that do not vary in pitch or intensity as the tuning is varied.
2. Those that vary in intensity only as the tuning is varied.
3. Those that vary in pitch and intensity as the tuning is varied.

1. This type of whistle is usually due to L.F. feedback. This may be caused by capacity coupling between stages or by a common anode impedance. In battery sets a 2-μF condenser across the H.T. supply will sometimes effect a cure. In general it may be necessary to decouple one or more anode circuits and sometimes, in extreme cases, grid circuit decoupling may stabilize an L.F. amplifier.

Low-frequency oscillation is sometimes caused by acoustic feedback from the loudspeaker to a microphonic valve or even to the vanes of a variable condenser. This can be stopped by applying acoustic damping. Valve sockets may be mounted on springs or on rubber while a small felt or rubber cap over the valve often effects a cure. Fortunately feedback to a condenser is very rare. It can only be cured by a major alteration in the set, either replacing the condenser by another type or mounting it on rubber.

2. This type of whistle is rarely due to the receiver. It may be due to a carrier modulated with the tone observed or may be due to heterodyne notes between adjacent stations. The latter can be suppressed by the use of rejector circuits in the low-frequency part of the receiver, tuned to nine kilocycles.

3. This type of whistle may be due to a number of causes:

(a) Second-channel interference.

(*b*) Mixed-channel interference.

(*c*) Other channel interference due to stations separated from any harmonic of the oscillator by a frequency distance equal to the I.F. frequency.

(*d*) Other channel interference due to stations separated from the oscillator frequency by a half or a third of the I.F. frequency. This occurs only when the mixer produces an output frequency equal to two or three times the normal difference frequency (beat frequency).

(*e*) Feedback of harmonics of the I.F. frequency to the R.F. stages.

(*a*) can be cured by a rejector circuit in the aerial tuned to the unwanted station.

(*b*) can be cured by a rejector circuit in the aerial circuit tuned to either of the mixed channels.

(*c*) and (*d*) may be cured by changing the mixer valve but are probably inherent in the type of valve and incurable unless another type of mixer valve can be used in the same circuit. Fortunately this type of whistle is rare.

(*e*) can be eliminated by a low-pass filter after the detector, which although designed primarily to suppress harmonics of the I.F. frequency may conveniently be adjusted to have a cut-off at half of the I.F. frequency.

Interference may occur from a station operating at or near the I.F. frequency. This may give rise to a whistle or to spark interference if from a spark station and also to cross-talk from the programme modulating the station in question. This is cured by a rejector in the aerial circuit tuned to the I.F. frequency.

A whistle of type 3 may occur in a set due to the fact that one or more of the R.F. stages are oscillating. Shunt a small condenser (e.g. 0·00005 μF) across the aerial input circuit. If whistle reduces in amplitude but does not change pitch it is due to second channel; if whistle changes pitch there is an R.F. oscillation in the R.F. circuit. Check decoupling condensers in R.F. circuit. If these are in order it may be necessary to introduce screening between the output of any R.F. valve or the mixer and any previous part of the circuit. The screen should be earthed. To find points between which coupling occurs try touching various parts of the circuit and note whether oscillation is stopped or increased. The most important parts are those which are not already screened, such as the anode or grid caps of valves. See that all screen grids are tied to ground through a proper capacity, e.g. not less than about 0·01 μF.

An oscillation in the R.F. circuit may sometimes be cured by using a longer aerial.

Motor-Boating. This is a low-frequency oscillation observable in the loudspeaker as a modulation of the amplitude of programme and noise at a few cycles per second. It may be due to:

(1) A back coupling in the L.F. part of the circuit giving rise to a normal oscillation at about two or three cycles per second.

(2) A supersonic or R.F. oscillation in any part of the circuit which builds up to a large value, paralyses the grid of a valve anywhere in the circuit, causing the oscillation to collapse, and then builds up again, repeating the process two or three times per second.

1. This type of oscillation is due to inadequate anode decoupling or to too high an impedance of the H.T. supply source, or both. It is very hard to cure by legitimate methods, which consist either in shunting the final smoothing condenser with an electrolytic condenser of large capacity or in shunting one of the existing decoupling condensers by a large electrolytic condenser. It may of course be due to one of the condensers having failed, in which case replacement by a good condenser may clear the trouble. A simple method of curing the trouble is to introduce a small series condenser in the A.F. part of the circuit so as to attenuate frequencies below about twenty cycles per second. Such a condenser should have a reactance equal to the circuit impedance at 20 c/s, e.g. if the circuit impedance is 100,000 ohms a condenser of $1 \times 159/100{,}000 \times 1{,}000/20 = 0{\cdot}0795$, say, $0{\cdot}1\ \mu F$ should be used.

2. This can be cured by the method given for eliminating an incipient or supersonic oscillation given under 2 (*c*) in the section " Output from Loudspeaker Distorted ". If an R.F. oscillation, it can be cured by the method given under 5 in the section on " whistles ".

Another frequent cause of motor-boating is an open grid.

Noise in the Loudspeaker. Hum has already been dealt with. Every set has a minimum value of background noise which appears as a low-level steady rushing noise. If this appears unduly exaggerated it may be due to an inefficient aerial coupling circuit or a fault in the aerial coupling circuit, a disconnected aerial, or a faulty valve.

Irregular scratches and clicks may be due to a loose connection either in a soldered joint, internally in components including valves, or in connecting wires. Frequent sources of such noises are bad switch contacts, badly fitting valve pins (in their sockets) and failing electrolytic condensers. Touching condenser vanes sometimes gives the same effect, as do also old dry-cell batteries used for H.T. Before condemning a set, always check that any suspected noise is not incoming to the aerial. This can usually be done by removing the aerial, and if necessary shorting the aerial and earth terminals.

A source of noise which sounds exactly like a loose joint sometimes occurs in audio-frequency amplifiers owing to sporadic ultra-short wave oscillations occurring due to the proximity of grid and anode or grid and cathode loads. These can be cured either by separating the leads, or by putting stopper resistances of about 5,000 ohms in the grid leads and about 100 ohms in the anode leads. Sometimes even it may be necessary to put a stopper resistance in the cathode lead. All stopper resistances must be located as close as possible to the electrode concerned.

Miscellaneous Notes. The most common faults in radio receivers are due to failure of electrolytic condensers, bad switch contacts and failure of valves. Remember that a bad joint looks exactly like a good joint unless the wire has actually come off, and a bad component looks like a good one unless you are fortunate enough to find a burnt-out resistance.

In the event of the A.C. mains supply failing to arrive at the mains transformer, remember to look for fuses built into the receiver.

If H.T. voltage of a mains set is low, check that mains transformer is on proper taps. If mains transformer is on proper taps, never change to a lower tap to compensate for a mains supply voltage which is temporarily low owing to peak load conditions.

An abnormally high valve current does not always mean a faulty valve : it may mean lack of grid bias, or may be due to a leaky condenser in the grid circuit resulting in the anode potential of the previous stage being applied to the grid.

The voltage of dry-cell batteries should always be tested when delivering a current equal to the current which they are called upon to supply in the set. In practice, the simplest way to do this is to measure the voltage of the battery when connected to the set with the set switched on.

CHAPTER XX

MEASURING EQUIPMENT

1. Ammeters, Milliammeters and Microammeters.

THESE are all instruments for measuring current. Although there is no definite standard, ammeters may be regarded as instruments measuring currents above one ampere, milliammeters from 1 mA to 1,000 mA, microammeters below 1 mA. The above figures refer to the full-scale deflections of the instruments. In practice the designation of an instrument near the boundaries of these ranges may depend on the whim of the manufacturer.

D.C. meters are usually operated by the effect of the current passing through a moving coil located between the pole pieces of a horseshoe magnet.

Low frequency A.C. meters operate :

(*a*) By the mechanical expansion of an A.C. wire carrying the current : *hot-wire meters.*

(*b*) By the magnetic effect of a coil carrying the current on a moving piece of iron or *armature* : *moving-iron meters.*

(*c*) By the force between a fixed and a moving coil each carrying the current : *dynamometer meters.*

(*d*) By the force exerted by a coil or coils inducing eddy currents in a disc, and the disc : *induction meters.*

Audio-frequency meters are constituted largely by thermocouples in conjunction with millivoltmeters, which may be associated in one assembly or may be separate. More recently copper oxide rectifiers have come into use in conjunction with milli- or microammeters. The latter usually have a high resistance, e.g. 1,000 to 2,000 ohms, and are not always of constant sensitivity over the audio-frequency range.

Radio-frequency meters are usually either of the hot-wire or thermocouple type.

Although the resistance of a D.C. microammeter may be over 500 ohms *all ammeters, milliammeters and microammeters must be regarded as zero resistance devices and must never be put in a circuit unless the resistance in the circuit is sufficient to limit the current to a value within the range of the meter,* having regard to the voltage effective in the circuit. This applies whether the circuit is D.C., A.C., or R.F. In using microammeters care must be taken that the resistance of the meter is not appreciable compared to the resistance in the remainder of the circuit, otherwise the meter reading will not indicate the current which flows when the meter is removed from the circuit.

Nearly all D.C. meters are magnetic devices and it must always be assumed that their calibration is affected by the presence of large masses of iron in their neighbourhood. For this reason, *when ordering meters it should be specified whether they are intended for use on a mounting panel of magnetic material or otherwise.*

The *calibration* of an indicating instrument refers both to the numbers on the scale indicating the quantity the instrument is measuring and to the process of adjusting the instrument or marking the scale so that it reads correctly.

A.C. ammeters are used for measuring 50-cycle alternating current. R.F. ammeters are used for measuring R.F. currents and care must be taken not to confuse the two. R.F. and A.F. milliammeters are used for measuring R.F. and A.F. currents respectively, and again care must be taken not to confuse the two. A meter which is suitable for R.F. may be suitable for A.F., but the reverse is generally not the case. *R.F. measurements of currents suffer from an increasing drop in accuracy as the frequency is increased ;* the accuracy at frequencies below 2 megacycles should never be considered as being better than $\pm 5\%$, while above that frequency R.F. ammeters should only be used to indicate that a particular current is constant : provided the circuit is not changed, there is no reason why the accuracy of the meter should change.

The inaccuracy of radio-frequency ammeters is due to the flow of radio-frequency current through the heater of the meter and the capacity of the meter to ground when the meter is at a high radio-frequency potential above ground. Since this current is dependent on the magnitude of the potential the error is not constant but varies as the potential is changed. The current is in quadrature with the true line current.

Fig. 1 shows a method of screening the meter to reduce the capacity current, the screen being connected to the side of the meter connected to the driving source. This precaution is only a palliative, and for accurate measurements the capacity current should be measured by disconnecting terminal B and noting the capacity current, which should then be subtracted vectorially from subsequent readings of the meter : i.e.

$$\text{line current} = \sqrt{(\text{observed current})^2 - (\text{capacity current})^2}.$$

Radio-frequency meters may be calibrated against D.C. meters by passing the same D.C. current through both meters. D.C. meters are calibrated by means of standard meters. Standard meters are calibrated by means of measurements involving the chemical effect of currents in passing through solutions of certain salts, by comparison with instruments designed to permit fundamental calculations of their sensitivity and by involved comparisons involving fundamental standards of resistance and e.m.f.

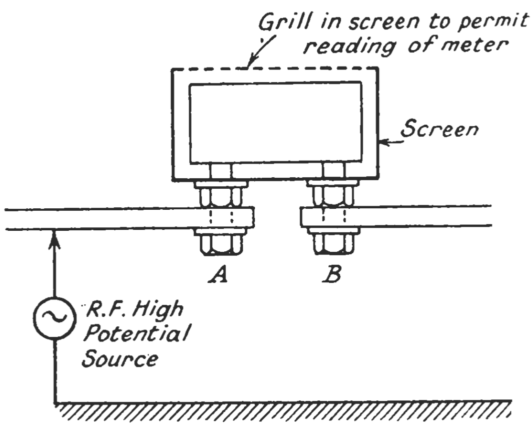


FIG. 1/XX:1.—Method of Screening Radio Frequency Meter to reduce Capacity Current.

The range of ammeters and milliammeters is often extended by the use of shunt resistances, which may be external or may be internal and controlled by a switch. *Care must be taken to see that the meter is always on the right range.*

2. Voltmeters and Millivoltmeters.

A voltmeter usually consists of a milliammeter or microammeter in series with a limiting resistance which determines the current which flows for a given voltage across the voltmeter terminals. A voltmeter can therefore be connected directly across any source of voltage which is less than the scale reading of the voltmeter, and, in the case of multirange instruments corresponds to the range in use. The range of voltmeters is extended by the use of a number of alternative series resistances which may be external or may be internal and controlled by a switch. *A multirange voltmeter should always be kept on its maximum range of voltage, only being adjusted* *for lower ranges when across the voltage source; immediately after use it should be returned to the maximum range.*

When using a voltmeter across a high-impedance circuit, such as a grid circuit, care must be taken that the resistance of the voltmeter is high compared to the resistance of the circuit: e.g. if it is 10 times the resistance of the circuit an error of 10% will occur. Most commercial voltmeters have a resistance in the neighbourhood of 1,000 ohms per volt, e.g. a meter to measure 200 volts has a resistance of 200,000 ohms. Where a commercial voltmeter has insufficient resistance, on low-voltage circuits it is a simple matter to measure a voltage by means of a microammeter and a high series resistance.

All meters which are used for measuring voltages or currents of critical value should be calibrated at regular intervals.

Most meters mounted on equipment in regular service are not measuring critical values and need only be calibrated when the performance of the equipment is unsatisfactory and the accuracy of a meter is suspected.

3. Electrostatic Voltmeters.

In these the needle is mechanically connected to one plate of a condenser, the plate being free to move so as to vary the capacity of the condenser. When a voltage is applied between the fixed plate and the moving plate of the condenser the plates become oppositely charged and attract one another. If a linear control spring is used, the deflection is proportional to the square of the applied voltage.

Electrostatic voltmeters for commercial use are not constructed for voltages below 300 to 400 volts. The upper limit is determined by practical dimensions, and electrostatic voltmeters can be constructed up to 10,000 or 15,000 volts, or even higher.

Such voltmeters can be used for measuring D.C. or A.C., including A.F., provided they are used in a circuit where their capacity reactance is not too low. With care they can be used for R.F. but should not be used for this purpose unless they have been calibrated at the frequency in question.

Electrostatic voltmeters are usually provided with a linear control spring and calibrated for use on D.C., they therefore read the R.M.S. value of any A.C. voltage applied across them.

4. Peak Voltmeters.

These provide an accurate means of measuring radio-frequency peak voltages and constitute the only accurate means of measuring high radio-frequency voltages. When the circuit impedance is known they provide the only accurate means of measuring radio-frequency currents and power. They are simple to construct and require no calibration other than the provision of an accurate D.C. meter. This statement requires qualifying by the statement that such instruments must be designed to give the required accuracy. The design is discussed below.

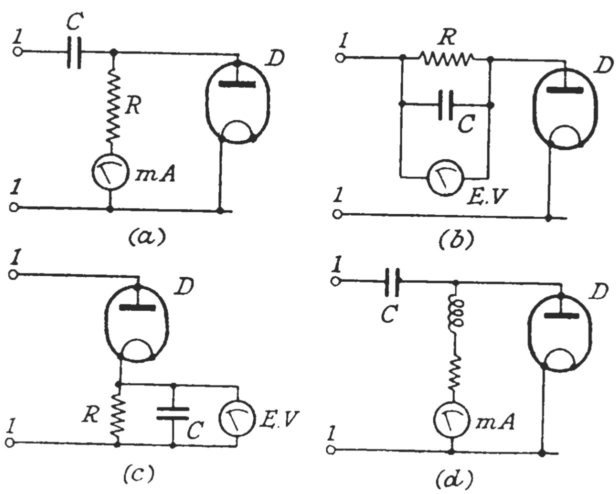


FIG. 1/XX:4.—Alternative Forms of Peak Voltmeter.

Alternative forms of circuit are shown in Fig. 1. The circuit at (*a*) has the advantage that any D.C. at the measuring-point is held off the valve by condenser *C*. Both circuits (*a*) and (*b*) have the disadvantage that the stray capacities of *R* and *C* to ground are in shunt across the diode : in this respect (*a*) is better than (*b*) and circuit (*b*) appears to have no advantages, so that the normal choice appears to lie between (*a*) and (*c*). (*c*) has the advantage that the stray capacities of *R* and *C* do not shunt the diode ; it requires, however, an insulated means of heating the filament. (*c*) is the preferred form for measuring very high frequencies.

In (*a*) the voltage across *R* is measured by means of a milliammeter (or microammeter) in series with *R*, while in (*b*) and (*c*) the voltage is measured by an electrostatic voltmeter *EV* across *R*.

Either arrangement may, however, be used equally satisfactorily in each of the circuits except that, on account of its capacity and because it would be subject to high R.F. voltages, an electrostatic voltmeter cannot be used in circuit (*a*). The use of an electrostatic voltmeter has the very large advantage, as will appear below, that R can be made very large since it is not necessary to obtain enough current through it to operate a microammeter. The use of a large value of R increases the accuracy of the peak voltmeter, *and if R is greater than* 100 *megohms the correction for the ratio of R to the diode resistance given below can be neglected.* In some peak voltmeters R is made as large as 20,000 megohms.

The diode D is sometimes constituted by a triode with the grid strapped to the anode, since, for high voltages, e.g. 10,000 volts, suitable diodes are not always available. For voltages up to 7,000 volts an MR4/E620 diode may be used ; such a valve has given satisfactory operation on voltages up to 10,000 volts, and provided therefore that the circuit is arranged so that breakdown of the valve can do no harm, such a valve may be used up to 10,000 volts, if no other type of suitable valve is available.

The value of R should be as high as possible. The error introduced by any given value of R is determined below.

The peak voltmeter indicates the peak volts applied across the input terminals 1,1, as given by $R\bar{\imath}_R$, where $\bar{\imath}_R$ is the mean current flowing through the resistance R as observed on the D.C. meter in series with R, or electrostatic voltmeter across R.

The operation of the peak voltmeter is that of a simple diode detector. Positive pulses of voltage applied to the anode of the valve give rise to currents flowing to ground through the diode resistance, which is very low compared to R. (By comparison the currents which flow through R due to positive voltages are negligible.) Negative pulses of voltage give rise to currents flowing to ground through resistance R only. The net result is that the anode assumes a mean negative potential $\bar{v}_a$ very nearly equal to the peak value of the voltage applied at 1,1. The condenser C is then charged to a mean voltage equal to $\bar{v}_a$.

An inductance is sometimes inserted at the top end of the resistance to protect the resistance from R.F. voltages, as in Fig. 1 (*d*). This is not always satisfactory, however, and it is sometimes better to make the top element of the resistance of sufficient dissipation to carry the R.F. current. Stray capacity to ground diverts R.F. current from the lower resistances.

Correction Factor for Diode Capacity. If the diode has a capacity C_d between anode and cathode, a capacity potentiometer is constituted by C and C_d and the voltage effective across the diode is reduced in the ratio

$$\frac{\frac{1}{C_d}}{\frac{1}{C_d}+\frac{1}{C}} = \frac{1}{1+\frac{C_d}{C}}.$$

The reading of the electrostatic voltmeter (or the result obtained by multiplying the current meter reading by R) should therefore be multiplied by $\left(1 + \frac{C_d}{C}\right)$, and by second correction factor.

4.1. Relation between the Value of *R*, the Diode Resistance, the Total Emission of the Diode and the Second Correction Factor of the Peak Voltmeter.

4.11. Case 1. Linear Diode. The case of an ideal diode, in which the anode current is directly proportional to the voltage across it, is considered first, because it serves as a useful introduction to the use of practical diodes.

The argument applies to any of the circuits of Fig. 1.

Conventions.

R_D = anode cathode resistance of linear diode.

$\hat{v} \sin \omega t$ = voltage to be measured, applied across terminals 1,1, in Fig. 1.

$\bar{v}_a$ = mean negative potential of diode anode (with regard to its cathode).

i_a = current flowing through diode ; this is given by $i_a = (\hat{v} \sin \omega t - \bar{v}_a)/R_D$ for positive values of i_a only : the diode cannot pass current in the opposite direction.

$\bar{i}_a$ = the mean value of i_a.

θ = the angle of current flow through the diode.

i_p = the peak value of i_a.

g = $\bar{i}_a/i_p$ and may be read from Fig. 1/X:22 by entering the value of θ.

$\bar{i}_R$ = the mean D.C. current through R, as read on a D.C. meter in series with R.

R_C = internal impedance of source, applying voltage under measurement to terminals 1,1.

Then $\bar{i}_a = g i_p = \dfrac{g(\hat{v} - \bar{v}_a)}{R_D}$.

(This neglects the fact that a small current flows through R so that the voltage across the diode is not always exactly equal to $\hat{v}\sin\omega t - \bar{v}_a$. The error consequent on this assumption is small and is discussed below.)

Since no direct current can flow through condenser C,

$$\bar{i}_a = \bar{i}_R, \text{ which is evidently equal to } \frac{\bar{v}_a}{R}.$$

$$\therefore \frac{g(\hat{v} - \bar{v}_a)}{R_D} = \frac{\bar{v}_a}{R}$$

$$\therefore gR\hat{v} - gR\bar{v}_a = \bar{v}_a R_D$$

$$\therefore \frac{\bar{v}_a}{\hat{v}} = \frac{gR}{gR+R_D} = \frac{g}{g+\frac{R_D}{R}} \quad . \quad . \quad . \quad (1)$$

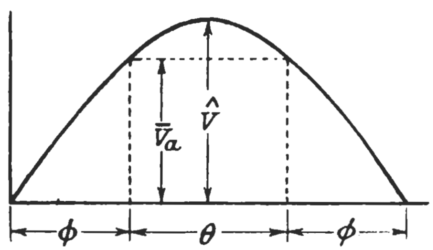


FIG. 2/XX:4.—Relation of θ and V_a during Positive Half Cycle.

The angle of current flow may be determined by reference to Fig. 2 as follows:

$$\bar{v}_a = \hat{v}\sin\phi = \hat{v}\sin\left(90^\circ - \frac{\theta}{2}\right) = \hat{v}\cos\frac{\theta}{2}$$

$$\therefore \frac{\bar{v}_a}{\hat{v}} = \cos\frac{\theta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2)

$$\frac{g}{g+\frac{R_D}{R}} = \cos\frac{\theta}{2}$$

$$\therefore g = g\cos\frac{\theta}{2} + \frac{R_D}{R}\cos\frac{\theta}{2}$$

$$\therefore \frac{R}{R_D} = \frac{\cos\frac{\theta}{2}}{g\left(1 - \cos\frac{\theta}{2}\right)} \quad . \quad . \quad . \quad (3)$$

By substituting in equations (2) and (3) corresponding values of g and θ as determined from Fig. 1/X:22 the corresponding values of $\frac{R}{R_D}$ and $\frac{\bar{v}_a}{\hat{v}}$ can be determined.

This operation is carried out in the table below and the results are plotted in Fig. 3.

$\frac{R}{R_D}$	θ	g	$1 - \cos\frac{\theta}{2}$	$\frac{\bar{v}_a}{\hat{v}} = \cos\frac{\theta}{2}$	Correction Factor $F = \frac{\hat{v}}{\bar{v}_a}$	Peak to mean ratio of diode current $= \frac{1}{g}$
114,200	5°	0·0092	0·00095	0·99905	1·00095	108·7
79,600	6°	0·011	0·0014	0·9986	1·0014	90·91
28,270	8°	0·0147	0·0024	0·9976	1·0024	68·0
14,140	10°	0·0184	0·0038	0·9962	1·0038	54·3
8,370	12°	0·022	0·0055	0·9945	1·0055	45·45
1,800	20°	0·0360	0·0152	0·9848	1·015	27·7
515	30°	0·0550	0·0341	0·9659	1·035	18·2
211	40°	0·0740	0·0603	0·9397	1·064	13·5
104·6	50°	0·0925	0·0937	0·9063	1·104	10·8

The above analysis has assumed that the peak voltmeter does not drop the voltage under measurement at any point in the R.F. cycle. In practice this is not true, but as the following argument shows, the resulting change in reading is usually negligible.

When the diode is passing current the effective voltage across the diode (i.e. $\hat{v} \sin \omega t - \bar{v}_a$) is dropped in the ratio $R/_D(R_D+R_C)$. Evidently the same diode current would flow if the voltage were kept constant and the diode resistance were increased to R_D+R_C. In other words, the value of $R/(R_D+R_C)$ should be entered in Fig. 3 instead of the value of R/R_D. When the value of R_C, the impedance of the circuit in which the voltage is being measured, is known, this may be done. The value of R_C is, however, difficult to determine in practice. Fortunately it is usually determined by a condenser carrying a charge of such magnitude that the voltage across the condenser is not appreciably reduced by the current taken by the peak voltmeter. This condition obtains, for instance, when the peak voltmeter is used to determine the anode peak volts of an amplifier.

As an example of the use of Fig. 3, if $R_D = 2{,}000$ ohms and $R = 1$ megohm, $R/R_D = 500$. Entering this value in Fig. 3, the correction factor is 1·035, which means that the value of $R\bar{i}_R$ must be multiplied by 1·035 to give the true reading. Fig. 3 can therefore be used to provide a correction factor when the value of diode resistance is known. For practical purposes the square law case should, however, be assumed (see below).

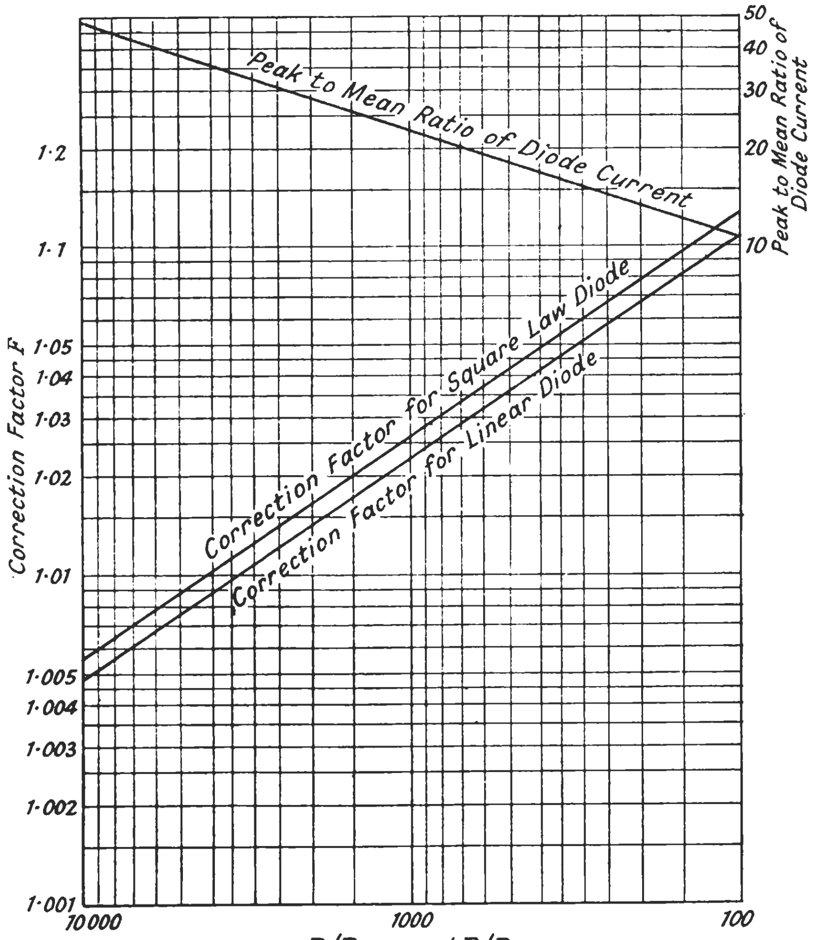


FIG. 3/XX:4.—Second Correction Factor of Peak Voltmeter and Peak-to-Mean Ratio of Diode Current.

On the basis of a linear diode, which approaches close to the performance of a practical diode, it will now be evident why it is important to keep the resistance R as high as possible. Since the effective resistance of a practical diode may be taken as about 1,000 ohms, it is evident that the value of R should be well over a megohm. A similar argument reaching the same conclusion can be applied in the case of the practical diode, which will now be considered.

4.12. Case 2. Practical Diode. In practice, linear diodes are not available. In a linear diode the anode current i is directly proportional to the voltage V between anode and cathode, that is,

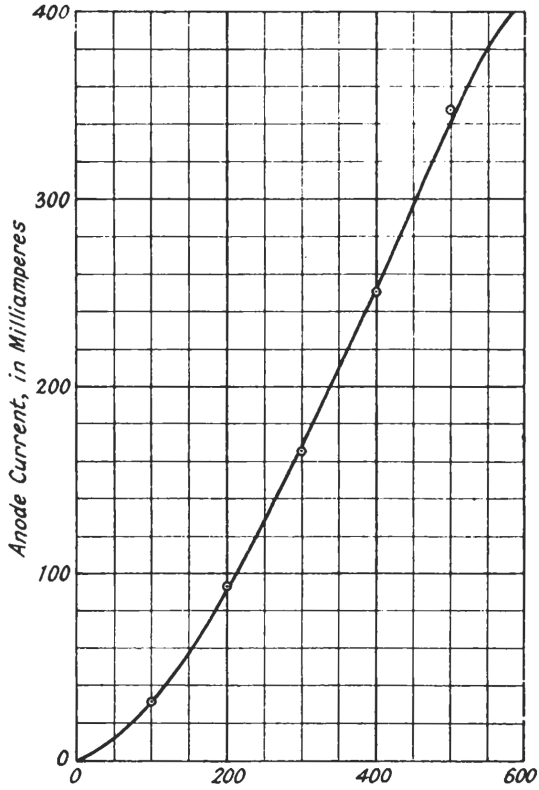


FIG. 4/XX:4.—Anode Voltage Anode Current Characteristic of MR_4 Valve.
[By courtesy of the B.B.C.]

$i = \dfrac{V}{R_D}$, where R_D is a constant and is the resistance of the diode.

In diodes which are available the anode current i is determined very closely by the relation $i = aV^n$, where a and n are constants and V is the voltage between anode and cathode. For instance, in an MR4/E620 valve the diode current below saturation is given very closely by $i = 4{\cdot}47 \times 10^{-2}V^{1{\cdot}44}$.

In Fig. 4 the full-line curve is the characteristic of a typical

MR4 valve, and the points in circles are plotted from the above equation.

In other valves the value of the index n varies between 1·4 and 2. It will appear later that the error introduced by assuming the index to be 2 is small, and this assumption will be made.

It should be noted that the diode resistance is given by

$$R_D = \frac{V}{i} = \frac{V}{aV^n} = \frac{2}{aV^{n-1}}$$

if $n = 2$, $R_D = \dfrac{1}{aV}$.

If a varying voltage is applied to the diode the diode resistance varies continuously, its minimum value being $R_{min.} = \dfrac{1}{a\hat{v}^{n-1}} = \dfrac{1}{a\hat{v}}$, when $n = 2$, $\hat{v}$ being the peak voltage *effective across the diode.*

Using the same conventions as in the first approximation, when the diode is in use in the circuits of Fig. 1 and a voltage $\hat{v}\sin\omega t$ is applied, the diode current is:

$$i_a = aV^2 = a(\hat{v}\sin\omega t - \hat{v}_a)^2$$
$$i_p = a(\hat{v} - \bar{v}_a)^2.$$

$$\text{The diode resistance} = \frac{V}{i_a} = \frac{V}{aV^2} = \frac{1}{aV}.$$

$$\text{The minimum diode resistance } R_{min.} = \frac{1}{a(\hat{v} - \bar{v}_a)}.$$

The mean current

$$\bar{i}_a = g''i_p = g''a(\hat{v} - \bar{v}_a)^2 = \frac{g''}{R_{min.}}(\hat{v} - \bar{v}_a) = \frac{\bar{v}_a}{R}$$

where g'' is the mean to peak ratio of the wave form of the current through the diode for the case of a sinusoidal voltage (applied to a square law valve) with an angle of current flow θ. (The value of g'' is shown on Fig. 6/VIII:1, and, for any value of θ, is very nearly equal to $\dfrac{\pi}{4}g$, the degree of approximation being about equal to the probable error in reading the curve.)

Hence, by reference to the derivation of equations (2) and (3),

$$\frac{\bar{v}_a}{\hat{v}} = \cos\frac{\theta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

and

$$\frac{R}{R_{min.}} = \frac{\cos\frac{\theta}{2}}{g''\left(1 - \cos\frac{\theta}{2}\right)} \quad . \quad . \quad . \quad . \quad (3a)$$

These equations are identical in form with equations (2) and (3), except that g is replaced by g'', and R_D by $R_{min.}$, the minimum value of R_D which occurs at the instant of peak current.

If a table is constructed as before, writing down the values of $\frac{R}{R_{min.}}$, g, $\frac{\bar{v}_a}{\hat{v}}$, the correction factor and the peak-to-mean ratio of diode current, the values of $\frac{R}{R_{min.}}$ are equal to the corresponding values of $\frac{R}{R_D} \times \frac{4}{\pi}$, while the values of peak-to-mean current ratio are multiplied by $\frac{4}{\pi}$. Hence the curve plotted between peak-to-mean current ratio and $\frac{R}{R_{min.}}$ is the same as the curve plotted in Fig. 3 between peak-to-mean current ratio and $\frac{R}{R_D}$. The curve plotted between $\frac{R}{R_{min.}}$ and the correction factor is, however, different from that between $\frac{R}{R_D}$ and the correction factor since the values of $\frac{R}{R_{min.}}$ are greater than those of $\frac{R}{R_D}$ in the ratio $\frac{4}{\pi}$. This curve has also been plotted in Fig. 3 and differs little from the curve for a linear diode, i.e. for the case where $n = 1$. It will now be clear that variations in the index n in the range 1·5 to 2 have no effect on the curve of peak-to-mean current ratio and only a small effect on the correction factor.

It must be remembered that in the case of non-linear diodes the value of $R_{min.}$ is the value of resistance at the peak current, and as the value of diode resistance varies very widely with current amplitude it is essential that the true value of $R_{min.}$ should be used.

4.13. To ensure that a valve of adequate current-carrying capacity is provided the circuit should be adjusted so that the peak current does not exceed about 0·8 of the total emission. The minimum permissible value of $R_{min.}$ is therefore the value of the diode resistance with an anode current equal to 0·8 of the total emission i_t. The peak-to-mean ratio $\left(= \frac{1}{g''}\right)$ is then read off from Fig. 3 by entering the value of $\frac{R}{R_{min.}}$, a tentative value of R being assumed. The mean current is then given by $\bar{i}_a = g''i_p$, and the maximum peak voltage the instrument is capable of reading without the peak current exceeding 0·8i_t is then given by $\bar{i}_a R$. If this peak voltage is not high enough, then either R must be increased or a valve with a higher total emission must be used.

4.14. To apply the correction factor to a peak voltmeter using a square law diode, by the use of Fig. 3, a more complicated procedure is necessary. This is because the value of $R_{min.}$ varies with the applied peak voltage, and depends on the peak current, which cannot be determined until the peak-to-mean current ratio is known. This in turn depends on $\frac{R}{R_{min.}}$.

It should be noted that the correction factor is different for each value of applied peak voltage. The most satisfactory procedure is to plot a curve of correction factor against peak volts as follows. Determine the value of diode resistance for each of a series of values of peak diode currents, and tabulate the resistance under a heading $R_{min.}$ and the currents under a heading i_p. For each value of $R_{min.}$ so chosen determine the peak-to-mean current ratio by entering the value of $\frac{R}{R_{min.}}$ in Fig. 3. By dividing the values of i_p by the peak-to-mean current ratios, obtain the corresponding values of $\bar{i}_a$. Then tabulate the values of $\bar{v}_a = R\bar{i}_a$ and the values of the correction factor F obtained by entering the values of $\frac{R}{R_{min.}}$ in Fig. 3. Finally, plot F against $\bar{v}_a$. The true value of peak volts can then be obtained from the observed value of $\bar{v}_a$ (whether measured by an electrostatic voltmeter across the diode load or by multiplying the value of the diode load R by the observed current through R) by multiplying $\bar{v}_a$ by F.

Correction Factor of a Peak Voltmeter Using one MR4/620 Valve and a Diode Load *R* of one Megohm. The table below shows the steps in the application of the above procedure to this case. Columns 1 and 2 are obtained from the characteristic of Fig. 4 by choosing values of voltage v_p effective across the diode and reading off the corresponding currents i_p. As the curve is difficult to read at low values of voltage the two lowest sets of values are obtained from the analytical expression for the curve given in XX:4.12. It is evident that all the values might have been so obtained, but it is quicker to read from the curve. Column 3 is obtained by dividing column 2 by column 1, and column 4 by dividing one megohm by column 3. Column 5 is obtained by entering the values in column 4 in Fig. 3. Column 6 is obtained by dividing column 1 by column 5, and column 7 by multiplying

To obtain true peak volts multiply load D.C. Volts ($\bar{V}_a$) by correction factor.

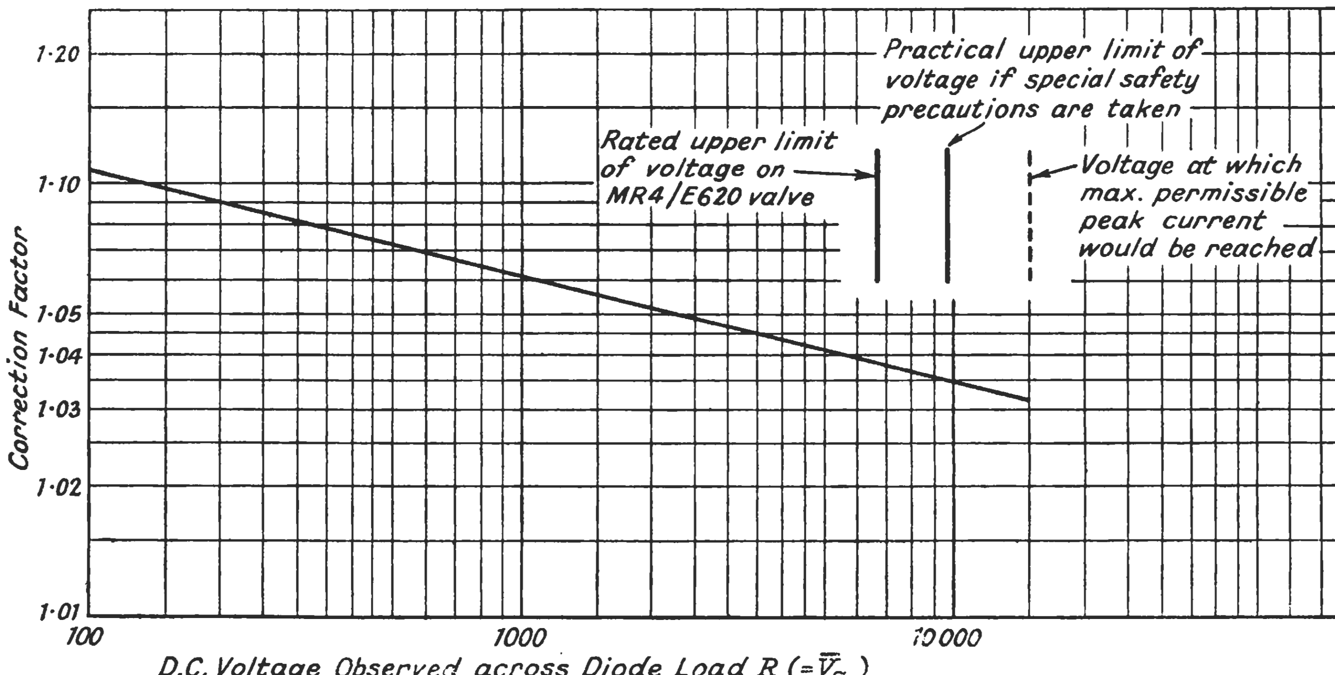


FIG. 5/XX:4.—Second Correction Factor of Standard Peak Voltmeter using one MR4/E620 Valve and Diode Load *R* of one Megohm.

column 6 by one megohm. The correction factor in column 8 is then obtained by entering the values in column 4 in Fig. 3. For the last stage it is assumed that the correction factor is given by a curve halfway between the square law curve and the linear curve (since the index 1·44 is about halfway between 1 and 2, which are respectively the indices of a linear and a square law curve); this approximation is evidently a close one.

Columns 7 and 8 then give corresponding values of observed D.C. voltages across the diode load and the correction factor F. These are plotted in Fig. 5, which serves as a calibration of a type of peak voltmeter in very general use in the B.B.C. This calibration applies to any of the circuits of Fig. 1, provided they use one MR4/620 valve and a diode load of one megohm.

1	2	3	4	5	6	7	8
i_p mA.	v_p	$R_{min.}$	$\frac{R}{R_{min.}}$	$\frac{\text{Peak}}{\text{Mean}}$	i_a mA.	$\bar{v}_a = R\bar{i}_a$	$F = \frac{\hat{v}}{\bar{v}_a}$
1·23	10	8,130	123	11·3	0·109	109	1·101
4·48	25	5,600	179	12·7	0·3525	352·5	1·079
12	50	4,170	240	14·15	0·848	848	1·064
32	100	3,125	320	15·5	2·065	2,065	1·0527
90	200	2,222	450	17·5	5·13	5,130	1·044
168	300	1,785	560	18·9	8·9	8,900	1·036
296	425	1,435	698	20	14·8	14,800	1·032

It is perhaps worth while calling attention to the use of log-log paper and the choice of scales which, by reducing the curves to straight lines, afford invaluable aid to the ham handed and simplify both plotting and checking.

4.2. Determination of the Value of the Condenser *C* in the Peak Voltmeter. The value of C depends on the value of R and the frequency of the applied voltage. It should be such that the value of the anode voltage does not decay appreciably during the time that the input voltage is less than $\bar{v}_a$. If $\theta = 5°$ this time is equal to $\frac{180 - 5}{180}$ of the duration of one cycle, and it is simplest to assume that it is equal to one cycle. If the amount of decay is restricted to 1%, for instance, the reduction of $\bar{v}_a$ below $\hat{v}$ will be less than 1%. By the principle of superposition the current driven through R by the voltage under measurement flows independently of the discharge current. The rate of discharge can therefore be considered independently of the effect of the voltage under measurement. This is actually a very close approximation, which becomes an exact representation of fact when the decay is zero.

If v_0 is the voltage between anode and cathode at the time i_a ceases to flow, the voltage at any subsequent time before i_a starts to flow again is:

$$v = v_0 e^{-\frac{1}{RC}t}, \text{ where } C \text{ is in Farads.}$$

After an interval $t = \frac{1}{f}$ = one period of the applied voltage, where f is the frequency

$$v = v_0 e^{-\frac{1}{RCf}} = 0{\cdot}99 v_0 \text{ if the decay is not to be less than } 1\%.$$

$$\therefore \; -\frac{1}{RCf} \times \log_{10} e = \log_{10} 0{\cdot}99.$$

$$\therefore \; \frac{0{\cdot}4343}{RCf} = 0{\cdot}00436.$$

$$\therefore \; RC = 100 \times \frac{1}{f} \text{ very nearly or } \frac{1}{C\omega} = \frac{R}{200\pi} \qquad (4)$$

Similarly it may be shown that for an error of 2%, $RC = 50 \times \frac{1}{f}$ and for an error of 5% $RC = 20 \times \frac{1}{f}$ very nearly.

In practice it is usually possible to make the value of C so large that the decay of voltage is entirely negligible.

4.3. Voltages in Peak Voltmeters. The voltage developed across the condenser C depends on the applied R.F. voltage and whether there is any D.C. difference of potential between the two terminals 1,1, due, for instance, to the H.T. anode voltage of a valve to which terminals 1,1 are connected. If there is no D.C. potential difference between the two terminals 1,1, then the maximum voltage across C is equal to $\hat{v}+\bar{v}_a$ or approximately $2\hat{v}$. If there is a D.C. voltage across 1,1 such that the terminal 1 connected to the condenser is positive with regard to the other terminal 1, the maximum voltage across C is equal to the D.C. voltage$+2\hat{v}$. The condenser C must be chosen to withstand the appropriate voltage.

The voltage between anode and cathode of the diode is equal to twice the peak volts ($2\hat{v}$) and a diode must be used capable of withstanding such a peak voltage.

On wavelengths below 100 metres the construction of peak voltmeters for high voltages is rendered difficult on account of the capacity, and dielectric losses, in the valve used for the diode. In certain cases special valves for use on short waves have been constructed but have not been found very satisfactory. If accurate measurements of peak voltage are required on a short-wave transmitter the voltage may be broken down by means of a capacity potentiometer, the peak voltmeter being bridged across the series condenser in the potentiometer which has one of its plates at ground potential. Valve capacity still introduces a difficulty, however, because it lowers the input impedance of the peak voltmeter, and this must be taken into account in designing the capacity in the potentiometer across which the peak voltmeter is bridged. In other words, this capacity, after being designed in the normal way for a given voltage ratio, must be reduced by the input capacity of the peak voltmeter.

4.4. Design of Peak Voltmeters. The design of a peak voltmeter therefore involves the following steps:

1. Choose a diode capable of withstanding a voltage between anode and cathode equal to twice the peak volts to be measured, and make a preliminary determination as in 2 below to check that the peak emission is adequate. If not, choose another diode or use two diodes in parallel.
2. As described in 4.12 and 4.13, determine the value of R necessary to measure the highest value of peak volts it is required to observe, in conjunction with the diode it is proposed to use. An (uncorrected) accuracy of 2% should be aimed at, which means that R should be at least 1,000 times the lowest value of $R_{min.}$ (which occurs on the lowest values of peak volts). This represents an ideal which cannot always be realized. The value of R to be used is made equal to the higher of the two values determined respectively by the requirements of maximum permissible peak current and accuracy.
3. For condenser C choose the largest condenser capable of withstanding the voltage across it which is determined as in 4.3.
   The limit on accuracy imposed by the valve of C is then found from 4.12.
4. If the design accuracy is not adequate, plot a correction curve as in 4.14.

5. Valve Voltmeters.

Although the peak voltmeter is one form of valve voltmeter, many other forms exist for use on low power. In one form, an amplifier with its gain stabilized by feedback, delivers its output into an anode-bend detector, into a full-wave rectifier circuit or into a thermocouple.

In another, a diode detector (in effect a peak voltmeter) is arranged so that the voltage developed in its anode circuit varies the bias of a valve of which the anode current gives an indication of the input voltage. This type is particularly suitable for use on short waves.

All valve voltmeters depend on the constancy of valve parameters and in certain cases calibrating means are included in the apparatus. These consist in means for applying a known voltage at some predetermined frequency, the validity of the calibration depending on the fact that the instrument has uniform sensitivity over the band of frequencies for which it is designed. A response curve of the instrument is sometimes supplied for extending its use above the frequency range for which it is designed.

The requirements of a valve voltmeter are a high input impedance, a stable and uniform response at all frequencies, and an adequate range of amplitude. This last is sometimes secured by the insertion of calibrated potentiometers or attenuators.

6. The Transmission Measuring Set.

This is an audio-frequency instrument designed for measuring the attenuation of lines and the gain of amplifiers.

It consists of a sending circuit supplying a known level of *tone* (single-frequency current) and a receiving circuit capable of measuring a received level. The levels are expressed in terms of decibels above or below one milliwatt.

Sending Circuit. One type of sending circuit is shown in Fig. 1 (*a*) and contains a screened repeating coil or transformer R, balanced on its secondary side, which is connected through a variable resistance R_1 and a thermocouple T to a resistance R_2, which is of the order of 40 ohms. Resistances R_3 and R_4 build up the output impedance to its required value. When this is 600 ohms as is usually the case

$$R_3 = R_4 = \tfrac{1}{2}(600 - 40) = 260 \text{ ohms.}$$

An A.F. oscillator is connected to the input terminals 1,1 of the repeating coil, and by adjustment of R_1 the current through the thermocouple and R_2 is adjusted until the e.m.f. in R_2 is 1·549 volts (assuming a 600-ohm sending circuit), in which case when a resistance equal to 600 ohms is connected across 2,2, one milliwatt (zero level) is supplied to it. When a circuit having any other impedance than 600 ohms is connected to 2,2 a lower value of power is supplied to it, the power loss corresponding exactly to the reflection loss (see VII:10) between the impedance in question and 600 ohms. This

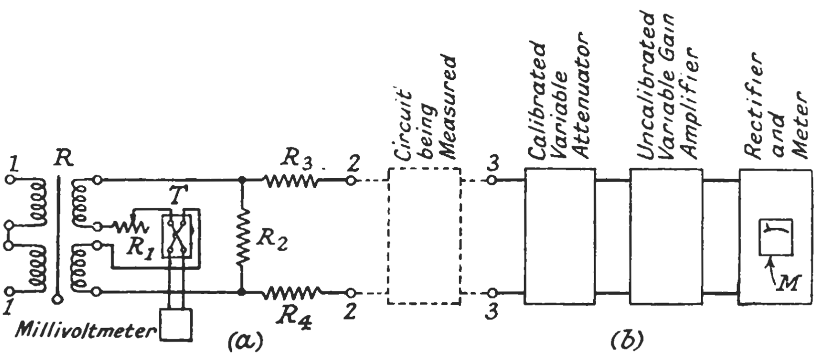


FIG. 1/XX:6.—(*a*) Sending Circuit of T.M.S. (*b*) Receiving Circuit of T.M.S.

reflection loss is considered to be part of the circuit loss of the apparatus under measurement and is measured automatically, as will appear below. Means not shown are incorporated in the set for calibrating the thermocouple by passing a known direct current through it.

Receiving Circuit. This is shown in Fig. 1 (*b*). It consists essentially of a variable calibrated attenuator, usually adjustable in steps of 0·5 db., connected to the input of a variable gain amplifier, the output of which is connected to a rectifier of some convenient form, or a valve used as an anode-bend detector associated with a meter *M*.

In practice the receiving circuit is not exactly as shown in Fig. 1 (*b*). The input of the circuit is usually a balanced and double-screened input transformer with an impedance ratio of 20,000 : 30,000 ohms, with a 30,000-ohm potentiometer across it calibrated in fairly large steps and connected to the grid of the first valve of an amplifier. A later calibrated potentiometer in the amplifier provides fine adjustment of loss. The input impedance is reduced to 600 ohms by means of a shunt 600-ohm resistance, for loss measurements. For " level " measurements (see below), the 600-ohm resistance is removed.

Calibration. The sending circuit is connected directly to the receiving circuit and adjusted to send one milliwatt, and the attenuator in the receiving circuit is adjusted to a known loss N_1 db. The gain of the receiving amplifier is then adjusted to bring the needle of the receiving rectifier meter to a marked position, usually at mid-scale. For calibration and all measurements, except measurement of level, the receiving circuit is adjusted to have an impedance of 600 ohms (or any other value corresponding to the nominal impedance of the circuit under measurement).

Measurement of Attenuation. The circuit to be measured is connected between sending and receiving circuit as shown in Fig. 1 and the sending circuit is adjusted to send one milliwatt. The receiving attenuator is then adjusted to bring the rectifier meter in the receiving circuit to mid-scale. If the number of decibels loss in the receiving attenuator is then N_2, which evidently must be less than N_1, the loss of the circuit under measurement is given by

$$L = N_1 - N_2 \text{ db.} \quad \quad (1)$$

Measurement of Gain. The procedure is identical with that for measurement of loss except that N_3 the setting of the receiving attenuator is now greater than N_1 and the gain is given by

$$G = N_3 - N_1 \text{ db.} \quad \quad (2)$$

Loop Measurement. If the two ends of the circuit to be measured are both in the same building, as, for instance, is the case when an amplifier gain is being measured, a single T.M.S. can be used for the measurement. In this case the sending and receiving circuits in Fig. 1 belong to the same T.M.S.

Straightaway Measurements. When the two ends of the circuit to be measured are not in the same building, as, for instance, is the case when a line between two places is to be measured, two T.M.S.s are required. In this case the sending circuit in Fig. 1 belongs to one T.M.S. and the receiving circuit to the other T.M.S.

The loss or gain measured by a T.M.S. is evidently the *insertion* loss or gain (see VII:12) and includes the reflection loss at each end of the circuit.

Measurement of Level. If after calibration as above the Receiving Circuit is used to *terminate* a line, with the 600-ohm bridging resistance in circuit, and the calibrated attenuator is adjusted to bring the rectifier meter to mid-scale, the attenuation removed from the attenuator gives the number of decibels by which the power level is lower than 1 milliwatt. In previous terms the power level is $N_1 - N_2$ db. below zero level, that is, below 1 milliwatt. If to bring the meter to mid-scale it is necessary to add attenuation, the amount of added attenuation gives the number of decibels by which the received power level is above zero level.

When it is required to know the power level in a 600-ohm line which is already terminated in 600 ohms due, for instance, to a piece of apparatus normally terminating the line, or constituted by a line going to a distant piece of apparatus, the shunt 600-ohm resistance in the Receiving Circuit is removed. Evidently measurements of level made exactly as above will now give the power level delivered to the terminal apparatus or line which now takes the place of the 600-ohm resistance.

In the case of a circuit terminated in an impedance $Z/\underline{\phi}$ other than 600 ohms, if $\pm L_0$ is the observed level in decibels above or below zero level ($+L_0$ db. $= L_0$ db. above zero level ; $-L_0$ db. $= L_0$ db. below zero level : 1 milliwatt) the true received level is given by

$$L = L_0 + 10 \log_{10} \left[\frac{600}{Z} \cos \phi\right] \quad . \quad . \quad . \quad (3)$$

Note that if $Z > 600$, the sign before the log is negative.

Example 1. $L_0 = -10$ db. $\quad Z = 1{,}200\ \Omega \quad \phi = 0$

$$L = -10 + \log_{10}\left[\frac{600}{1{,}200} \times 1\right]$$
$$= -10 + 10 \times \bar{1}\cdot699 = -10 - 10 \times 0\cdot301$$
$$= -13\cdot01 \text{ db.}$$

Example 2. $L_0 = -20$ db. $\quad Z = 300\ \Omega \quad \phi = 30°$

$$L = -20 + \log_{10}\left[\frac{600}{300} \times 0\cdot866\right]$$
$$= -20 + \log_{10} 1\cdot732$$
$$= -20 + 2\cdot39 = -17\cdot61 \text{ db.}$$

It will have been realized that, in its last use, the transmission measuring-set has been used like a voltmeter. Owing to the variation of the input impedance of cables with frequency and the consequent frequency distortion introduced by the variation of the reflection loss with frequency, a vogue exists among telephone engineers involving the use of low-impedance amplifiers as sources of power driving the lines and voltmeters bridged across the lines to obtain an arbitrary measurement of level. This technique has reference to steady tone measurements as well as to measurements of varying speech power ; see VII:6. When this technique is applied to the measurement of items of equipment having impedances of constant resistance, the relation to normal conceptions of power gain and loss is simple and obvious ; where this is not the case it is necessary to establish simple conventions to express level, gain and loss.

7. Measurement of Resistance.

7.1. Ohmmeters. There are a number of battery-operated devices on the market with which it is possible to obtain a direct reading of resistances, up to 100,000 ohms, on a meter scale. The principle on which most of these operate is shown in Fig. 1. A battery B is connected through a switch S to a variable potentiometer R_1 which supplies a fraction of the battery voltage to the circuit constituted by R_2, meter M and the external circuit through terminals 2,2. The device is originally calibrated by shorting terminals 2,2 and adjusting R_1 until the meter gives full-scale deflection. After this different known values of resistance are placed across 2,2 and the meter needle position marked with the value of resistance.

For subsequent use of the apparatus, by varying R_1, the meter is adjusted to full-scale deflection with 2,2 shorted, after which the meter indicates the value of external resistance connected across 2,2.

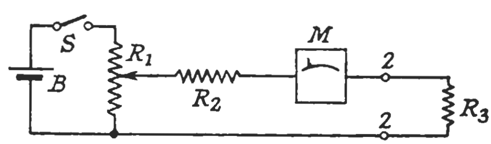


FIG. 1/XX:7.—Ohmmeter Circuit.

These instruments usually have three ranges, 0–1,000 ohms, 0–10,000 ohms and 0–100,000 ohms. A particular commercial type of instrument operating on this principle is the Avometer.

7.2. Megger. This is designed for measuring high resistances, and in particular insulation resistances from a few megohms up to 1,000 megohms. It consists of a hand-driven generator and a direct-reading ohmmeter operating on a different principle from the above. This consists of two coils, with an angular displacement between them, fixed together, and free to rotate in a magnetic field. One coil is connected in series with the generator and a fixed resistance internal to the megger. The other coil is connected in series with the generator and the external resistance to be measured. The position taken up by the two coils is therefore dependent on the ratio of the currents in the coils (and so on the ratio of the internal and the unknown resistance) and is independent of the e.m.f. of the generator. Meggers are usually made to apply 500 or 1,000 volts to the resistance under measurement and are graduated in resistances from 1 to 100 megohms.

The megger is particularly valuable for revealing faulty insulation which only breaks down when a high voltage is applied.

7.3. Resistance Bridges. These are sometimes called Wheatstone Bridges after the original inventor.

The circuit of a resistance bridge is illustrated in Fig. 2. R_1 and R_2 are fixed known resistances, R_3 is a calibrated variable resistance, and R_4 is the unknown resistance to be measured. S_1 and S_2 are switches and G is a sensitive current-indicating device such as a *galvanometer*, which is nothing more than an uncalibrated milliammeter or microammeter, except in exceptional cases where it is more sensitive even than a microammeter, i.e. it will give an indication on a very small fraction of a microampere.

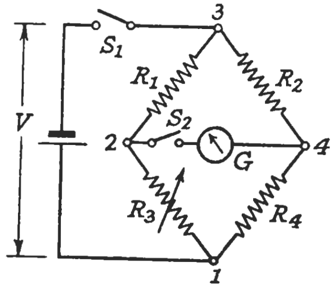


Fig. 2/XX:7.—Resistance Bridge.

If V is the voltage of the battery, with switch S_1 closed and S_2 open, the P.D. between 1 and 2 is $\frac{R_3}{R_1+R_3}V$ and the P.D. between 1 and 4 is $\frac{R_4}{R_2+R_u}V$. As a mnemonic make $R_4 = R_u$.

When
$$\frac{R_3}{R_1+R_3}V = \frac{R_u}{R_2+R_u}V \quad . \quad . \quad . \quad (1)$$

the potential of points 2 and 4 is the same, so that when S_2 is closed no current flows through G.

In this condition the bridge is said to be balanced, and at balance, from equation (1):

$$\frac{1}{\frac{R_1}{R_3}+1} = \frac{1}{\frac{R_2}{R_u}+1}$$

$$\therefore \frac{R_1}{R_3} = \frac{R_2}{R_u}$$

$$\therefore R_u = \frac{R_2}{R_1}R_3 \quad . \quad . \quad . \quad . \quad (2)$$

Hence by adjusting the bridge to balance (by varying R_3), the value of R_u can be obtained. In order to prevent inductive kicks from giving a false reading on the galvanometer, S_1 should be closed before S_2, and arrangements are usually incorporated in the bridge to do this.

The resistances R_1 and R_2 are called the ratio arms, and bridges are usually designed so that it is possible to give these resistances any of the values 1, 10, 100 or 1,000 ohms. By this means it is theoretically possible to measure resistances having values varying from $\frac{1}{1000}$ to 1,000 times any resistance in the range of resistances covered by R_3. The sensitivity of the bridge is a maximum when $R_1 = R_2 = R_u$, and in practice, when possible, equal ratio arms should be used of value in the neighbourhood of R_u: the approximate value of R_u can always be found by a preliminary measurement. When it is necessary to make R_1 and R_2 very different from one another it will probably be necessary to increase the voltage of the battery to obtain increased sensitivity. When this is done care should be taken not to exceed the current rating of the resistances in the bridge.

It can be shown by a similar argument to that used above that the positions of battery and galvanometer can be interchanged without changing the conditions for balance.

8. Impedance Bridges.

These exist in a variety of forms of which Fig. 1 is the simplest. Z_1 and Z_2 constitute the ratio arms, Z_3 an adjustable impedance of known values, and Z_u the unknown impedance. T_1 and T_2 are

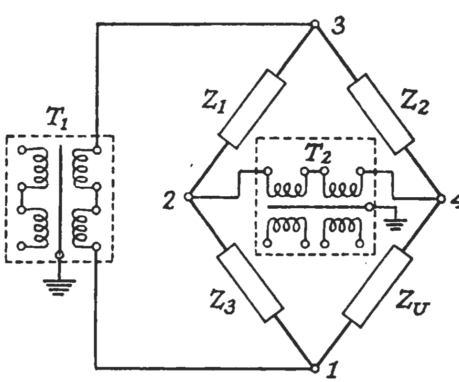


FIG. 1/XX:8.—Simple Impedance Bridge.

screened transformers with the windings facing the bridge balanced for inductance and for impedance to the screen and case. If an oscillator is connected to the input of transformer T_1 it can be shown by an argument identical with that used for the resistance bridge, that balance is obtained when $\frac{Z_1}{Z_3} = \frac{Z_2}{Z_u}$, in other words, when this condition exists there is no potential difference between points 2 and 4, and no e.m.f. induced in the secondary of T_2. Hence at balance

$$Z_u = \frac{Z_2}{Z_1} Z_3 . \qquad (1)$$

If $Z_u = R_u + jX_u$ and at balance $Z_3 = R_3 + jX_3$

$$Z_u = R_u + jX_u = \frac{Z_2}{Z_1} Z_3 = \frac{Z_2}{Z_1} R_3 + j\frac{Z_2}{Z_1} X_3 \qquad (2)$$

In the simplest forms of impedance bridge Z_1 and Z_2 are usually resistances, so that $\frac{Z_2}{Z_1}$ is a numeric (a simple arithmetical ratio).

In this case, since two impedances can be equal only if their real parts are equal to one another and their imaginary parts are equal, it follows from (2) that

$$R_u = \frac{Z_2}{Z_1} R_3 \text{ and } X_u = \frac{Z_2}{Z_1} X_3 . \qquad (3)$$

The balance condition is observed by connecting a pair of headphones to the output of T_2. An amplifier in the output of T_2 may be used to increase the sensitivity of the bridge, in which case care must be taken that no transmission path exists between the oscillator and the amplifier (e.g. due to stray capacity, mutual impedance, etc.) other than through the bridge.

8.1. Audio-Frequency Bridges.

8.11. Capacity Bridge. Fig. 2 shows an audio-frequency bridge for measurement of capacity, constructed from two repeating coils, making use of a calibrated variable condenser box for C_3, the known calibrated capacity. Such a bridge can be used for measurement of capacity to give values correct to about $\pm 5\%$ over the range of the calibrated condenser. On account of the capacity to screen and ground of T_1 this bridge should always be used with R_1 equal to R_2. If this is done the transformer capacities balance out, provided, as is essential, the winding of T_2 facing the bridge is balanced for inductance, and for capacity to screen and to case.

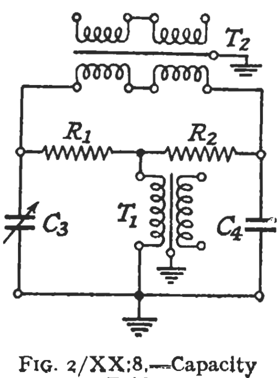


FIG. 2/XX:8.—Capacity Bridge.

At balance
$$\frac{1}{jC_u\omega} = \frac{R_2}{R_1}\cdot\frac{1}{jC_3\omega}$$
$$\therefore \quad C_u = \frac{R_1}{R_2}C_3 \quad . \quad . \quad . \quad . \quad (4)$$

Hence if $R_1 = R_2$, $C_u = C_3$.

8.12. Capacity and Conductance Bridge. Fig. 3 shows additions to Fig. 2 to eliminate any capacity unbalance due to T_2, and to provide for measurement of the leakage of the unknown condenser. The winding of T_2 facing the bridge is separately screened, and the screen connected to one end of the winding. This locates the capacity of this winding to ground, via the second screen of T_2, to one end of the winding, and makes it substantially independent of frequency. As an initial adjustment, C_b is adjusted to balance out his capacity with C_3 and C_u removed.

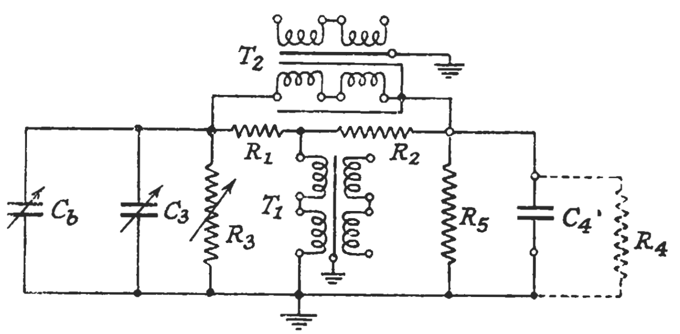


FIG. 3/XX:8.—Capacity and Conductance Bridge.

A known fixed resistance R_s is connected in parallel with C_u and therefore in parallel with R_u the leakage resistance of C_u.

At balance
$$C_u = C_3 \quad . \quad . \quad . \quad . \quad (5)$$

and
$$\frac{R_sR_u}{R_s+R_u} = R_3$$
$$\therefore R_sR_u = R_3R_s + R_3R_u$$
$$\therefore R_u(R_s - R_3) = R_3R_s$$
$$\therefore \quad R_u = \frac{R_3R_s}{R_s - R_3} \quad . \quad . \quad . \quad . \quad (6)$$

A suitable value of R_s is 10,000 ohms, and assuming this value to have been used, if R_3 = 9,990 ohms

$$R_u = \frac{9{,}990 \times 10{,}000}{10{,}000 - 9{,}990} = 9{\cdot}99 \text{ M}\Omega.$$

Evidently, this bridge can be used without modification, for the measurement of any impedance with a negative reactance. By transferring C_3 from the unknown side to the known side the bridge can be used for measuring impedances with positive reactance.

8.13. General-Purpose Impedance Bridge. Fig. 4 shows a simple practical form of general-purpose impedance bridge in which

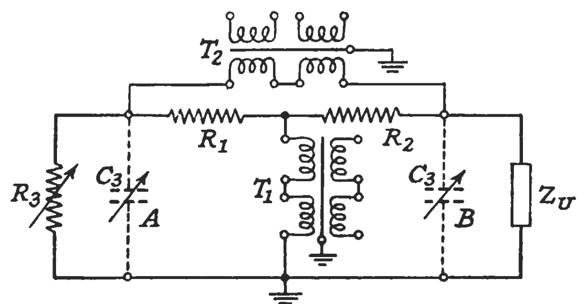


FIG. 4/XX:8.—Practical Impedance Bridge.

the capacity C_3 is located at A if Z_u has a negative reactance and at B if Z_u has a positive reactance. On account of transformer capacities this bridge should also be used only with equal ratio arms.

This bridge expresses the unknown impedance in the form $R_u//jX_u$, i.e. at balance $R_u = R_3$ and $X_u = \pm \frac{1}{C_3\omega}$.

For the means of conversion to the series form of representation of impedance see V:16 and Fig. 3/V:16.

Remember that if, for balance, C_3 has to be placed on the Z_u side of the bridge, the reactance of Z_u is positive, and if on the other side, the reactance of Z_u is negative.

8.14. Inductance Bridge. There is little difficulty in devising either a series or a parallel resonance bridge to measure inductance. This bridge, however, involves a fairly accurate knowledge of the frequency since the value of inductance involves the square of the frequency. Further, radio engineers are often

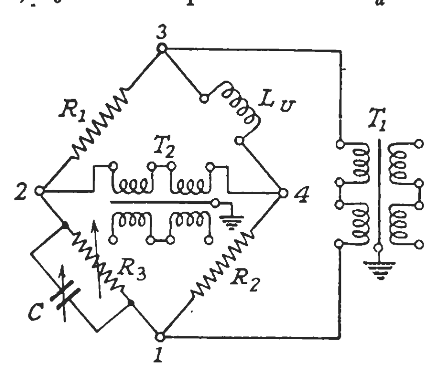


FIG. 5 XX:8.—Maxwell Inductance Bridge.

interested in measuring the inductance of radio-frequency coils which cannot be resonated at audio frequency with practical values of capacity.

The Maxwell inductance bridge shown in Fig. 5 is designed to overcome these difficulties, but introduces its own difficulties owing to the location of transformer capacities. R_1 and R_2 are equal and are fixed during final balance of the bridge. The bridge is balanced by adjustment of C and R_3.

At balance

$$\frac{R_1}{\dfrac{R_3}{1+jR_3C\omega}} = \frac{R_u+jL_u}{R_2}$$ where R_u is the resistance of the coil L_u.

$$\therefore \frac{R_1}{R_3}+j\frac{R_1R_3C\omega}{R_3} = \frac{R_u}{R_2}+\frac{jL_u\omega}{R_2}$$

$$\therefore R_u = \frac{R_1R_2}{R_3} \text{ and } L_u = R_1R_2C \qquad . \quad . \quad (7)$$

Example. At balance

$$R_1 = R_2 = 100\ \Omega,\quad R_3 = 10{,}000\ \Omega,\quad C = 0{\cdot}005\ \mu\text{F}$$

hence
$$R_u = \frac{100^2}{10{,}000} = 1\ \Omega$$

$$L_u = 100^2 \times 0{\cdot}005 \times 10^{-6} = 50\ \mu\text{H}.$$

Assuming R_u and L_u to be fixed, the effect of reducing R_1 and R_2 is to lower the impedance of R_3 and C at balance. This is an advantage since the effect of transformer capacities is reduced, but if it demands such a large value of C that the stray capacities of C are increased, because a larger size of variable condenser has to be used, it may become a disadvantage. If, however, L_u is a small coil, the earth point may be located at 1 or 2, in which case the stray capacities of C constitute part of the indicated value of C, since they were present during the calibration of C.

If, however, L_u is a large coil with bigger stray capacities than C, the earth will have to be located at point 3 or point 4, in which case the stray capacities of C become important.

If the oscillator is connected to T_1, the earth should be located at 1, if the C stray capacities are most serious, and at 3 if the L_u stray capacities are most serious. Provided R_1 and R_2 are not greater than about 100 ohms, the transformer capacities should be negligible, for measurements at 1,000 c/s, and little trouble is likely to be experienced due to the other stray capacities. As a compromise, if both sets of external stray capacities are equally important, the earth should be located at the midpoint of the winding of T_1 which faces the bridge.

8.2. Bridge Technique.

In Fig. 6 is shown the set-up for measuring a balanced unknown impedance.

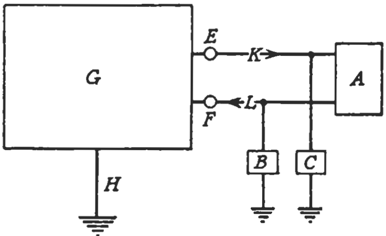


FIG. 6/XX:8.—Bridge and Load.

A is the unknown and is "balanced" to earth if either:

(1) The admittances *B* and *C* to earth are negligibly small.

(2) The admittances *B* and *C* to earth are equal.

G is the measuring bridge. If the bridge is equally sensitive to the currents at *E* and *F* it is a "symmetrical" bridge.

If not, it is an unsymmetrical bridge. The bridge measures the balanced unknown correctly if the earth current through *H* is zero, i.e. the currents at *K* and *L* are equal and opposite. These currents will be equal and opposite with a balanced unknown if the voltages (to earth) at *E* and *F* are equal and opposite.

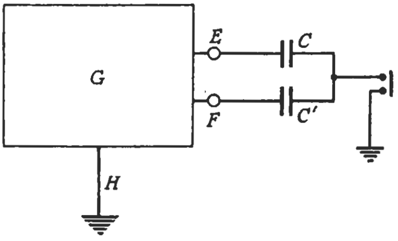


FIG. 7/XX:8.—Check for Balance.

This may be secured in two ways:

(1) If the bridge admittance to earth is negligibly small.

(2) If the bridge admittances to earth are equal on the two poles of the bridge.

With a modern type of bridge the bridge will be balanced, but not symmetrical, i.e. it will be sensitive to current in only one arm, E. The check for balance is shown in Fig. 7.

Two equal condensers, *C* and *C'*, are measured in series and the measurement noted. Their centre point is then earthed and the measurement repeated. If the bridge is balanced the two measurements will agree.

With such a bridge measurement technique is as follows. One measurement is taken of the balanced unknown and the leads are then interchanged. If the readings remain the same the unknown is accurately balanced and the readings are correct. If the readings are different the unknown is not accurately balanced, and if necessary further measurements will give complete data. (An unbalanced unknown is a three-terminal network.) If the unknown has one pole earthed, then the bridge will give the correct measurement if correctly poled.

Symmetrical Bridge. The symmetrical bridge is not thereby balanced and therefore does not necessarily give correct measurements of a balanced unknown. It gives no means of checking the balance of the unknown and cannot be used to measure unbalanced unknowns.

The above summary is due to C. G. Mayo.

9. Radio-Frequency Bridges. (See B.B.C. Patent No. 561,693 and Application No. 3212/44.)

The principles of the audio-frequency bridges described above apply equally to radio-frequency bridges, with the qualifications and limitations consequent on the large admittances of stray capacities at radio frequencies. Radio-frequency bridges require careful screening, the importance of the screening increasing with frequency. (For accurate measurements at audio frequency, particularly when the output of the bridge is amplified, screening is also necessary.)

The input to a radio-frequency bridge is provided by means of a radio-frequency oscillator in a metal box constituting a complete screen : a frequency accuracy as good as can be obtained with normal commercial methods of manufacture is usually adequate. The output of the oscillator which should have a low impedance, e.g. about 200 ohms, should be balanced and should be completely enclosed in a heavy screen of braided copper sleeving which makes good contact at each end with the screens of oscillator and bridge. The input and output to the bridge should be constituted by a transformer (e.g. T_1 and T_2, Fig. 1) with both primary and secondary windings balanced, and the input and output impedances should be about 200 ohms. The balanced output from the bridge should be led to the receiver through a pair of wires enclosed in a heavy screen of braided copper sleeving making good contact with screens of bridge and receiver. Both bridge and receiver should be contained in a metal box constituting a complete screen. In practice, owing to the difficulty of obtaining plugs and sockets suitable for making connections in balanced screened circuits, unbalanced circuits are often used between oscillator and bridge, and between bridge and receiver. In this case mutual impedance exists between the connecting circuits at input and output of the bridge and the length of the connections should not exceed a couple of feet, otherwise the bridge will be partly by-passed at high frequencies so that a false balance will be obtained.

9.1. Unbalanced Radio-Frequency Bridge. Fig. 1 shows a circuit of a typical R.F. bridge for measuring impedance in unbalanced circuits.

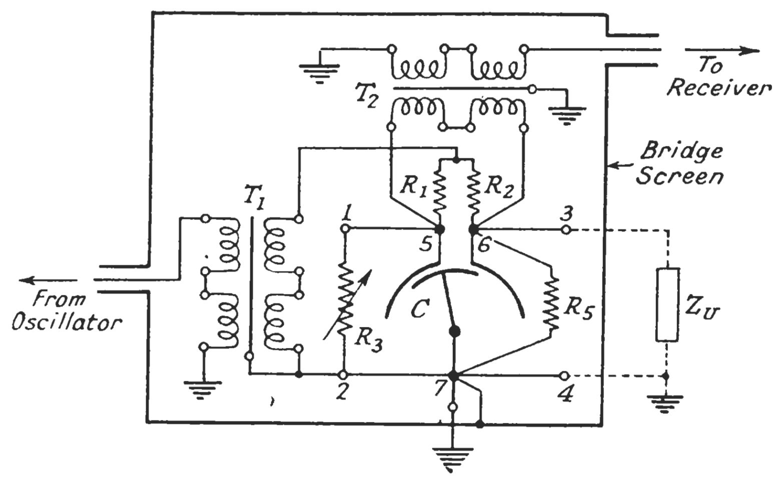


FIG. 1/XX:9.—Unbalanced R.F. Bridge.

R_1 and R_2 are equal ratio arms and R_5 and R_3 fulfil the same function as in Fig. 3/XX:8. T_1 and T_2 correspond to T_1 and T_2 in Fig. 3/XX:8 but are designed to operate over a band of *radio* frequencies. The double screening of T_2 has been omitted since it is not always used on account of mechanical difficulties of construction : it can, however, be introduced with advantage. C is a differential condenser which, on being rotated in a clockwise direction, for instance, reduces the capacity between 1 and ground and increases the capacity between 3 and ground. C is calibrated in terms of the difference in capacity introduced between 1 and ground and between 3 and ground.

Before use the bridge capacities are balanced out by balancing the bridge with terminals 3 and 4 open circuited. The reading of

C is noted and this reading must be subtracted from all subsequent readings of C obtained during measurement. On a bridge in which T_2 is properly balanced, the bridge should balance on open circuit with $R_3 = R_s$, but if a bridge is found to give a poor capacity balance with $R_3 = R_s$ an extra variable balance resistance is sometimes introduced across one side of the bridge, cancelling a fixed resistance on the other side of the bridge. For instance, a fixed resistance of 100,000 Ω may be placed in parallel with R_s and a 200,000-Ω variable resistance in parallel with R_s. The bridge is then initially adjusted for balance and capacity unbalance simultaneously with R_3 set equal to R_s.

R_3 is not calibrated in terms of its own resistance values but in terms of the external resistance across 3, 4, which will give balance with each setting of R_3, e.g. if r_3 is any particular value of R_3 it is marked with the value $R_p = \dfrac{r_3 R_s}{R_s - r_3}$.

The range of C is sometimes extended by the addition of fixed capacities which can be connected either side of the bridge according to whether Z_u is inductive or reactive. If this is done, care must be taken to see that the length of connections 1–5, 6–3, 2–7 and 7–4 is not unduly increased, otherwise the different shunt elements are in effect spaced out along a transmission line and their effective impedance respectively at 1, 2 and 3, 4, may depart appreciably from their actual value. This difficulty is not normally serious until frequencies above 10 Mc/s are reached when lengths of two or three inches begin to become important.

9.2. Balanced Radio-Frequency Bridge. Fig. 2 shows a circuit of a typical R.F. bridge for measuring impedance in balanced circuits.

Comparison with Fig. 1 shows that this bridge is a balanced equivalent of the unbalanced bridge just described. R_1, R_2 are one pair of equal ratio arms and R_1' and R_2' an identical pair. C_1 and C_2, which are physically ganged on the same shaft to rotate in the same direction, are electrically connected so that effectively, in Fig. 2, C_1 moves in a clockwise direction when C_2 moves in a counter-clockwise direction, and vice versa. W_1 and W_2 are equal balanced windings on transformer T_2, both screened from the output winding, and connected in such relative sense that an e.m.f. injected *into* the output winding would cause aiding e.m.f.s to flow round the circuit 1, 2 . . . 7, 8.

The operation of the bridge is otherwise exactly as described for the unbalanced bridge.

9.3. Performance of R.F. Bridges. The frequency range of R.F. bridges such as the above is determined firstly by the pass range of the transformers T_1 and T_2, and secondly by the physical dimensions of the bridge. A long- and medium-wave bridge has been designed, for instance, to operate from 150 kc/s to 2 Mc/s, and a short-wave bridge to operate from 2 to 30 Mc/s. Above 30 Mc/s

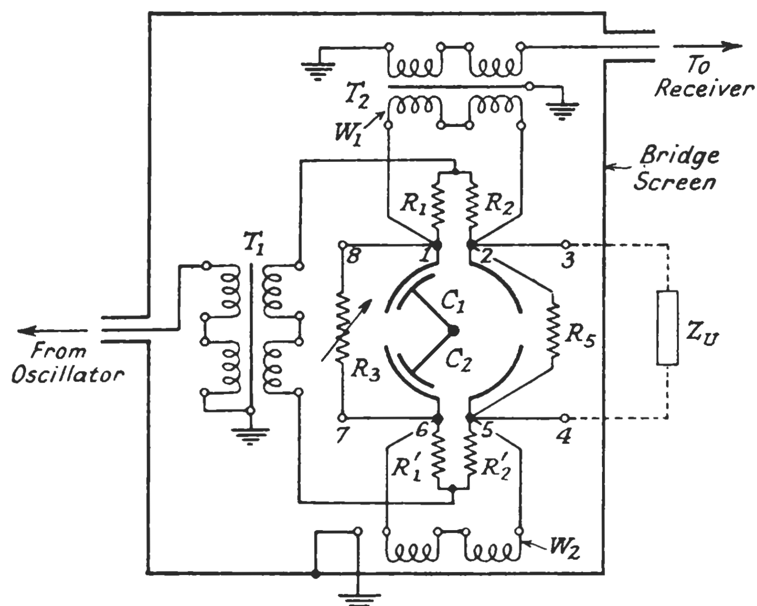


FIG. 2/XX:9.—Balanced R.F. Bridge.

difficulties begin to appear owing to the physical dimensions of the bridge. The accuracy of measurement is about $\pm 5\%$ on both reactance and resistance.

9.4. Improved Types of R.F. Bridge. These have been made possible by the production of high permeability radio-frequency closed cores, which make possible the construction of highly balanced windings at radio frequencies. A particular core which has been found suitable is the R2821 Rhometal tape ring core made by the Telegraph Construction and Maintenance Company.

The circuit of a simple type of bridge which has been constructed in the B.B.C., by C. G. Mayo, to cover the frequency range 1–40 Mc/s is shown in Fig. 3. In this the ratio arms are constituted by the balanced windings (3–4) and (5–6) on transformer T_2. These are each constituted by two turns of copper tape. The known impedance is constituted by the variable condenser C_3 and the variable resistance R_3.

Owing to the close coupling existing between the two ratio arm windings it is found in practice that this bridge can be used to measure balanced or unbalanced circuits without any modification to the circuit.

A balancing condenser, not shown, is added on one side of the bridge to balance out stray capacity.

Fig. 4 shows a bridge designed by Mayo to extend the range of impedances which can be measured. In this bridge, windings (1–2) and (3–4) of T_2 have 10 turns, winding (3–5 and 3–5′) has 1 turn, winding (6–7) of T_1 has 10 turns and winding (6–8) has 1 turn.

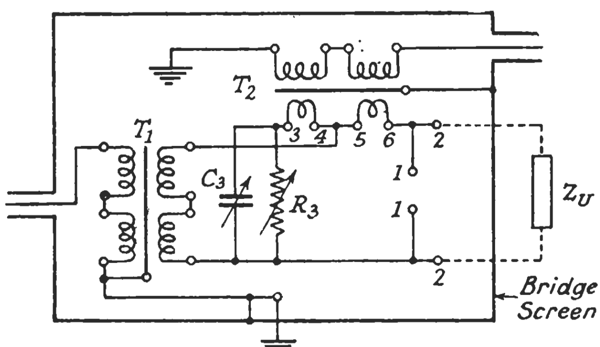


FIG. 3/XX:9.—Balanced and Unbalanced R.F. Bridge.

Switch S_1 is arranged to connect the lower end of the known impedance constituted by R_3 and C_3 either to bus-bar C connected to terminal 8 on T_1 or to bus-bar D connected to terminal 7 on T_1. This makes it possible to change the voltage across the known impedance by a ratio of 10 times and so to change the current through the known impedance and winding (1–2) by a ratio of 10. This in turn changes the value of Z_u, the unknown impedance required to give balance, by a ratio of 10 times.

Similarly the unknown impedance can be connected between A and C or A and D, so changing the value of the known impedance necessary for balance in a ratio of 10. Finally, by changing the upper end of the unknown impedance from A to B the value of unknown impedance required to give balance is changed in the ratio 10 on account of the change in the value of the ratio arm.

The net result is summarized in the table below, assuming that the known impedance covers a range of resistance from 1,000 Ω to 50,000 Ω and a range of capacity from 0 μμF to 150 μμF. This

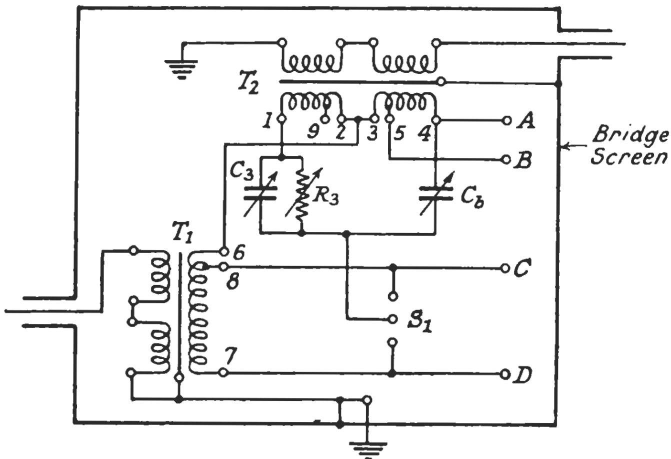


FIG. 4/XX:9.—Balanced and Unbalanced R.F. Bridge with Extended Impedance Range.

table gives the impedance range of the bridge for each of the possible combinations of the position of S_1 and output terminals.

Accepting the principle that the volts on the unknown impedance (which, if an aerial circuit, may be noisy) should be a maximum, to reduce interference, conditions 2, 3, 4 and 6 should be used.

Condition	S_1 on	Z_u Terminals	Resistance Range Ohms	Capacity Range μμF
1	*D*	*AC*	100–5,000	0–1,500
2	*D*	*AD*	1,000–50,000	0–150
3	*D*	*BC*	10–500	0–15,000
4	*D*	*BD*	100–5,000	0–1,500
5	*C*	*AC*	1,000–50,000	0–150
6	*C*	*AD*	10,000–500,000	0–15
7	*C*	*BC*	100–5,000	0–1,500
8	*C*	*BD*	1,000–50,000	0–150

Condenser C_b is an initial balancing condenser to balance out the zero capacity of C_3 and any unbalance capacities in the bridge.

A bridge of this type constructed for a frequency range from 150 kc/s to 2 Mc/s was found to measure any resistance in the range 10 Ω to 100,000 Ω and any capacity in the range 10 μμF to 10,000 μμF with an accuracy of $\pm 5\%$, whether balanced or unbalanced, i.e. whether terminals *C* or *D* were earthed or unearthed.

By inserting a tap 9 in winding (1–2) a tenth of the winding from terminal 2, the range of the bridge may be still further extended if the standard C_3, R_3, is connected to 9 instead of to 2 : the bridge will then have a resistance range from 0·1 to 5 megohms, and capacity range from 0 to 1·5 $\mu\mu$F.

It is important to note that, in a certain region of frequency and impedance where residual inductance becomes important, some advantage in accuracy of standards may be obtained by using a ratio bridge so that the standard is of higher impedance than the unknown.

A bridge of the type shown in Fig. 4 is being manufactured to operate from 15 kc/s to 5 Mc/s with an accuracy of 2%. Another bridge of this kind is being made to operate up to 100 Mc/s and to measure resistances down to 10 ohms and capacities up to 250 $\mu\mu$F.

10. The Cathode-Ray Oscillograph.

This consists essentially of an electron gun in an evacuated glass tube which projects a narrow beam of electron against a fluorescent screen at one end of the tube. Two pairs of deflecting plates are

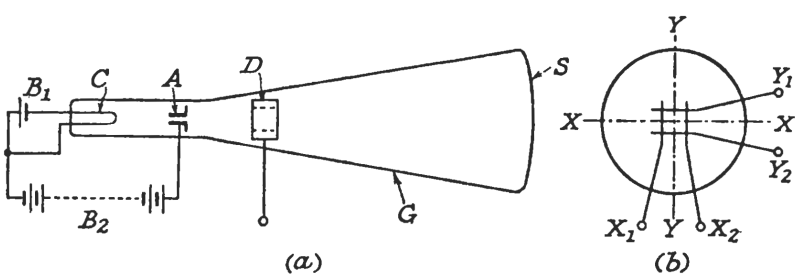


FIG. 1/XX:10.—Cathode-Ray Oscillograph.

located mutually at right angles on each side of the beam. By applying a potential difference between either pair of plates the beam can be deflected along the line normal to the plates. The arrangement is illustrated in Fig. 1. *G* in Fig. 1 (*a*) is a glass envelope containing at its narrow end a cathode *C* which may be indirectly heated, but is shown directly heated by battery B_1. *A* is a cylindrical anode maintained at a high positive potential with regard to the cathode by means of battery B_2, as a result of which a stream of electrons flows from cathode to anode, some of which acquire such a high velocity that they emerge from the far side of the anode in a beam. Special means of concentrating this beam are provided which differ in different types of tube. Plates D in Fig. 1 (*a*) are shown in Fig. 1 (*b*), which is an end view of the tubes, as X_1, X_2 and Y_1, Y_2. These are insulated and lead out to terminals outside the envelope. They constitute deflecting plates : the beam normally strikes the centre of the end of the envelope, at the intersection of lines XX and YY. The end of the envelope, or screen, is covered with fluorescent material which glows under the impact of the electron beam so that a small spot of light appears where the beam strikes the screen. If a P.D. is applied between plates X_1 and X_2 the beam is deflected in a direction parallel to XX and sense depending on the sense of the applied P.D. Similarly a P.D. between Y_1 and Y_2 causes deflection along the line YY. In practice the leads from the electrodes are usually brought out through a cap at the narrow end of the tube and not through the envelope as shown.

A number of commercial C.R.O. equipments are on the market which usually comprise the following :

(1) A.F. and/or wide band R.F. amplifiers for supplying inputs to the deflecting plates.

(2) A *time base* which can be switched so as to supply a wave of saw-tooth form, at any required frequency in the range provided, to the X plates of the C.R.O. This is provided with an input circuit to which any periodic wave form, applied to the Y plates for examination, can also be applied, so that the saw-tooth wave is held in synchronism with the wave under examination.

(3) An A.C. power pack for providing all necessary power supplies for the C.R.O., input amplifiers and time base.

Great care should be taken to see that the input amplifiers are not overloaded, as this is a frequent source of misleading results.

The applications of the C.R.O. are almost infinite. It can be used for the exact comparison of two different frequencies, for observation of the form of any periodic wave, for the indication and measurement of transient voltages of too short duration to read on a meter, for the measurement of phase difference, percentage modulation, and very small time intervals. It provides a means of checking the stability of high-power valve amplifiers throughout each instant of their operating cycle, and hence a means of detecting parasitic oscillations. It presents a direct picture of the non-linear relation between output and input in any circuit due to overloading or any other cause. In conjunction with appropriate accessory equipment, a C.R.O. can be made to represent an instantaneous picture of a response curve : this feature is particularly useful when ganging receivers. A response curve set is marketed by Cossor under the name of " Ganging Oscilloscope ". See XX:15.

10.1. Frequency Comparison. Two methods of frequency comparison are in use. The first is most convenient for small simple ratios between the two values of frequency, such as 1 to 2, 1 to 3, 1 to 4, 2 to 3, 2 to 5, or for ratios which approximate very closely to such simple ratios. In this method one frequency is applied to one pair of deflecting plates and the other frequency to the other pair of plates, as the result of which the C.R.O. spot traces a path in the form of what is called a " Lissajou " figure. When the frequency ratio is simple (see qualification below) the figure is fixed, its form depending on the time relations of the two frequencies. There are therefore an infinitely graded series of possible figures corresponding to any one simple ratio and some skill is required in recognizing any ratio : this can only be acquired by experience : by the observation of known ratios. If the frequency ratio is not an exact simple ratio, but approximates to it, the figure passes periodically and progressively through all the configurations applying to the simple frequency ratio. If the whole cycle of change lasts T seconds and the approximate simple ratio is $1 : n$ such that $f_2 - nf_1$, where n is integral, i.e. $n = 1, 2, 3$, etc., the relation between the two frequencies given by

$$f_2 = nf_1 \pm \frac{1}{T} \quad . \quad . \quad . \quad . \quad (1)$$

(When n is non-integral equation (2) applies : see Multiple Traces below.)

To determine whether the plus or minus sign must be taken, change one of the frequencies. If increasing or decreasing f_1 makes the figure rotate more slowly f_1 is respectively less than or greater than $\frac{1}{nf_2}$ and the plus and the minus signs must be taken respectively. If increasing or decreasing f_1 makes the figure rotate faster, the minus and plus signs respectively must be taken. If increasing or decreasing f_2 makes the figure rotate more slowly the minus and plus signs respectively must be taken, and so on.

Proof of Equation (1). If any period of time T (less than the time of persistence of the C.R.O. screen and the eye) exists such that a number of complete cycles Tf_1 of frequency f_1 occur while a number of complete cycles Tf_2 of frequency f_2 occur, the figure is retraced every T seconds and is stationary. If T is greater than the persistence time and if, as is the case in practice when the ratio approximates to a simple ratio, the persistence time is only great enough to show simultaneously half a dozen to a dozen cycles of the lower frequency, the figure progressively changes through all the configurations of the nearest fixed ratio. The time T taken for each cycle of change is the shortest time during which both frequencies execute an integral number of cycles. If f_2 is greater than f_1 and approximately equal to nf_1, T is the time during which f_1 does Tf_1 cycles and f_2 does Tnf_1 cycles plus or minus one cycle. But f_2 does Tf_2 cycles in time T

$$\therefore \; Tf_2 = Tnf_1 \pm 1.$$

Alternatively T is the time during which f_2 does Tf_2 cycles, mid f_1 does $T\frac{f_2}{n}$ cycles plus or minus one cycle, but f_1 does Tf_1 cycles in time T

$$\therefore \; Tf_1 = T\frac{f_2}{n} \pm 1$$

$$\therefore \; Tf_2 = Tnf_1 \mp 1$$

$$\text{or } f_2 = nf_1 \mp \frac{1}{T}, \text{ which is equation (1).}$$

In this case the inversion of the $\pm$ sign is unimportant and this is therefore merely an alternative form of proof, which is, however, necessary to show that no second condition exists.

Some care is required in observing the period T since a figure may repeat the same configuration twice in one cycle, so that if this configuration is chosen as the beginning of the cycle it also appears once in the middle of the cycle as well as at the end of the cycle. The configurations which lie in time between recognized configurations must therefore be examined to see whether they differ. As an example, Fig. 2 shows some of the series of progressive configurations corresponding to a ratio which is very nearly 1 : 1, the time T of one complete cycle being indicated.

When a particular " Lissajou " figure is unrecognizable it is sometimes helpful to make it progress through all its forms, since some forms of one ratio are more easily recognizable than others. An obvious method if a variable oscillator is available is to try to reproduce the figure with known frequency ratios. To make recognition of figures more easy the two frequencies should always be applied to the plates at approximately equal amplitude.

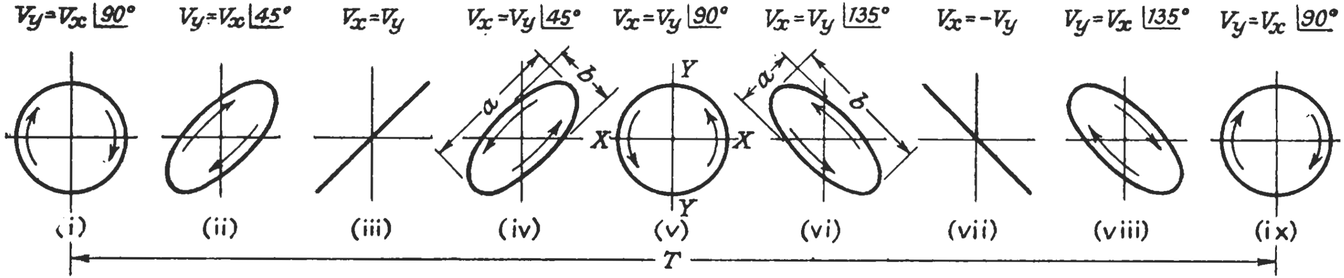


FIG. 2/XX:10.—Series of Progressive Configurations corresponding to Approximate 1 : 1 Frequency Ratio. Arrows indicate direction of motion of spot.

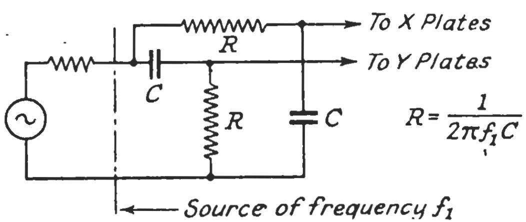


FIG. 3/XX:10.—Circuit for Feeding X and Y Plates in Quadrature.

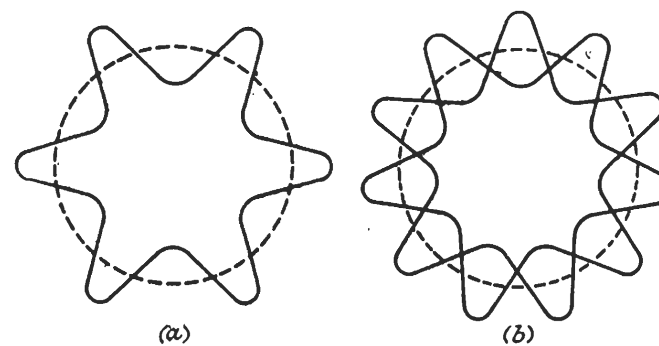


FIG. 4/XX:10.—Frequency Comparison by Cathode-Ray Oscillograph.

(*a*) Frequency Ratio = 1 : 6. (*b*) Frequency Ratio = 2 : 11.

The second method of frequency comparison is really a laboratory method, since commercial C.R.O. equipments are not fitted either with phase shift devices or with means for applying a sinusoidal voltage in series with the anode battery (i.e. B_2 in Fig. 1). In this method the lower frequency for comparison is supplied in quadrature (i.e. 90° relative phase shift) to the two pairs of plates. In practice this can generally be achieved very simply by feeding one pair of plates through a series condenser and a shunt resistance and the other pair of plates through a series resistance and a shunt condenser. The reactances of the condensers should be equal to the values of the associated resistances at the applied frequency, the exact arrangement being shown in Fig. 3. The result is that the cathode-ray spot describes a circle corresponding to one of the conditions in Fig. 2. It is not very important that a circle should be produced very exactly; in other words, it is not important that the voltages applied respectively to each pair of plates should be exactly in quadrature and exactly of equal amplitude.

The higher frequency is then applied through two blocking condensers across a choke in series between the anode battery and the anode of the C.R.O. The result is to produce a sinusoidal variation in the electron velocity which causes a sinusoidal variation in the control effected by the deflecting plates, and results in the second frequency appearing as a small amplitude sine wave described with the periphery of the circle as axis.

Single Trace. When the resultant configuration is fixed and consists of a single trace the frequency ratio n is given by the number of cycles (including fractions of a cycle) of the superimposed frequency which can be counted in one traverse of the circle. Fig. 4 (*a*) shows a single-trace diagram corresponding to a frequency ratio of 1 to 6. The dotted circle indicates the path followed by the spot when the lower frequency only is applied.

When the figure consists of a single trace which rotates, equation (1) applies, where T is the time taken for the rotating diagram to move forward one cycle, and n is the number of cycles on the trace. Single traces always occur when the frequency ratios are of the form $1 : n$, where n is an integer.

Multiple Traces. When the frequency ratio in its simplest form is $2 : n$, $3 : n$, $4 : n$, etc., there are respectively 2, 3, 4, etc., traces. This will be clear by reference to Fig. 4 (*b*), which shows the trace resulting from a frequency ratio of $2 : 11$.

If the figure is stationary the frequency ratio is given by $m : n$, where m is the number of traces and n is the total number of cycles of the higher frequency in the whole diagram, i.e. in the *m* traces. This can be checked by reference to Fig. 4 (*b*).

When the figure consists of *m* traces, say, the frequency ratio when the figure rotates may be determined as below, where *n* is the number of cycles in the complete figure, i.e. in *m* traces.

In the time of one circuit of the tracing spot, i.e. in $\frac{1}{f_1}$ seconds, f_2 completes $\frac{f_2}{f_1}$ cycles. In the same time a frequency $\frac{n}{m}f_1$ applied in place of f_2 would complete $\frac{nf_1}{mf_1}$ cycles and the figure would remain stationary. When f_2 is applied the figure therefore advances $\pm\left(\frac{f_2}{f_1} - \frac{n}{m}\right)$ cycles in $\frac{1}{f_1}$ seconds. Hence the figure will advance *n* cycles in time

$$T = \pm\frac{\frac{n}{f_1}}{\frac{f_2}{f_1} - \frac{n}{m}} = \pm\frac{n}{f_2 - \frac{n}{m}f_1}$$

$$\therefore\ f_2 = \frac{n}{m}f_1 \pm \frac{n}{T} \quad . \quad . \quad . \quad . \quad (2)$$

T is then the period of rotation of one tooth round the whole figure. It will be understood that to traverse the whole figure a tooth must make *m* circuits. For Lissajou figures *T* is the time to pass through all configurations once.

10.2. Measurement of Phase Difference. If two equal amplitude sinusoidal voltages V_x and V_y are applied in phase respectively to the *X* and *Y* plates of a C.R.O. a straight line is obtained as at (iii) in Fig. 2. If V_x is now made to lead on V_y by 45° condition (iv) results, and increase of lead of V_x to 90° gives condition (v), and so on. The phase relations corresponding to each condition are indicated immediately above it, while the arrows indicate the direction of rotation of the spot.

In this diagram the convention has been adopted that two voltages applied to the *X* and *Y* plates are in phase when a positive increase in each voltage makes the C.R.O. spot move in a positive direction along both axes, i.e. "upwards" along the *Y* axis and to the right along the *X* axis.

Fig. 5 shows an ellipse produced by two sinusoidal voltages of equal amplitude *A* and with a mutual phase shift equal to ϕ, less than 90°. In this case the axis a is the major axis and axis b is the minor axis. When ϕ is greater than 90° axis b is the major axis and a is the minor axis, see Fig. 2, (iv) and (vi).

Fig. 6 shows the relation between $\frac{a}{A}$ and $\frac{b}{A}$ and ϕ, which is derived in CIV.

Hence to find the phase shift between two sinusoidal voltage waves apply one wave to the X plates and adjust to give a swing A not greater than $\frac{1}{\sqrt{2}}$ times the maximum swing the tube will take. Remove the voltage from the X plates and apply the other voltage

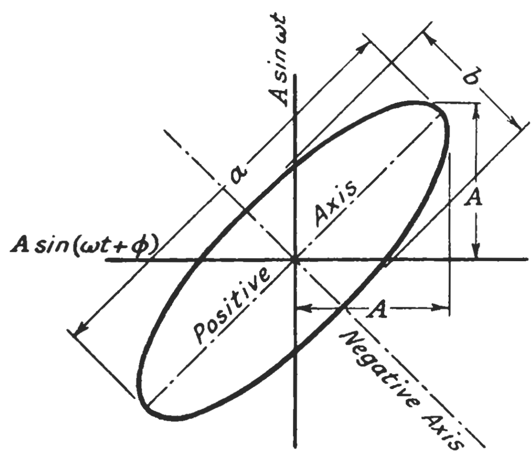


FIG. 5/XX:10.—Ellipse formed by Two Waves: $A \sin(\omega t + \phi)$ applied to X Plates and $A \sin \omega t$ applied to Y Plates.

to the Y plates adjusting to the same amplitude A. Then apply both waves together and measure a and b. The mutual phase shift ϕ can then be determined by entering the value of $\frac{a}{A}$ or $\frac{b}{A}$ in Fig. 6 and reading off the value of ϕ on the appropriate curve. It is probably best to use the reading taken from the major axis since the thickness of the trace then introduces less chance of error.

10.3. Wave Forms Resulting from the Presence of Odd and Even Harmonics. Fig. 7 shows at $A1$ and $B1$ a fundamental sine wave and a 30% second harmonic, with two different relations in time between the fundamental and harmonic. The composite wave resulting from adding the ordinates of fundamental and harmonic is shown for $A1$ at $A2$ and for $B1$ at $B2$. It will be noted that in each case the composite wave form is asymmetrical.

At $C1$ and $D1$ are shown a fundamental sine wave and a 30% third harmonic with different time relations between fundamental and harmonic. The corresponding composite waves are shown respectively at $C2$ and $D2$. It will be noted that in each case the composite wave form is symmetrical.

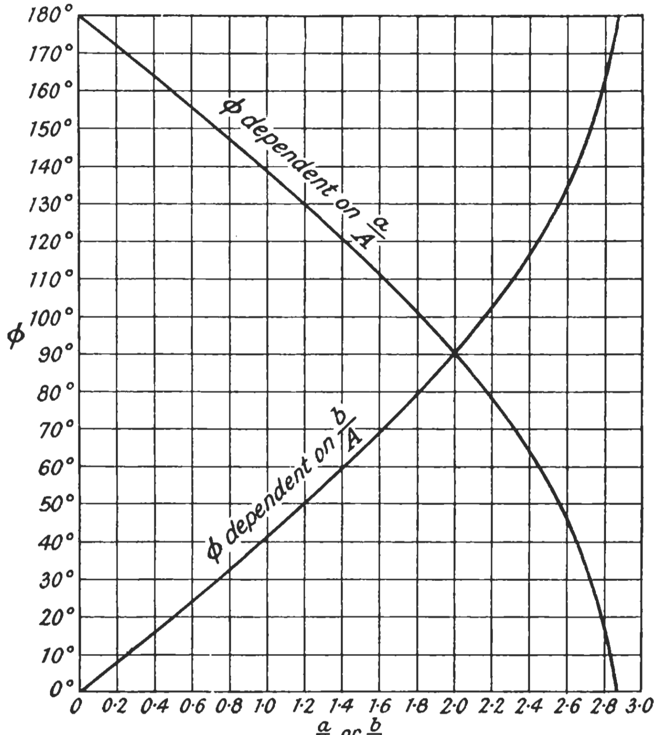


FIG. 6/XX:10.—Relation between ϕ and $\frac{a}{A}$ or $\frac{b}{A}$.

In general, even-order harmonics produce asymmetry and odd-order harmonics give rise to a symmetrical wave form. It should be noted in particular that $A2$ is really an asymmetrical wave form although it appears to have a kind of skew symmetry.

When harmonics are sufficiently large, by inspecting the wave form at the output of an amplifier driven with a sinusoidal input, it is possible to obtain an idea of what harmonics are present. More important, perhaps, examination of the output wave form may give an indication of the type, and hence the source of distortion. This may be made clearer by reference to Fig. 8, which shows the distortion of a sinusoidal input resulting from anode limitation and bottom bend of the valve characteristics. In Fig. 8, the curves at *A*1 and *B*1 are obtained by applying the output wave of the

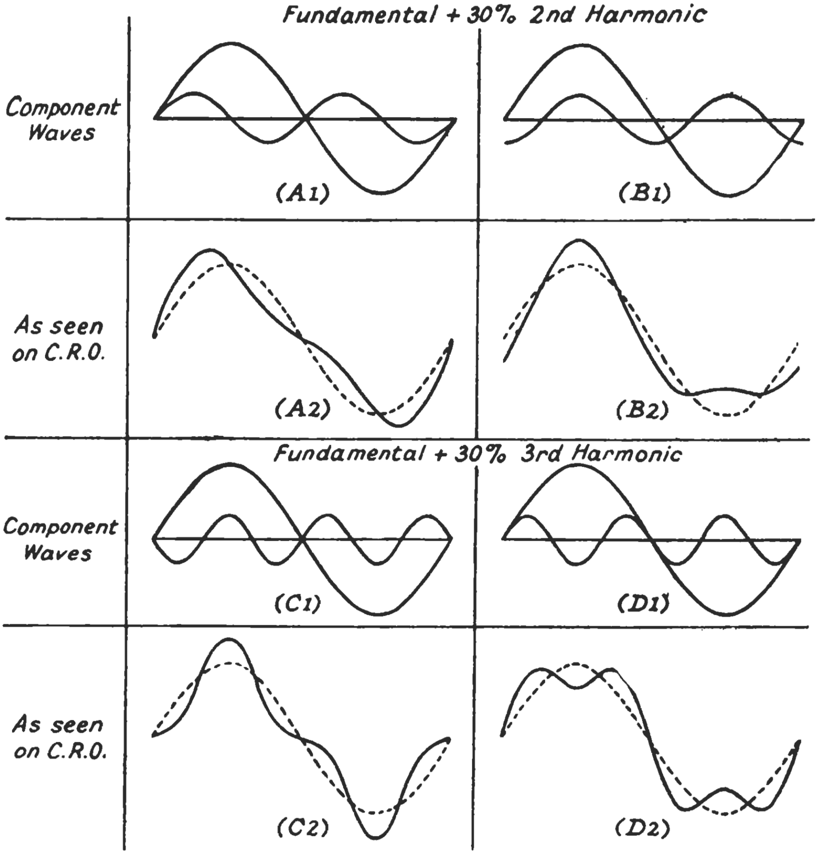


FIG. 7/XX:10.—Component Waves and Corresponding Composite Waves as seen on C.R.O.

amplifier to the *Y* plates of the C.R.O. and the time base of the C.R.O. to the *X* plates. The curves at *A*2 and *B*2 are obtained by applying the output wave as before to the *Y* plates, and the input wave to the *X* plates, *adjusting the voltages on the X plates to the same phase as the output wave and equal amplitude.* The resultant curve in the second case will be called an Input-Output diagram. It is particularly useful because it presents a direct picture of the non-linearity of the amplifier, which is much more easily seen than in the curves at *A*1 and *B*1, because departure from a straight line is more easy to detect by eye than departure from a sine wave.

It will be found that, in a single-sided (i.e. non-push-pull) amplifier, anode limitation and/or bottom bend gives rise to odd and even order harmonics. In a push-pull amplifier anode limitation gives rise to odd-order harmonics.

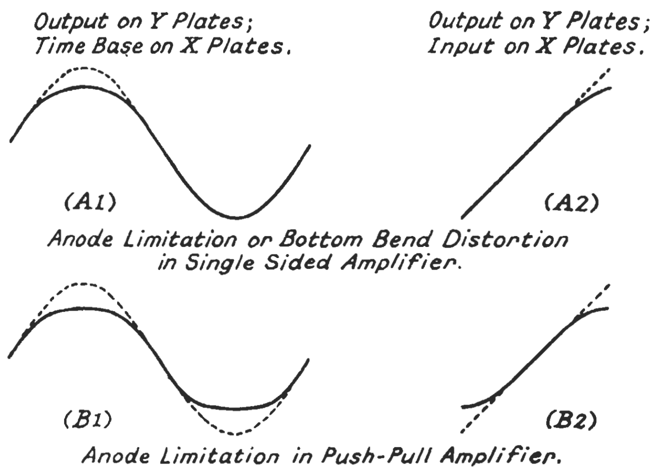


FIG. 8/XX:10.—Types of Curves seen on C.R.O. connected to Amplifier with Sinusoidal Input.

10.4. Trapezium Diagrams. Fig. 9 shows a number of modulated waves and their corresponding Input-Output diagrams which are usually called " Trapezium " diagrams. The trapezium diagram is derived by applying to the *X* plates the modulating (envelope) frequency derived from some point at or before the input to the modulator and adjusted to be in phase with the envelope of the modulated wave, the modulated wave being applied to the *Y* plates. The effect, on the trapezium diagram, of phase shift between the input wave and the envelope, is shown on the extreme right of Fig. 9 (*a*).

Fig. 9 (*a*) shows a 100% modulated wave and Fig. 9 (*b*) a wave modulated less than 100%.

Fig. 9 (*c*) illustrates a case of over-modulation, which would occur in an anode-modulated amplifier, for instance, if the amplitude of the audio-frequency swing applied to the anode of the modulated amplifier were larger than its steady H.T. voltage. In this case during part of the negative modulation half-cycle the anode would stay negative for an appreciable time and there would be no output from the modulated amplifier during this time. Needless to say,

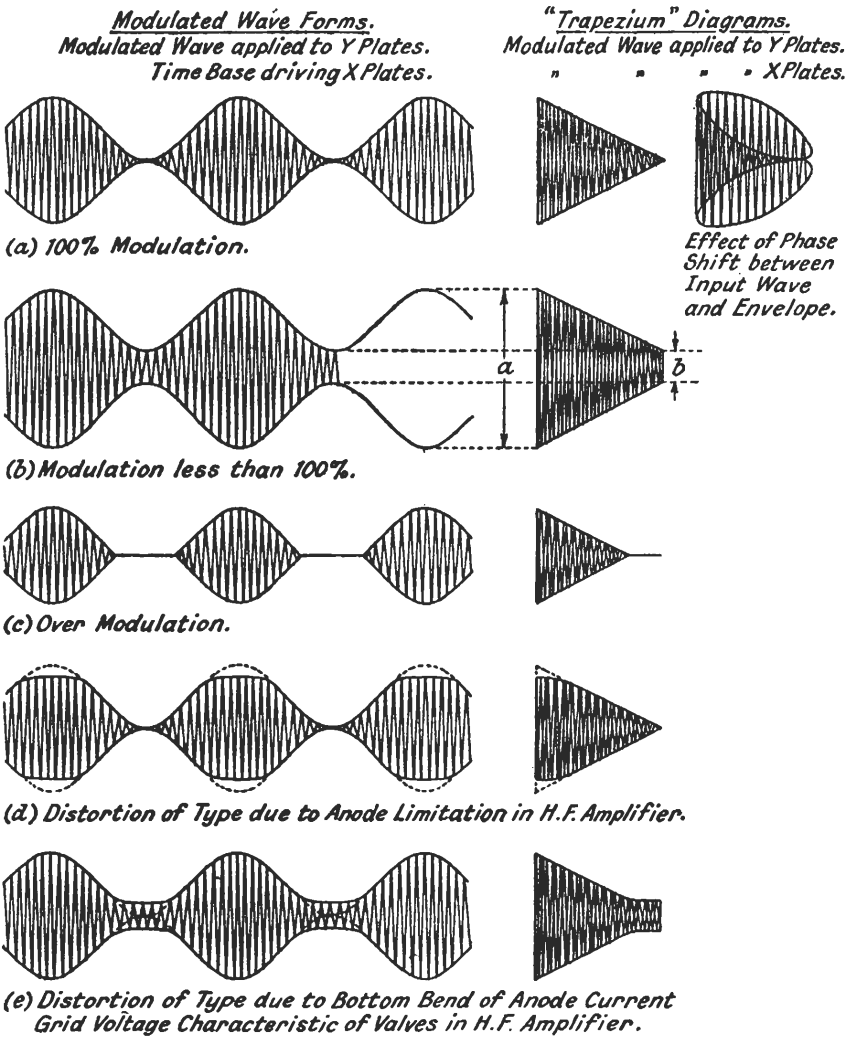


FIG. 9/XX:10.—Modulated Waves Viewed with C.R.O. under Different Conditions.

the degree of over-modulation indicated is very much greater than would be tolerable in practice.

Fig. 9 (*d*) shows a wave such as would be produced at the output of a high-frequency amplifier driven with a normally modulated wave and running into anode limitation or filament saturation. The last condition is, however, unusual.

Fig. 9 (*e*) illustrates a case where the bias on an H.F. amplifier, amplifying the modulated wave, is too large and bottom bend distortion occurs. If the bias is made still larger the output will fall to zero during some of the time and the resultant wave might be mistaken for over-modulation as illustrated at (*c*).

10.5. Measurement of Percentage Modulation. This can be determined either from the true wave form seen on the C.R.O. as shown in Fig. 9 (*b*) or from the corresponding trapezium diagram also shown. The trapezium diagram has no advantages for this purpose except that the trapezium diagram can be obtained on a C.R.O. which is not fitted with a time base. If in the wave in Fig. 9 (*b*), C is the amplitude of the carrier frequency and S the sum of the amplitudes of the sidebands, the modulation $m = \frac{S}{C}$.

Also $$a = 2C + 2S$$
and $$b = 2C - 2S$$

$$\therefore C = \frac{a+b}{4} \text{ and } S = \frac{a-b}{4}$$

$$\therefore m = \frac{S}{C} = \frac{a-b}{a+b} \quad . \quad . \quad . \quad . \quad (3)$$

10.6. Location of Instability in H.F. Amplifier. A cathode-ray oscillograph is practically the only means of locating instability which only occurs during a limited part of the R.F. cycle. (See, however, XI:13.) It may show up on the trapezium diagram as a small kink or wriggle in the curve, but may be difficult to see on account of the speed at which the spot moves. A method that is sometimes used to check stability on an R.F. amplifier is to drive the amplifier with a 50-c/s wave derived from the mains and to examine the Input-Output Diagram, with the output taken across the output-tuned circuit. Since this circuit is fed through a small condenser having a high reactance at 50 c/s, and since the impedance of the output circuit at 50 c/s is negligible, the resultant diagram will be substantially a horizontal straight line unless any parasitic oscillation occurs, in which case the line may appear broken, or of increased breadth, in the region of input amplitude where the oscillation occurs.

11. Harmonic Analyser.

This is a device for the measurement of the amplitude of each frequency component of a complex wave.

The simplest form of harmonic analyser is a tuned circuit arranged so that the current through an associated indicating device reaches a maximum at a frequency determined by the setting of the tuned circuit. Such a simple arrangement is not usually sufficiently selective, its input impedance varies with frequency and may upset the operation of the measured equipment at some frequency other than that to which the circuit is tuned, and finally, the range of tuning on one coil range is limited. A high input impedance can be secured by using an input amplifying valve, while improved selectivity can be obtained by using a number of tuned amplifying stages before the detecting device. Means of calibrating the system at each frequency must be provided, e.g. by supplying a known voltage at the input, from the output of an attenuator supplied with a known current or voltage at its input.

Where high selectivity is required, the superheterodyne principle is used ; this also increases the range of tuning on one swing of the frequency controlling condenser. The voltage wave under examination is modulated with a frequency derived from a calibrated oscillator, and as the oscillator frequency is varied the band of difference frequencies resulting from modulation is progressively shifted through the pass range of a highly selective fixed frequency filter circuit following the modulator. The oscillator is marked at each setting with the value of frequency supplied at the input of the analyser which, when modulated with the oscillator frequency, will produce a frequency in the pass range of the filter. Such a device can be constructed to read directly the amplitude of each component frequency in volts or millivolts.

When designed for the audio-frequency range it is very convenient for measuring the harmonic distortion of audio-frequency amplifiers and transmitters with sinusoidal input waves (fundamental frequencies).

An analyser may be designed, for instance, to read the amplitude of any frequency in the range 100 c/s to 15,000 c/s. By supplying fundamental frequencies in the range 100 to 7,500 c/s, any harmonic below the 150th harmonic of 100 cycles can then be measured, while in the case of 7,500 c/s only the second harmonic can be measured. On intermediate fundamental frequencies an increasing range of harmonics can be measured as the frequency is reduced below 7,500 c/s. Such a device is marketed by General Radio.

If the R.M.S. sum of the harmonics is required it must be calculated by taking the square root of the sum of the squares of the individual harmonic amplitudes.

12. Direct Measurement of R.M.S. Harmonics.

By using a low-pass filter and a high-pass filter, each with the same cut-off frequency, which is located between the fundamental input frequency and its second harmonic, all the harmonics can be separated from the fundamental frequency. The R.M.S. sum of the harmonics can then be measured by using a square law detecting device such as a thermocouple, or a valve with a square law anode-current/grid-voltage characteristic.

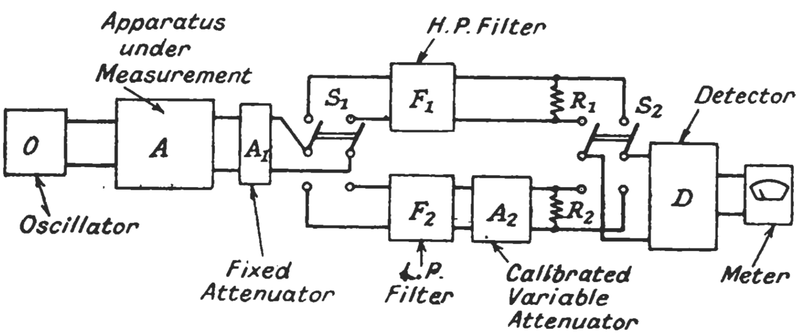


FIG. 1/XX:12.—Measurement of R.M.S. Harmonics.

One form of arrangement is shown in Fig. 1. The oscillator O supplies the fundamental frequency to A, the apparatus under measurement, the output of which is connected through a fixed T or π attenuator A_1 (or the balanced equivalent), with not less than 10 db. loss, to the input of the H.P. filter F_1 or the L.P. filter F_2, selection being made by switch S_1. A variable attenuator A_2 is connected in series with F_2, the L.P. filter, to adjust the R.M.S. amplitude of the fundamental frequency to the same amplitude as the R.M.S. harmonic amplitude as measured on the detector D, which is switched from one circuit to the other by means of the switch S_2. The loss in the variable attenuator then gives in decibels the difference in level of the R.M.S. harmonics and the fundamental frequency, after allowance has been made for the loss in the pass ranges of F_1 and F_2. R_1 and R_2 are equal resistances equal to the characteristic impedances of the H.P. and L.P. filters, and variable attenuator, which should all have the same image impedance.

The object of A_1 is to mask the impedance presented by A to F_1 and F_2 and by F_1 and F_2 to A, and to present to A, F_1 and F_2 their required impedances; if the loss of A_1 is 10 db., its input impedance cannot differ from its characteristic impedance by more than 20%. A_1 may also be an unequal ratio attenuator if the nominal output impedance of A differs from the image impedances of F_1 and F_2. If at any point it is necessary to change the circuit from a balanced circuit to an unbalanced circuit, repeating coils should be inserted at the point of transition. It is more simple to build unbalanced filters than balanced filters, and for this reason the whole circuit to the right of A_1 may be unbalanced, in which case a repeating coil should be inserted in the output of A_1. Care should be taken not to associate a balanced attenuator in the position of A_2 with an unbalanced filter F_2. This may, however, be done if repeating coils are placed each side of A_2 and allowance made for their loss. As already indicated, in accurate measurements, allowance should be made for the losses in the pass ranges of F_1 and F_2; for this purpose the mean loss in the pass range of F_1 should be used, while for F_2 the loss at the fundamental frequency should be taken.

13. Modulation Meter.

This is a device giving a direct indication of percentage modulation. Various arrangements are employed for this purpose. In one type of instrument separate measurements are made of the amplitude of rectified carrier and detected envelope, and the ratio between the two gives the percentage modulation. The accuracy of this measurement depends on the accuracy of the two separate measuring trains and the accuracy of two observations, while the accuracy is impared if the carrier level varies between the two sets of measurements. In the preferred circuit in use in the B.B.C. these difficulties are overcome by comparing electrically the amplitude of modulation with the mean carrier amplitude, before applying the result to an indicating circuit.

The carrier is first rectified and smoothed so that a steady potential, equal to the mean peak value of the carrier, is obtained, with a potential following the modulation envelope superimposed. These two components are separated out and the modulation frequency is rectified again, giving a potential equal to the peak value of the modulation envelope. This potential is then subtracted from that arising from the first rectification, and the difference is measured, a convenient stable and accurate voltmeter for this purpose being a valve arranged as a " cathode follower ", with a meter for reading cathode current. This arrangement has been developed in the B.B.C. by C. G. Mayo and H. McD. Ellis.

Fig. 1 is a circuit showing this arrangement in its simplest form. The modulated wave is applied at terminals 1,1. V_1 is a diode which passes the rectified wave through its cathode circuit. In the cathode circuit the low-pass radio filter constituted by L, C_1 and C_2 removes the radio frequency, leaving a D.C. current corresponding to the carrier amplitude, plus an audio-frequency alternating current corresponding to the modulation depth. These two currents flow through resistance R_1, producing corresponding voltages across it. Resistance R_2 and condenser C_3, which is a large condenser, e.g. $4\mu F$, constitute a filter removing the audio frequency, so that across C_3 appears a voltage corresponding to the carrier amplitude. The voltage across R_2 is, therefore, the difference between the voltage

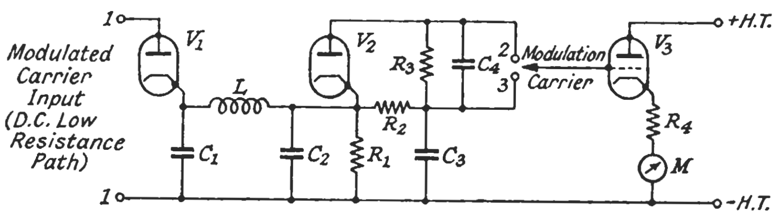


FIG. 1/XX:13.—Circuit showing Principle of Direct Reading Modulation Meter.

across R_1 and the voltage across C_3, that is, it is the voltage corresponding to the modulation depth. Bridged across R_2 is what is in fact a peak voltmeter constituted by a diode V_2 and the parallel resistance R_3 and condenser C_4. A D.C. voltage is, therefore, developed across R_3 proportional to the modulation depth. With the diode V_2 in the sense shown in Fig. 1 the sense of the voltage across R_3 corresponds to the depth of the negative modulation peak, so that if the negative modulation peaks down to zero radio frequency current, the volts across R_3 are equal and opposite in sign to the volts developed across C_3.

The result is that while the voltage developed between terminal 3 and ground is proportional to the carrier amplitude, the voltage developed between 2 and ground with the negative modulation peak reaching 100%, is zero. For lower values of modulation the voltage between 2 and ground is equal to the carrier level minus the amplitude of the negative modulation peak. By measuring the voltage between 3 and ground and between 2 and ground, it is, therefore, possible to measure the percentage of modulation. V_3 in conjunction with its cathode resistor R_4 and meter M constitutes a high impedance valve voltmeter, which can be switched either to point 2 or to point 3. The meter M is a normal D.C. meter calibrated with 0% modulation at full-scale deflection and 100% modulation at zero deflection. For purposes of measurement the voltmeter is connected to terminal 3 and the amplitude of the input wave adjusted until the meter M reads full-scale deflection ; owing to the presence of the audio-frequency filter constituted by R_2 and C_3, this deflection is independent of the presence of modulation. If the voltmeter is connected to terminal 2, meter M will then read percentage modulation direct.

The full circuit is shown in Fig. 2. T_1 is an R.F. input transformer supplying the valve V_1 which fulfils the same function as valve V_1 in Fig. 1. The R.F. filter is constituted by C_1, C_2 and R_1. The audio-frequency filter is constituted by the winding 1–2 of transformer T_2 and condenser C_3, so that a D.C. voltage proportional to the carrier amplitude is developed across C_3 and therefore between contact 3 of switch S and ground. Resistances R_2, R_3 and inductance L_3 are provided to constitute in conjunction with the impedance presented by terminals 1–2 of transformer T_2 and condenser C_3, a resistance termination to the radio-frequency filter, which is constant and independent of frequency (cf. XIX:6).

The audio frequency flowing through windings 1–2 of transformer T_2 induces audio-frequency voltages in the two secondary windings proportional to the depth of modulation. Dependent on the sense of the secondary windings and the connections to the two anodes of the double diode V_2, the voltages developed across R_4 and R_5 correspond respectively to the negative peak and positive peak modulation amplitudes. The remainder of the circuit operates in exactly the same way as Fig. 1.

Since, however, modulation lasting less than 5 milliseconds is substantially unnoticeable by ear, the time constants of the second rectifier, i.e. the two peak voltmeter circuits of V_2, are arranged so that peaks of modulation of shorter duration than 5 milliseconds do not fully deflect the instrument, while peaks of longer duration are fully recorded. This follows the practice adopted with the peak programme meter, but the actual value of the time constant is made half the value of that in the peak programme meter since the modulation monitor operates on only one-half of the modulation cycle.

T_1 is constituted by two air-cored coils, L_1 and L_2, with variable

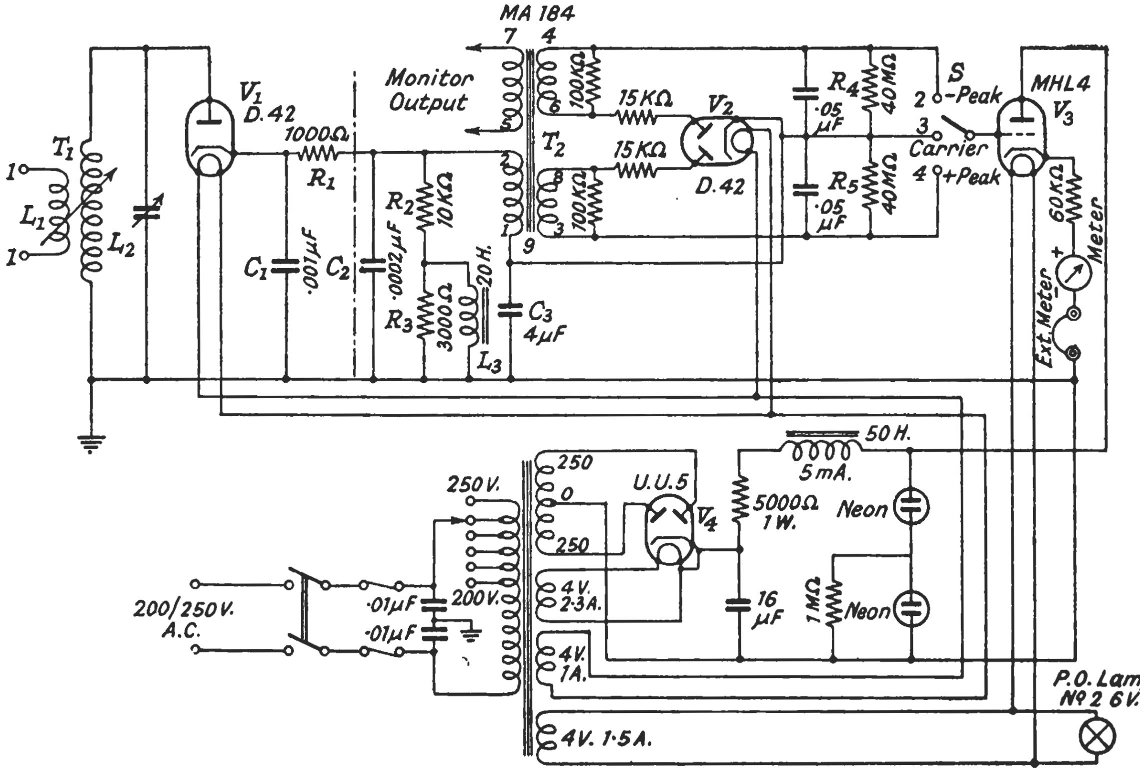


FIG. 2/XX:13.—Circuit of Modulation Meter.
(By courtesy of the B.B.C.)

coupling between them ; the values of L_1 and L_2 for various wave-length ranges being as follows :—

Wave Range Meters	L_1	L_2
12·3–53	0·6 μH	1·6 μH
24–100	1·6 μH	6·0 μH
150–600	77·0 μH	207 μH

The variable condenser tuning L_2 has an effective range of capacity from 28 to 525 μμF.

A modulator winding is fitted to transformer T_2, giving audio-frequency output at zero level (1 milliwatt) on 40% modulation, which may be used instead of the usual check receiver associated with a transmitter. The output of this winding is approximately 11 ohms and it may be loaded with any impedance of not less than 600 ohms without appreciably upsetting the operation of the device.

14. Peak Programme Meter.

The peak programme meter is a type of valve voltmeter with a high impedance input designed to measure the peak voltage in a circuit-carrying programme, and to represent this level on a meter having a logarithmic scale. In other words, equal intervals on this scale represent equal changes of level measured in decibels.

Fig. 1 shows the circuit of a typical B.B.C. peak programme meter. T_1 is the input transformer which has its primary bridged across the circuit in which the level is to be measured. The secondary of this transformer supplies voltage to the potentio-meter P_1, the slide of which is connected to valve V_1, which is a normal audio-frequency amplifier valve, resistance coupled in its anode circuit to transformer T_2. The secondary of T_2 drives the push-pull rectifier circuit constituted by the two diodes V_2 and V_3, as a result of which a D.C. voltage of intensity proportional to the peak programme level is built up across resistance R_4. This voltage changes the bias of valve V_4, a variable μ valve, which by good fortune provides a logarithmic relationship between its grid voltage and its anode current, when suitable values of cathode resistance and screen volts are used.

The meter in the anode circuit of V_4 then indicates the level of the voltage applied at the input of the device ; on a logarithmic basis. This meter has seven divisions on it, numbered from 1 to 7, and each division corresponds to a change in input level of 4 db. For any one setting of sensitivity controls at the input, the meter therefore covers a level range of 28 db. Taps are provided on the input transformer, in order to extend the range of sensitivity of the

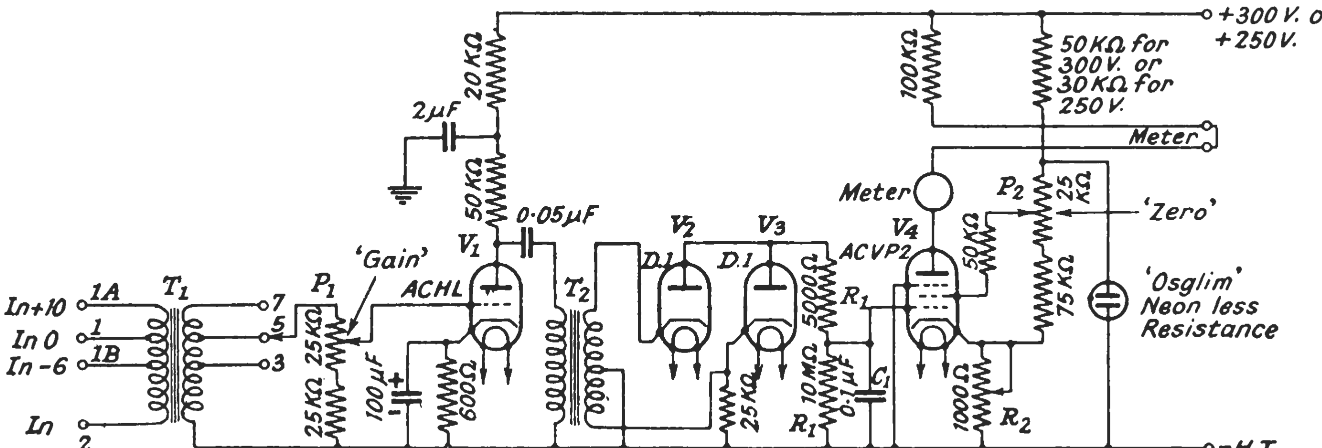


Fig. 1/XX:14.—Peak Programme Meter.

(By courtesy of the B.B.C.)

device. In Fig. 1, for instance, connection to terminals 1 and 2 provides for measurement of a circuit carrying zero level, while connection to *1a* and *1b* instead of to 1 provides respectively for reception of levels of plus 10 db. and minus 6 db. These taps are typical only and do not represent all cases met in practice.

Potentiometer P_1 is not calibrated since the whole device is always calibrated by transmitting line-up tone through the circuit at a known level, and adjusting P_1 until a predetermined deflection is obtained on the programme meter. This predetermined deflection is normally up to division 5 on the output meter. Having set the sensitivity in this way on line-up tone, control operators regulating the outgoing level of programmes from studios continually adjust their gain controls so that the outgoing level makes the programme meter peak up to division 7. It is essential that the programme should not peak up above division 7, otherwise overloading of transmitters fed by the programme will occur.

Normally the programme is not allowed to fall below a level at which the programme meter peaks up to division 2. This means that the outgoing range of levels covers a variation in level of 20 db. (= plus or minus 10 db.).

Since the programme meter is adjusted on line-up tone to give a deflection up to division 5, and since the programme level is adjusted to make the programme meter peak up to division 7, it is possible to adjust transmitters to 40% modulation on line-up tone, and so to be certain that, on programme, 100% modulation will be obtained (since the difference between 40% modulation and 100% modulation = 8 db. = 2 divisions on the programme meter). See XV:4.2.

15. Ganging Oscilloscope.

This is really a trade name for a piece of apparatus designed to portray response characteristics (particularly of radio receivers) on the screen of a cathode-ray oscillograph. It consists of a cathode-ray oscillograph and time base, and a variable frequency oscillator whose frequency at any instant during the working stroke of the time base, is directly proportional to the displacement of the cathode-ray spot (in a horizontal direction on the screen). The amplitude of the frequency swing of the variable frequency oscillator is usually about ± 15 kc/s and the mean frequency, about which the oscillator frequency swings, can be adjusted to any required frequency (usually covering the medium- and long-wave broadcast bands).

The result is that, if the output of the oscillator is applied to the aerial and earth terminals of a radio receiver, and the output from the receiver diode detector is connected to the *Y* plates of the cathode-ray oscillograph (sometimes through an amplifier incorporated in the equipment), the time base being, of course, connected to the *X* plates, a response curve on a voltage basis will appear on the cathode-ray screen.

It is evident that this useful piece of apparatus reproduces instantaneously response curves which would otherwise have to be plotted from a series of laborious measurements. It is therefore particularly valuable for adjusting the ganging and tracking of radio receivers, since it is possible to see at a glance whether the response of the side-bands is symmetrical about the carrier frequency. See XIX:19.6.

16. Absorption Wavemeter.

An absorption wavemeter is the simplest possible form of wavemeter. It consists of a tuned circuit and a device indicating when the current in the tuned circuit is a maximum. This device may be a small lamp lit by the R.F. current flowing in the circuit, a thermocouple in conjunction with meter or any other device which does not disturb the tuning of the circuit.

An absorption wavemeter is used by bringing it near any coil carrying an R.F. current whose frequency is to be determined, and adjusting the tuning of the wavemeter condenser until maximum current flows in the tuned circuit as shown by the indicating device. The frequency, or the wavelength, is then either read off directly from the condenser dial or from a calibration chart which converts the reading of the condenser dial to frequency or to wavelength.

The name of the wavemeter is derived rather obviously from the fact that it absorbs power from the source of the frequency to be determined. It is, however, rather hard to conceive a wavemeter which operated without absorbing power.

CHAPTER XXI

RESPONSE ADJUSTMENT AND EQUALIZER DESIGN

1. Causes of Variation in Response; Response Distortion.

THE variation with frequency of the response of a piece of apparatus or circuit is due to the presence of reactive elements usually in series with or in shunt across the circuit.

Fig. 1/V:17 shows the reactance of all practical values of inductance and capacity. The reactance of an inductance is proportional to frequency, while the reactance of capacity is inversely proportional to frequency. The presence of a series inductance or a shunt capacity in an otherwise resistive circuit, as for instance in Figs. 1 (*a*) and 1 (*b*) respectively, therefore gives rise to a response which falls with increase of frequency. At any one frequency, reducing the value of inductance or capacity respectively, reduces the circuit loss.

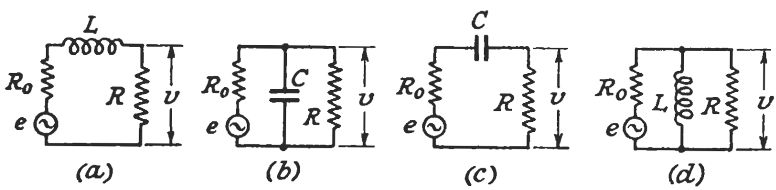


FIG. 1/XXI:1.—Illustrating Effect of Series and Shunt Inductances and Capacities.

Similarly, the presence of series capacity or shunt inductance, as in Figs. 1 (*c*) and 1 (*d*) respectively, gives rise to a response which falls with decrease of frequency. Increasing the value of capacity or inductance respectively reduces the circuit loss at any one frequency. In the case of shunt elements the circuit loss is evidently small when the reactance of the shunt element is large compared to the ***mean circuit impedance*** as defined below, while in the case of series circuit elements the loss is evidently small when the reactance of the element is low compared to the ***impedance sum*** as defined below. For this reason shunt inductance and series capacities usually are effective in determining the lower frequency cut-off of the circuit, while series inductance and shunt capacity are usually effective in determining the upper cut-off frequency of the circuit.

It will be appreciated that in the simple cases described the circuit does not cut off sharply but the response falls away progressively. The exact point at which the cut-off may be said to occur in such a case is entirely a matter of convention ; a convenient figure to take for the cut-off frequency in such a case is the frequency at which the response has fallen 3 db. below the midband response. This is because a fall of 3 db. is about the largest that can be permitted without it being noticeable by ear in an audio-frequency circuit. Many figures have been suggested for the cut-off point and 3 db. has only the disputable merit claimed here.

As a corollary of the fact that series and shunt reactance elements introduce response distortion, it follows that suitable combinations of reactance and resistance can be used to correct the response of a circuit to a level response, or to adjust it to any required response characteristic. Special assemblies of reactance and resistance elements for response adjustment are called ***equalizers*** because they were originally used to make the overall attenuation of transmission lines equal at all frequencies. In general, however, equalizers may be used to convert a response characteristic from any one form to any other.

The adjustment of response curves requires a combination of art and science and is a particular field where use of a little artfulness saves a large amount of systematic labour. The art consists partly in the application of a variety of circuits and circuit modifications and partly in the proper choice of the type of structure which is capable of being designed to provide a required response curve. The science consists of the method of design of chosen structures to give the required response characteristic. This involves the calculation of the response of prescribed forms of circuit.

2. The Means Used for Response Adjustment may be conveniently classified as follows :

(1) *Circuit Modification.* Response characteristics can be adjusted by modifying the values of circuit elements and sometimes by the addition of extra elements. Generally, but not always, this method has the advantage that no loss of gain or little loss of gain, results over parts of the frequency range where it is not required to change the response. This statement suffers from ambiguity since the response is defined by a curve of relative efficiency of reproduction. It loses its ambiguity if it is taken to refer only to cases where the response falls off at low or high frequencies

due to a specific cause for which compensation can be provided.

(2) *Series and Shunt Equalizers.* Series or shunt impedance elements made up of the combination of reactance and resistance, are added at any convenient point in the circuit. This method has the advantage of simplicity, but the disadvantage that the consequent change in circuit impedance makes its behaviour difficult to predict.

(3) *Open-Circuit Equalizers.* These are four-terminal networks, designed to work from a generator of zero internal impedance into an open circuit. In general they can only be inserted between valves where the required open-circuit impedance terminating the network is approximated by the grid circuit of a following valve. The internal impedance of the driving source is considered as part of this network. The only advantage which may exist in the use of such circuits is in cases where they give simply characteristics which would otherwise be difficult to obtain. This type of equalizer is not considered separately, although examples occur incidentally throughout the discussion of other types. Examples are shown in Figs. 5, 6, 10 and 11/XXI:5.

(4) *Variable Impedance Equalizers.* These are four-terminal networks consisting of assemblies of reactances and resistances with image impedances which vary over the frequency range. These also are usually but not always limited to the cases where they can be inserted between valves, i.e. constituting the coupling between the input of one valve and the output of the next. These, also, are not discussed because no useful specific design instructions can be given ; in the cases where they are used they are best adjusted by measurements of overall response.

(5) *Constant Resistance Equalizers.* These consist of four-terminal networks (fourpoles) built up of assemblies of reactances and resistances in such a way that their image impedances are equal to a resistance of constant value, i.e. independent of frequency. This type of equalizer is the one which has the most general application, since by adjusting the respective image impedances of such an equalizer to be equal to the (resistive) impedance facing *either* its input or output, or both, the insertion of the equalizer introduces no new reflection loss, and the insertion

loss of the equalizer is equal to its loss when terminated with its image impedances. Since this loss is determinable as a specific characteristic of the equalizer, such equalizers can be used in a variety of circuits to introduce known values of loss varying over a range of frequencies.

Contrary to the performance of constant resistance equalizers, equalizers of types 2 and 4 can only give a predicted performance when working between resistive impedances at both input and output, while open-circuit equalizers normally require a resistive impedance (constituting part of this equalizer) at the input and an open circuit impedance at the output, in order to give a predicted performance.

3. Response Adjustment by Circuit Modifications.

The high-frequency response of a circuit may be improved by reducing the value of shunt capacities due either to straight capacities, deliberately inserted circuit elements, or valve capacities. The low-frequency response may be improved by increasing the size of shunt chokes or series condensers, such as blocking condensers in resistance coupled amplifiers or stopping condensers, as they are sometimes called. If the response of a circuit is limited by the performance of transformers, usually the most satisfactory thing to do is to get a better transformer. There are, however, one or two measures which can be adopted to improve the performance of an existing transformer.

Fig. 1 (*a*) shows a unity ratio transformer working between equal impedances R. At (*b*) is shown the equivalent circuit of this transformer where C_1 and C_2 are the self-capacities of the windings. M is the mutual inductance and $L - M$ is half the leakage inductance. The leakage inductance is the inductance looking into one winding with the other winding short circuited. $2(L - M)$ is therefore only an approximate expression for the leakage inductance since the inductance looking into one winding of a transformer with the secondary short circuited is $(1 - k^2)L_1$, where L_1 is the primary inductance. Since k is usually nearly unity this expression is very nearly equal to $2(1 - k)L_1$, and in a unity ratio transformer, where $L_1 = L_2 = L$ say, the expression reduces into $2(L - M)$, see Fig. 16/XXIV:6. At low frequencies the reactances of C_1 and C_2 are so high that they can be neglected, while the reactance of $L - M$ is so small that it can also be neglected. The circuit therefore degrades to that shown at (*c*). The generalized response characteristic of the structure at (*c*) is given by the dotted curve on

Fig. 9/XXI:5. This is the insertion loss consequent on the introduction of the shunt inductance M between resistances R. If a condenser of capacity $C = \frac{2M}{R^2}$ is inserted in the position shown in Fig. 1 (*d*) the resulting response curve is given by the full line curve of Fig. 9/XXI:5.

The use of generalized network characteristics is fully explained in XXI:5, which should preferably be read before proceeding further. It may, however, be helpful here to give a short explanation of the frequency scale in Fig. 9/XII:5 which, it will be noted, is marked $0{\cdot}01f_0$, $0{\cdot}1f_0$ and f_0, etc. In all cases f_0 is the frequency at which the reactance of one element becomes equal to the reactance or

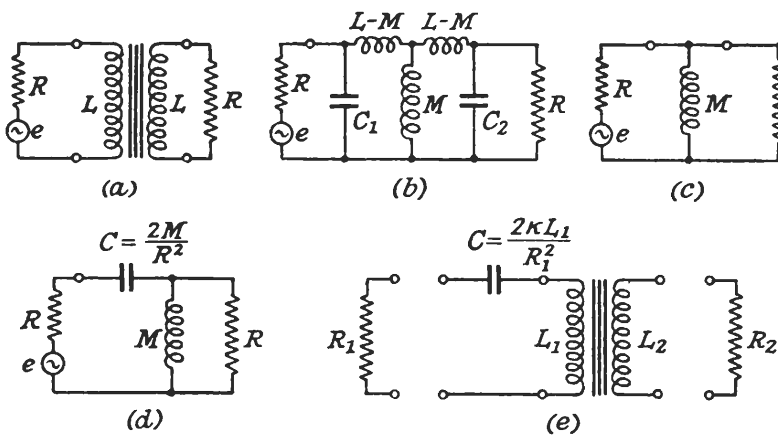


FIG. 1/XXI:3.—Equivalent Circuits of Transformer and Use of Bumping Condenser.

resistance of another element. In Fig. 9/XXI:5, for instance, f_0 is determined by the frequency at which the reactance of L ($= M$ in Fig. 1) becomes equal to the resistance R. For instance, if L is 3 henrys and R is 600 ohms, f_0 = frequency at which the reactance of L is equal to R = 32 c/s. The point marked f_0 on the scale of abscissae on Fig. 9/XXI:5 should in this case, therefore, be marked with 32 c/s, and the point marked $10f_0$ corresponds to 320 c/s. The final-response curve on Fig. 9/XXI:5 therefore has fallen 0·5 db. at 32 c/s, 3·5 db. at 16 c/s, and 11·8 db. at 8 c/s. It should be noted that the improvement in the response obtained due to the insertion of the capacity is very small. It will be evident from inspection of Fig. 9/XXI:5 that the dotted curve can be shifted bodily to the left by reducing the value of R. Even if it is not possible to reduce R both sides, an improvement in response may be obtained at the low end by shunting the transformer with a resistance. This may, however, have a disastrous effect at the upper end of the characteristic. Inspection of Fig. 1 (*b*) shows that at high frequencies the reactance of M is so high as to be negligible, and in practice a further simplification leading to the elimination of either C_1 or C_2 (on the lower impedance side) can be effected because most audio-frequency transformers are step-up transformers and the only capacity of importance is that on the high impedance side. This reduces the circuit to the form shown on Fig. 7/XXI:5, where the full line curve shows the response for a correctly designed transformer. If by changing R the parameter n is changed from unity to any other value, either 0·5 or 2·0, the response is degraded as shown. Further curves for different values of n may be calculated from the formula given on

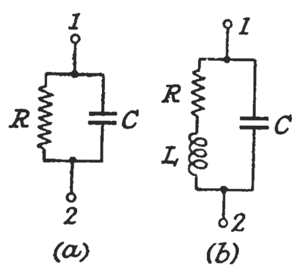


FIG. 2/XXI:3.—Use of Shunt-Capacity-Compensating Inductance.

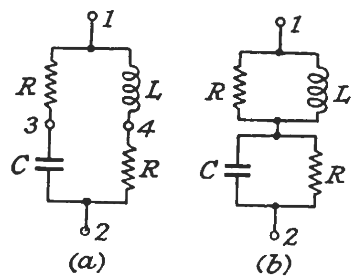


FIG. 3/XXI:3.—Constant Resistance Networks.

Fig. 7/XXI:5. Inspection of Fig. 7/XXI:5 shows that a poor response at the upper end may sometimes be improved by changing the value of the resistances facing the high impedance side of the transformer. The obvious way to see whether such a measure is effective is to try it.

Fig. 2 (*a*) shows a resistance R in parallel with a capacity such, for instance, as might represent a grid leak shunted by the input capacity of a valve. The reactance and resistance components of the impedance observed between terminals 1 and 2, are shown in generalized form on Fig. 2/XXI:5, and it is seen that at f_0 the value of resistance and reactance are both equal to half R, where f_0 is the frequency at which the reactance of the condenser equals the reactance of the resistance. The magnitude of the impedance at this frequency is therefore $R/\sqrt{2}$. By inserting an inductance $L = R^2C$ in series with the resistance, the impedance observed between 1 and

2 is transformed to that represented by the reactance and resistance curves of Fig. 3/XXI:5. It will be seen that the impedance keeps up to a value greater than R, at least up to the frequency at which the reactance of the condenser is equal to the reactance of the resistance. The improvement in response obtained is again small, but this circuit has achieved quite a considerable degree of popularity and is certainly worth while using because, although the improvement is small, its cost is cheap. The limit to the use of this arrangement is usually the unfortunate fact that the required inductance is too large to be realized without such a large self-capacity, that at f_0 the inductance no longer presents an inductive reactance, but behaves as a capacity.

Fig. 3 (*a*) shows a circuit which when $L = R^2C$ presents between terminals 1 and 2 a constant resistance equal to R at all frequencies. Since, when a voltage is applied across 1,2 the potential between points 3 and 4 is zero, these points may be joined without affecting the performance of the network. The network then assumes the form shown in Fig. 3 (*b*). This network is seen to consist of the network on Fig. 1/XXI:5 in series with the network on Fig. 2/XXI:5, and inspection of the resistance and reactance curves for these two networks shows that when the value of $\omega_0 = 2\pi f_0$ is the same for the two networks and the resistances in each are equal to R, the reactances cancel and the effective resistances add up to R.

Fig. 4 shows an application of the network of Figs. 2 (*b*) and 3 (*b*) to a cathode follower valve in which the filament is directly heated with A.C. T_1 is an audio-frequency transformer supplying the input to the grid. T_2 is a power transformer supplying heating to the filament and fed by a second power transformer T_3. If winding W_2, of T_2, were connected directly to the filament it would introduce a capacity to ground via T_3 and the mains. The difficulty is overcome by inserting two screens, S_1 and S_2, in the transformer between the two windings, connecting S_1 to the midpoint of W_1 and ground, and S_2 to the midpoint of W_2 and connecting resistance R_4 between the two screens, which constitutes, with the capacity between the screens, one-half of a circuit corresponding to the lower half of Fig. 3 (*b*). The equivalent of the upper half of Fig. 3 (*b*) is provided by resistances R_2 and the balanced inductance L_2 effective in both legs of the filament leads. Neglecting L_1, therefore, the impedance to ground of the filament is in this way built up to constant resistance. The output lead is taken away from the two centre-pointing resistances R_1 to feed the output load. Since the latter in general has, and is assumed in this case to have, a certain amount of incidental capacity C_s to ground, the balanced inductance L_1 is inserted to constitute in conjunction with the constant resistance already built up, and C_3, a structure of the form shown in Fig. 2 (*b*). Inductances L_1 and L_2 are both in series opposing and therefore in parallel aiding.

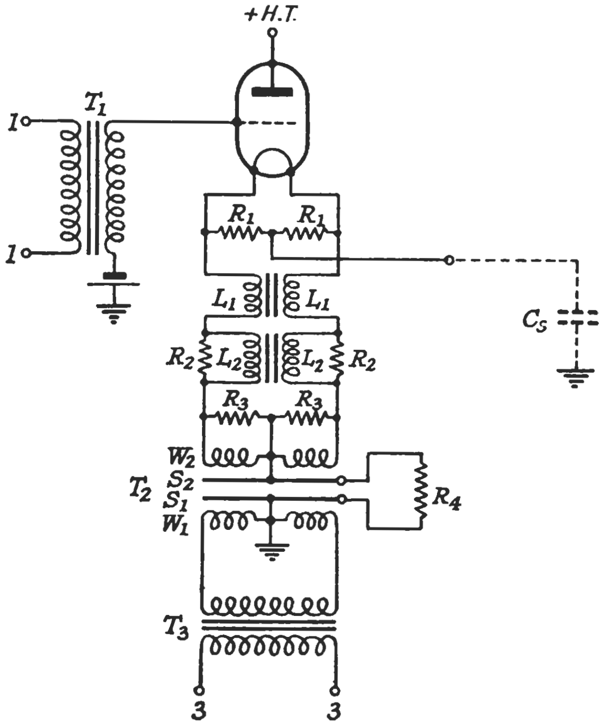


FIG. 4/XXI:3.—Cathode Follower Circuit for Directly Heated Valve, driving Circuit with Capacity C_s to Ground.

The adjustment of the response of radio-frequency circuits in transmitters is dealt with under VII:14, while the adjustment of aerial circuits for response purposes is dealt with under XVI:12. Response in radio receivers is discussed in XIX:8, 11, 12, 14 and 19.

4. Series and Shunt Equalizers and Constant-Resistance Equalizers.

The application of these types of equalizer may be said to constitute nearly the whole of the systematic methods of response adjustment. Before considering these in detail, it is necessary to give some definitions.

Circuit Impedance. While the term impedance is one designed expressly for the purpose of embracing complex impedances, it is used also to describe the input and output impedances of apparatus which, over the range of frequencies for which they are designed, are

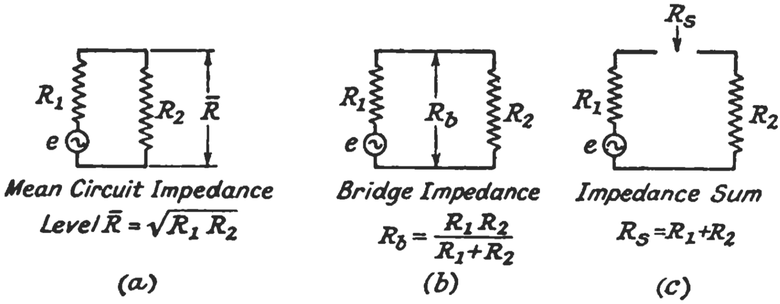


FIG. 1/XXI:4.—Impedance Conventions.

substantially impedances of zero angle. Since these impedances are not constituted by resistance, it is not customary to call them resistances, but to refer to them as impedances of value R or R_1, R_2, etc.

Fig. 1 shows a generator of internal impedance R_1 feeding a load of impedance R_2, three circuits being drawn to explain three different impedance conventions.

Definitions :

The Mean Circuit Impedance Level $\bar{R} = \sqrt{R_1 R_2}$.

The Bridge Impedance $R_b = \dfrac{R_1 R_2}{R_1+R_2}$; see Fig. 1 (*b*).

The Impedance Sum $R_s = R_1+R_2$; see Fig. 1 (*c*).

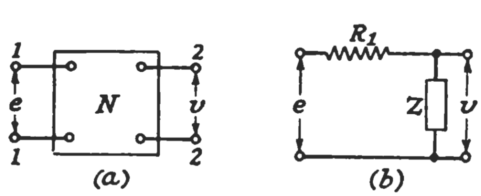


FIG. 2/XXI:4.—Illustrating Voltage Transfer Loss.

Fig. 2 (*a*) shows a four-pole with an input e.m.f. applied to terminals 1,1 and a resultant voltage V generated across its open-circuited output terminals 2,2.

The network of Fig. 2 (*b*) shows a specific example where N is constituted by a series resistance R_1 and a shunt resistance Z. In practice it may sometimes be convenient to consider the internal impedance of a sending circuit as constituting a series element in the network N, e.g. corresponding to R_1 in Fig. 2 (*b*).

The Voltage Transfer Ratio (V.T.R.) of a network, such as N in Fig. 2 (*a*), is the ratio between the open-circuit output voltage and the input voltage: the V.T.R. of the networks in Fig. 2 (*a*) and (*b*) is $\frac{v}{e}$.

The Voltage Transfer Loss (V.T.L.) of a network, such as N in Fig. 2 (*a*), is the decibel equivalent corresponding to the V.T.R.

That is, V.T.L. $= 20 \log_{10} \frac{v}{e} = 20 \log_{10}$ V.T.R

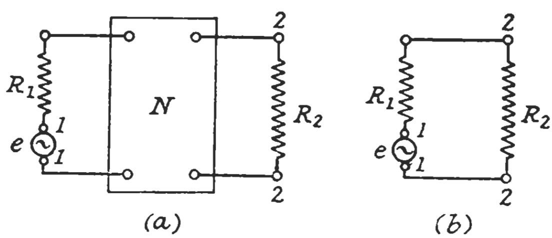


FIG. 3/XXI:4.—Illustrating Insertion Loss.

Fig. 3 (*a*) shows a network N inserted between impedances R_1 and R_2 and the definition of the resultant insertion loss has already been given. It is, however, useful to give a new definition of insertion loss consistent with the previous definition.

The Insertion Loss of network N in Fig. 3 (*a*) is equal to the difference between the V.T.L. of the whole circuit from 1,1 to 2,2 with N inserted and the V.T.L. with N removed as in Fig. 3 (*b*).

Forms of Constant-Resistance Networks. The series of structures in Fig. 4 provide a very simple line of approach to the design of shunt and series equalizers and constant-resistance equalizers. The networks at (*d*) to (*k*) inclusive shown between terminals 1,1 and 2,2 are constant-resistance structures, those at (*d*) and (*e*) having constant resistance looking into terminals 1,1 when terminals 2,2 are closed through a resistance R and the rest from (*f*) to (*k*) inclusive, having constant resistance looking into either pair of terminals when the other pair are closed through an impedance R.

The V.T.R. of (*a*) is seen by inspection to be

$$\frac{Z_b}{R_b+Z_b} = \frac{1}{1+\dfrac{R_b}{Z_b}}.$$

The V.T.L. of (*a*) is therefore

$$20 \log_{10} \left| \frac{1}{1+\dfrac{R_b}{Z_b}} \right| \quad . \quad . \quad . \quad . \quad (1)$$

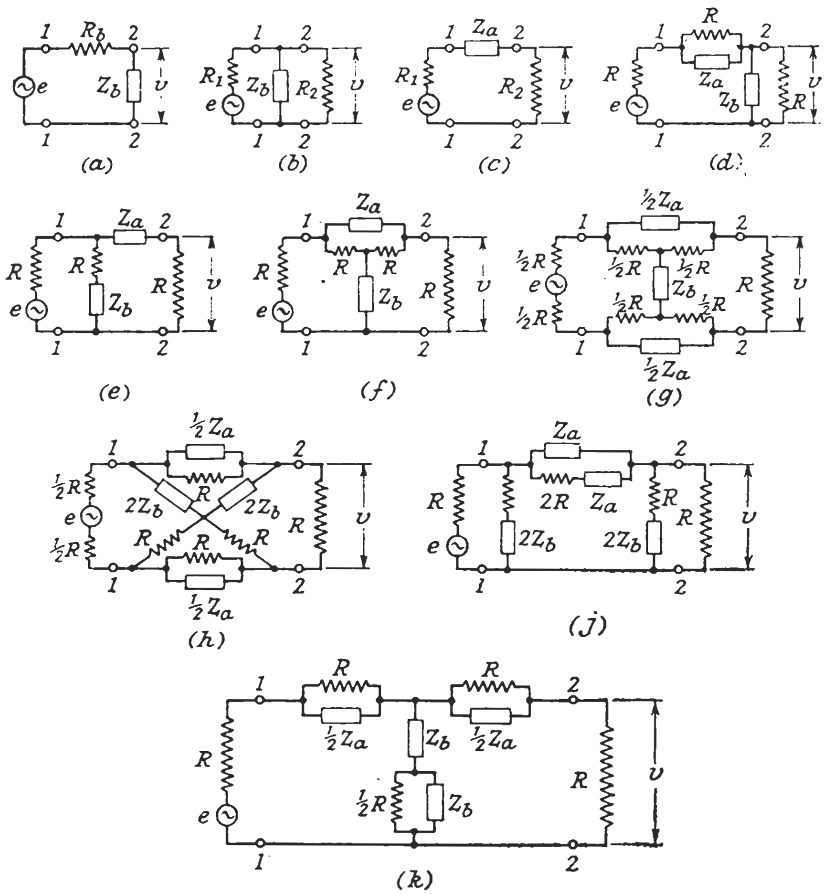


FIG. 4/XXI:4.—Constant Resistance Equalizers and Related Networks.

When (i) $Z_a = \frac{\bar{R}^2}{Z_b}$, (ii) Z_b has the same value in each structure, (iii) $\bar{R} = \sqrt{R_1 R_2} = R$, and (iv) $R_b = \frac{R_1 R_2}{R_1 + R_2} = \frac{1}{2}R$, the insertion Losses of Structures (*b*) to (*k*) inclusive are the same, and equal to the Voltage Transfer Loss of Structure (*a*). The Structure in each case is the Fourpole between 1,1 and 2,2.

The V.T.R. of (*b*) can similarly be shown to be

$$\frac{R_2}{R_1+R_2} \times \frac{1}{1+\dfrac{R_1R_2}{R_1+R_2} \times \dfrac{1}{Z_b}}.$$

Since $\dfrac{R_2}{R_1+R_2}$ is the value of $\dfrac{v}{e}$ for this network with the shunt removed, the insertion loss due to the shunt in (*b*) is

$$20 \log_{10} \left| \frac{1}{1+\dfrac{R_1R_2}{R_1+R_2} \times \dfrac{1}{Z_b}} \right| \quad . \quad . \quad . \quad (2)$$

Similarly, the insertion loss due to the series impedance Z_a in (*c*) is

$$20 \log_{10} \left| \frac{1}{1+\dfrac{Z_a}{R_1+R_2}} \right|$$

and, if $Z_a = \dfrac{\bar{R}^2}{Z_b}$ where $\bar{R} = \sqrt{R_1R_2}$ = the mean impedance level, the insertion loss in case (*c*) is

$$20 \log_{10} \left| \frac{1}{1+\dfrac{R_1R_2}{R_1+R_2} \times \dfrac{1}{Z_b}} \right| \quad . \quad . \quad . \quad (3)$$

Finally, for all the structures from (*d*) to (*k*) inclusive, if $Z_a = \dfrac{R^2}{Z_b}$ the loss due to the insertion of the four terminal networks between the sending and receiving impedances R is

$$20 \log_{10} \left| \frac{1}{1+\dfrac{R}{Z_b}} \right| = -\,20 \log_{10} \left| 1 + \frac{Z_a}{R} \right| \quad . \quad . \quad (4)$$

The method of demonstrating the equality of insertion loss of networks (*d*) to (*k*) is given in Example 3 of XXIV:6.10.1.

Hence the insertion losses of the networks (*b*) to (*k*) inclusive are equal to the voltage transfer loss of the network at (*a*) when the following conditions hold :

Network (*b*) : $\dfrac{R_1R_2}{R_1+R_2} \times \dfrac{1}{Z_b} = \dfrac{R_b}{Z_b}$. It is evidently not necessary that the values of Z_b in the two networks should be equal, but only that this relation should hold. In other words, it is not necessary that the impedance level of the two circuits should be equal. The same remark applies in all the other cases below . . . . (5)

Network (c) : $\frac{Z_a}{R_1+R_2} = \frac{R_b}{Z_b}$. In other words

$$Z_a = \frac{(R_1+R_2)R_b}{Z_b} \quad . \quad . \quad . \quad . \qquad (6)$$

Networks (d) to (k) inclusive : $\frac{R}{Z_b}$ as defined by the elements in

networks (d) to (k) $= \frac{R}{Z_b}$ as defined by the elements in network (a) . . . . (7)

Similarly network (c) has the same insertion loss as (b) when

$\frac{Z_a}{R_1+R_2}$ as defined by the elements in network (c)

$= \frac{R_1R_2}{Z_b(R_1+R_2)}$ as defined by the elements in network (b) . (8)

Networks (d) to (k) have the same insertion loss as (b) when

$\frac{R}{Z_b}$ as defined by the elements of networks (d) to (k)

$= \frac{R_1R_2}{Z_b(R_1+R_2)}$ as defined by the elements in network (b) . (9)

Networks (d) to (k) have the same insertion loss as (c) when

$\frac{R}{Z_b}$ as defined by the elements in networks (d) to (k)

$= \frac{Z_a}{R_1+R_2}$ as defined by the elements of network (c) . (10)

In order, therefore, to design any of the networks (b) to (k) inclusive, it is only necessary to find the type of shunt in network (a) which gives the required response characteristics expressed by the V.T.L. of network (a) and then to make the transformations defined by (5), (6) or (7). This leads directly to the values of the elements of any of the other networks in order that their insertion loss characteristic (when inserted respectively between impedances R_1 and R_2 or between equal impedances R) shall be the same as the chosen V.T.L. of (a).

The whole point of this is that the V.T.R., and hence the V.T.L. of (a), is easy to determine by calculation or measurement on the simple type of network (a). See end of section XXI:4.3.

Alternatively, of course, if the insertion loss of any of the other networks is known, or can be easily determined, if it does not constitute the network it is required to use, the characteristics of the chosen network can be determined by making use of any of the transformations (8), (9) or (10).

4.1. Inverse Networks. Impedances Z_a and Z_b in networks (*d*) to (*k*) are called inverse impedances and they are said to be inverted about the resistance *R*. The inversion is mutual :

$$Z_a = \frac{R^2}{Z_b}, \text{ so that } Z_b = \frac{R^2}{Z_a}.$$

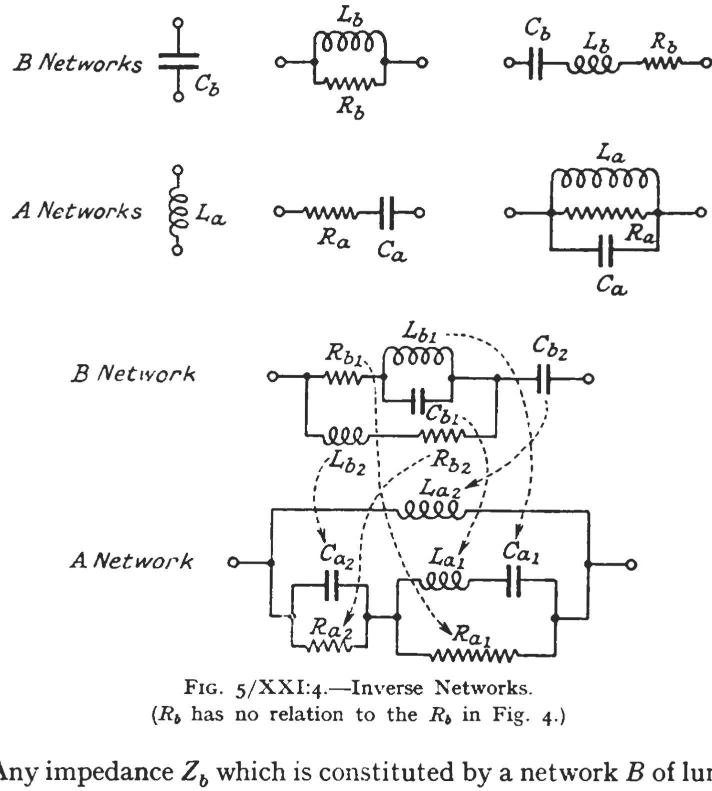


FIG. 5/XXI:4.—Inverse Networks.
(R_b has no relation to the R_b in Fig. 4.)

Any impedance Z_b which is constituted by a network *B* of lumped impedances may have an inverse impedance Z_a realized by an inverse network *A*.

The method of inverting a network *B* is very simple. In the inverse network *A* :

1. Every parallel arrangement of elements in *B* is replaced in *A* by a series arrangement of elements, and vice versa.

2. Every inductance L_b in B is replaced in A by a capacity $C_a = \frac{L_b}{R^2}$.
3. Every inductance C_b in B is replaced in A by an inductance $L_a = R^2 C_b$.
4. Every resistance R_b in B is replaced in A by a resistance $R_a = \frac{R^2}{R_b}$.

It is evident therefore that, once the form of the network constituting Z_b is known, the form of network constituting Z_a may be determined as above, or vice versa. Equalizer design is therefore primarily concerned with the determination of the form and the element values of one two-terminal network.

Fig. 5 shows corresponding inverse networks, inverted about an impedance R.

4.2. Determination of Loss Curve of an Equalizer. Recapitulating: since the loss curves of all the networks in Fig. 4 are the same, any one network can be chosen for the purpose of finding a loss curve, and for approximate estimations of loss the network of Fig. 4 (*a*) is the simplest to consider. The *form* of the network may, therefore, be chosen by considering Fig. 4 (*a*) and its loss characteristics be determined from equation (4) as follows:

The insertion loss of networks (*d*) to (*k*)

$$= I.L = 20 \log_{10} \left| \frac{1}{1+\frac{R}{Z_b}} \right| = -20 \log_{10} \left| 1+\frac{R}{Z_b} \right| = -20 \log_{10} \left| 1+\frac{Z_a}{R} \right|$$

and if $Z_a = a + jb$

$$I.L = -20 \log_{10} \left| 1+\frac{a}{R}+j\frac{b}{R} \right| = -20 \log_{10} \sqrt{\left(1+\frac{a}{R}\right)^2+\left(\frac{b}{R}\right)^2}$$

$$= -10 \log_{10} \left[\left(1+\frac{a}{R}\right)^2 + \frac{b^2}{R^2} \right] \text{ db.} \quad . \quad . \quad . \quad (11)$$

This gives the loss as a negative number of decibels which is in accordance with common convention in use in fundamental transmission-line theory; it is, however, more convenient to reverse this convention and so to reverse the sign in equation (11), and this is often done.

In this case, from equation (11)

$$\left(1+\frac{a}{R}\right)^2+\left(\frac{b}{R}\right)^2 = \text{antilog}_{10} \frac{I.L}{10} . \quad . \quad . \quad . \quad (12)$$

If a plane is chosen, with values of $\frac{a}{R}$ plotted along the axis of abscissae and values of $\frac{b}{R}$ along the axis of ordinates, equation (11) is the equation of a series of circles with centre at the point $-1,0$ and radius r where $r^2 = \text{antilog}\ \frac{I.L}{10}$.

The circles corresponding to insertion losses from 1 db. to 45 db. are shown in Figs. 6 (*a*) and (*b*).

The curves on Figs. 6 (*a*) and (*b*) provide a rapid means of determining the equalizer loss at any frequency, once the value of Z_a is known at that frequency; that is once the values of a and b are known.

For instance, if $a/R = 6$ and $b/R = 3{\cdot}8$, from Fig. 6 (*a*), the insertion loss = 18 db. If $a/R = 100$ and $b/R = 148$, from Fig. 6 (*b*) the insertion loss = 45 db. Alternatively, if the required insertion loss is known at any frequency, an infinitely graduated range of corresponding values of a and b is given.

4.3. Practical Equalizer Design. The above method of approach has provided a means of determining the loss characteristics of a variety of structures suitable for equalization, once the form of the impedance Z_a and the value of its components have been determined.

Unfortunately the problem of equalization always appears in the inverse form: instead of wanting to know the loss curve corresponding to a certain form of network (i.e. constituting Z_a or Z_b), it is always required to determine the form of network, and the values of its components which will provide a certain loss curve. There is no satisfactory general method for doing this. In practice, it is therefore desirable to have available a series of loss curves for the most common types of shunt elements in use. Figs. 5, 6 and 8/XXI:5 show the kind of curves which should be prepared. It will be noted that Figs. 5 and 6/XXI:5 use one set of curves to describe the voltage transfer loss of one type of network corresponding to Fig. 4 (*a*) and the insertion-loss curves of two other types of network corresponding respectively to Fig. 4 (*b*) and (*c*). *Although not specifically so described, all the curves are, of course, directly applicable to all the circuits of Fig.* 4, *provided the transformations of equations* (5), (6) *and* (7) *are used.* It should be noted that these curves are plotted as response curves, the loss being plotted as a negative quantity.

Since the general form of equalizers required for different purposes is very varied, it is impossible to show here characteristics of

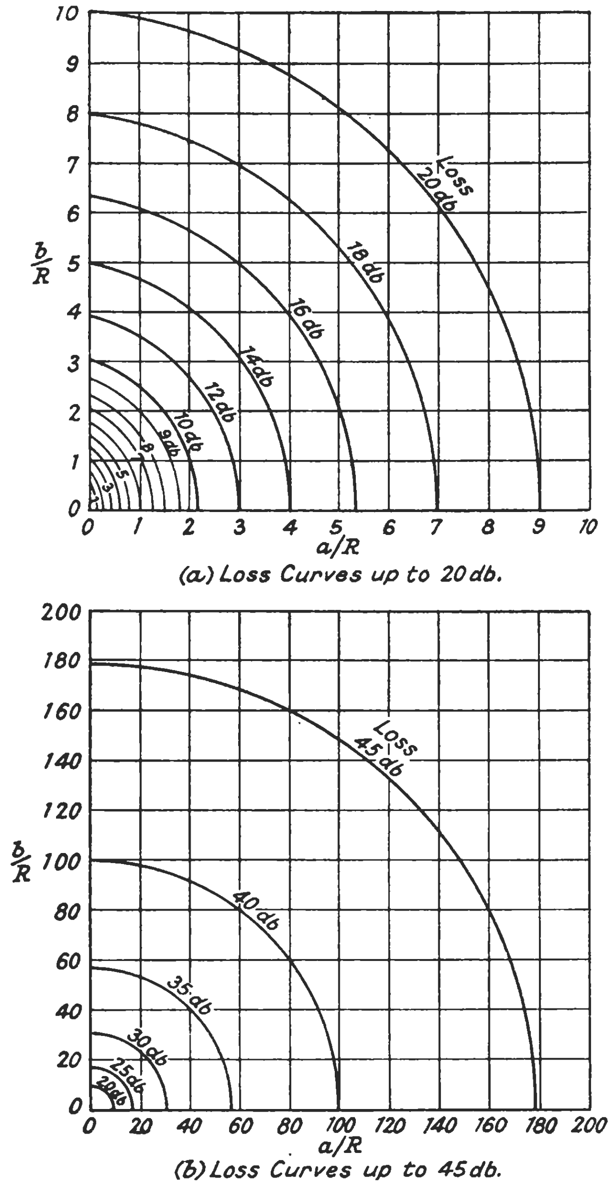


FIG. 6/XXI:4.—Dependence of Equalizer Insertion Loss on $\frac{a}{R}$ and $\frac{b}{R}$ where a and b are defined by $a+jb = Z_e$; and R is Impedance Level of Circuit in which Generator Impedance = Load Impedance = R.

a sufficient range of types to be of general utility. It is, therefore, necessary for each type of equalizer to be designed for its particular purpose, and this involves as much art as science. Inspection of the required loss curve that the equalizer has to provide gives a clue to the general form of equalizer. In cases where the required form is not provided by any of the basic curves of Figs. 5 and 6/XXI:5, a very useful form of equalizer is shown in Fig. 7 at A_1 and B_1 which show respectively the forms of network constituting Z_a and Z_b (either or both of which may be used according to the type of structure employed). If A_1 is a series element it can be made to

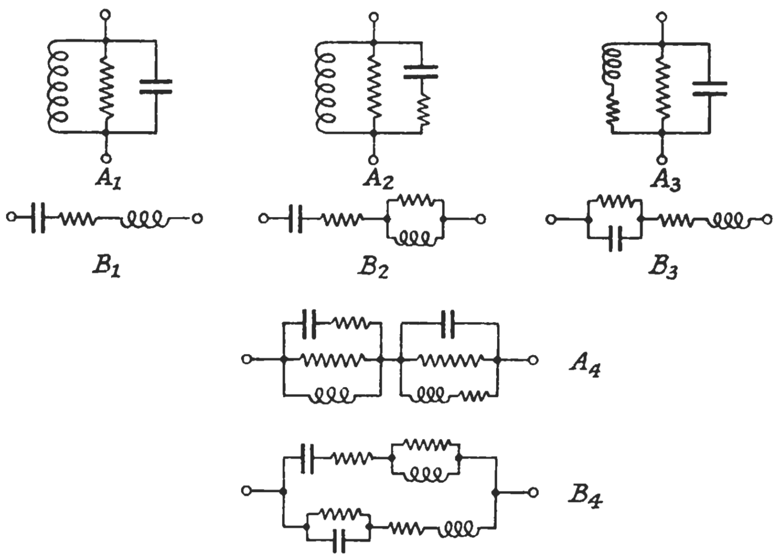


FIG. 7/XXI:4.—Impedance Elements for Inserting Losses in Different Parts of the Frequency Spectrum.

introduce a fall in the response curve of width in the frequency range which can be varied by changing the L–C ratio : and of magnitude which can be varied by changing the value of R. The form of the loss curve which this type of network is capable of introducing is shown on Fig. 12/XXI:5, which also gives the formula for the overall V.T.R., from which the insertion loss can be calculated for any particular value of Q and n. See definition of insertion loss in XXI:4. Fig. 13/XXI:5 shows an insertion loss curve for the same network when $Q = 45$, which normally is not of any interest from the point of view of equalization, but illustrates the performance of this circuit when used as an acceptor circuit for purposes of eliminating an unwanted frequency. If A_1 is a shunt element (in which case B_1 is the corresponding series element) it can be made to introduce a rise in the response curve, of width in the frequency range controlled by the L–C ratio, and of magnitude controlled by the magnitude of the resistance.

Assuming that a network corresponding to A_1 and B_1 has been built to introduce a rise or fall in response at the low frequency end of the pass range, if the rise or fall of loss respectively in the middle of the range is too small, it can be increased by inserting a resistance in series with the condenser as shown at A_2 with a corresponding resistance in shunt with the inductance in B_2 if used.

If, on the other hand, the network of a type at A_1 has been used to insert a rise or fall of response at the top end of the frequency range, and if the rise or fall is not great enough in the middle of the band, it can be increased by inserting a resistance in series with the inductance as shown at A_3 with a corresponding resistance in parallel with the capacity in B_3 if used. See Figs. 5 and 6/XXI:5.

By putting in series, networks of the types shown at A_2 and A_3, a rise in response may be introduced at each end of the pass range. Such a network is shown at A_4 with its corresponding inverse network at B_4. In this case, of course, A_4 is a shunt element and B_4 is a series element. If series resonance of L of A_2 with C of A_3 gives too high loss, two separate equalizers may be necessary.

It should be noted that where the required rise in response at each end of the pass band is not sharp, this can be achieved by a shunt element of the type shown at B_1, which may then be regarded at the low-frequency end as constituting a network of the type shown on Fig. 5/XII:5 (shunt impedance consisting of series condenser and resistance) and at the upper end of the frequency range as constituting a network of the type shown on Fig. 6/XXI:5 (shunt impedance consisting of series inductance and resistance). If the required curves at the bottom end of the range and the top end of the range correspond respectively to any corresponding curves on Figs. 5 and 6/XXI:5, providing the values of f_0 for the two networks are sufficiently far apart, the values of inductance and capacity can be determined separately from these two figures. The reason for specifying corresponding curves on the two figures is to introduce the limitation that each end of the networks demands the same value of r_c or r_L. This network may be regarded as a case of the network on Fig. 12/XXI:5 with a very low value of Q, e.g. $Q = 0{\cdot}1$ or less.

Broadly, therefore, when it is not possible to choose a network from prepared characteristics, a series of intelligent trials have to be made with networks of different form and different values of elements until one is found which gives the required characteristic. For the purpose of these trials the response curves may be determined by calculation as described above or better still by measurement, using the types of components which are finally going to be employed. This is quite an important point, because in practice the variation of the impedance of inductances with frequency does not always correspond to that of a pure inductance. For the purpose of such a trial, the circuit of Fig. 8 may be used. O is an oscillator supplying an input resistance R_b in series with the impedance Z_b, representing the impedance under measurement. V is a valve voltmeter with a high impedance input capable of being switched so as to measure either the voltage across Z_b or the voltage output from the oscillator. By this means the V.T.R. can be measured directly and so can be

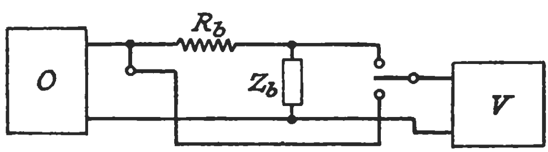


FIG. 8/XXI:4.—Circuit for Measuring Voltage Transfer Ratio.

obtained the V.T.L. and the insertion loss for any equivalent structure. If a transmission measuring set with a high impedance input is available this may, of course, be used.

In the equalization of landlines and in similar cases where equalizers have to be designed comparatively frequently to match or correct response or loss characteristics of a particular kind, it is worth while developing a standardized technique of plotting equalizer characteristics and line characteristics. This means little more than the adoption of standard scales of frequency and loss : suitable scales are shown in Fig. 5/XXI:5 ; loss or response is plotted in decibels and frequency is plotted against a logarithmic scale. Generalized response or loss characteristics are then plotted on transparent paper for each type of equalizer which is found to be most useful : a characteristic or a family of characteristics is plotted for each of a chosen set of values of each variable parameter which changes the form of the response curve : e.g. parameter a in Fig. 5/XXI:5.

By placing these transparencies or masks over loss curves plotted to the same scales of frequency and loss or response (the frequency scale in this case being absolute, however) they may be moved about until one curve on the mask coincides sufficiently closely with the curve to be matched or equalized. The frequency opposite the unity frequency on the mask then gives the value of ω_0 from which the values of the components can be calculated, using the value of parameter written against the curve.

As an example suppose that, using a mask having the form of Fig. 5/XXI:5, the curve marked $a = 0{\cdot}5$ was found to match the required loss curve sufficiently accurately when the unity point of the frequency scale was opposite 1,000 c/s on the frequency curve of the curve to be matched. Suppose, then, that it was required to make a bridged-T equalizer of the kind shown at (*f*) in Fig. 4/XXI:4 to work between impedances of 600 ohms, i.e. $R = 600$ ohms so that R_b is 300 ohms. The elements of Z_b are then $C = \dfrac{1}{2\pi f_0 R_b}$, where $f_0 = 1{,}000$ c/s, and $r = aR_b$ where $a = 0{\cdot}5$. The elements of Z_a then follow by inverting C and r about 600 ohms.

When constant-resistance equalizers are always being used it is more useful to label the masks directly in terms of the elements of either the Z_a or the Z_b impedance arm.

Fig. 9 shows such a mask with the Z_a arm indicated at the top right corner. In this case no scale of frequency is normally provided, although a relative one has here been shown to indicate what scale is used for plotting the characteristics to be matched. The calibration of the frequency scale is obtained from the frequencies of zero loss which correspond to the resonant frequencies of L and C. Evidently if the value of L/C is known and the value of the resonant frequency of L and C is known the values of L and C are determined. These values and the value of the resistance R_a correspond to a value of R equal to 600 ohms ; in other words, this equalizer is intended to be used in a circuit of 600 ohms impedance.

In the opinion of the present writer, the method of presenting generalized characteristics used in Fig. 5/XXI:5, and neighbouring figures, is preferable to that shown in Fig. 9.

Whether equalizer characteristics are plotted as response curves, the loss being plotted as a negative quantity as in Fig. 5/XXI:5 and neighbouring figures, or whether they are plotted as loss curves as in Fig. 9, depends only on the convention being used for plotting the characteristics of the structure being equalized. If these are plotted as loss curves, then it may be considered preferable to plot equalizer characteristics as response curves, and vice versa.

This point will be made clear by reference to Fig. 10, which shows how equalizer response characteristics, when plotted to the

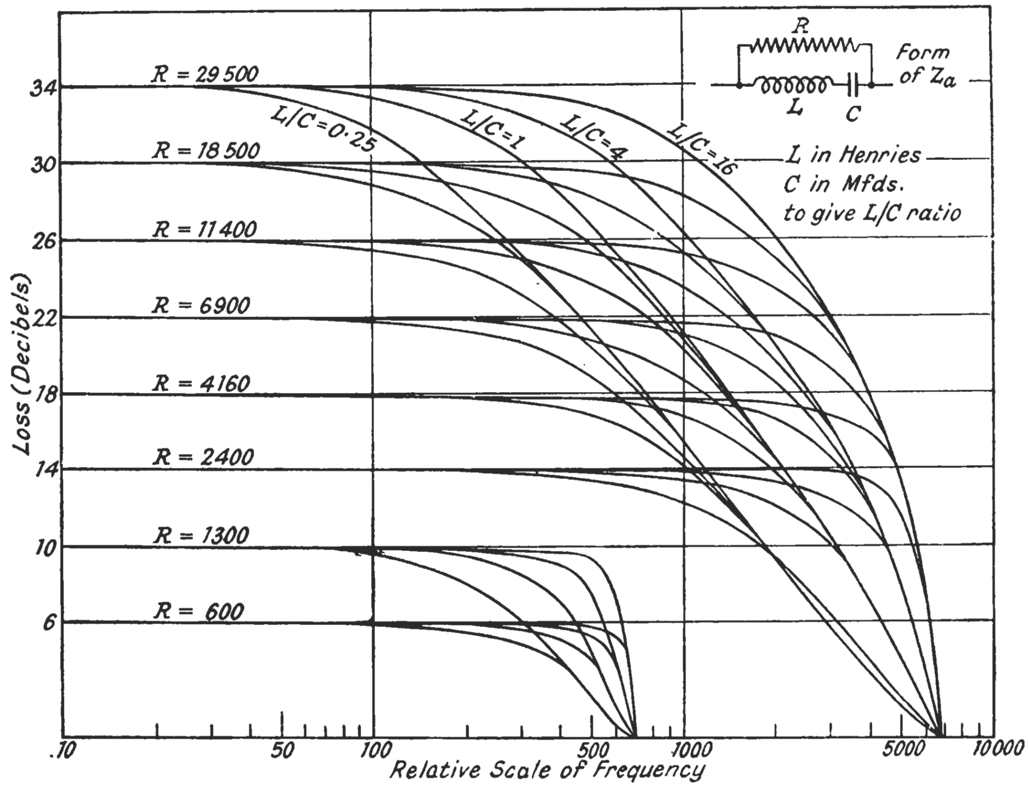


FIG. 9/XXI:4.—Typical Mask for Constant Resistance Equalizer.
(By courtesy of the B.B.C.)

convention shown, may be added to line-loss characteristics to give an overall flat response as indicated by the chain-dotted curve which is assumed to be drawn on the same paper as the structure (e.g. line) characteristic. Similarly, equalizer loss characteristics may be subtracted from structure response characteristics to give an overall

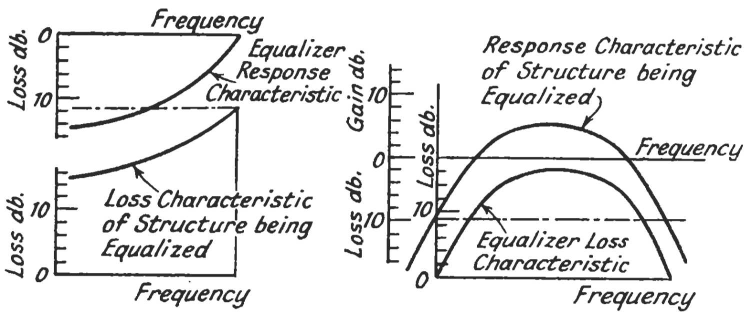


FIG. 10/XXI:4.—Use of Loss and Response Characteristics.

flat response. In Fig. 10 it is assumed that the equalizer mask is in the position it would be just before it was slid so that the two characteristic curves coincide.

The use of masks for equalization has been practised in the B.B.C. for a considerable time.

5. Generalized Network Characteristics.

The advantage of algebra over arithmetic is the facility it affords for presenting generalized solutions to problems. These are expressed in terms of symbols to which values may be assigned corresponding to the arithmetical values which occur in any practical problem to which the solution applies. If a voltage V is applied across a resistance R a current flows of value $I = \frac{V}{R}$, and if in any particular case $V = 24$ and $R = 12$, then $I = \frac{24}{12} = 2$ amps. $I = \frac{V}{R}$ is therefore a generalized solution of the problem as to what happens when any voltage is applied across any resistance. It describes what happens independently of the particular values of V and R.

Similarly, generalized network characteristics are drawn so that the performance of networks of resistance, inductance and capacity is described independently of the particular values of resistance, inductance and capacity and independently of the absolute value of frequency. A simple transformation of the scales of the axes of abscissae and co-ordinates is then used for each particular case which calibrates these axes in terms of absolute values of the elements and of frequency.

The characteristics of networks which can be portrayed in this way are :

For two-terminal networks : Impedance.

For four-terminal networks (four-poles) : Voltage transfer constant, voltage transfer loss, insertion loss and attenuation (the attenuation of a network is its insertion loss when placed between a generator and load respectively, having impedances equal to the image impedances of the network at the terminals which face them).

Figs. 1 to 13 show a number of generalized network characteristics. Their method of calibration and method of use for any practical case will be made clear by consideration of the network of Fig. 1. This is a two-terminal network and the curves represent generalized values of the series reactance and resistance, observed at the terminals of the network, plotted against a generalized scale of frequency. The way in which these curves are plotted will serve as an introduction to their method of use.

The impedance presented by an inductance L in parallel with a resistance R is

$$Z = \frac{jRL\omega}{R+jL\omega} = \frac{jRL\omega(R-jL\omega)}{R^2+L^2\omega^2}$$
$$= \frac{RL^2\omega^2+jR^2L\omega}{R^2+L^2\omega^2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If ω_0 is the angular frequency at which the reactance of L is equal to R, then

$$L\omega_0 = R \text{ and } L = \frac{R}{\omega_0} \quad . \quad . \quad . \quad (2)$$

Substituting (2) in (1)

$$Z = \frac{\frac{\omega^2}{\omega_0^2}R^3+j\frac{\omega}{\omega_0}R^3}{R^2+\frac{\omega^2}{\omega_0^2}R^2} = R\left[\frac{\frac{\omega^2}{\omega_0^2}}{1+\frac{\omega^2}{\omega_0^2}}+j\frac{\frac{\omega}{\omega_0}}{1+\frac{\omega^2}{\omega_0^2}}\right]$$
$$= R\left[\frac{\frac{f^2}{f_0^2}}{1+\frac{f^2}{f_0^2}}+j\frac{\frac{f}{f_0}}{1+\frac{f^2}{f_0^2}}\right] \quad . \quad . \quad . \quad . \quad . \quad (3)$$
$$= R(r+jx) \text{ say} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $$r = \frac{\dfrac{f^2}{f_0^2}}{1+\dfrac{f^2}{f_0^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$x = \frac{\dfrac{f}{f_0}}{1+\dfrac{f^2}{f_0^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\omega_0 = 2\pi f_0 \text{ and } \omega = 2\pi f.$$

On Fig. 1 is plotted the value of r against $\dfrac{\omega}{\omega_0} = \dfrac{f}{f_0}$ from equation (5) and the value of x against $\dfrac{\omega}{\omega_0} = \dfrac{f}{f_0}$ from equation (6).

To calibrate graphs for any particular value of R and L it is therefore only necessary to multiply the scale of ordinates by R and the scale of abscissae by f_0, where $f_0 = \dfrac{R}{2\pi L}$.

Example 1. An inductance of 10 millihenrys is shunted with a resistance of 1,000 ohms.

The value of f_0 is given by $f_0 = \dfrac{1{,}000}{2\pi \times 10 \times 10^{-3}} = 15{,}900$ c/s.

Hence on the scale of abscissae, against $\dfrac{\omega}{\omega_0} = 1$, write 15,900 c/s, against $\dfrac{\omega}{\omega_0} = 0{\cdot}1$ write 1,590 c/s, against $\dfrac{\omega}{\omega_0} = 10$ write 159,000 c/s, etc. The values of r and x read off from the curves must then be multiplied by 1,000 to give the values of reactance and resistance.

Example 2. What is the reactance of the above parallel combination of inductance and resistance at 12,000 c/s? 12,000 c/s corresponds to a value of $\dfrac{f}{f_0} = \dfrac{12{,}000}{15{,}900} = 0{\cdot}755$. Entering this on the $\dfrac{f}{f_0}$ scale at 0·755 the values of r and x are $r = 0{\cdot}365$, $x = 0{\cdot}482$. Hence the impedance is given by $Z = 365+j482$ ohms.

Fig. 2 shows the generalized impedance curves for a resistance and capacity in parallel.

Figs. 3 and 4 show generalized impedance curves for two R, L, C combinations in which in each case the reactance of both inductance and capacity reaches the value R at the same frequency f_0, so that $L\omega_0 = \dfrac{1}{C\omega_0} = R$. It is only necessary, therefore, to specify

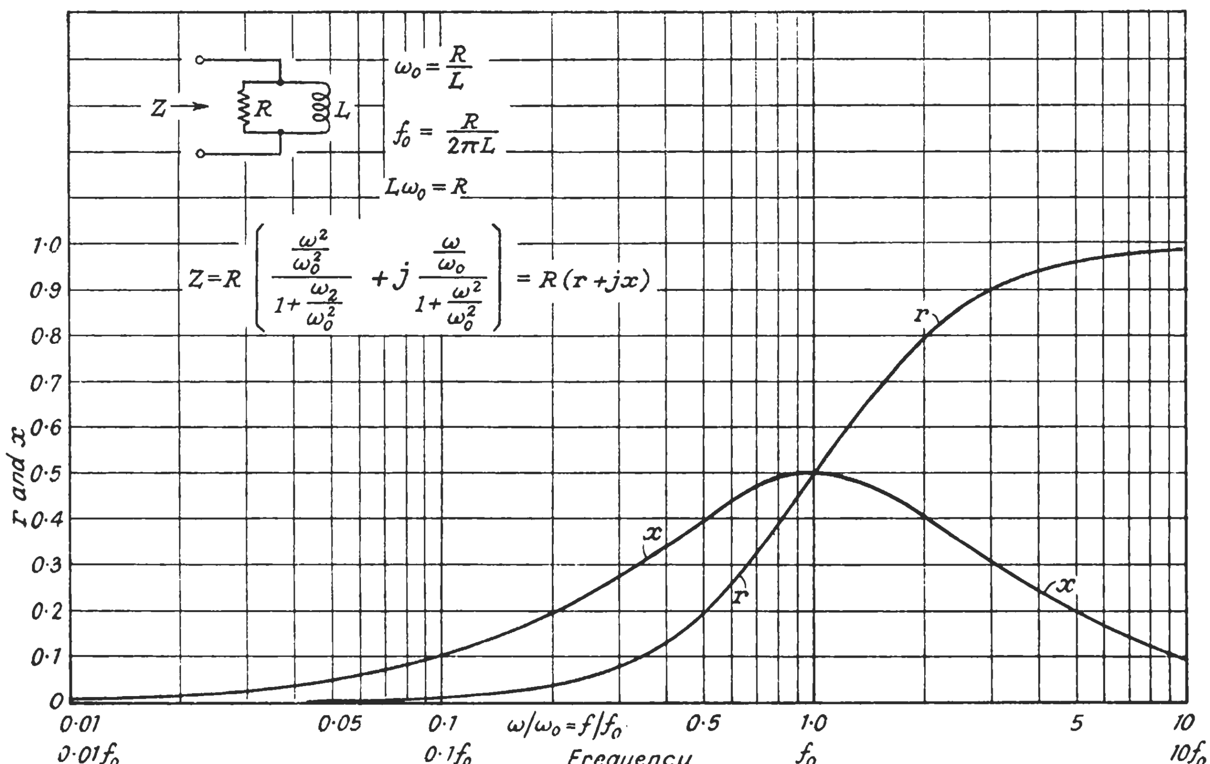


FIG. 1/XXI:5.—Generalized Resistance and Reactance Curves for Network Shown.
(By courtesy of S.T. & C. and *Wireless Engineer*.)

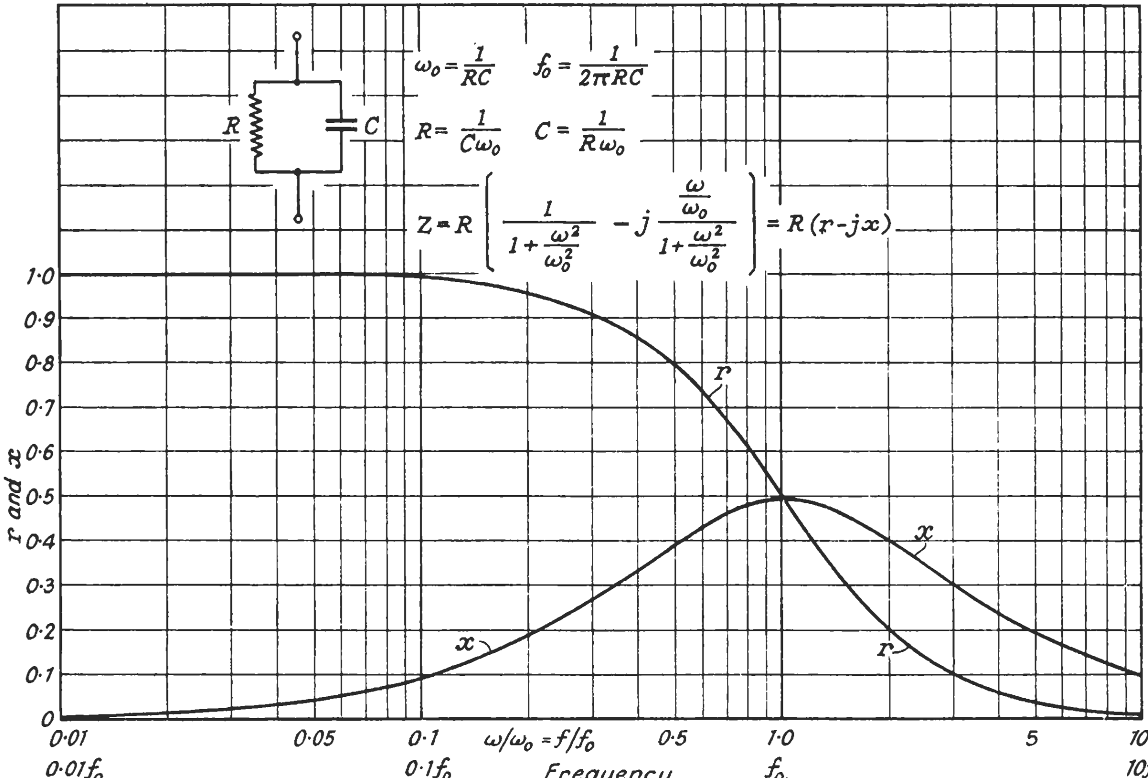


FIG. 2/XXI:5.—Generalized Resistance and Reactance Curves for Network Shown.
(By courtesy of S.T. & C. and *Wireless Engineer*.)

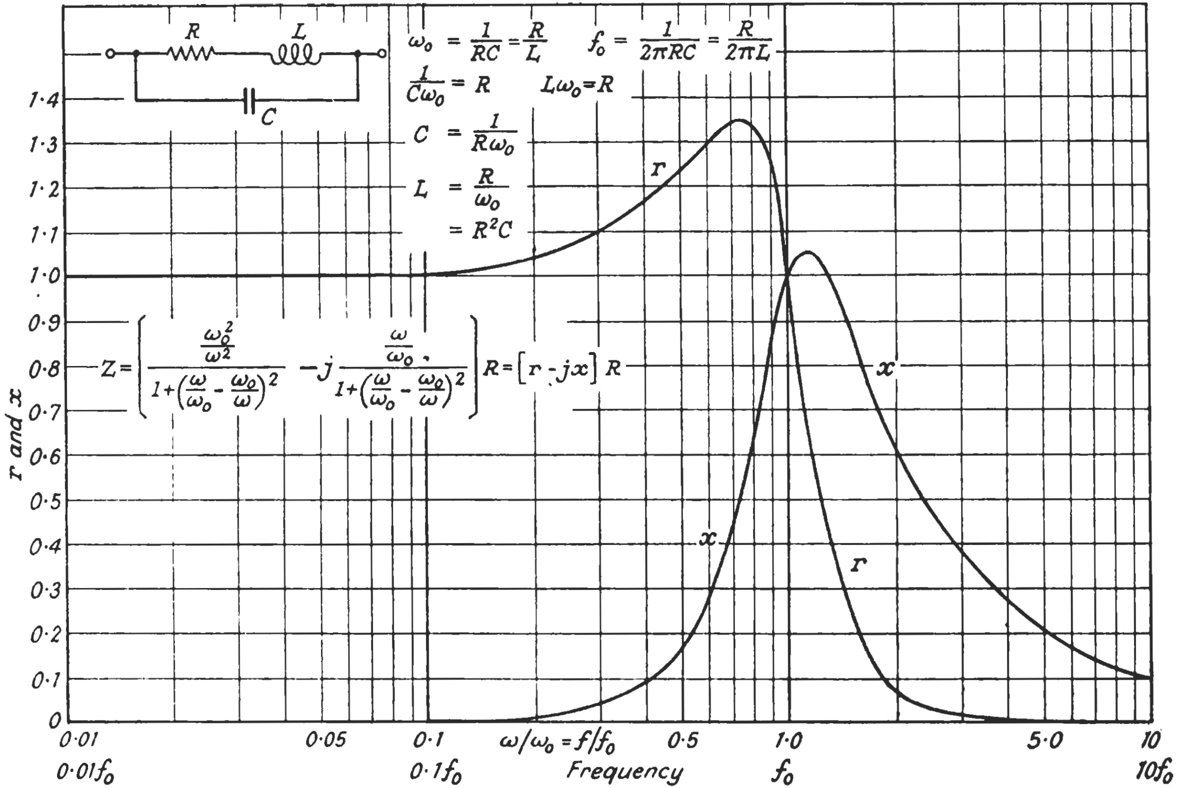


Fig. 3/XXI:5.—Generalized Resistance and Reactance Curves for Network Shown.
(By courtesy of S.T. & C. and *Wireless Engineer*.)

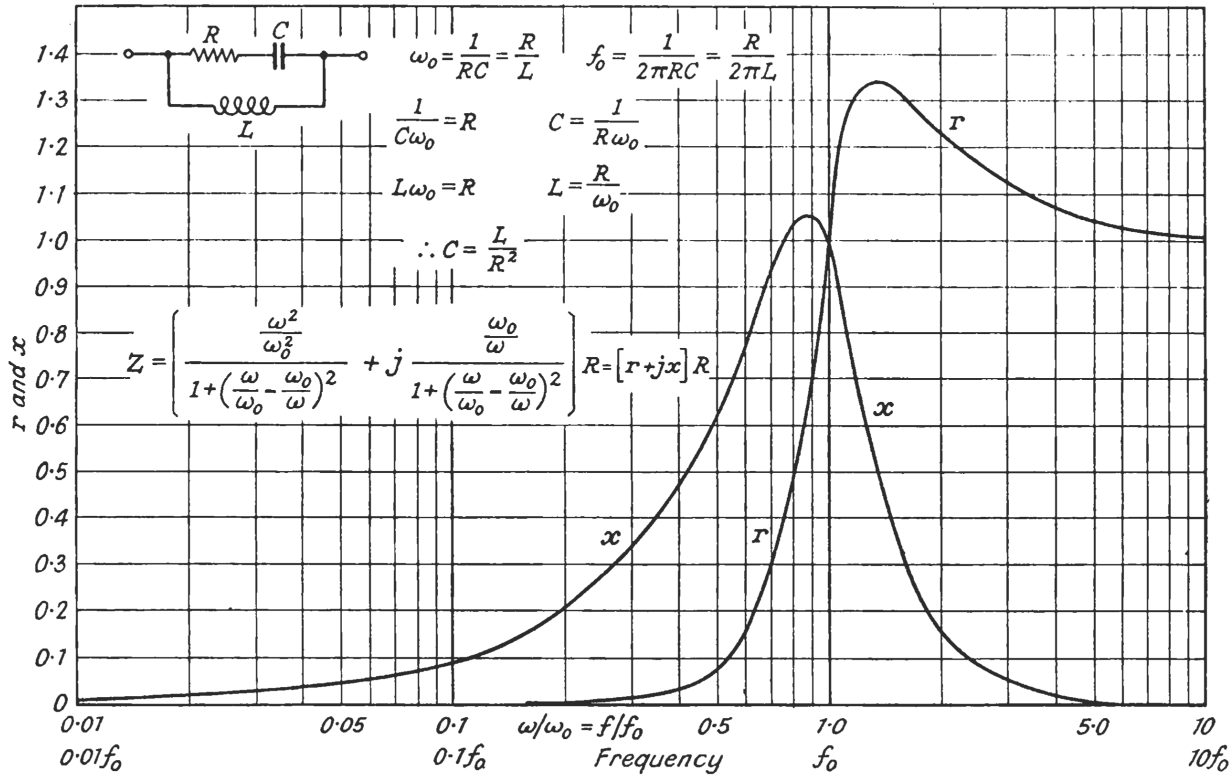


FIG. 4/XXI:5.—Generalized Resistance and Reactance Curves for Network Shown.
(By courtesy of S.T. & C. and *Wireless Engineer*.)

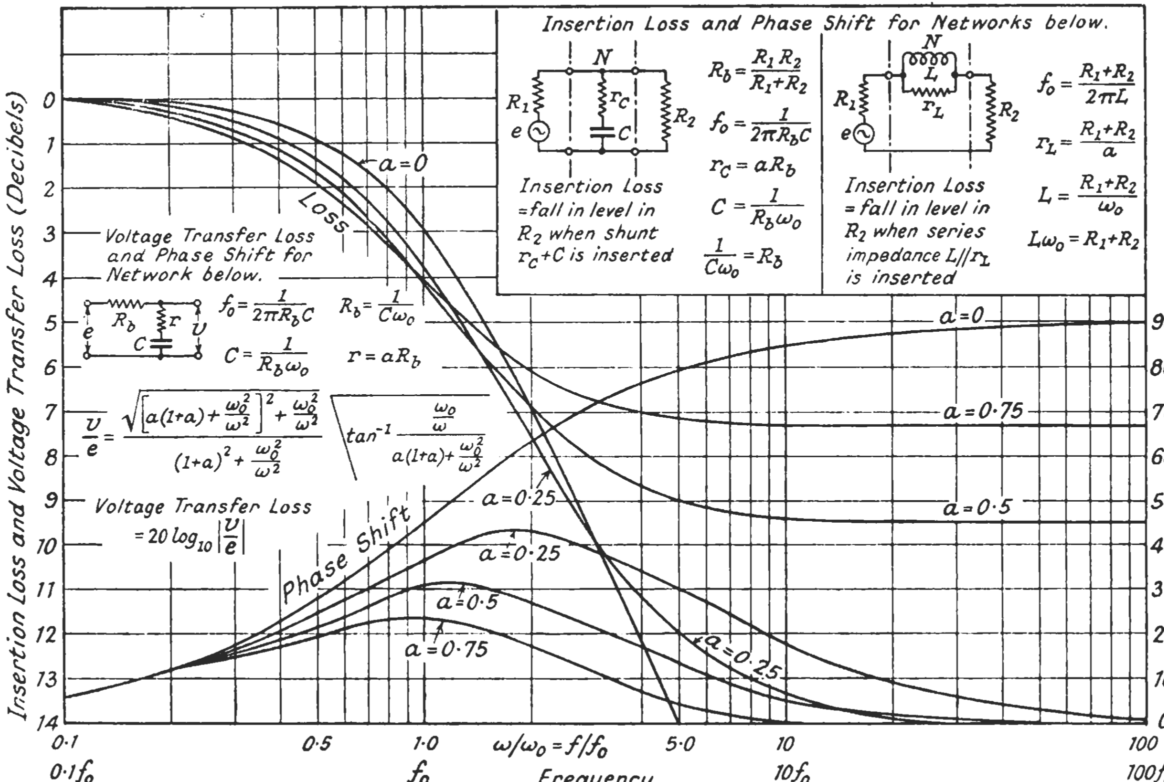


FIG. 5/XXI:5.—Loss and Phase Characteristics of Networks Shown, under Conditions specified, and of Equivalent Constant Resistance Structures.

(By courtesy of S.T. & C. and *Wireless Engineer*.)

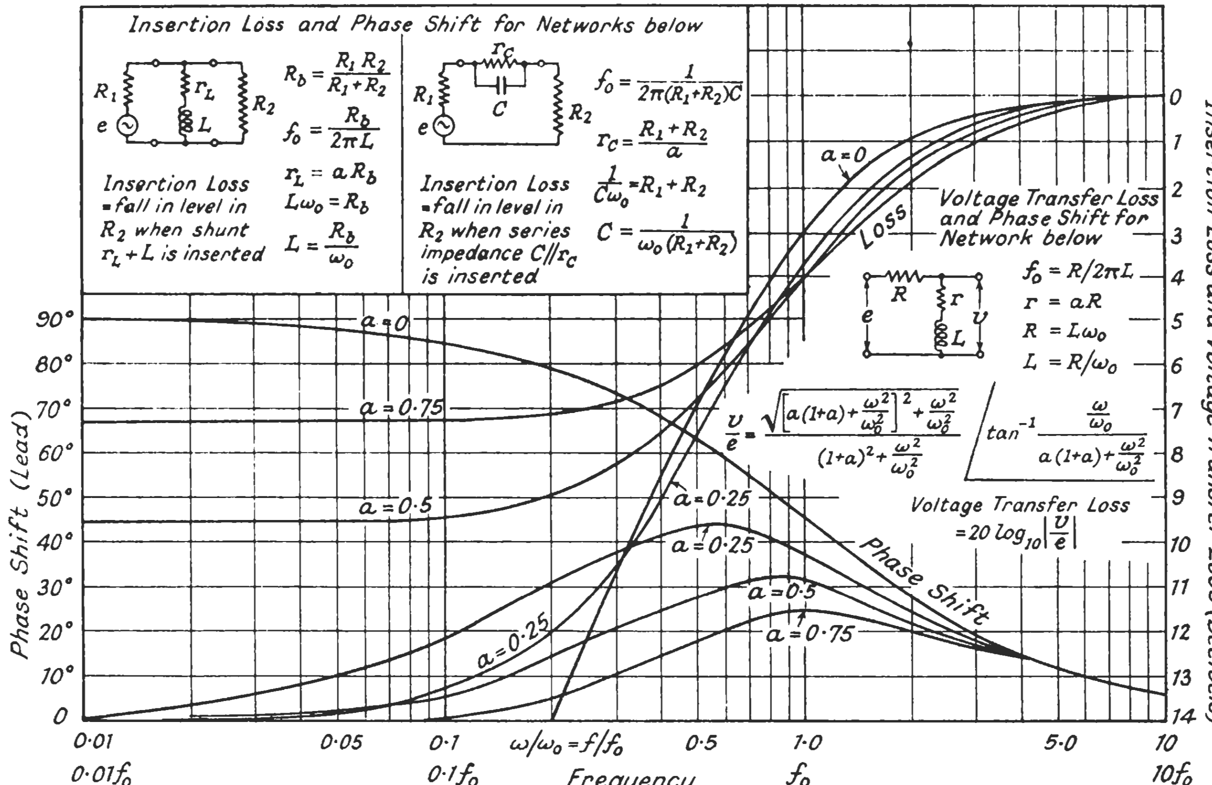


FIG. 6/XXI:5.—Loss and Phase Characteristics of Networks Shown, under Conditions specified, and of Equivalent Constant Resistance Structures.

(By courtesy of S.T. & C. and *Wireless Engineer*.)

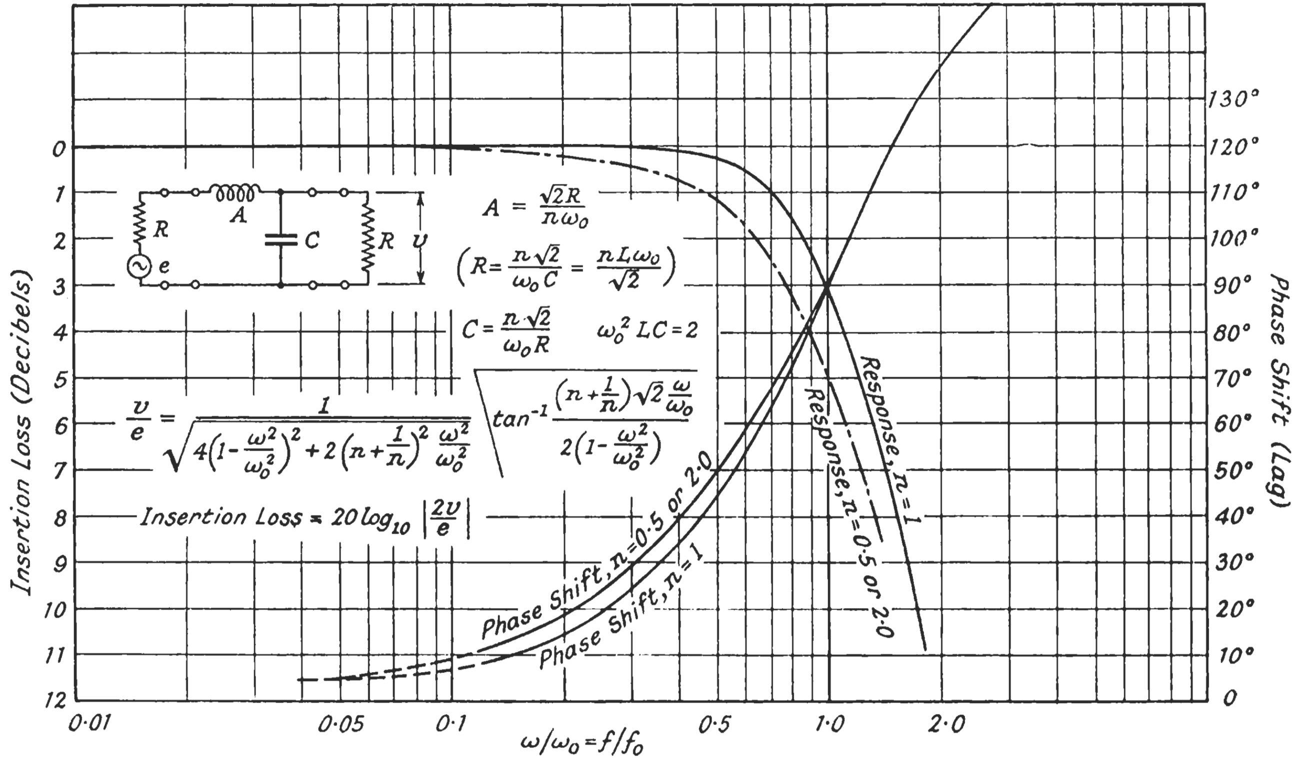


FIG. 7/XXI:5.—Insertion Loss and Phase Shift of Network Shown.

(By courtesy of S.T. & C. and *Wireless Engineer*.)

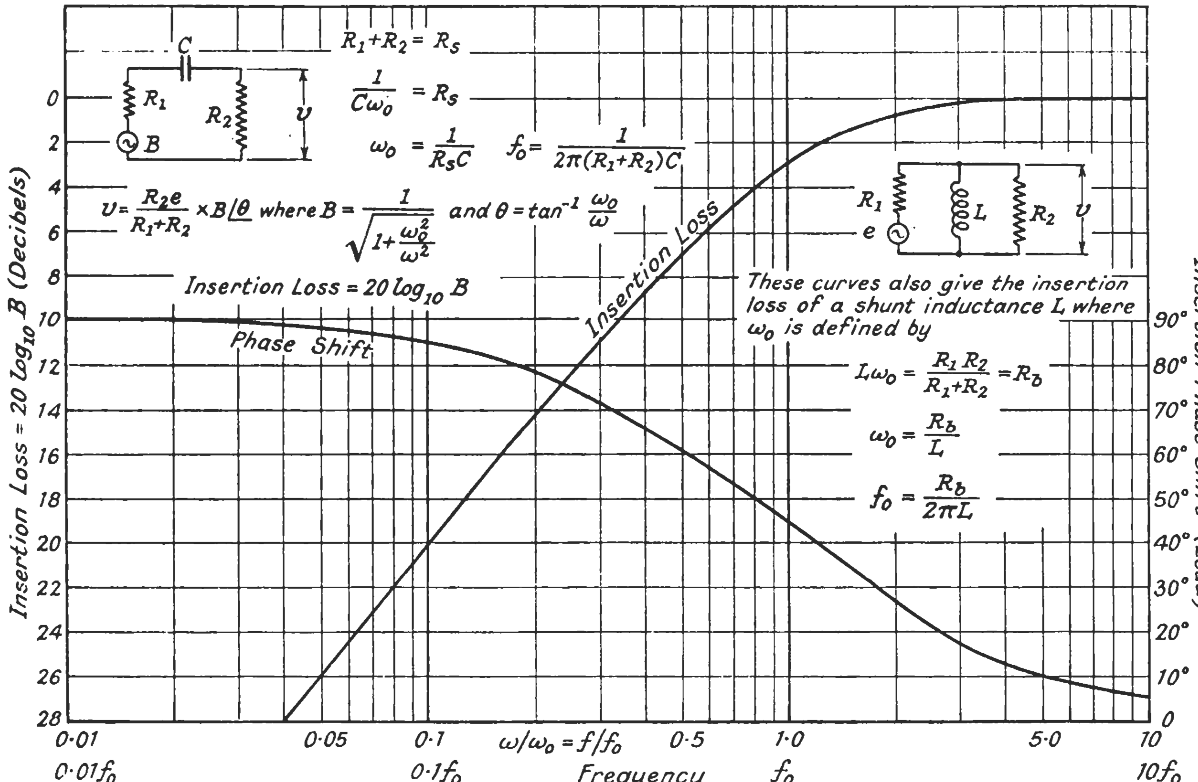


FIG. 8/XXI:5.—Insertion Loss and Phase Shift of Series Capacity and Shunt Inductance between Resistances R_1 and R_2.
(By courtesy of S.T. & C. and *Wireless Engineer.*)

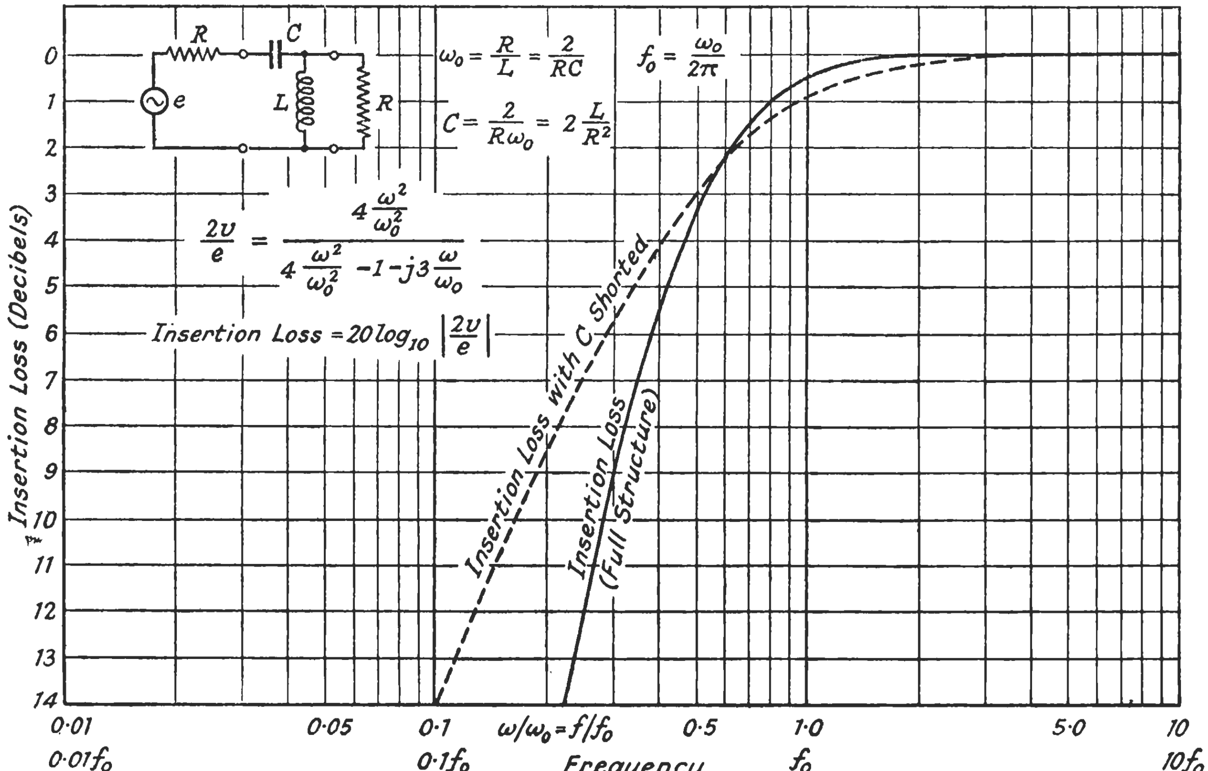


FIG. 9/XXI:5.—Insertion Loss of Network Shown.
(By courtesy of S.T. & C. and *Wireless Engineer*.)

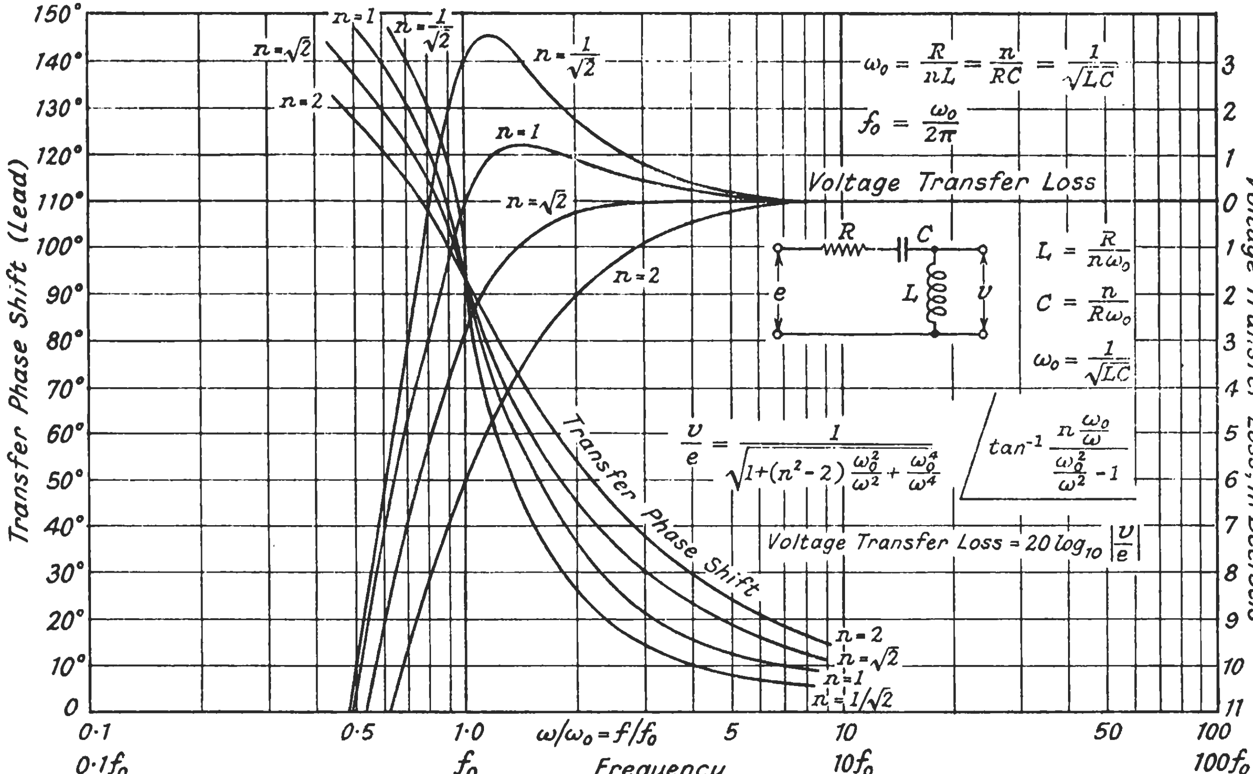


FIG. 10/XXI:5.—Voltage Transfer Loss and Phase Shift of Network Shown.

(By courtesy of S.T. & C. and *Wireless Engineer.*)

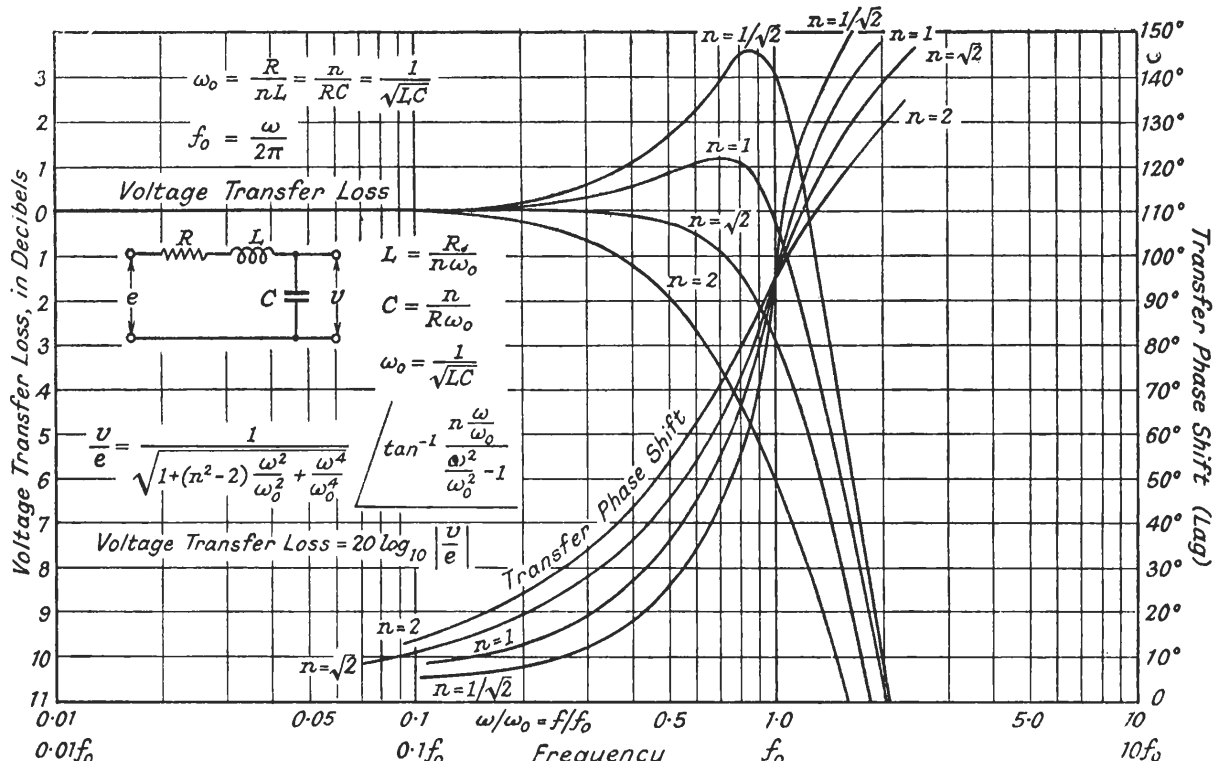


FIG. 11/XXI:5.—Transfer Loss and Phase Shift of Network Shown.
(By courtesy of S.T. & C. and *Wireless Engineer*.)

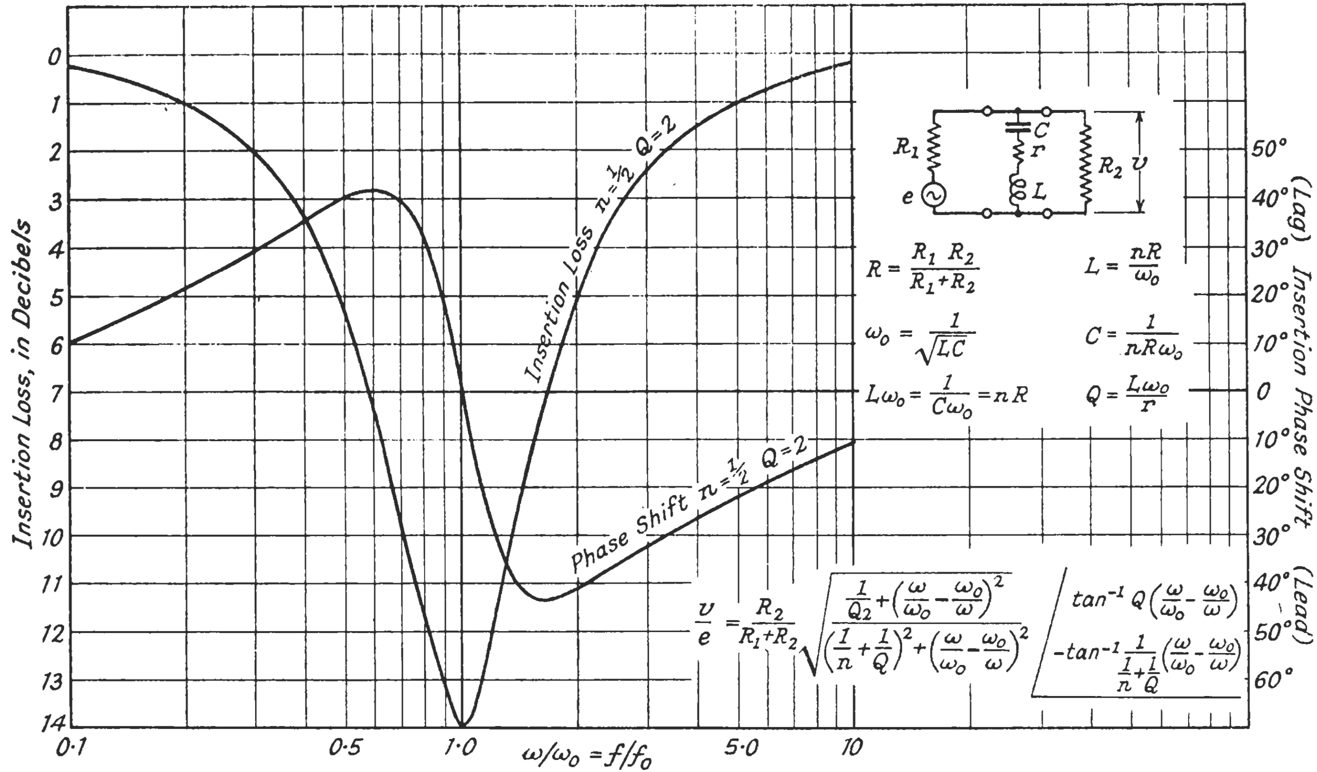
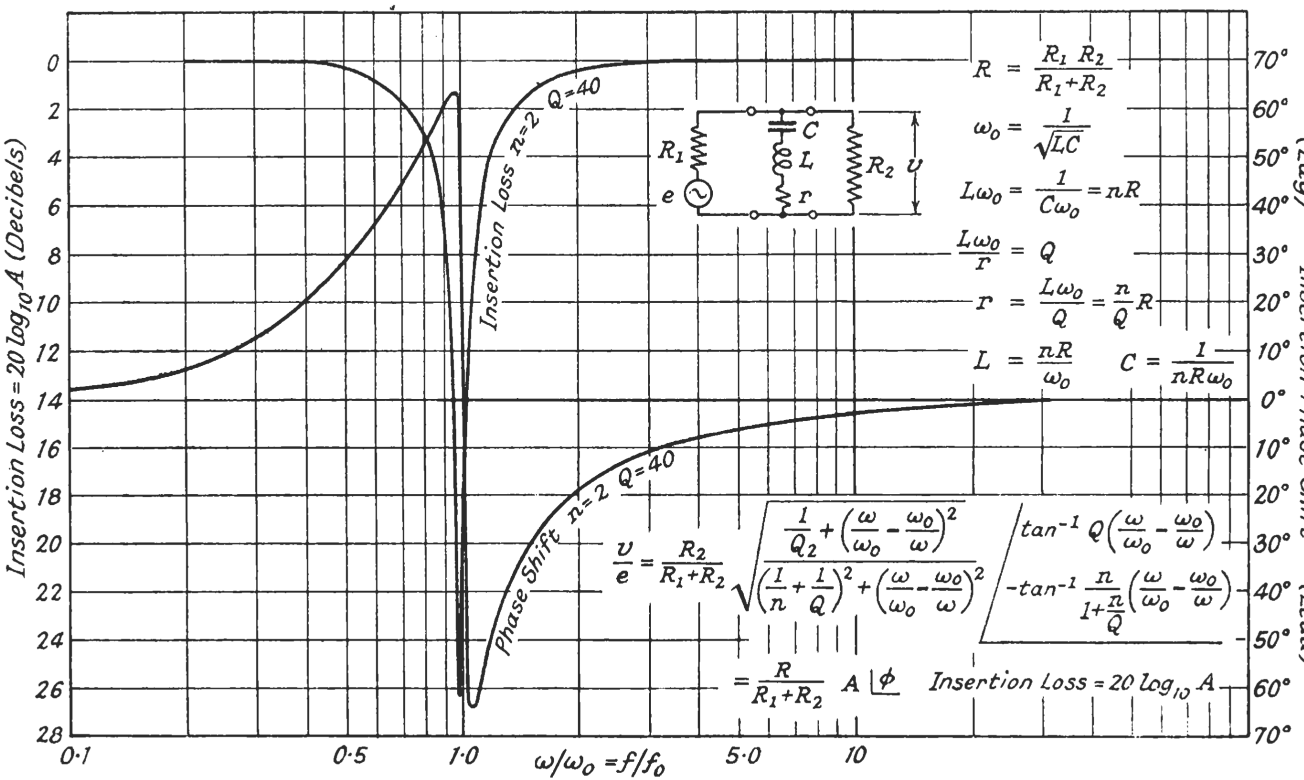


FIG. 12/XXI:5.—Insertion Loss and Phase Shift of Network Shown.

(By courtesy of S.T. & C. and *Wirelcss Engineer.*)



$$\frac{v}{e} = \frac{R_2}{R_1+R_2}\sqrt{\frac{\frac{1}{Q_2}+\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}{\left(\frac{1}{n}+\frac{1}{Q}\right)^2+\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}} \Bigg/ \tan^{-1} Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right) - \tan^{-1}\frac{n}{1+\frac{n}{Q}}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$

$$= \frac{R}{R_1+R_2} A \underline{\big|\phi}$$

FIG. 13/XXI:5.—Insertion Loss and Insertion Phase Shift of Network Shown.
(By courtesy of S.T. & C. and *Wireless Engineer.*)

two of the quantities L, R, C or ω_0 in order to specify completely one physical embodiment of the network and to obtain the proper calibrations for the scales.

The process of disembodiment or generalization of any network containing n elements consists therefore in the following steps :

(1) Calculate the characteristic of the network of interest in terms of L, C, R and ω. This characteristic may be impedance, V.T.R., V.T.L., or insertion loss.

(2) Equate the reactance $L\omega_0$ or $\frac{1}{C\omega_0}$ of one inductance or capacity to the value of one resistance R in the network at an arbitrary angular frequency ω_0.

(3) Equate the reactance, at ω_0, of any other inductances or capacities, or the value of any other resistance in the network to any constant times R where the constant may have any positive value. There are therefore $n - 2$ constants or parameters. Every different value of each parameter defines a different network of the same form.

(4) Plot the characteristics of the network so calculated against $\frac{\omega}{\omega_0}$.

An example of the case of step 3 is given in Figs. 5 and 6. In Fig. 5 the resistance r_c in series with the condenser is defined by $r_c = aR_b$ and characteristics are plotted for each of a number of values of a. These characteristics define the V.T.L. of one type of network and the insertion loss of two other types of network. Each of the three inserts in Fig. 5 define the conditions under which the characteristics of Fig. 5 are applicable to the network in question. Similar remarks apply to Fig. 6. See also XXIV:2.23, Example 2.

Fig. 7 shows the insertion loss due to the leakage inductance and self-capacity of a transformer, and so defines the response at the high-frequency end of any transformer of high-impedance ratio, working between primary and secondary impedances R_1 and R_2 such that $\frac{R_1}{R_2} = \frac{L_1}{L_2}$. A then represents the leakage inductance as determined by measuring, at a frequency in the middle of the pass range of the transformer, the inductance looking into the high-impedance winding of the transformer with the low-impedance winding short circuited. The capacity C represents the self-capacity of the high-impedance winding and can be determined by measuring the capacity looking into the high-impedance winding at a frequency equal to 10 or 20 times the upper cut-off frequency of the transformer. The value of the resistances R then represent respectively the impedance facing the high-impedance winding, and the impedance referred or transferred from the low-impedance to the high-impedance winding, which is evidently of the same magnitude.

Fig. 8 is of very general application since it determines the low-frequency response characteristics of resistance-capacity couplings, and also of the low-frequency end of the pass range of transformers.

Fig. 9 has already been discussed in XXI:3.

Fig. 11 represents the response at the high-frequency end of a transformer working into an open circuit or a grid capacity. R is then the impedance looking back into the transformer with the primary terminated in its working impedance, L is the leakage inductance observed on the open-circuited side, and C is the self-capacity of the transformer plus grid-input capacity of any valve driven by the transformer. Separate networks of this kind may be used as open-circuit equalizers for correcting response characteristics at the high-frequency end of the pass range of a circuit.

Similarly, the network on Fig. 10 may be used as an open-circuit equalizer to correct the response characteristics at the low-frequency end of the pass range of a circuit.

Fig. 12 has already been discussed in XXI:4.3.

Fig. 13 shows how the characteristics of the network of Fig. 12 may be sharpened by changing the values of the parameters n and Q.

Example 2 in XXIV:2 gives the derivation of the generalized formula for the impedance of a parallel resonant circuit.

CHAPTER XXII

LEVEL RANGE COMPRESSION AND EXPANSION AND LEVEL LIMITATION

1. Objects.

It has already been pointed out that the range of levels occurring in speech and music in a studio is very large and may be as much as 100 db. For reasons connected with noise, and with normal listening conditions as well as with the performance of apparatus, the level range of practical importance for broadcasting is less than this, and appreciable advantage is obtained by compressing this range by manual operation of a volume control at the control position. The control operator attempts to hold the volume within a range of 20 db., but is not always successful in doing this. Further, even if the volume as indicated by a peak-programme meter is kept within this range it is certain that the absolute change of level is very much greater than 20 db., and it is in fact desirable that this should be the case. This is because the decay time constant of the peak-programme meter is such that the reading falls by 20 db. in two to three seconds after cessation of signal. Low levels of duration less than this which occur between periods of higher level are therefore not indicated at their true level, but at a higher level.

For these and other reasons automatic means of volume compression have been proposed, but there are serious objections to such schemes and so far they have not been put into general use. It is therefore not proposed to consider in detail any arrangements for level range compression, but merely to describe diagrammatically what is involved in order to distinguish between level-range compression and level limitation.

Since the control operator does not always succeed in preventing the volume from exceeding the level which will fully load the transmitter or transmitters being fed, limiting devices are often fitted in shunt across part of the transmitter circuit to prevent the occurrence of unduly high voltages and the consequent risk of flashover. These devices usually consist of some circuit element such as neon lamp which does not pass any current until the voltage reaches a certain value, and when this voltage is reached they instantaneously start to pass current and constitute a shunt across the circuit, preventing any further rise in voltage, and may even drop the voltage. This introduces non-linear distortion, which is undesirable. A supplementary type of limiter has therefore been recently introduced and inserted in the audio-frequency circuits at the input of a transmitter. This is designed so that when the volume of speech or music reaches the level which is just capable of fully loading the transmitter, the gain of an amplifier in the circuit is progressively reduced at such a ratio and in such a way that the level fed to the transmitter rises no further, but remains substantially constant until the level fed to the limiting device has dropped below the level which fully loads the transmitter.

2. Characteristics of Level-Range Compressors, Expanders and Limiters.

The simplest way of showing the performance of level-range compressors, expanders and limiters is by means of a curve relating the input and output levels of the device. It may be of some assistance in understanding curves illustrating the performances of such devices to consider first the equivalent curves relating the input and output levels of an amplifier or an attenuator.

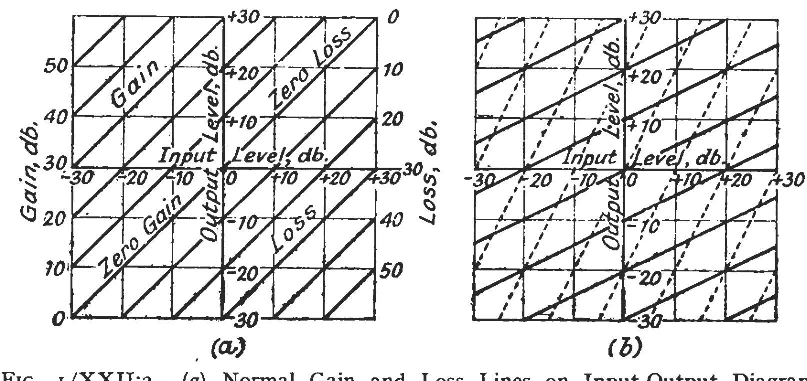


FIG. 1/XXII:2.—(*a*) Normal Gain and Loss Lines on Input-Output Diagram. (*b*) Volume Compression Curves (full line) and Volume Expansion Curves (dotted line).

Fig. 1 (*a*) shows the input-output characteristics corresponding with a number of values of straight gain or loss, respectively, of an amplifier or an attenuator. Each characteristic is simply a plot of the relation between input and output level expressed in decibels relative to one milliwatt.

It is seen that these characteristics constitute a series of straight lines at 45°. In Fig. 1 (*b*) the full lines show typical input-output characteristics for level-range compressors with a compression ratio of 2 : 1. This means that the output-level range is half that of the input-level range. It is evident that to change from one line to another it is only necessary to insert gain or loss before or after the compressor. Also it may be noted that the slope of the line determines the range-compression ratio. The dotted lines show typical input-output characteristics for level-range expanders with an expansion ratio of 2 : 1.

Fig. 2 shows typical input-output characteristics for limiters. It will be noted that for low levels the limiter behaves exactly as an amplifier or attenuator with a certain gain or loss, but that when a certain level *R* is reached, which will be called the roof of the limiter, as the input level is increased further, the output level rises no further or rises very little. Ideally, the output level should rise no further, but in practice it is not possible to achieve the ideal performance and a small rise occurs. The input level at the point *R* may be called the input roof and the output level at the point *R* may be called the output roof.

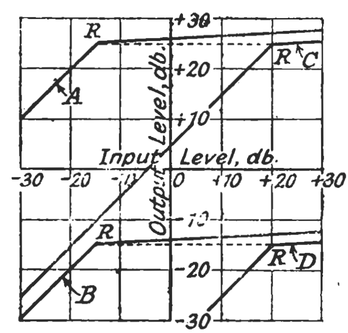


Fig. 2/XXII:2.—Typical Limiter Characteristics.

It may be noted that to change from curves *A* or *B* to *C* or *D* respectively, attenuation must be added (or gain removed) before the limiter, while to change from curves *A* and *C* to *B* and *D* respectively, attenuation must be added (or gain removed) after the limiter. This makes it evident that the level of the input roof can be adjusted independently of the output roof, and vice versa.

Level-range compressors and expanders have not yet come into general use, although it seems probable that they will be introduced in the future ; limiters are in common use.

3. Methods of Using a Limiter.

There are two ways of using a limiter. The first and most obvious method is to adjust it so that the output roof corresponds with the level which fully loads the transmitter or transmitters, while the input roof is adjusted to the maximum level which the control operator is supposed to supply to the limiter. In this case the limiter will never operate unless the control operator allows the level to go too high. The function of the limiter in this case is, therefore, merely to supplement the control operator.

The second method has particular application to short-wave or other circuits in which noise seriously affects reception, so that an increase of signal-to-noise ratio at the expense of volume contrast becomes worth while. In this case the limiter is first adjusted exactly as in the previous case. The gain before the limiter is then increased (or alternatively the attenuation before the limiter is reduced) by an amount which represents the best compromise between noise suppression and contrast, and which is usually between 4 and 8 db. Assume that in a particular case the gain is increased by 6 db. The effect of this is that all levels lower than 6 db. below the peak-programme level (which is supposed to be maintained by the control operator) are increased in level by 6 db. All levels above this are transmitted at the same level, i.e. the roof level. If, in such a case, the frequency of occurrence of programme peaks above the roof level is such that they occupy a comparatively small fraction of the whole programme time, the effect of the use of the limiter is equivalent to increasing the power of the transmitter by the ratio corresponding to 6 db., that is in the ratio of 4 : 1. This is a very large increase indeed.

4. Circuit of Practical Limiter.

Fig. 1 shows the essentials of a limiter similar to that in general use in the B.B.C. (B.B.C. Patent No. 509309). The input is applied through the variable constant-resistance attenuator marked H, through transformer T_1 to the two valves V_1 and V_2 in push-pull. The output of these valves is taken through transformer T_2 and winding (3–4) of this transformer. The above constitutes an ordinary amplifier except that valves V_1 and V_2 are variable-μ valves of which the bias, and therefore the gain, is controlled by the output level as observed on winding (5–6) of transformer T_2. The amplifier described above is called the main amplifier or circuit and the circuit about to be described which provides the bias for the main amplifier valves is called the side circuit. This starts with the potentiometer P controlling the level supplied to the valve V_3, which is an amplifier valve applying rectified input to valve V_4 through transformer T_3 and the copper oxide rectifiers *CuR*. V_4 is a straight amplifier valve supplying the rectified wave form through condenser C_4 to the diode circuit constituted by V_6, R_9, C_5 and the timing resistances. The object of inserting the rectifiers *CuR* is, firstly, to ensure that any sudden incoming high-amplitude pulse operates immediately to reduce the gain of the main amplifier, regardless of its sense, and secondly, to act as a precaution against feedback and tendency to

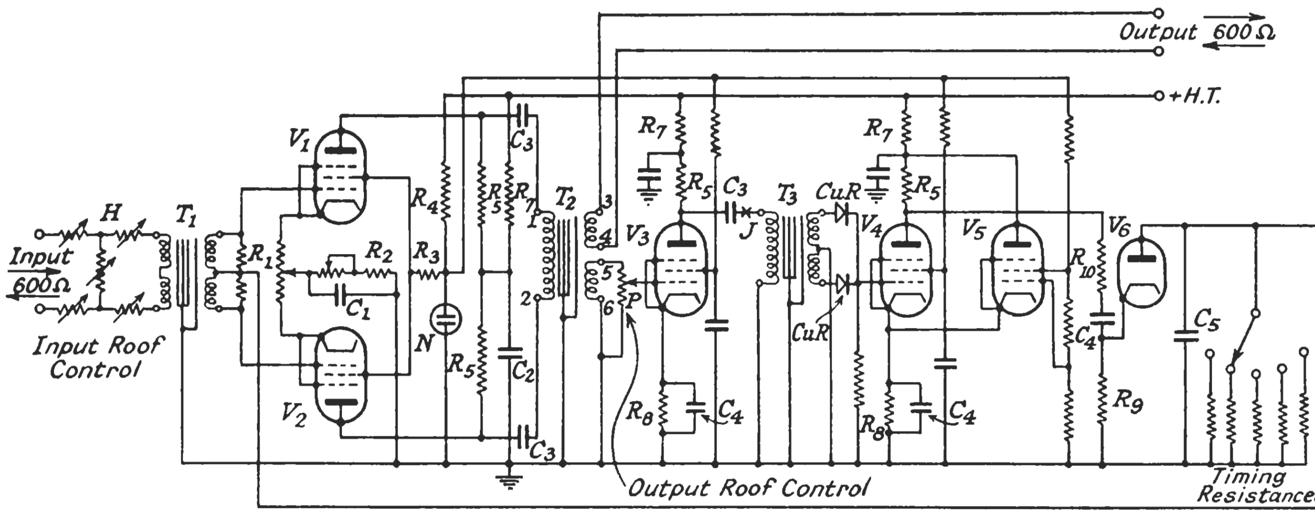


FIG. 1/XXII:4.—Circuit of Practical Limiter.
(By courtesy of the B.B.C.)

sing round the path constituted by the side circuit and main amplifier. The cathode of V_4 is maintained at about 20 volts positive by V_5 in order that the side circuit, and so the limiting action, shall not start to operate until the required input and output roofs are reached. When the input and output roofs are reached or exceeded the output voltage developed by the diode V_6 is fed to the midpoint of transformer T_1 and so reduces the gain of the main amplifier.

On the occurrence of a sudden overload the grid of V_4 becomes positive, and to prevent the condenser C_5 from being too rapidly charged a series resistance R_{10} is inserted. This has the effect of reducing the speed of operation, but even with this reduction the pick-up time, or charge time, of condenser C_5 is of the order of one or two milliseconds and has been made slightly faster than that of the peak-programme meter, so that a peak-programme meter following the limiter will register no overloads. In any case, overloads which do not exceed 5 milliseconds in duration are undetectable by ear. The discharge time of condenser C_5 can be varied by means of the timing resistances to any of the following alternative values: 0·1, 0·5 second, 2, 10 and 40 seconds. It is normally employed on the 0·5-second position. The provision of such a range of time constants was to some extent experimental, the longer time constants being intended for use when the equipment is employed as a straight limiter, i.e. with its input roof equal to the peak-programme level as maintained by the control operator. The voltages on the screen grid of the valves are stabilized by means of the neon lamp N in order to secure constancy of performance.

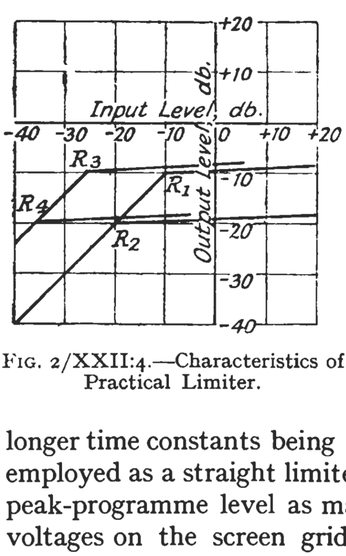


FIG. 2/XXII:4.—Characteristics of Practical Limiter.

Distortion introduced by this limiter owing to the non-linearity of the variable-μ valves, is very small, being represented by under 1% harmonic content when the limiter is driven 20 db. into limitation, i.e. with an input level 20 db. above the roof; for smaller degrees of overdrive it is correspondingly less.

The performance of this limiter is such that with maximum loss in the attenuator H the gain at all levels below the roof is zero. By means of the potentiometer P, the output roof can be adjusted to any point between the levels minus 10 and minus 20 db. With maximum loss in the attenuator H, since the gain is then zero, the input roof is always equal to the output roof. By removing attenuation from the attenuator *H*, the input roof is lowered by the amount of attenuation removed from *H* without affecting the level of the output roof. For an increase of input level of 20 db. above the roof, the output level rises by just under 1 db.

The two right-hand characteristics of Fig. 2 therefore represent the limits of performance of this limiter with *H* at maximum loss. The maximum loss of *H* is 16 db., so that removing all the loss from *H* shifts the performance characteristics to levels 16 db. lower. The two left-hand characteristics therefore represent the limits of performance of the limiter with *H* at minimum loss.

5. Line-up of Limiter for Programme.

Fig. 1 shows the bare essentials of a programme input circuit using a limiter. The line normally enters through a repeating coil, and possibly an attenuator and equalizer not shown, and then is fed into the input of an amplifier shown as amplifier 1, which has a peak-programme meter connected across its output and is connected through the fixed attenuator H_2 to the input of the limiter. The

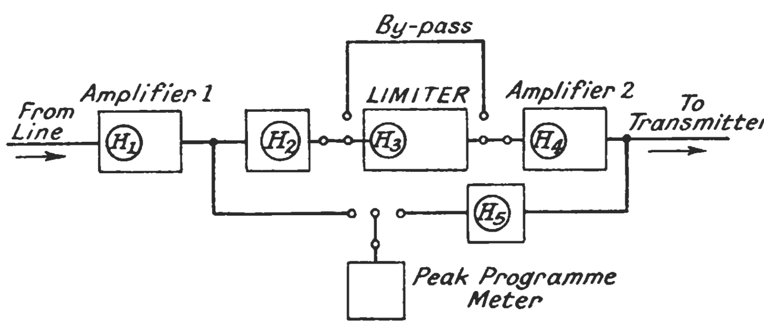


FIG. 1/XXII:5.—Typical Limiter Operating Circuit.

output of the limiter is connected to the input of amplifier 2, which drives the transmitter or transmitters, possibly through trap valves not shown. Break-jacks in the input and output of the limiter enable it to be by-passed in the event of a fault in the limiter.

This circuit is normally lined up so that the level at the output of amplifier 1 is zero level, and the output roof of the limiter is adjusted to minus 15 db. This level—minus 15 db.—is chosen merely because it is the halfway between the possible limits of output roof, and so can easily be reproduced at any time. The fixed attenuator H_2 is made to have a loss of 23 db. Hence, with H_3 at maximum loss, the zero level output from amplifier 1, which produces minus 23 db. at the input to the limiter, will produce a level of minus 23 db. at the output of the limiter. Since the programme peak voltage at any point is 8 db. above the line-up tone peak voltage, zero volume line-up tone at the output of amplifier 1 will produce a level 8 db. below the roof of the limiter, while zero volume programme at the same point will produce a level which will just reach the roof of the limiter.

If, then, with zero volume of programme at the input of H_2, the loss in H_3 is reduced by n db., the limiter is driven n db. into limitation. That is to say, all levels of programme which are lower than a level n db. below peak-programme level will be amplified normally by the limiter ; levels of programme which are n db. below peak-programme level, and all higher levels, will be delivered at the output of the limiter at the level of the output roof, that is, at minus 15 db. The gain of amplifier 2 is therefore adjusted so that a level of minus 15 db. at its input modulates the transmitter to a depth of 100%, or to a depth departing from 100% by an amount which gives what is considered to give the best compromise between loudness and distortion at the receiving end. It will be evident that the gain of amplifier 2 is determined by the level of the output roof and the level required to modulate the transmitter ; it is independent of any other level and also of the value of n.

As has already been indicated, the value of n may be zero, or may have any value between zero and 8 or 10 db. Higher levels of n are usually not practicable since too much loss of contrast results. It may be remarked that a limiter used in this way behaves like a compressor, which reduces the top n db. of programme-level range to zero.

Detailed Procedure for Line-Up. The procedure for line-up is designed to produce a standard level at the output of amplifier 2. This standard level is normally either zero level or plus 4 db., but may be any level between zero and plus 6 db. In order to enable a programme meter calibrated to read zero level to indicate the level at the output of amplifier 2, the variable potentiometer H_5 is adjusted to have a loss equal to the amount by which the required output level of amplifier 2 exceeds zero level. A peak-programme meter is necessary and the instruction below applies to the use of such a programme meter.

The line-up procedure is as follows :

1. The programme meter is lined up. This means that its sensitivity is adjusted so that it deflects up to 5 on zero level tone.

2. Potentiometer P in Fig. 1/XXII:4 is set to its maximum loss position which reduces the feedback bias to a minimum and adjusts the output roof to as high a level as possible. H_3 is adjusted to maximum loss and the timing switch is adjusted to stud 2.

3. Tone is sent from the studio centre at normal level and H_1 is adjusted until zero level is obtained at the output of amplifier 1 as indicated by a deflection up to 5 on the programme meter connected at the output of amplifier 1.

4. The gain of H_4 is adjusted until the transmitter is modulated to a depth equal to 0·4 times the maximum permissible modulation.

5. The programme meter is connected to the output of H_5 and the loss of H_5 is adjusted until the programme meter reads 5.

6. The input to the transmitter (assumed to be of 600 ohms impedance) is disconnected and replaced by a 600-ohm resistance.

7. The loss of H_3 is reduced by 12 db.

8. The loss of potentiometer P is reduced until the peak-programme meter connected to the output of H_5 reads 7.

9. The loss of H_3 is restored to maximum. If it is required to drive the limiter n db. into limitation, the loss of H_3 is then reduced by n db.

10. The 600-ohm resistance is removed from the output of amplifier 2 and the input to the transmitter is reconnected.

The circuit is then ready for service on programme.

CHAPTER XXIII

FEEDBACK

(This chapter is published by courtesy of *The Wireless Engineer*.)

THE idea of using positive feedback, or reaction as it was first called, to increase the magnification of a radio-frequency amplifier, appeared comparatively soon after the discovery of the three-electrode valve. The advantages to be gained from the sacrificing of amplification by the application of negative feedback were not appreciated until much later. Most of the credit for pointing out these advantages belongs to two members of the Bell Telephone Laboratories, H. S. Black and H. Nyquist.

The practical achievements of negative feedback up to date are many and various. At one end of the scale it has contributed stable precision instruments for measuring very small varying or alternating currents and voltages; at the other end of the scale it has simplified the problem of building high-efficiency radio transmitters delivering hundreds of kilowatts to the aerial. It has made possible the construction on an economic basis of single-core cable circuits capable of carrying hundreds of speech channels, while the development of television has been considerably facilitated by its use.

Conventions.

A feedback circuit is considered to be *passive* when the amplifier has infinite loss instead of a gain, the input and output impedances of the amplifier proper being unaffected.

Positive Feedback is said to occur when the phase shift round the feedback path is zero; the amplification is then increased by the feedback.

Negative Feedback is said to occur when the phase shift round the feedback path is 180°; the amplification is then reduced by the feedback. (180° phase shift may be introduced by a commutation or by a network.)

No rigorous terminology exists for defining the limits of positive and negative feedback as the phase shift round the loop path is varied. Evidently, if the phase shift is nearly zero or nearly 180°, the feedback would be referred to as being positive and negative respectively; when the phase shift is exactly zero or exactly 180°, the feedback is sometimes referred to as being pure positive and pure negative feedback respectively. A useful alternative definition is that the feedback is positive when it acts to increase the effective gain, and negative when it acts to reduce the gain.

μ = ratio between input voltage of amplifier proper and internal e.m.f. in output (except in first part of XXIII:1).

β = ratio between e.m.f. in amplifier output circuit and feedback voltage effective on its input (except in first part of XIII:1) = the feedback ratio.

$\mu\beta$ = loop amplification = $|\mu\beta|\underline{/\theta}$ where θ is the loop phase shift.

R_1 = input impedance of amplifier.

R_2 = output impedance of amplifier.

R_3 = input impedance of feedback circuit.

R_4 = output impedance of feedback circuit.

G = internal impedance of generator driving amplifier.

L = impedance of load into which amplifier works.

V_1 = input voltage to complete circuit = terminal voltage of driving generator.

V_g = voltage effective on input of amplifier under specified conditions.

V_f = feedback voltage effective on input of amplifier.

e_0 = internal e.m.f. in output of amplifier.

V_0 = output voltage of amplifier.

i_1 = input current to complete circuit = output current of driving generator.

i_g = input current to amplifier under specified conditions.

i_0 = output current from amplifier.

Z_{1p} = passive (i.e. with no feedback) input impedance at terminals 1,1.

Z_{1f} = active (i.e. with feedback) input impedance at terminals 1,1.

Z_{2p} and Z_{2f} = passive and active output impedances at terminals 2,2.

1. Series–Parallel Feedback.

In order to introduce the subject as simply as possible the circuit of Fig. 1 (*a*) will be discussed ; this represents rather an ideal condition in that it contains an amplifier with an infinite input impedance, or at least an input impedance so high that it can be regarded as infinite in comparison with the rest of the circuit. This is then reduced to its practical equivalent. After examining the two other practical examples which follow, engineers will have little difficulty in evolving the relations for any practical embodiment of this circuit.

Consider an amplifier in which the output is not a faithful enlargement of the input. The difference between the output, and a magnified replica of the input of amplitude equal to the output, can be referred to loosely as distortion products.

Suppose, further, that in the circuit of Fig. 1 (*a*) the impedance level at all points is finite and resistive, i.e. of zero angle, except the input impedance of the amplifier *A* which is infinite.

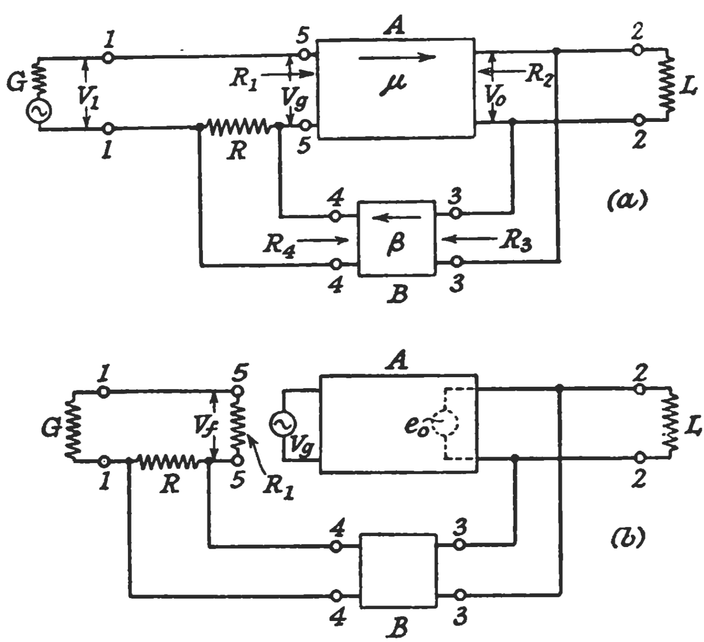


FIG. 1/XXIII:1.—Series Parallel Feedback.

In the circuit of Fig. 1 (*a*) *A* is an amplifier and *B* is a resistance attenuator. *G* is a generator of internal impedance *G*, and *L* is a load of impedance *L*.

If the effective sinusoidal input voltage to the amplifier is V_g, the amplification and distortion are defined by saying that the output voltage V_0 is $\mu V_g + D$, where $\mu = |\mu| \underline{/\theta}$, a complex quantity, is the voltage amplification of the amplifier including the effect of the load *L* across terminals 2,2, and *D* is the complex system of voltages representing the distortion products.

Suppose now that there is fed back, through the network *B*, a fraction β of the output voltage, in series with the input circuit as shown in Fig. 1, giving rise to a feedback voltage $V_f = \mu\beta V_g + \beta D$. (The network B is assumed to contain no commutation and to introduce no phase shift.)

In order to provide the voltage V_g, a voltage $V_1 = V_g - V_f$ is required (assuming in accordance with the usual convention, that V_f aids V_1 when $\theta = 0$, i.e. that positive feedback occurs).

$$\therefore\ V_1 = V_g - \mu\beta V_g - \beta D$$

$$\therefore\ V_g(1 - \mu\beta) = V_1 + \beta D$$

$$\therefore\ V_g = \frac{V_1 + \beta D}{1 - \mu\beta}$$

and the output voltage $V_0 = \mu V_g + D$.

$$\therefore\ V_0 = \frac{\mu V_1}{1 - \mu\beta} + \frac{\mu\beta D}{1 - \mu\beta} + D$$

$$= \frac{\mu V_1}{1 - \mu\beta} + \frac{D}{1 - \mu\beta} \quad . \quad . \quad . \quad (1)$$

The gain and the distortion products are therefore both multiplied by the factor $1/(1 - \mu\beta)$.

If the feedback is positive, i.e. when $\theta = 0$, provided $\mu\beta$ is less than unity the effect of feedback is to increase the gain and to increase the distortion products. If $\mu\beta$ is greater than unity at a frequency at which pure positive feedback occurs the circuit will normally oscillate, although a special case exists where this will not happen, see XXIII:6. If the feedback is negative, i.e. when $\theta = 180°$, the gain and the distortion products will be reduced by the ratio $1/(1 + |\mu|\beta)$.

When θ is not equal to zero but varies with frequency the gain is modified by the factor $1/(1 - |\mu|\beta\underline{/\theta})$, where θ is the phase shift through the amplifier at each driving frequency as represented by V_1. Each component frequency of the distortion products is reduced in the same ratio *where μ, β and θ are the values of these quantities at the frequency concerned.*

It is to be noted that $\mu\beta$ is the voltage (or current) amplification round the feedback path.

This quantity is spoken of as the loop amplification vector, or loosely but conveniently, as the loop gain (gain should strictly be expressed in decibels, of course).

These relations are discussed by H. S. Black in an article, "Stabilized Feedback Amplifiers", in the *Bell System Technical Journal* for January, 1934, page 1. He gives there a useful chart of the value of $1 - \mu\beta$ as a function of $|\mu|\beta$ and θ. When $\theta = 0$

this defines the conditions for positive feedback : the condition for negative feedback is obtained by making $\theta = 180°$.

It will now be apparent, from the form of the expressions derived, that, although μ has been assumed to be a complex quantity and β has been assumed to be a numeric, the quantity θ in the expression $\mu\beta = |\mu|\beta\underline{/\theta}$ may be used to take account of any phase shift in the feedback path. In other words, it is permissible to write $\mu = |\mu|\underline{/\theta_1}$, $\beta = |\beta|\underline{/\theta_2}$, and $\mu\beta = |\mu\beta|\underline{/\theta_1+\theta_2} = |\mu\beta|\underline{/\theta}$, where $\theta = \theta_1+\theta_2$.

It is to be noted that the effective amplification is $\frac{V_0}{V_1}$, and, neglecting distortion products, this is a fraction $\frac{1}{1-\mu\beta}$ of the amplification without feedback.

Each frequency in the distortion product D has also been reduced in the same proportion, with μ and β given the values effective at each harmonic frequency, while the phase shift through the amplifier at the driving frequency has been changed from

$$\theta_1 \text{ to } \theta_1+\tan^{-1}\left(\frac{|\mu\beta|\sin\theta}{1+|\mu\beta|\cos\theta}\right),$$

where $\theta = \theta_1+\theta_2$. With negative feedback this gives a reduction of phase shift.

Note that when $\mu\beta$ is very large compared with unity, V_0 tends towards

$$\frac{\mu}{\mu\beta}V_1+\frac{D}{\mu\beta} = \frac{1}{\beta}V_1+\frac{D}{\mu\beta} \quad . \quad . \quad . \quad (2)$$

The important point to note is that the amplification tends to be equal to $1/\beta$, and therefore independent of variations in μ. The first effects of negative feedback are therefore to reduce or substantially eliminate :

(i) Distortion products, such as combination tones and harmonics, due to valve non-linearity.

(ii) Distortion products, such as noise, e.g. due to anode noise in valves, hum picked up from A.C. heating of filaments, cross-talk from other amplifiers, etc.

(iii) Distortion due to variation of the amplification factor with frequency.

(iv) Change of amplification due to change of μ with time, e.g. due to battery or supply mains variation.

(v) Phase distortion.

Practical Values of μ and β. The circuit of Fig. 1 (*a*) is an entirely practical circuit for the application of feedback, but of course in any practical circuit a generator of finite internal impedance would be connected to terminals 1,1, and the input impedance of amplifier A would be finite. The performance of such a circuit can be analysed in exactly the same way as above, but it is then convenient to define μ and β in a slightly different way:

μ = the ratio: internal e.m.f. e_0 in the output of A divided by the input voltage V_g.

β = the ratio: V_f divided by e_0, when the circuit of Fig. 1 (*b*) is set up. This is the circuit of Fig. 1 (*a*) with the input to amplifier A broken at 5,5, terminals 5,5 terminated in in an impedance equal to R_1 the input impedance of A, and terminals 1,1 closed through an impedance equal to G the impedance of the generator normally connected to terminals 1,1. V_f is then the voltage generated across terminals 5,5, when an input voltage V_g is applied to the input of the amplifier, e_0 being the corresponding internal e.m.f. in the output of A, which of course may be measured as the open-circuit output voltage of A.

In general terms, in all feedback circuits, μ is the ratio between the input voltage to the amplifier and the internal e.m.f. in its output circuit, while β is always the ratio between the e.m.f. in the output circuit of the amplifier and the resultant feedback voltage applied to the input of the amplifier with the amplifier supposed to be passive, i.e. non-amplifying. In case it is difficult to conceive an amplifier which is non-amplifying and which has an e.m.f. in its output circuit, the output circuit may be considered to be replaced by a generator of the same internal impedance (R_2) as the output circuit and with the same internal e.m.f.

The value of $\mu\beta$ is then the loop voltage amplification. It is also the loop current amplification, since the loop amplification is always measured from any point in the loop circuit back to the same point: the impedance at the beginning of the loop is the same as the end of the loop, because the beginning of the loop is also the end of the loop.

The relations derived above apply to all practical feedback circuits, provided $\mu\beta$ always represents the loop amplification with the generator treated as a passive impedance equal to its internal impedance.

In all *simple* practical feedback circuits *the effect of feedback is to modify the input and output impedances of the amplifier*. This is the second important effect of feedback. When required, special means may be introduced to eliminate this effect. See XXIII:5 and 11 below. On the other hand, feedback can be used to provide an amplifier with a low impedance output ; which can be useful.

2. Parallel-Parallel Feedback.

This is sometimes called *voltage feedback.*

Fig. 1 shows the basic circuit of parallel-parallel feedback ; the resistance R provides the feedback path. A represents the amplifier. This circuit operates by feeding back a current which adds to or subtracts from the current supplied by the driving generator.

The reduction of gain and the reduction of distortion products are given by the same expressions as were derived for the case of series-parallel feedback ; see equations (1) and (2) of the last section. μ and β have the meanings described at the end of the last section and given in the list of conventions at the beginning of the chapter.

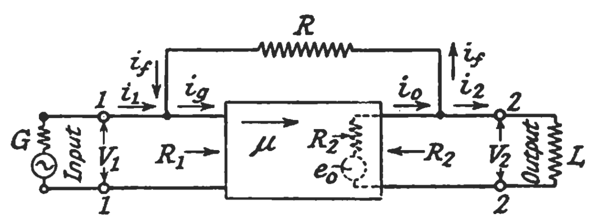


FIG. 1/XXIII:2.—Parallel-Parallel Feedback.

For instance, the value of β is given by :

$$\beta = \frac{\dfrac{L\left[R+\dfrac{R_1G}{R_1+G}\right]}{L+R+\dfrac{R_1G}{R_1+G}}}{R_2+\dfrac{L\left[R+\dfrac{R_1G}{R_1+G}\right]}{L+R+\dfrac{R_1G}{R_1+G}}} \times \frac{\dfrac{R_1G}{R_1+G}}{R+\dfrac{R_1G}{R_1+G}}$$

This is derived by the method of inspection described in XXV:2.1. Simplifying :

$$\beta = \frac{\dfrac{LR_1G}{(L+R)(R_1+G)+R_1G}}{R_2+\dfrac{L(R_1R+RG+R_1G)}{(L+R)(R_1+G)+R_1G}}$$

$$= \frac{LR_1G}{R_2(L+R)(R_1+G)+R_1R_2G+L(R_1R+RG+R_1G)}$$

Note that this expression involves both G and L.

2.1. Modification of Input Impedance with Parallel-Parallel Feedback. Refer to Fig. 1 and define μ as $\frac{e_0}{V_1}$, where V_1 is the input voltage. Then, if a voltage V_1 applied at 1,1 causes a current i_1 to enter terminals 1,1, the current i_g entering the amplifier input is equal to i_1+i_f, where i_f is the current fed back through R:

$$i_g = i_1 + i_f \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also
$$e_0 = \mu V_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$i_0 = \frac{e_0 - V_0}{R_2} = i_f + i_2 \quad . \quad . \quad . \quad (3)$$

where i_2 = the current flowing through the load L.

$$V_2 = i_2 L \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and
$$i_f = \frac{V_2 - V_1}{R} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Ignore i_0 and substitute into (3): for i_2 from (4), and for i_f from (5).

$$\therefore \frac{e_0 - V_2}{R_2} = \frac{V_2 - V_1}{R} + \frac{V_2}{L}$$

$$\therefore RLe_0 - RLV_2 = R_2LV_2 - R_2LV_1 + R_2RV_2$$

$$\therefore V_2 = \frac{RLe_0 + R_2LV_1}{R_2L + R_2R + RL}$$

$$= \frac{\mu RL + R_2L}{R_2L + R_2R + RL} V_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\therefore i_f = \frac{V_2 - V_1}{R} = \frac{V_1}{R}\left[\frac{\mu RL + R_2L}{R_2L + R_2R + RL} - 1\right] \quad . \quad . \quad (7)$$

also
$$i_g = \frac{V_1}{R_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From (1), (7) and (8),

$$i_1 = i_g - i_f = V_1\left[\frac{1}{R_1} - \frac{\mu RL + R_2L}{R(R_2L + R_2R + RL)} + \frac{1}{R}\right] . \quad (9)$$

$$\therefore Z_{1f} = \frac{V_1}{i_1} = \frac{1}{\frac{1}{R_1} + \frac{1}{R} - \frac{\mu RL + R_2L}{R(R_2L + R_2R + RL)}} \quad . \quad . \quad (10)$$

$$= \frac{RR_1(R_2L + R_2R + RL)}{R(R_2L + R_2R + RL) + R_1(R_2L + R_2R + RL) - \mu RLR_1 - R_1R_2L}$$

$$= \frac{R_1(RL + R_2R + R_2L)}{R_2L + R_2R + R_1R_2 + R_1L + RL - \mu R_1L} \quad . \quad . \quad . \quad . \quad (11)$$

This is equal to the passive input impedance divided by $1 - \mu\beta$, where β is the feedback ratio with G infinite.

i.e.
$$Z_{1f} = \frac{Z_{1p}}{1 - \mu\beta} \quad . \quad . \quad . \quad . \quad (12)$$

Proof.

By inspection, the value of β with G infinite is

$$\beta = \frac{R_1}{R_1+R} \times \frac{\dfrac{L(R_1+R)}{L+R_1+R}}{R_2+\dfrac{L(R_1+R)}{L+R_1+R}}$$

$$= \frac{R_1L}{R_2L+R_2R+R_1R_2+R_1L+RL} \quad . \quad . \quad (13)$$

The passive input impedance is

$$Z_{1p} = \frac{R_1\left(R+\dfrac{LR_2}{L+R_2}\right)}{R_1+R+\dfrac{LR_2}{L+R_2}}$$

$$= \frac{R_1(RL+R_2R+R_2L)}{R_2L+R_2R+R_1R_2+R_1L+RL} \quad . \quad . \quad (14)$$

From (12), (13) and (14), the impedance with feedback is :

$$Z_{1f} = \frac{\dfrac{R_1(RL+R_2R+R_2L)}{R_2L+R_2R+R_1R_2+R_1L+RL}}{1 - \dfrac{\mu R_1L}{R_2L+R_2R+R_1R_2+R_1L+RL}}$$

$$= \frac{R_1(RL+R_2R+R_2L)}{R_2L+R_2R+R_1R_2+R_1L+RL - \mu R_1L} \quad . \quad . \quad (15)$$

which is identical with equation (11).

Similarly
$$Z_{2f} = \frac{Z_{2p}}{1 + \mu\beta} \quad . \quad . \quad . \quad . \quad (16)$$

where β is the feedback ratio with the output load infinite.

Fig. 1 and therefore equations (12) and (16) correspond to positive feedback when $\theta = 0$.

3. Series-Series Feedback.

This has also been called *current feedback*. The circuit is shown in Fig. 1.

It will be seen that the impedance R is in series in both input and output circuits, so that any current through the output circuit induces a voltage in series with the input circuit. The value of β is still defined as the ratio between the internal e.m.f. in the output of the amplifier proper and the resultant voltage induced across 4,4.

3.1. Modification of Output Impedance with Series–Series Feedback. Referring to Fig. 1, μ is the relation between V_g, the input voltage to A and e_0 the internal e.m.f. in the output of A. G is the input impedance facing terminals 1,1. R_1 and R_2 are respectively the input and output impedances of A. R is the current feedback impedance.

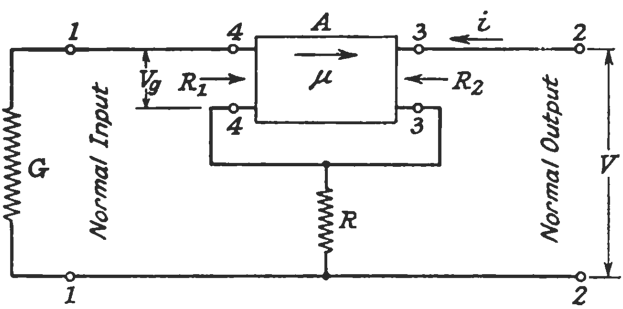


Fig. 1/XXIII:3.—Series-Series Feedback.

To determine the impedance looking into terminals 2,2, assume that a voltage V applied across terminals 2,2 causes a current i to flow into the output of the amplifier returning via R in parallel with G and R_1 in series. The voltage appearing across R is evidently

$$\frac{R(R_1+G)}{R+R_1+G}i \text{ and the voltage}$$

$$V_g = \frac{R_1}{R_1+G} \times \frac{R(R_1+G)}{R+R_1+G}i$$

$$= \frac{RR_1}{R+R_1+G}i,$$

so that the internal e.m.f. in the output of A is

$$\frac{RR_1\mu}{R+R_1+G}i$$

of such sense as to aid V in generating i.

The total e.m.f. in the output mesh is therefore

$$V+\frac{RR_1\mu}{R+R_1+G}i$$

whence

$$i=\frac{1}{\dfrac{R(R_1+G)}{R+R_1+G}+R_2}\left[V+\frac{RR_1\mu}{R+R_1+G}i\right] \quad . \quad . \quad (1)$$

$$\therefore \left[\frac{R(R_1+G)}{R+R_1+G}+R_2\right]i=V+\frac{RR_1\mu}{R+R_1+G}i$$

Hence the output impedance

$$Z_{2f}=\frac{V}{i}=\frac{R(R_1+G)}{R+R_1+G}+R_2-\frac{RR_1\mu}{R+R_1+G}$$

$$=\left[\frac{R(R_1+G)}{R+R_1+G}+R_2\right]+\left[1-\mu\frac{\dfrac{RR_1}{R+R_1+G}}{\dfrac{R(R_1+G)}{R+R_1+G}+R_2}\right]$$

$$=\left[\frac{R(R_1+G)}{R+R_1+G}+R_2\right]\times\left[1-\mu\frac{RR_1}{R(R_1+G)+R_2(R+R_1+G)}\right] \quad . \quad (2)$$

$$=[1-\mu\beta]Z_{2p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $\mu\beta$ is the loop voltage amplification with terminals 2,2 shorted, i.e. with the output load zero.

The sign before $\mu\beta$ shows that this case is one of positive feedback when $\theta=0$. By inserting a commutation in A, the case of negative feedback at $\theta=0$ is obtained.

Proof that the loop voltage amplification $=\mu\dfrac{RR_1}{R(R+G)+R_2(R+R_1+G)}$.

Assume 1 volt applied at the input of A: i.e. $V_g=1$ and let μ now represent the ratio of the internal e.m.f. in the output of the amplifier to the input voltage of the amplifier.

Then the internal e.m.f. in the output of $A=\mu$. The impedance facing the output of A with terminals 2,2 shorted is

$$\frac{R(R_1+G)}{R+R_1+G}=A\text{, say.}$$

The voltage across R is $\dfrac{A}{A+R_2}\mu$ and the voltage across R_1 is $\dfrac{R_1}{R_1+G}\cdot\dfrac{A}{A+R_2}\mu$ and thus is evidently equal to the loop voltage amplification.

Substituting for A, the loop voltage amplification is

$$= \frac{RR_1}{R+R_1+G} \cdot \frac{1}{\frac{R(R_1+G)}{R+R_1+G}+R_2}\mu$$

$$= \frac{RR_1}{R(R_1+G)+R_2(R+R_1+G)}\mu \quad . \quad . \quad . \quad (4)$$

Similarly, it may be shown that

$$Z_{1f} = (1 - \mu\beta)Z_{1p} \quad . \quad . \quad . \quad . \quad (5)$$

where β is now the value of the feedback ratio with terminals 1,1 shorted, i.e. with the generator impedance equal to zero.

4. Summary of Factors by which Input and Output Impedances are Multiplied.

It will be appreciated that pure positive or negative feedback can only occur when $\theta + 0$, or 180° respectively.

	General Case	*Negative Feedback*
Parallel-Parallel Feedback (Input and Output Impedances)	$\frac{1}{1-\mu\beta}$	$\frac{1}{1+\lvert\mu\beta\rvert}$
Series-Series Feedback (Input and Output Impedances)	$1-\mu\beta$	$1+\lvert\mu\beta\rvert$
Series-Parallel Feedback (Input Impedance)	$1-\mu\beta$	$1+\lvert\mu\beta\rvert$
(Output Impedance)	$\frac{1}{1-\mu\beta}$	$\frac{1}{1+\lvert\mu\beta\rvert}$

It will be realized that while the values of μ are the same in each of the above formulae, for any given amplifier, the values of β vary according to whether input impedance, output impedance, or gain and harmonic reduction are being calculated. As the variation is constituted by changes in the values of generator and load impedances from their actual values G and L, to zero or infinity, the simplest way of indicating the value of β is to indicate the values of G and L to be used in calculating β. This is done in the table below.

Values of Generator and Load Impedance to be Used in the Calculation of β.

Quantity being Determined		*Generator Impedance*	*Load Impedance*
Gain Reduction: all cases		G	L
Z_{1f}:	S-S and S-P Feedback	0	L
	P-P Feedback	Infinity	L
Z_{2f}:	P-P and S-P Feedback	G	Infinity
	S-S Feedback	G	0

The distortion reduction is of course the same as the gain reduction at any given frequency.

5. Means for Making Input and Output Impedances Independent of Feedback.

It is evident that where an amplifier has two input circuits with infinite attenuation between them, feedback into one cannot affect the impedance looking into the other. A simple case of this discussed below is the cathode follower valve.

In the case of an ordinary amplifier, simple bridge circuits can be devised which have the twofold effect of preventing the modification of the input and output impedances of the amplifier in the way

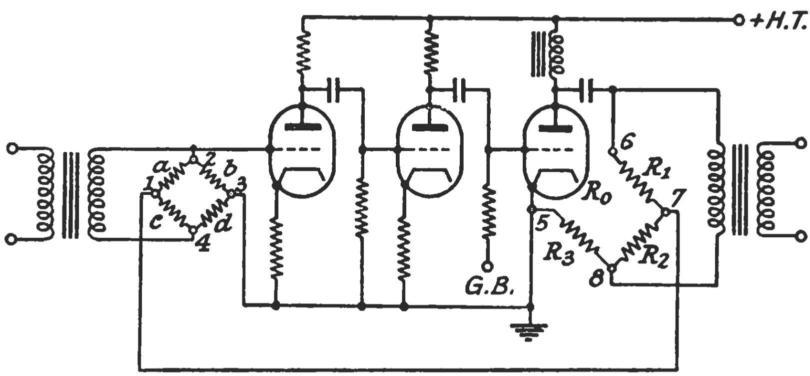


FIG. 1/XXIII:5.—Feedback Amplifier with Input and Output Bridges.

described above, and also of eliminating any phase shift round the loop path due to reactances introduced into the feedback path due to the input or output circuits.

Fig. 1 shows a circuit of an amplifier with bridges for this purpose in both input and output circuits. The bias arrangements for the output valve are omitted for clearness.

One of the arms of the output bridge is constituted by R_0, the anode impedance of the valve, the bridge being balanced so that $R_1/R_0 = R_2/R_3$.

Considering the output bridge first : any voltage applied at the output terminals produces a voltage across terminals 6 and 8, but no voltage across terminals 5 and 7, and so supplies no voltage to the feedback circuit. The presence of the feedback circuit therefore does not modify the output impedance.

Considering the input bridge : the energy supplied by the feedback circuit produces a voltage across terminals 1 and 3, but no voltage across terminals 2 and 4, and so feeds no energy back into the input circuit. The presence of the feedback circuit therefore does not modify the input impedance. Further, since no energy is supplied to the input circuit, any reactance shunted across terminals 2 and 4, due to the input circuit, does not introduce phase shift into the loop path. The bridge in the output circuit does not, however, serve to eliminate phase shift due to any shunt reactance introduced by the output circuit.

It should be noted that, if the secondary winding of the input transformer has appreciable stray capacity to ground, this constitutes a reactance across terminals 2 and 3 and so introduces phase shift into the loop path, which in certain cases may be very troublesome. For this reason the stray capacity of the transformer should be kept as low as possible and the value of resistance b should be made as small as possible.

6. Stability of Feedback Circuits.

It is convenient here to introduce the term " differential phase shift " to describe, in any pass band, the increase of lag or lead with deviation of frequency from f_0, the frequency at which pure negative or positive reaction is obtained according to the sense of commutation. Similarly, the term " differential attenuation " will refer to the decrease of amplification with deviation of frequency from the frequency of maximum loop amplification.

The differential phase shift will be represented by θ_d.

As before θ is the loop phase shift.

The stability of an amplifier with negative feedback is normally determined by the loop gain (amplification) at the frequency at which the loop phase shift as defined below reaches zero. If this loop gain (amplification) is less than zero (unity) the circuit is stable ; if it is greater than zero (unity) the circuit will usually oscillate.

In practice, every amplifier is a band-pass structure. In the case of D.C. amplifiers the band extends down to zero frequency, but if we consider the case where this is not so, which is the more difficult case, the application to the simpler case of D.C. amplifiers will be evident.

Fig. 1 shows the amplification and phase shift characteristics of a hypothetical resistance capacity or transformer coupled amplifier. (The feedback path is assumed to introduce no phase shift and to have an attenuation ratio β, constant and independent of frequency.)

It will be seen that in the middle of the band a frequency f_0 occurs at which the phase shift θ_d is zero. At this frequency the feedback constitutes either pure negative ($\theta = \theta_d \pm 180°$) or pure positive ($\theta = \theta_d$) feedback according to the sense of commutation. If this sense is such that negative feedback occurs at frequency f_0, then it is usual to say that negative feedback has been applied. At frequencies f_1 and f_2 at which θ_d reaches respectively + and − 180°, pure positive reaction occurs, and if at these frequencies the loop amplification $|\mu\beta|$ is greater than unity, in the case shown, the amplifier will sing, the oscillation building up to such an amplitude

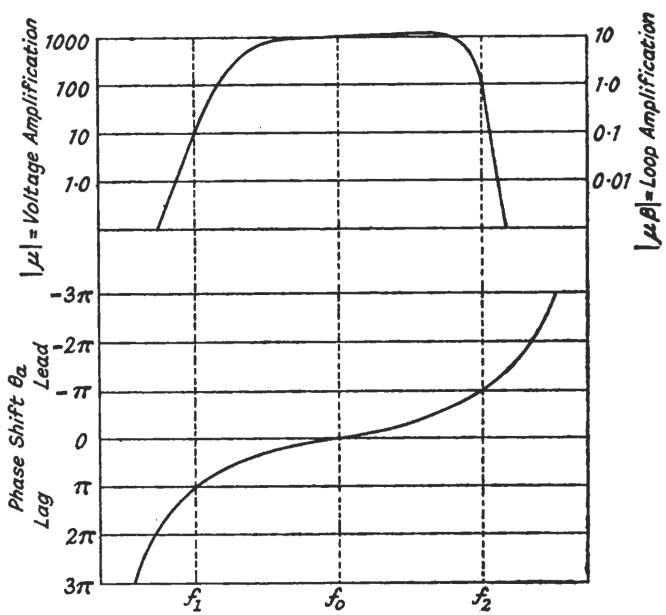


FIG. 1/XXIII:6.—Amplification and Loop Amplification and Phase Shift of a Typical Feedback Amplifier.

that the loop amplification falls to unity. It is hardly necessary to explain that the gain of an overloaded amplifier falls below its value when not overloaded.

The value of $\mu\beta$, effective for determining the degree of stability of a feedback circuit, is the same as that which is effective in producing gain reduction.

A simple rule for stability in negative feedback amplifiers can now be established: *the loop amplification at the frequency or frequencies at which the loop phase shift reaches zero shall not exceed unity.* At these frequencies the differential phase shift reaches $\pm\pi$. These frequencies will therefore be called *the π frequencies,* and the differential attenuation at these frequencies, *the π attenuation.* Strictly, this condition is sufficient but not necessary. This point is discussed below.

In the case of Fig. 1 the amplification $|\mu|$ at f_1 is 10, while at f_2 it is 100. This means that for stability the value of β must be $\frac{1}{100}$ or less. Assuming the marginal condition of stability to have been chosen (i.e. $\beta = \frac{1}{100}$), then at f_0, $|\mu\beta| = 1{,}000 \times \frac{1}{100} = 10$. In general, the maximum permissible value of loop amplification is equal to the fall in amplification from its maximum value (usually occurring in the neighbourhood of f_0) to its value at the π frequency. When there are two frequencies (respectively at each end of the pass band) at which the phase shift θ_d reaches π (θ reaches zero), evidently the one which prescribes the lower value of loop gain provides the determining limitation. *Hence limiting loop amplification* $= \pi$ *attenuation.*

The *singing margin* $= \pi$ attenuation minus maximum loop amplification. Evidently the practical condition for stability is that the singing margin shall be positive and greater than any possible variation in loop gain.

In the case cited, where $\beta = \frac{1}{100}$, the curve of Fig. 1 represents the value of $|\mu\beta|$ when the right-hand scale is used, which has ordinates equal to $\frac{1}{100}$ of the value of $|\mu|$, portrayed by the left-hand scale. The value of $\mu\beta$ is now represented by the two curves of Fig. 1, since, as no phase shift has been introduced in the feedback path, the loop phase shift is equal to 180° plus the phase shift through the amplifier.

The value of $\mu\beta$ can be portrayed in another way. The case for a system commutated for negative feedback is shown in Fig. 2 (*a*), where the contour drawn on the complex plane is formed by plotting at all frequencies the value of the complex quantity $\mu\beta$, considered as a vector, and joining the tips of the resultant vectors.

In consistence with the condition for stability formulated above, it appears clear that if the contour embraces the point $\underline{1/0}$ as shown in Fig. 2 (*b*), the system will sing because the loop gain is greater than unity at a frequency at which the loop phase shift is zero.

It was formerly believed that if at any frequency the loop phase shift is zero and the loop amplification greater than unity, the system must sing. Nyquist * has shown, however, that as long as the contour described according to the above convention does not embrace the point $\underline{1/0}$, the system is stable : he gives rules for

* H. Nyquist, " Regeneration Theory ", *Bell S. Tech. Journ.*, January, 1932.

determining whether a contour embraces the point $1\angle 0$ in certain cases where this is not immediately obvious. The system of Fig. 2 (*c*) is stable, although the amplification is greater than unity at a frequency at which the absolute phase shift is zero. This system has the rather odd characteristic that if the amplification is either increased *or reduced* sufficiently, singing will result. Such a system has been built and was found to be stable, but its practical applications have so far been almost nil on account of the difficulty of usefully modifying an unstable circuit of the type shown, for instance, at Fig. 2 (*b*) to a stable circuit as shown at Fig. 2 (*c*).

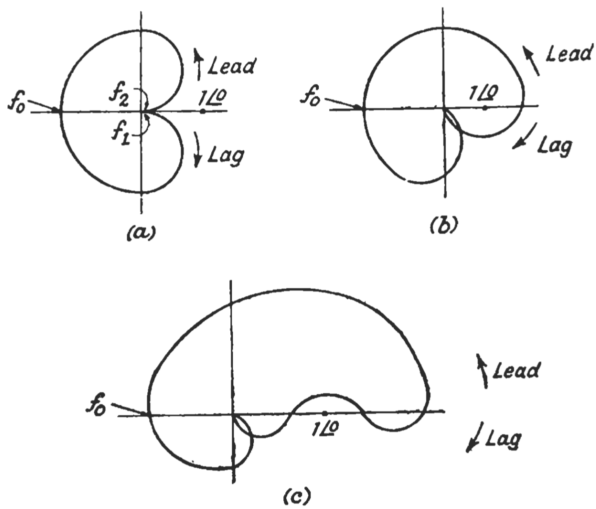


FIG. 2/XXIII:6.—Nyquist Diagrams of Typical Feedback Amplifiers.

The chief problem in applying feedback is that of stability, and in practice it is usual to think in terms of systems of the type shown in Figs. 2 (*a*) and 2 (*b*), which, incidentally, have a single pass band only.

The limitations on loop gain, which are imposed by the requirement of stability, are dependent on a variety of factors, chief among which are the phase shift at infinite frequency and at zero frequency and the differential attenuation when $\theta = 0$: the π attenuation.

It is a fact, perhaps not obvious, that if the differential phase shift at infinite and zero frequency is not more than 180° the system can never sing when negative feedback is applied, however high the loop amplification. This is because if 180° differential phase shift does occur at zero and/or infinite frequency, the loop attenuation will be also infinite at those frequencies. If 180° differential phase shift is never reached, positive feedback can never occur. Equally, if there is zero differential phase shift at zero frequency, no singing can be introduced at the low-frequency end when negative feedback is used.

It is approximately a true representation of fact to say that when a system sings it starts to sing at the frequency at which the absolute phase shift round the loop is zero. It does not necessarily continue to oscillate at this frequency, since as the amplitude of the oscillation builds up the loop phase shift may change owing to non-linearity of the elements, while in general the oscillation must be regarded as a relaxation oscillation. In practice, however, it is usually found that the oscillation frequency is very close to the frequency of zero loop phase shift.

The magnitude of the π attenuation is determined by the type or types of circuit contributing phase shift and attenuation, by their number, and by the relative locations of their phase shift and attenuation curves in the frequency spectrum.

The magnitude of the π attenuation can be considerably increased by staggering the cut-off frequencies of the different circuits contributing phase shift, that is by making them occur at frequencies as wide apart as possible. A patent in the author's name covers this point.

The following treatment due to C. G. Mayo shows the point very clearly. Consider an amplifier with feedback which contains three resistance-capacity coupled circuits of conventional type in the total loop path. Valves of internal resistance G_1, G_2, G_3 feed through condensers C_1, C_2 and C_3 into loads R_1, R_2 and R_3. The loss ratio introduced by these circuits with relation to the loss ratio at infinite frequency is:

$$r = \frac{R_1}{R_1+G_1-j\frac{1}{C_1\omega}} \cdot \frac{R_2}{R_2+G_2-j\frac{1}{C_2\omega}} \cdot \frac{R_3}{R_3+G_3-j\frac{C_3\omega}{C_3\omega}} \cdot \frac{(R_1+G_1)(R_2+G_2)(R_3+G_3)}{R_1.R_2.R_3}$$

Putting $R_1+G_1 = A_1$, $R_2+G_2 = A_2$, etc.

$$r = \frac{A_1A_2A_3}{A_1A_2A_3 - \frac{A_1}{C_2C_3\omega^2} - \frac{A_2}{C_1C_3\omega^2} - \frac{A_3}{C_1C_2\omega^2} + j\left[\frac{1}{C_1C_2C_3\omega^3} - \frac{A_2A_3}{C_1\omega} - \frac{A_1A_3}{C_2\omega} - \frac{A_1A_2}{C_3\omega}\right]}$$

The phase angle of r will be zero when the reactive term is zero, i.e. when

$$\omega^2 = \frac{1}{A_1A_2C_1C_2 + A_1A_3C_1C_3 + A_2A_3C_2C_3}$$

$\frac{\omega}{2\pi}$ is the π frequency since at $\omega = \infty$ the phase shift is zero, and at $\omega = 0$ the phase shift is 270°. Whence at the π frequency, substituting in the expression for r the value of ω above, the π attenuation ratio,

$$r = \frac{A_1A_2A_3}{A_1A_2A_3 - \left[\frac{A_1}{C_2C_3} + \frac{A_2}{C_1C_3} \times \frac{A_3}{C_1C_2}\right] \times [A_1A_2C_1C_2 + A_1A_3C_1C_3 + A_2A_3C_2C_3]}$$

$$= \frac{A_1A_2A_3}{A_1A_2A_3 - A_2A_1^2\frac{C_1}{C_3} - A_1^2A_3\frac{C_1}{C_2} - A_1A_2A_3 - A_1A_2^2\frac{C_2}{C_3} - A_1A_2A_3 - A_2^2A_3\frac{C_2}{C_1} - A_1A_2A_3 - A_1A_3^2\frac{C_3}{C_2} - A_2A_3^2\frac{C_3}{C_1}}$$

$$= -\frac{1}{2 + \frac{A_1}{A_3}\cdot\frac{C_1}{C_3} + \frac{A_1}{A_2}\cdot\frac{C_1}{C_2} + \frac{A_2}{A_3}\cdot\frac{C_2}{C_3} + \frac{A_2}{A_1}\cdot\frac{C_2}{C_1} + \frac{A_3}{A_2}\cdot\frac{C_3}{C_2} + \frac{A_3}{A_1}\cdot\frac{C_3}{C_1}}$$

$$= -\frac{1}{2 + \frac{A_2C_2 + A_3C_3}{A_1C_1} + \frac{A_1C_1 + A_3C_3}{A_2C_2} + \frac{A_1C_1 + A_2C_2}{A_3C_3}}.$$

If the cut-off frequencies are all equal then $A_1C_1 = A_2C_2 = A_3C_3$

and

$$r = \frac{1}{2+2+2+2} = \frac{1}{8}$$

If, for instance, $A_1C_1 = A_2C_2 = 10A_3C_3$, then

$$r = \frac{1}{2 + \frac{11}{10} + \frac{11}{10} + 20} = \frac{1}{24} \text{ approximately.}$$

In other words, by staggering one of the circuits so that it cuts off at a frequency ten times as high as the other two, an increase in the π attenuation of nearly 10 db. is obtained, with a resultant increase in permissible loop gain.

The maximum loop amplification would, therefore, be just under 24 (voltage ratio). It is hardly necessary to mention that a feedback amplifier would not be operated without a margin of safety. If, for instance, this were 6 db., the permissible loop amplification would only be 12 in this case. It should be pointed out that at the π frequencies, positive reaction occurs, so that distortion products in that region are ***increased*** by the factor $F = \frac{1}{1 - |\mu\beta|}$, where $\mu\beta$ is the loop amplification at the π frequency. If the factor of safety is 6 db., $\mu\beta$ at the π frequency $= \frac{1}{2}$ and $F = 2$, if the factor of safety is 1 db., $\mu\beta = 0{\cdot}89$ and $F = 9{\cdot}0$, and so on. The importance of an adequate stability margin is, therefore, twofold.

Similar advantages apply to circuits introducing increase of attenuation and increase of lag with increase of frequency, such as are constituted by the effects of shunt capacity across the circuit due to valve input and output capacities, which introduce high frequency cut-offs.

In all cases the π attenuation can be increased by staggering the circuit cut-offs.

It is quite a simple matter in practice to build one resistance capacity coupling with a drop of 6 db. at 20 c/s and another with a drop of 6 db. at 1 c/s or even lower. I have chosen 6 db. drop as the point defining cut-off. Evidently any other value of decibels might have been chosen for comparison purposes. In this way it is usually found possible to render low-frequency phase shift comparatively innocuous from the point of view of introducing singing, while most of the low-frequency phase shift can be relegated to frequency regions where no useful frequency components occur. In cases where it is necessary to amplify down to zero frequency no low-frequency phase shift occurs and the problem does not arise.

Phase shift at the high-frequency end of the spectrum is usually a much more serious problem, because it is nearly always impossible to stagger the cut-off frequencies, of the several circuits concerned, in any ratio sufficiently large to be useful. The best thing to do in practice, is to make the cut-off of all circuits as high as possible, and then to introduce a circuit to increase the π attenuation. Such a circuit must be one which introduces attenuation at the π frequency while introducing a minimum of lag : at the same time it must not introduce appreciable lag inside the range of transmitted frequencies, since this would reduce the distortion reduction.

Fig. 5/XXI:5 shows the attenuation and phase-shift characteristics of a suitable circuit ; as shown, it consists of a shunt made up of a condenser in series with a resistance.

These are generalized characteristics (see XXI:5) plotted to a universal frequency scale. All frequencies are represented by their ratio to the frequency at which the reactance of the shunt condenser equals the impedance of the circuit across which it is bridged. The parameter *a* defines the ratio of the resistance in series with the condenser to the circuit impedance (see XXI:5). The effect of this circuit is to introduce an attenuation which, as the frequency is increased, increases asymptotically to a limiting value, and a phase shift which rises to a maximum just about the arbitrary unit frequency as defined by the scale and then falls away asymptotically to zero.

In order that the circuit may be of appreciable use it is desirable to fit the range of frequencies, where the lag rises to a maximum, between the highest useful frequency to be transmitted and the π frequency. This means that it is desirable that the π frequency should be at least 10 times the top frequency, e.g. to transmit a range up to 10,000 c/s a π frequency range up to 100 kc/s is required. It will be appreciated that if appreciable lag is introduced in the neighbourhood of the π frequency, the π frequency will shift to a lower frequency where the attenuation introduced by the existing circuit is lower, and some of the extra attenuation introduced at the π frequency will be lost. For this reason it is generally better to err on the side of increasing the lag at the wanted frequencies.

Until someone has developed a simple technique of doing it, any attempt at evading the effect of the phase shift by converting the attenuation phase-shift vector diagram to the form shown in Fig. 2 (*c*) is not recommended. If carried out, the circuit would be entirely stable, provided the loop gain were kept up. If for any reason the loop gain fell appreciably, the circuit would sing.

7. Discontinuities in Amplitude Characteristics.

It is self-evident that if an amplifier has complete limitation of amplitude characteristics above a certain amplitude, and is driven into limitation so introducing harmonics, no amount of feedback can reduce these harmonics since the system cannot transmit the correcting frequencies fed back in opposite phase. In general, any kind of discontinuity in amplitude characteristic cannot be corrected, while limitation effects can be corrected only in inverse proportion to their sharpness.

8. Feedback of Rectified Radio Frequency.

It requires no proof to show that the complete system comprised by the speech input circuits to a radio transmitter, its modulator and high-frequency amplifiers, and a detector delivering the original audio frequency at its output, can be considered as one example of an amplifier.

If the output of such an amplifier is fed back to its input the system obeys all the laws given above for predicting its performance in terms of its amplitude, gain and phase characteristics.

The only point of interest which really arises is how radio-frequency circuits affect the gain and phase characteristics of the overall audio-frequency path.

Only radio-frequency systems having characteristics symmetrical about the carrier frequency will be considered. For such systems the variation of audio-frequency response is a replica of the variation of the response of the R.F. circuits to the sidebands: each audio frequency is amplified or attenuated relatively to other frequencies by the same amount that the pair of sidebands which " carry " that audio frequency is amplified or attenuated with regard to other pairs of sidebands. This is so well known that it requires no proof. In a system free from phase distortion (i.e. having a phase-shift characteristic linear with frequency) the audio-frequency phase shift is equal to the upper sideband phase shift minus the phase shift at the carrier frequency.

The proof of this is very simple, as follows:

An amplitude-modulated wave may be represented by

$$(1+m \sin vt) \sin ct$$

$$= \sin ct + \frac{m}{2} \cos (c - v)t - \frac{m}{2} \cos (c+v)t$$

where c and v are respectively the angular frequencies of carrier and modulating frequency.

If the wave passes through a system which shifts the carrier by ϕ_0 degrees lag, the lower sideband by $\phi_0 - \alpha$ lag, and the upper sideband by $\phi_0+\alpha$ lag, without changing their amplitudes, the emergent wave is

$$\sin (ct+\phi_0) + \frac{m}{2} \cos (ct - vt+\phi_0 - \alpha)$$

$$- \frac{m}{2} \cos (ct+vt+\phi_0+\alpha)$$

$$= \sin (ct+\phi_0) + m \sin (ct+\phi_0) \sin (vt+\alpha)$$

$$= [1+m \sin (vt+\alpha)] \sin (ct+\phi_0) \quad . \quad . \quad . \quad (1)$$

That is, the carrier is shifted by ϕ_0 and the audio frequency or modulating frequency is shifted by an angle equal to the upper sideband phase shift minus the carrier phase shift (i.e. by α).

In phase-shift problems it is often the custom to use the convention that lag is positive. This has been done above, and is consistent with defining network transfer vectors as ratio input voltage divided by output voltage, see XXIV:4. Since the delay is equal to the differential of the phase shift with regard to angular frequency, the delay only appears as a positive quantity if lag is regarded as positive.

9. Examples of Important Feedback Circuits.

9.1. The Cathode Follower. Fig. 1 (*a*) shows the circuit which has been called the Cathode Follower, because the cathode follows the grid in its excursions about earth potential. It is not to be confused with Fig. 1 (*b*).

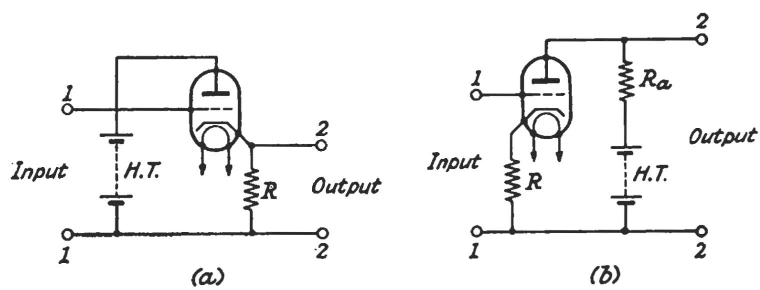


FIG. 1/XXIII:9.—(*a*) Cathode Follower. (*b*) Simple Cathode Feedback Circuit.

Conventions.

g_m = mutual conductance of valve.
R_0 = anode impedance of valve.
$\mu = R_0 g_m$ = voltage amplification factor of valve.
μ_p = forward amplification factor neglecting effect of feedback.
β = attenuation ratio of feedback path.
μ_e = effective overall amplification factor of circuit.

Evidently $\mu_p = \dfrac{\mu R}{R+R_0}$ and $\mu\beta = -\dfrac{\mu R}{R+R_0}$; this value of $\mu\beta$ is effective for determining both gain reduction and output impedance modification.

$$\mu_e = \frac{\mu_p}{1-\mu\beta} = \frac{\dfrac{\mu R}{R+R_0}}{1+\dfrac{\mu R}{R+R_0}} = \frac{\mu R}{R_0+(1+\mu)R} \quad . \quad . \quad (1)$$

Evidently μ_e can never exceed unity.

The output impedance of the circuit is R in parallel with R_0, divided by $(1 - \mu\beta)$: although it looks like series feedback, this is a case of parallel feedback :

$$\therefore \quad Z_{2f} = \frac{\dfrac{RR_0}{R+R_0}}{1+\dfrac{\mu R}{R+R_0}} = \frac{R_0}{1+\mu+\dfrac{R_0}{R}} \quad . \quad . \quad . \quad (2)$$

$$= \frac{R}{\dfrac{R}{R_0}+1+\dfrac{R}{R_0}\mu} \quad . \quad . \quad (2a)$$

When $\qquad R = \text{infinity}$

$$Z_{2f} = \frac{R_0}{1+\mu}, \text{ so that } \textit{approximately}$$

$$Z_{2f} = \frac{R_0}{\mu} = \frac{1}{g_m} = \frac{1}{\text{valve mutual conductance}}$$

It is evident that, since the anode is at earth potential, normal Miller capacity (see section below) is absent : the static grid-anode capacity is, however, effective between grid and earth. The grid-cathode capacity is reduced by the following action of the cathode in the ratio of the voltage of the grid to cathode, to the voltage of the grid to earth.

The factor by which the capacity is multiplied is, therefore, $1 - \mu_e$.

If a load Z is connected across terminals 2,2, the voltage amplification from terminals 1,1 to terminals 2,2 is given by equation (1) with R replaced by $\dfrac{RZ}{R+Z}$. The value of Z_{2f} is, of course, unchanged and is still given by equation (2) or (2*a*).

It will be seen from equation (2*a*) that the value of Z_{2f} is always less than R, and, as a result, a cathode follower valve is very useful for providing a low-impedance driving source. One instance of the use of a cathode follower valve is for driving the grid circuit of an audio-frequency valve in which the grid runs positive so that a large grid current flows. By driving from a low impedance source, the non-linearity which would otherwise occur is substantially avoided.

A cathode follower valve therefore has three advantages :

(1) Low input capacity.
(2) Low output impedance.
(3) By virtue of the negative feedback introduced, the effects of non-linearity in the cathode follower valve itself are reduced.

9.11. Simple Cathode Feedback Circuit. This is shown in Fig. 1 (*b*), and constitutes a case of series feedback. Observing the same conventions as before :

$$\mu_p = \frac{\mu R_a}{R_0+R+R_a}$$

$$\mu\beta \text{ (for gain reduction)} = -\frac{\mu R}{R_0+R+R_a}$$

$$\mu\beta \text{ (for impedance modification)} = -\frac{\mu R}{R_0+R}$$

$$\mu_e = \frac{\mu_p}{1-\mu\beta} = \frac{\mu R_a}{R_0+R_a+(1+\mu)R} \quad . \quad . \quad . \quad (3)$$

For the purpose of determining the output impedance, the resistance R_a is regarded as a shunt external to the amplifier, so that

$$Z_{2f} = Z_{2p}(1-\mu\beta) = (R+R_0)\left(1+\frac{\mu R}{R_0+R}\right) \quad . \quad . \quad (4)$$

The impedance looking back into the amplifier is then $\dfrac{RZ_{2f}}{R+Z_{2f}}$.

9.2. Input Impedance of Triode : Miller Capacity. The formula established above for the input impedance of an amplifier with feedback can be used to determine the input impedance of

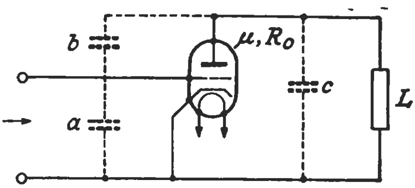


FIG. 2/XXIII:9.—Valve with Internal Capacities and Anode Load *L*.

a triode taking into account the feedback through the grid anode capacity. A triode is shown in Fig. 2 working into a load *L*. Negative parallel feedback occurs through the anode grid capacity.

Conventions with Reference to Fig. 2.

a = impedance of grid-cathode capacity.
b = impedance of anode-grid capacity.
c = impedance of anode-cathode capacity.
R_0 = anode impedance of valve.
L = anode load impedance.

The input impedance $Z_{1f} = \dfrac{Z_{1p}}{1 - \mu\beta}$

where Z_{1p} is the input impedance in the passive condition.

$$Z_{1p} = \frac{a\left[b+\dfrac{1}{\dfrac{1}{R_0}+\dfrac{1}{c}+\dfrac{1}{L}}\right]}{a+b+\dfrac{1}{\dfrac{1}{R_0}+\dfrac{1}{c}+\dfrac{1}{L}}}$$

$$= \frac{a\left[\dfrac{b}{R_0}+\dfrac{b}{c}+\dfrac{b}{L}+1\right]}{\dfrac{a+b}{R_0}+\dfrac{a+b}{c}+\dfrac{a+b}{L}+1}$$

$$= \frac{a(bcL+bR_0L+bR_0c+R_0cL)}{(a+b)cL+(a+b)R_0L+(a+b)R_0c+R_0cL}$$

$$= \frac{N_1}{D}, \text{ say}$$

$$\beta = \frac{-a}{a+b}\cdot\frac{\dfrac{1}{\dfrac{1}{a+b}+\dfrac{1}{c}+\dfrac{1}{L}}}{R_0+\dfrac{1}{\dfrac{1}{a+b}+\dfrac{1}{c}+\dfrac{1}{L}}}$$

$$= \frac{-\dfrac{a}{a+b}}{\dfrac{R_0}{a+b}+\dfrac{R_0}{c}+\dfrac{R_0}{L}+1}$$

$$= \frac{-acL}{R_0cL+(a+b)LR_0+(a+b)cR_0+(a+b)cL}$$

$$= \frac{-N_2}{D}, \text{ say}$$

$$\therefore\ 1 - \mu\beta = 1+\mu\frac{N_2}{D} = \frac{D+\mu N_2}{D}$$

$$\therefore\ Z_{1f} = \frac{Z_{1p}}{1-\mu\beta} = \frac{N_1}{D}\Big/\frac{D+\mu N_2}{D} = \frac{N_1}{D+\mu N_2}$$

$$= \frac{a[bcL+bR_0L+bR_0c+R_0cL]}{(a+b)cL+(a+b)R_0L+(a+b)R_0c+R_0cL+\mu acL}$$

$$= \frac{a[R_0L(b+c)+bc(L+R_0)]}{(a+b)(cL+R_0L+R_0c)+cL(R_0+\mu a)} \quad . \quad (5)$$

To simplify this expression for the case where a and c are neglected put

$$a = c = \text{infinity}.$$

Then

$$Z_{1f} = \frac{a[R_0Lc+bc(L+R_0)]}{a(cL+R_0c)+cL\mu a}$$

$$= \frac{R_0L+bL+bR_0}{L+R_0+L\mu}$$

$$= \frac{R_0L+b(R_0+L)}{R_0+L(1+\mu)} \quad . \quad . \quad . \quad . \quad (6)$$

If b is constituted by a small capacity C_b of impedance large compared to R_0, Z_{1f} becomes a capacitive impedance corresponding to a capacity C_m such that, *approximately*,

$$Z_{1f} = \frac{1}{jC_m\omega} = \frac{R_0+L}{R_0+L(1+\mu)}\cdot\frac{1}{jC_b\omega}. \quad . \quad . \quad . \quad (7)$$

$$\therefore\ C_m = \frac{R_0+L+L\mu}{R_0+L}C_b$$

$$= \left[1+\frac{L}{R_0+L}\mu\right]C_b \quad . \quad . \quad . \quad (8)$$

The effective input capacity of the grid circuit is therefore the normal grid-cathode capacity (represented by a in Fig. 2) plus C_m. The effective capacity C_m is called Miller-capacity and is given by multiplying the valve anode grid capacity by one plus the effective voltage amplification of the valve from anode to grid.

It is to be noted that if, in equation (7), we put $L = jx$ corresponding to an inductive load

$$Z_{1f} = \frac{R_0+jx}{R_0+jx(1+\mu)}\cdot\frac{1}{jC_b\omega}$$

$$= \frac{x - jR_0}{C_b\omega[R_0+jx(1+\mu)]}$$

This will have a negative resistance component when

$$\tan^{-1}\frac{R_0}{x}+\tan^{-1}\frac{x(1+\mu)}{R_0} > 90°$$

in which case singing will result, unless the grid is loaded with an appropriate resistance.

10. Measurement of Phase Shift.

It will have become apparent that in investigating amplifying circuits to which feedback is applied it is often necessary to know what are the phase-shift characteristics of these circuits, in addition to their amplification characteristics, expressed as a function of frequency.

The simplest form of phase-shift measuring circuit is shown in Fig. 1.

A is the amplifier being measured, R_L is the load impedance into which it is intended to work. $R_1 = R_2$ and $R_3 = R_4$ are resistances large enough to reduce the loading of the circuit to substantially negligible proportions, B_1 and B_2 are attenuating networks, R_5 is

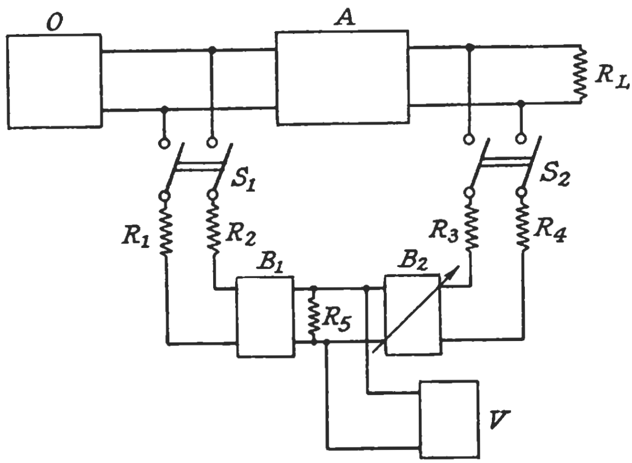


FIG. 1/XXIII:10.—Phase-Shift Measuring Circuit.

a resistance of convenient value terminating the two attenuating networks. V is a voltmeter. B_1 and B_2 must have at all times a combined attenuation which is large compared to the amplifier gain less the reflection loss introduced by the resistances R_1, R_2, R_3 and R_4. S_1 and S_2 are double-pole break switches. Further, the attenuation of B_2 is variable and the individual attenuations of B_1 and B_2 should preferably never be less than 20 db., so that the terminal conditions on their inputs (facing R_1, R_2, R_3 and R_4) do not appreciably affect their output impedances.

The method of measurement consists in first observing the voltage V_1 across R_5 with S_1 closed and S_2 open. S_1 is then opened and S_2 closed and attenuator B_2 adjusted until the voltage V_2 across R_5 is equal to V_1. S_1 is then closed and V_3, the voltage resultant from the two paths feeding into R_5, is observed.

If ϕ is the phase difference between the two equal voltages V_1

and V_2, it is obvious that their sum $V_3 = 2V_1 \cos \frac{\phi}{2}$, so that $\phi = 2 \cos^{-1} \frac{V_3}{2V_1}$.

An improved form of circuit is shown in Fig. 2, in which a bridge circuit B has been introduced to remove all restrictions on the attenuation in B_1 and B_2 other than those imposed by the require-

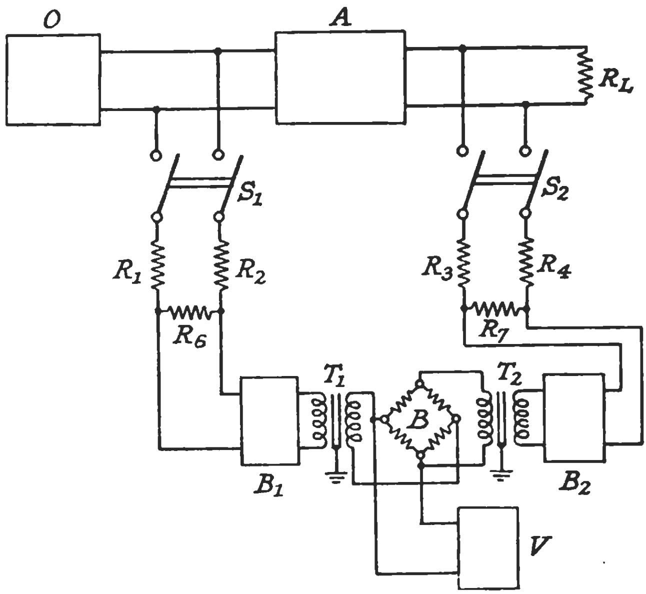


FIG. 2/XXIII:10.—Improved Phase-Shift Measuring Circuit.

ment of equality of V_1 and V_2. Identical wide-range repeating coils T_1 and T_2 have been introduced to eliminate any unbalance contributed by the bridge and voltmeter, or to enable unbalanced circuits to be used at input and output of the amplifier under test.

Resistances R_6 and R_7 have been added to limit the impedances facing the transformers to a constant value. Attenuator B_2 must now be of the constant impedance type, the circuit constant being adjusted so that T_1 and T_2 always work between the same impedances as one another.

More elaborate methods are described in a paper by Peterson, Kreer and Ware in the *Bell System Technical Journal* for October, 1934 : " Regeneration Theory and Experiment ".

11. Use of Series–Series and Parallel–Parallel Feedback in Combination.

By using series-series feedback in combination with parallel-parallel feedback, with the same loop again in each of the two feedback parts, a feedback circuit is constituted in which the input and output impedances are unmodified by the presence of feedback.

Such a circuit is shown in Fig. 1 (*a*). The detailed analysis of this circuit is rather complicated, but a good idea of its behaviour can be obtained from a comparatively superficial examination.

In Fig. 1 (*b*) the feedback path is redrawn. If the loop gain in each of the component paths of Fig. 1 (*b*) is the same the feedback

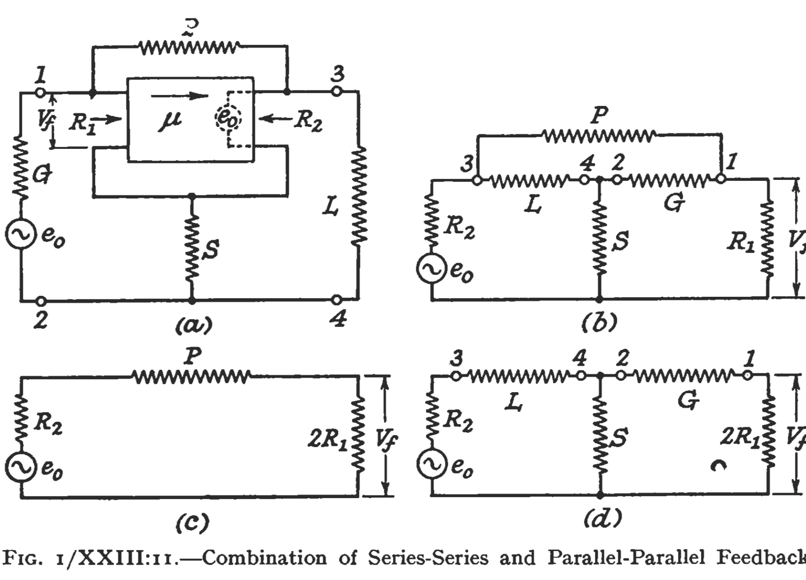


FIG. 1/XXIII:11.—Combination of Series-Series and Parallel-Parallel Feedback.

currents driven through the input of the amplifier by each path are equal. Each path therefore finds itself terminated in an impedance equal to twice R_1. The component paths are redrawn in Figs. 1 (*c*) and 1 (*d*). The value of β can therefore be calculated for each path, according to the methods established above for each type of feedback, entirely independently, by assuming that the value of R_1 is doubled.

Assuming that it is required to make the input impedance independent of feedback, the value of β for the parallel feedback path is calculated assuming $S = 0$, while the value of β for the series feedback path is calculated neglecting the presence of P, the value of input impedance of the amplifier being assumed to be $2R_1$ in each case. The value of generator and load impedance are given values as in the Table at foot of page 241. From these calculations the values of P and S can be determined to give the same required value of $\mu\beta$.

Since the feedback voltages due to the two feedback paths add in the input of the amplifier, the net feedback voltage is the sum of the two feedback voltages, and the gain reduction with such a system corresponds to the fraction:

$$\frac{1}{1 - \mu(\beta_1+\beta_2)},$$

where β_1 and β_2 are the values of β_1 and β_2 in each path calculated with generator and load impedance given their actual values.

Since neither L nor G is symmetrically disposed with regard to the formula for β with parallel feedback and the formula for β with series feedback, it follows that a circuit adjusted to give no input impedance modification due to feedback, will not be entirely free from output impedance modification due to feedback, and vice versa.

In practice, of course, it is seldom necessary to make accurate calculations of feedback, but only to adjust the one or both of the two feedback paths, by varying P and/or S until the desired result is obtained. Evidently, from the point of view of gain reduction there is no need to adjust the values of $\mu\beta$ (controlling the gain reduction) to exact equality, so that the fact that this cannot be done is unimportant.

12. Degradation of Response due to Application of Small Amounts of Feedback.

Although the application of stable negative feedback improves the response characteristic in the frequency region where the value of loop gain is large, the effect of a low loop gain, in conjunction with variation of phase shift, is to degrade the response of the amplifier.

Consider an amplifier with a flat response characteristic ($|\mu|$ independent of frequency) and a varying phase shift (θ varying with frequency) to which uniform feedback is applied at all frequencies (β independent of frequency). And let $|\mu\beta|$ be less than unity. It is convenient here to define μ and β as at the beginning of XXIII:1.

The effective amplification is:

$$\mu_e = \frac{\mu}{1+|\mu\beta|\underline{/\theta}} \quad . \quad . \quad . \quad . \quad (1)$$

Suppose that $|\mu\beta| = 0{\cdot}8$. Then when $\theta = 0$

$\mu_e = \mu/(1+0{\cdot}8) = \mu/1{\cdot}8$, and when $\theta = 180°$
$\mu_e = \mu/(1 - 0{\cdot}8) = 5\mu$.

The ratio between the two values of μ_e is nearly 10, corresponding to a degradation of response represented by a deviation of 20 db.

If θ varies from 0 to some angle less than 180°, a deviation of response will still occur. Suppose for instance that $|\mu\beta| = 1$ and θ varies from 0 to 90° ; μ_e then varies from $\frac{1}{2}\mu$ to $\mu/(1+j) = \frac{\mu}{\sqrt{2}}\overline{\backslash 45°}$ corresponding to a rise in response of 3 db.

The general-response characteristic is evidently defined by

$$20 \log_{10} \left|\frac{\mu_{ef}}{\mu_{e0}}\right| = \left|\frac{1+|\mu_0\beta_0|}{1+\mu_f\beta_f}\cdot\frac{\mu_f}{\mu_0}\right| \quad . \quad . \quad . \quad (2)$$

where the suffixes 0 denote the values of each quantity at the frequency at which $\theta = 0$ and the suffixes f denote the values of each quantity at any other frequency f.

It will be found that as $|\mu\beta|$ increases in magnitude the deviation in response is reduced. *It will be evident that at the ends of the pass band of any feedback amplifier the loop gain is low : there is a small amount of feedback. Hence this degradation of response always occurs in any feedback amplifier, and the design must be so arranged that it occurs outside the pass band of interest.*

12.1. Use of a Feedback Amplifier to Provide a Required Response Characteristic. It has already been pointed out that, when the loop gain is large, the effective amplification approximates to $1/\beta$. This means that the response of the amplifier is of the same form as the loss characteristic of the feedback path. If, therefore, it is required to equalize the response distortion of any line or network ; by providing an amplifier with a feedback path having the same loss characteristic as the network to be corrected, an equalizer with substantially the required response characteristic is constructed.

13. Overloading of Feedback Amplifiers.

Feedback operates by reducing the energy fed to the input of the amplifier proper. *The amplifier therefore operates at full gain from input to output.* If, therefore, the amplifier has been economically designed as a normal amplifier, the early stages will be operating with a considerable margin of safety when the output stage is fully loaded. Any overloading will therefore occur in the output stage as in a normal amplifier. Further, the frequency at which overloading tends to take place is determined as in a normal amplifier by the overall response characteristic (e.g. as given by equation 2/XXIII:12) and the spectrum of the input frequencies.

Apart from any modification of the response characteristic, the presence of feedback does not make an amplifier any more liable to overload. In fact, the distortion products for any given input level are reduced, and for this reason the feedback amplifier may be said to be less affected by mild overloading. Heavy overloads, which drive the amplifier into full limitation, produce as much distortion as if no feedback were present.

14. Method of Expressing Amount of Feedback.

The amount of feedback in a circuit is normally expressed by the gain reduction in decibels due to feedback at the frequency at which the phase shift round the feedback path is zero. This ignores the fact that the gain reduction at the ends of the frequency range may be less. Where possible, it should therefore be supplemented by a statement of the gain reduction at each end of the band of frequencies which is required to be transmitted.

Gain reduction should not be confused with the loop gain round the feedback path.

15. Negative Resistance Constituted by means of Feedback Circuits.

In XI:6 two forms of negative resistance are described, corresponding respectively to series-parallel and parallel-parallel (positive) feedback. These are constituted by the feedback circuits shown in Fig. 1/XI:6. The magnitude of the impedance looking into terminals 1,1 in this figure may be very easily calculated as follows.

Referring to Fig. 1 (*a*)/XI:6, $Z_{1p} = R_1+R_2$ and with positive feedback, $Z_{1f} = Z_{1p}(1 - \mu\beta)$, where μ and β are defined as in the list of conventions at the beginning of this chapter. Hence

$$Z_{1f} = \left(1 - \frac{\mu R_1}{R_1+R_2}\right)(R_1+R_2)$$

$$= R_1+R_2 - \mu R_1 \quad . \quad . \quad . \quad . \quad (1)$$

Z_{1f} is the impedance looking into terminals 1,1. In the case where R_1 and R_2 are pure resistances, and $\mu = |\mu|\underline{/0}$, Z_{1f} is a pure resistance R_N which is negative when μR_1 is greater than R_1+R_2, the exact value of R_N being given by equation (1).

Applying the normal criteria for stability to this circuit, it will evidently oscillate when R_N is greater than the positive resistance connected across 1,1.

Referring to Fig. 1 (*b*)/XI:6,

$$Z_{1p} = \frac{R_1R_2}{R_1+R_2}$$

and with positive feedback

$$Z_{1f} = \frac{Z_{1p}}{1 - \mu\beta} = \frac{\dfrac{R_1R_2}{R_1+R_2}}{1 - \mu\dfrac{R_1}{R_1+R_2}}$$

$$= \frac{R_1R_2}{R_1+R_2 - \mu R_1} \quad . \quad . \quad . \quad . \quad (2)$$

Again, when R_1 and R_2 are pure resistances and μ is of zero angle Z_{1f} is a negative resistance R_N when μR_1 is greater than R_1+R_2, the value of R_N being given by equation (2).

The stability criteria show that this circuit will oscillate when R_N is less than the positive resistance connected across 1,1.

15.1. Use of Feedback Circuits to Provide Special Forms of Reactance. Examination of equations (1) and (2) shows that if the angle of μ is 90°, and if μR_1 is much greater than $=R_1+R_2$, when R_1 and R_2 are pure resistances, the input impedance with feedback approximates to a pure reactance. This fact can be made use of for producing reactances of magnitude which can be modulated by including a variable-μ valve in the amplifier circuit and applying a modulating wave to the bias of this valve.

The reactance is negative in the series feedback case and positive in the parallel feedback case. If the angle of μ is changed from 90 to $-$ 90°, the sign of the reactance is changed.

Another use of such circuits is to provide reactances, which, over small frequency ranges, behave like negative inductances or negative capacities. A negative inductance has a positive reactance which is of magnitude inversely proportional to frequency, while a negative capacity has a negative reactance which is of magnitude proportional to frequency. Such reactances can evidently neutralize the reactances of real inductances and capacities over the frequency range for which the degree of simulation is adequate.

It will be evident that equivalent circuits can be constructed in which the output impedance of a feedback amplifier is used instead of its input impedance.

The practical limitation on the use of all circuits of this type for producing quaint reactances is due to the fact that such a circuit is only stable with small values of loop gain at the frequencies of interest. As a consequence it is not possible to obtain reactances which approach pure reactances: the resistance component is always appreciable.

CHAPTER XXIV

NETWORK THEORY

1. Scope.

NETWORK theory deals with the behaviour of various arrangements of linear impedance elements to which one or more e.m.f.s are applied.

A *passive* network is one which contains no internal sources of e.m.f. : all the e.m.f.s considered below should be regarded as being external to the network : in other words, the discussion deals with passive networks. A further limitation is that it relates to the steady state behaviour of networks under the influence of sinusoidal applied e.m.f.s : conditions are considered only after sufficient time has elapsed for the transients consequent on the initial application of the e.m.f.s to have died away. Although the principles established serve as a basis for determining also the behaviour of transients and the response of networks to e.m.f.s of form other than sinusoidal, the discussion of these rightly constitutes a separate subject and is not considered here.

Very broadly, steady state network theory may be divided into two parts : circuit analysis and circuit synthesis.

Circuit Analysis. This deals in detail with the behaviour of circuits, taking account of the contribution of every element in the circuit. Circuit analysis is capable of determining the behaviour of any circuit, but the conclusions of circuit analysis may sometimes be formulated in such a way that it leads to a useful classification of common types of circuit arrangement characterized by certain *parameters* which determine the behaviour of each type of circuit arrangement. This leads to circuit synthesis.

Circuit Synthesis. This consists in the grouping of impedance elements into regular configurations (e.g. the **T**, π, lattice and bridged-**D** networks), the determination of the parameters which define the performance of each network configuration, and finally, the determination of the performance of networks of known parameters when operating between known impedances. An extension of circuit synthesis determines the necessary relations between the impedance elements of networks of different form in order that they shall have the same performance. Finally, it determines the values of the impedance elements of minimal (the simplest possible) networks which have the same performance as more complicated and/or several networks of specific (but not necessarily the same) form.

Strictly, a parameter is a constant to which any one of an unlimited set of numerical values may be assigned ; for instance, an impedance in one part of a network may be defined as being equal to aZ, where Z is the value of another impedance element in the network and a is a parameter defining the value of the first element. It is, however, convenient to use the word parameter to denote any quantity defining the performance of a network, such, for instance, as the image impedances and the transfer vector, or the terms in the matrix of a network.

It is not always possible to draw a hard-and-fast line between circuit analysis and circuit synthesis, since the principles established by each are often helpful in developing the fundamental relations of the other. It has, however, been thought useful to adopt these headings with the reservation that the presentation of certain forms of circuit analysis will appear under the heading of circuit synthesis.

Broadly, two methods of circuit synthesis exist, which may be called respectively the English method and the German method. Since the English method owes a great deal to American engineers, the term English should be taken to refer only to the language in which the method was originally expounded.

The English method, in conjunction with circuit analysis, is most useful for determining the parameters which define the performance of each network configuration, and for determining the performance of networks of known parameters when operating between known impedances.

The German method is most useful for determining the relations between the elements of networks of different form in order to make them equivalent : in other words, so that their performance is defined by the same parameters. It is also particularly useful for resolving combinations of specific networks into single equivalent networks.

The English method has been so adequately set forth in three American books that to write a complete new exposition at the present time would be an unnecessary labour. These three books have been written respectively by A. E. Kennelly, K. S. Johnson and T. E. Shea ; they will be referred to by quoting the names of their authors ; their titles are given on the next page.

It has, however, been possible to present some of the conclusions of these works in a very much simplified form, which enables any engineer, who can handle complex algebra, to solve quickly any of the normal problems encountered. This form of presentation will be termed the simplified English method. Incidentally, it will be found to have a simplifying effect on the German method. It will, of course, be evident from the chapter on Filters and the chapter on Response Correction and Equalizer Design that these two subjects are special cases of the application and development of network theory. The three books previously mentioned are :—

Transmission Circuits for Telephone Communication, by K. S. Johnson.
Transmission Networks and Wave Filters, by T. E. Shea.
The Application of Hyperbolic Functions to Electrical Engineering, by A. E. Kennelly.

2. Circuit Analysis.

Circuit analysis involves the application of all the relations between voltage and current developed in IV and V, with special reference to Thévenin's Theorem, the Principle of Superposition, the Principle of Reciprocity and Kirchhoff's Laws. Examples of circuit analysis have appeared in VI and VII, XVI and XXI.

Kirchhoff's Laws provide a powerful method of solving complicated network problems, particularly those where more than one e.m.f. is present. A word of advice and warning is, however, apt : many problems to which these laws are applicable can be solved very much more simply by other means. Further, novices will often find themselves in difficulties with regard to the conventions to be assigned for directions of current flow. Finally, the solution of the equations resulting from the application of Kirchhoff's Laws is often extremely laborious. Examples of the application of Kirchhoff's Laws have been given in V:18 and these represent cases where Kirchhoff's Laws constitute the only practical method of solution. No further examples will be given here, but reference should be made to the above examples.

Where possible, therefore, other methods of solution should be used. One alternative method which is often very useful is the method of " Open and Shorts " (open-circuit and short-circuit impedances) developed in XXIV:4.13 and used in XXIV:5.1 and 5.2 for the determination of the image impedances of T and π networks respectively.

In this section another method will be described which will be called the Method of Inspection. This method is not of such general application as Kirchhoff's Laws, but is very much simpler and quicker to apply. In many cases the answer to a problem can be written down immediately. See also XXIV:2.4.

2.1. The Method of Inspection. This is best illustrated by examples.

Example 1.

Suppose that in Fig. 1 it is required to determine the impedance Z_s looking into terminals 0,0.

Let Z_1 be the impedance looking right from terminals 1,1.
Z_3 be the impedance looking right from terminals 3,3.
and Z_4 be the impedance looking right from terminals 4,4.

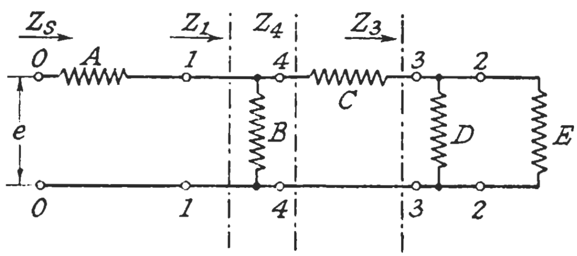


FIG. 1/XXIV:2.—Network Representing π Fourpole between Generator of Internal Impedance A and Internal e.m.f. e and a Load of Impedance E.

Then

$$Z_3 = \frac{DE}{D+E} \qquad (1)$$

$$Z_4 = C+Z_3 \qquad (2)$$

$$Z_1 = \frac{BZ_4}{B+Z_4} \qquad (3)$$

$$Z_s = A+Z_1 \qquad (4)$$

Without labouring the point further it is evident that the impedance Z_s is given by substituting the value of Z_3 from (1) into (2), the resultant value of Z_4 into (3) and the resultant value of Z_1 into (4). Hence, in practice the value of Z_s may be written down directly without forming equations (1) to (4) as :

$$Z_s = A + \frac{B\left[C+\dfrac{DE}{D+E}\right]}{B+C+\dfrac{DE}{D+E}}$$

$$= A + \frac{B[C(D+E)+DE]}{(B+C)(D+E)+DE} \qquad (5)$$

Example 2.

If in Fig. 1 an e.m.f. e is applied across terminals 0,0, what is the resultant voltage V_2 across terminals 2,2?

Let V_1 be the voltage across terminals 1,1. Then evidently element A and the impedance Z_1 presented at terminals 1,1 constitute a potentiometer across terminals 0,0, i.e. across e.

Hence
$$V_1 = \frac{Z_1}{A+Z_1}e \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where Z_1 is defined by equations (3), (2), and (1). In the same way element C and the impedance presented at terminals 3,3 constitute a potentiometer across terminals 4,4 and 1,1, and so across V_1.

Hence
$$V_2 = \frac{Z_3}{C+Z_3}V_1 \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where Z_3 is defined by equation (1).

Substituting the value of V_1 from (6) into (7)

$$V_2 = \frac{Z_3}{C+Z_3} \times \frac{Z_1}{A+Z_1}e \quad . \quad . \quad . \quad (8)$$

Writing in (8) the values of Z_3 and Z_1 in full,

$$V_2 = \frac{\dfrac{DE}{D+E}}{C+\dfrac{DE}{D+E}} \times \frac{\dfrac{B\left[C+\dfrac{DE}{D+E}\right]}{B+C+\dfrac{DE}{D+E}}}{A+\dfrac{B\left[C+\dfrac{DE}{D+E}\right]}{B+C+\dfrac{DE}{D+E}}}e$$

(This expression would normally be written down directly without forming equation (8), although this may, of course, be done if required.)

$C+\dfrac{DE}{D+E}$ cancels, and so, multiplying above and below by $B+C+\dfrac{DE}{D+E}$

$$V_2 = \frac{\dfrac{BDE}{D+E}}{AB+AC+\dfrac{ADE}{D+E}+BC+\dfrac{BDE}{D+E}}e$$

Multiplying above and below by $(D+E)$,

$$V_2 = \frac{BDE}{(AB+BC+CA)(D+E)+DE(A+B)}e \quad . \quad . \quad (9)$$

For purposes of evaluation this expression is handled as described in the next section. (XXIV:2.4 gives alternative method.)

2.2. Method of Evaluation of Expressions containing Complex Quantities. In general, the quantities A, B, C, D, and E in expressions (5) and (9) are complex, since they represent impedances of any magnitude and angle

$$\left.\begin{aligned} \text{Let } A &= R_a+jX_a = M_a\underline{/\phi_a} \\ B &= R_b=jX_b = M_b\underline{/\phi_b} \end{aligned}\right\}\text{etc.} \qquad (10)$$

(Evidently $M_a = \sqrt{R_a^2+X_a^2}$, $\phi_a = \tan^{-1}\dfrac{X_a}{R_a}$, $R_a = M_a \cos \phi_a$, $X_a = M_a \sin \phi_a$, etc.)

2.21. The Direct Substitution Method. In this method numerical substitution is made before rearrangement of the expression.

The first step is to express all impedances in the form suitable for the first operation to be performed on them.

If A is to be multiplied by B, write

$$AB = M_a\underline{/\phi_a} \times M_b\underline{/\phi_b} = M_aM_b\underline{/\phi_a+\phi_b} \qquad (11)$$

If A is to be divided by B, write

$$AB = \frac{M_a\underline{/\phi_a}}{M_b\underline{/\phi_b}} = \frac{M_a}{M_b}\underline{/\phi_a - \phi_b} \qquad (12)$$

In other words, for multiplication or division, express impedances in the form $M\underline{/\phi}$.

For addition or subtraction express impedances in the form $R+jX$.

e.g. $A \pm B = R_a+jX_a \pm (R_b+jX_b) = R_a \pm R_b + j(X_a \pm X_b)$.

For instance, equation (9) becomes

$$V_2 = \frac{M_bM_dM_e\underline{/\phi_b+\phi_d+\phi_e}}{[M_aM_b\underline{/\phi_a+\phi_b}+M_bM_c\underline{/\phi_b+\phi_c}+M_cM_a\underline{/\phi_c+\phi_a}] \times [R_d \times R_e \times jX_d \times jX_e] \times M_dM_e\underline{/\phi_d \times \phi_e}[R_a \times R_b \times jX_a \times jX_b]}$$

This is now of the form

$$V_2 = \frac{M_1\underline{/\phi_1}}{[M_2\underline{/\phi_2}+M_3\underline{/\phi_3}+M_4\underline{/\phi_4}][R_1+jX_1]+M_5\underline{/\phi_5}[R_2+jX_2]}$$

Where $M_1 = M_bM_dM_e$, $\phi_1 = \phi_b+\phi_d+\phi_e$, and so on.

Since $M_1\underline{/\phi_1}$ is going to be divided by the denominator it should be left in this form.

$M_2\underline{/\phi_2}$ and the other terms in the same bracket must be reduced to the form $R+jX$ by putting $R = M \cos \phi$ and $X = M \sin \phi$. The resistances and reactances must then be added together and the whole reduced to the form $M/\underline{\phi}$ for the purpose of multiplication with R_1+jX_1, which must also be reduced to the form $M/\underline{\phi}$ before multiplication. After multiplication the resulting vector must be converted to the form $R+jX$ for addition to the other term in the denominator.

The other term in the denominator must be treated in a similar manner.

The whole denominator is in this way finally reduced to the form $M/\underline{\phi}$ and divided into the numerator.

2.22. The Explicit Method. In this method, before substitution is made, the whole expression is transformed to the form $A+jB$ (or $M/\underline{\phi}$ according to which form is required), where A and B (or M and ϕ) are functions of R_a, R_b, etc., and X_a, X_b, etc.

For this purpose, in the present case, all impedances are inserted in the formula in the form $R+jX$.

Hence equation (9) becomes

$$
\left.
\begin{aligned}
V_2 &= \frac{(R_b+jX_b)(R_d+jX_d)(R_e+jX_e)}{\begin{array}{l}[(R_a+jX_a)(R_b+jX_b)+(R_b+jX_b)(R_c+jX_c)+(R_c+jX_c)\\ \quad (R_a+jX_a)]\times[R_d+R_e+j(X_d+X_e)]+(R_d+jX_d)\\ \quad (R_e+jX_e)[R_a+R_b+j(X_a+X_b)]\end{array}} \\
&= \frac{\begin{array}{l}R_bR_dR_e - X_bX_dR_e - X_dX_eR_b - X_eX_bR_d\\ \qquad +j(X_bR_dR_e+X_dR_eR_b+X_eR_bR_d - X_bX_dX_e)\end{array}}{\begin{array}{l}(R_aR_b+R_bR_c+R_cR_a)(R_d+R_e) - (X_aX_b+X_bX_c+X_cX_a)\\ \quad (R_d+R_e) - (R_aX_b+R_bX_a+R_bX_c+R_cX_b+R_cX_a\\ \quad +R_aX_c)(X_d+X_e)+R_dR_e(R_a+R_b) - X_dX_e(R_a+R_b)\\ \quad - R_dX_e(X_a+X_b) - R_eX_d(X_a+X_b)\\ +j\left[\begin{array}{l}(R_aR_b+R_bR_c+R_cR_a)(X_d+X_e) - (X_aX_b+X_bX_c+X_cX_a)\\ \quad (X_d+X_e)+(R_aX_b+R_bX_a+R_bX_c+R_cX_b+R_cX_a\\ \quad +R_aX_c)(R_d+R_e)+R_dR_e(X_a+X_b)+X_dX_e(X_a+X_b)\\ \quad +R_dX_e(R_a+R_b)+R_eX_d(R_a+R_b)\end{array}\right]\end{array}}
\end{aligned}
\right\} \quad (13)
$$

This expression is of the form

$$V_2 = \frac{R_1+jX_1}{R_2+jX_2} \quad . \quad . \quad . \quad . \quad (14)$$

where $R_1 = R_bR_dR_e - X_bX_dX_e - X_dX_eR_b - X_cX_bX_d$, and so on.

It may be transformed to the form $A+jB$ by multiplying above and below by $R_2 - jX_2$.

$$\therefore\ V_2 = \frac{R_1R_2+X_1X_2+j(R_2X_1 - R_1X_2)}{R_2^2+X_2^2}$$

$$= \frac{R_1R_2+X_1X_2}{R_2^2+X_2^2}+j\frac{R_2X_1 - R_1X_2}{R_2^2+X_2^2} \quad . \quad . \quad (15)$$

The advantage of the explicit method is that the final substitution involves no complex arithmetic and leads to greater safety in making approximations. In the example given it would, however, be simpler to use the direct substitution method on account of the laborious algebra and arithmetic involved in the explicit method. In many initially complicated practical cases where, owing to some of the impedances being equal, a number of terms cancel, the final expression is simplified. In this case, although the algebra is still (though less) laborious, the avoidance of complex arithmetic makes the explicit method worth while.

The explicit method is essential when it is required to produce generalized formulae ; see XXI:5.

2.23. Examples.

Example 1.

Evaluate Z_4 in Fig. 1 when $C = 500+j500$, $D = 2{,}000+j2{,}000$, and $E = 1{,}000 - j1{,}000$.

(*a*) *By Use of Direct Substitution Method.*

From (1) and (2), $Z_4 = C+\dfrac{DE}{D+E}$.

Since D and E are multiplied together in the numerator of the expression, they must be converted to the form $M/\underline{\phi}$ for use in the numerator only.

$$D = \sqrt{2{,}000^2+2{,}000^2}\,\big/\underline{\tan^{-1}\frac{2{,}000}{2{,}000}} = 2{,}828/\underline{45^\circ}$$

$$E = \sqrt{1{,}000^2+1{,}000^2}\,\overline{\big\backslash\tan^{-1}\frac{1{,}000}{1{,}000}} = 1{,}414\overline{\backslash 45^\circ}$$

$$\therefore\ Z_4 = 500+j500+\frac{2{,}828 \times 1{,}414/\underline{0}}{2{,}000+j2{,}000+1{,}000 - j1{,}000}$$

$$= 500+j500+\frac{2{,}828 \times 1{,}414/\underline{0}}{\sqrt{3{,}000^2+1{,}000^2}\,\big/\underline{\tan^{-1}\frac{1{,}000}{3{,}000}}}$$

$$= 500+j500+1{,}260\overline{\backslash 18^\circ\ 25''}$$
$$= 500+j500+1{,}260 \cos 18^\circ\ 25'' - j1{,}260 \sin 18^\circ\ 25''$$
$$= 500+j500+1{,}260 \times 0{\cdot}949 - j1{,}260 \times 0{\cdot}316$$
$$= 500+j500+1{,}200 - j398$$
$$= 1{,}700+j102$$

(*b*) *By Use of Explicit Method.*

Put $C = R_c+jX_c$, $D = R_d+jX_d$ and $E = R_e+jX_e$.

Then
$$Z_4 = C+\frac{DE}{D+E}$$

$$= R_c+jX_c+\frac{(R_d+jX_d)(R_e+jX_e)}{R_d+R_e+j(X_d+X_e)}$$

$$= R_c+jX_c+\frac{[R_dR_e - X_dX_e+j(R_dX_e+R_eX_d)][R_d+R_e-j(X_d+X_e)]}{(R_d+R_e)^2+(X_d+X_e)^2}$$

$$= R_c+jX_c+\frac{(R_dR_e - X_dX_e)(R_d+R_e)+(R_dX_e+R_eX_d)(X_d+X_e) +j[(R_d+R_e)(R_dX_e+R_eX_d)+(X_dX_e - R_dR_e)(X_d+X_e)]}{(R_d+R_e)^2+(X_d+X_e)^2}$$

$$= R_c+jX_c+\frac{R_dR_e(R_d+R_e)+R_dX_e^2+R_eX_d^2 +j[R_d^2X_e+R_e^2X_d+X_dX_e(X_d+X_e)]}{(R_d+R_e)^2+(X_d+X_e)^2}$$

Even in this simple case it is evident that the explicit method leads to a result which is far too complex to be of much practical use in competition with the direct substitution method.

Substituting in this formula makes

$$Z_4 = 500+j500+1{,}200-j400 = 1{,}700+j100$$

This agrees with the previous answer within slide-rule accuracy, having regard to the number of operations involved.

Lest it may appear from the above that the explicit method is of little value, it should be pointed out that it is of the utmost value, as reference to the generalized formulae and characteristics of XXI:5 will show. As has been explained, this is because in practice some simplification of the resulting expressions often occurs, and also because some or all of the impedance elements are either pure resistances or pure reactances.

An example will now be considered in which the explicit method is useful. This example also illustrates the use of circuit parameters and the means of deriving a generalized formula.

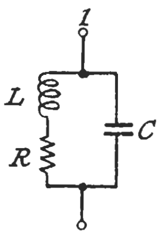


FIG. 2/XXIV:2. Parallel Resonant Circuit.

Example 2.

Derive the formula for the impedance of a parallel resonant circuit. This is the impedance looking into terminals 1,1 in Fig. 2.

Conventions.

L = the value of the inductance in henrys.
C = the value of the capacity in farads.
R = the internal resistance of the coil constituting the inductance, in ohms.
f = frequency.
$\omega = 2\pi f.$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

$$\omega_0 = 2\pi f_0. \qquad \therefore\ L\omega_0 = \frac{1}{C\omega_0}.$$

$$Q = \frac{L\omega_0}{R}.$$

Z = the impedance looking into terminals 1,1 $= R_e + jX_e$.

$$\text{Then } Z = \frac{\frac{1}{jC\omega}(R+jL\omega)}{R+j\left(L\omega-\frac{1}{C\omega}\right)} = \frac{\frac{1}{jC\omega}(R+jL\omega)\left[R-j\left(L\omega-\frac{1}{C\omega}\right)\right]}{R^2+\left(L\omega-\frac{1}{C\omega}\right)^3}$$

$$= -\frac{j(R+jL\omega)[RC\omega-j(LC\omega^2-1)]}{R^2C^2\omega^2+(LC\omega^2-1)^2}$$

$$= \frac{RLC\omega^2-R(LC\omega^2-1)}{R^2C^2\omega^2+(LC\omega^2-1)^2}+j\frac{L\omega(1-LC\omega^2)-R^2C\omega}{R^2C^2\omega^2+(LC\omega^2-1)^2}$$

$$= \frac{R}{R^2C^2\omega^2+(LC\omega^2-1)^2}+j\frac{L\omega-L^2C\omega^3-R^2C\omega}{R^2C^2\omega^2+(LC\omega^2-1)^2} \quad . \quad (16)$$

Equation (16) gives the values of effective resistance and reactance presented at terminals 1,1 and may be used to work out the value of Z over a range of frequencies if required. Such a procedure is, however, very tedious and liable to errors. The expression may be very much simplified as follows:

$$\text{Since } L\omega_0 = \frac{1}{C\omega_0},\quad Q = \frac{L\omega_0}{R} = \frac{1}{C\omega_0 R}.$$

$$\therefore\ C = \frac{1}{L\omega_0^2} \text{ and } R = \frac{L\omega_0}{Q} \quad . \quad . \quad . \quad (17)$$

Substituting the values of C and R from (17) into (16):

$$Z = \frac{\frac{L\omega_0}{Q}}{\left(1-\frac{\omega^2}{\omega_0^2}\right)^2+\frac{1}{Q^2}\frac{\omega^2}{\omega_0^2}}+j\frac{L\omega_0\frac{\omega}{\omega_0}\left(1-\frac{\omega^2}{\omega_0^2}-\frac{1}{Q^2}\right)}{\left(1-\frac{\omega^2}{\omega_0^2}\right)^2+\frac{1}{Q^2}\frac{\omega^2}{\omega_0^2}} \quad . \quad (18)$$

$$= L\omega_0 \left[\frac{\frac{1}{Q}}{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \frac{1}{Q^2}\frac{\omega^2}{\omega_0^2}} + j \frac{\left(1 - \frac{1}{Q^2} - \frac{\omega^2}{\omega_0^2}\right)\frac{\omega}{\omega_0}}{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \frac{1}{Q^2}\frac{\omega^2}{\omega_0^2}} \right] \quad . \quad (19)$$

2.3. Representation of a Generator in Circuit Analysis and Circuit Synthesis. A generator here refers to any source of electrical power whether a battery, a dynamo or alternator, an oscillator or an amplifier or any passive network to the input of which energy is supplied and which, by supplying power at its output, affects the circuit connected to its output exactly in the same way as a generator.

Thévenin's theorem states that when a generator is connected to a load impedance the current that will flow through the load is given by the open circuit voltage of the generator divided by the vector sum of the impedance looking back into the generator and the load impedance. The impedance looking back into the generator is called the internal impedance of the generator and the open circuit voltage of the generator is called the internal e.m.f. of the generator.

It follows that any generator, as far as its influence on any external circuit is concerned, can be represented by an e.m.f. equal to its internal e.m.f. in series with an impedance equal to its internal impedance. This is merely restating Thévenin's theorem.

2.31. Relation between Response, Total Loss and Insertion Loss of a Circuit. When a generator of internal e.m.f. e and internal impedance Z_g is connected to a load impedance Z_L a current flows of magnitude

$$I = \frac{e}{Z_g + Z_L} \quad . \quad . \quad . \quad . \quad (20)$$

Since in general Z_g and Z_L are impedances which vary with frequency, the value of I will vary with frequency.

The Response of the circuit at any frequency f is defined as equal to

$$-10 \log_{10} \frac{P_{fr}}{P_f} = -10 \log_{10} \frac{I_{fr}^2 R_{fr}}{I_f^2 R_f} = -20 \log_{10} \frac{I_{fr}\sqrt{R_{fr}}}{I_f\sqrt{R_f}}$$

where e is constant and

P_f = the power supplied to the load at frequency f
P_{fr} = the power supplied to the load at any chosen reference frequency
I_f = the current supplied to the load at frequency f

I_{fr} = the current supplied to the load at the reference frequency

R_f = the series resistance component of Z_L at frequency f

R_{fr} = the series resistance component of Z_L at the reference frequency.

The Response at the reference frequency is evidently zero.

By creating definitions for the Total Loss of a circuit, simple and more general definitions of Response can be obtained in terms of Reflection Loss and Insertion Loss which are readily calculable.

The *Total Loss* of the simple circuit described is equal to the Reflection Loss (see VII:10.1).

When a network is inserted between the generator and the load the Insertion Loss at any frequency is the decibel equivalent of the current reduction in the load consequent on the insertion of the network (see VII:12 and XXIV:4.14). Fig. 1 was originally designed to represent such a case. In this figure Z_g is represented by A, and Z_L by E.

The *Total Loss* of such a circuit is equal to the Reflection Loss *between A and E* with the network removed and terminals 1,1 connected to 2,2, plus the Insertion Loss.

The *Response* of such a circuit is equal to the Total Loss at each frequency, minus the Total Loss at the reference frequency, loss being plotted as a negative quantity.

It will now be clear that, while the characteristics of a network such as that shown between terminals 1,1, and 2,2 in Fig. 1 (see XXV:4 and 5 and 6) can be completely specified without reference to its terminating impedances, the performance of a network can only be specified by taking into account both the generator impedance and the load impedance.

2.4. The Method of Addition. This is a systematic method of expressing the currents and/or voltages at any point in a ladder structure in terms of the currents and/or voltages at any other point in the structure nearer the output. For this purpose all currents and voltages are expressed in terms of the voltage across the output load or last shunt element, or the current through this element.

The method is applicable to ladder-type structures only.

Fig. 3 shows a ladder structure with the currents through and the voltages across each element as indicated. The input is supposed to be on the left ; the element A can either be regarded as the load or as the final element of the structure.

Starting from the voltage across and the current through the element A, the other voltages and currents are then built up as follows by a simple process of addition.

$$I_b = I_a = \frac{V_2}{A}, \quad V_2 = V_a \quad . \quad . \quad . \quad (21)$$

$$V_c = V_a+V_b = V_2+I_bB = V_2+\frac{B}{A}V_2 = V_2\left(1+\frac{B}{A}\right) \quad . \quad . \quad (22)$$

$$I_d = I_b+I_c = \frac{V_2}{A}+\frac{V_c}{C} = \frac{V_2}{A}+\frac{V_2}{C}\left(1+\frac{B}{A}\right) \quad . \quad . \quad . \quad . \quad (23)$$

$$V_e = V_c+V_d = V_2\left(1+\frac{B}{A}\right)+V_2\left(\frac{D}{A}+\frac{D}{C}\left(1+\frac{B}{A}\right)\right)$$

$$= V_2\left[\frac{D}{A}+\left(1+\frac{D}{C}\right)\left(1+\frac{B}{A}\right)\right] \quad . \quad (24)$$

$$I_f = I_d+I_e = V_2\left[\frac{1}{A}+\frac{1}{C}\left(1+\frac{B}{A}\right)+\frac{D}{AE}+\frac{1}{E}\left(1+\frac{D}{C}\right)\left(1+\frac{B}{A}\right)\right] . \quad (25)$$

$$V_1 = V_e+V_f = V_2\left[\frac{D}{A}+\left(1+\frac{D}{C}\right)\left(1+\frac{B}{A}\right)+\frac{F}{A}+\frac{F}{C}\left(1+\frac{B}{A}\right)\right.$$

$$\left.+\frac{FD}{AE}+\frac{F}{E}\left(1+\frac{D}{C}\right)\left(1+\frac{B}{A}\right)\right]$$

$$= V_2\left[\left(1+\frac{B}{A}\right)\left\{1+\frac{D}{C}+\frac{F}{C}+\frac{F}{E}\left(1+\frac{D}{C}\right)\right\}+\frac{D+F}{A}+\frac{FD}{AE}\right]$$

$$= V_2\left[\left(1+\frac{B}{A}\right)\left\{\left(1+\frac{D}{C}\right)\left(1+\frac{F}{E}\right)+\frac{F}{C}\right\}+\frac{D}{A}+\frac{F}{A}\left(1+\frac{D}{E}\right)\right] \quad . \quad (26)$$

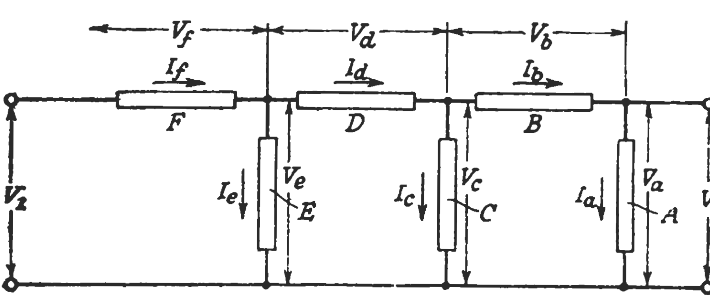


FIG. 3/XXIV:2.—Ladder Structure illustrating Method of Addition.

Each of the equations (22), (23), (24), etc., constitutes a solution applicable to a structure of corresponding form : e.g. equation (22) gives the relation between the input and output voltage of the structure constituted by elements A and B ; equations (23) and (24) apply to the structure formed by elements A, B, C and D, etc.

This method is due to C. G. Mayo.

3. Introduction to Circuit Synthesis.

Circuit Synthesis deals with the behaviour of *Fourpoles*.

A *fourpole* is a four-terminal network in which power is supplied at one pair of (input) terminals (1,1 in Fig. 1) and, after transmission through the network is supplied, attenuated in magnitude and shifted in phase (as regards its current and voltage components) to any load impedance connected to the output pair of terminals (2,2 in Fig. 1).

The general representation of a fourpole is shown in Fig. 1 at (*a*), while particular examples of fourpoles are shown at (*b*), (*c*), (*d*) and (*e*). An ordinary transmission line such as a feeder or a telephone circuit constitutes one very important example of a fourpole.

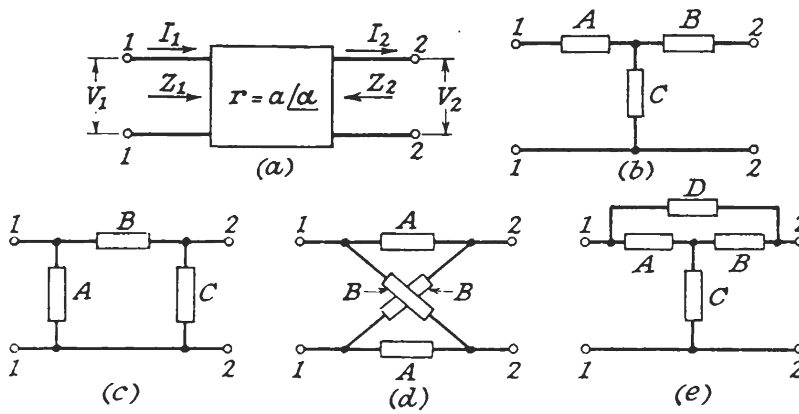


FIG. 1/XXIV:3.—Fourpoles.

The fourpoles at (*b*) and (*c*) will be recognized as T and π networks respectively, while those at (*c*) and (*d*) are respectively lattice and bridged-T structures.

As has been explained, the English method is most suitable for developing the transmission equations which determine the behaviour of fourpoles of known characteristics, while the German method is most suitable for determining the relations between different types of networks to make them equivalent.

Section 4 which follows will develop the English method in simplified form.

It should be stressed that the modification of the English method in no way limits its scope of application or its effectiveness in solving particular problems. For anyone with an elementary knowledge of complex algebra, the reverse is the case, on account of the fewer thought processes involved.

Section 5 will then show how to determine the characteristic parameters of specific types of fourpole (i.e. the image impedances and transfer vector ; see definitions in section 4).

Section 6 will then develop the German method and show its applications. It will appear that the German method has a rather more extensive use than is indicated by the bare statement above.

4. Circuit Synthesis : The Simplified English Method.

The English method normally uses Complex Hyperbolic Functions which makes the subject appear much more difficult than it really is. Since no satisfactory tables of Hyperbolic Functions exist and the necessary interpolation in existing tables is very cumbersome, the only practical value of the use of such functions is brevity in the derived formulae which can quite easily be achieved by other devices.

This advantage is in any case outweighed by the obscurity in which the subject is thereby wrapped in the eyes of those who are not mathematically inclined. For this reason, in the treatment below, formulae are derived first without reference to Hyperbolic Functions. *The formulae so derived are entirely adequate for all purposes* and no reference need be made to the subsequent correlations with the hyperbolic forms : these have, however, been inserted in order to relate the discussion to current literature.

Since this is a new exposition of transmission theory or network theory, new and simpler terms are used. The conventions and definitions which follow specifically define these terms. While it is instructive, and indeed essential, to glance through these before reading the development of the theory it will probably be found that the conventions and terms are better understood with reference to the theory. The method of developing transmission formulae should, therefore, be read, referring to the definitions and conventions to explain each symbol and term as it appears.

Conventions Relating to Fourpoles.

The *matched condition* of a fourpole is the condition in which terminals 1,1 (see Fig. 2) are faced with an impedance equal to Z_1, and terminals 2,2 are faced with an impedance equal to Z_2, regardless of the direction of transmission through the network. That is, regardless of whether terminals 1,1 or 2,2 constitute the input terminals.

Z_1 = image impedance looking into terminals 1,1.
Z_2 = image impedance looking into terminals 2,2.
${}_1V_1$ and ${}_1I_1$ = respectively the voltage applied at, and the current entering, terminals 1,1 in the matched condition, with terminals 1,1 constituting the input terminals.
${}_1V_2$ and ${}_1I_2$ = respectively the voltage appearing at, and the current leaving, terminals 2,2 in the matched condition, with terminals 1,1 constituting the input terminals.
${}_2V_2$ and ${}_2I_2$ = respectively the voltage applied at, and the current entering, terminals 2,2 in the matched condition, with terminals 2,2 constituting the input terminals.
${}_2V_1$ and ${}_2I_1$ = respectively the voltage appearing at, and the current leaving, terminals 1,1 in the matched condition, with terminals 2,2 constituting the input terminals.
V_1 = voltage applied across terminals 1,1 = input voltage at 1,1 under any condition of impedance termination.
V_2 = voltage appearing at terminals 2,2 under any condition of impedance termination.
I_1 = current entering terminals 1,1 under any condition of impedance termination.
I_2 = current leaving terminals 2,2 under any condition of impedance termination.
v_1 and v_2 = incident voltages at 1,1 and 2,2 respectively.
v_1' and v_2' = reflected voltages at 1,1 and 2,2 respectively.
i_1 and i_2 = incident currents at 1,1 and 2,2 respectively.
i_1' and i_2' = reflected currents at 1,1 and 2,2 respectively.
s = the impedance ratio of the fourpole $= \dfrac{Z_1}{Z_2}$, a vector.
r = the root impedance ratio of the fourpole $= \sqrt{s}$, a vector.
u = the (symmetric) transfer vector of the fourpole

$$= \frac{1}{r} \times \frac{{}_1V_1}{{}_1V_2} = \frac{1}{r} \times \frac{{}_2I_2}{{}_2I_1} = r \times \frac{{}_2V_2}{{}_2V_1} = r \times \frac{{}_1I_1}{{}_1I_2} = a\underline{/\alpha}$$

a = the attenuation ratio of the network $= |u|$, a numeric.
α = the (symmetric) phase shift of the network = the angle of u.

$S_n = \frac{1}{2}(u - 1/u)$, a vector . . . . (1)
$C_s = \frac{1}{2}(u + 1/u)$, a vector . . . . (2)

$$T_n = \frac{S_n}{C_s} = \frac{u - 1/u}{u + 1/u} = \frac{u^2 - 1}{u^2 + 1}, \text{ a vector} \quad . \quad . \quad . \quad (3)$$

$$C_t = \frac{1}{T_n} = \frac{u^2+1}{u^2-1}, \text{ a vector} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

L = the image loss of the fourpole = the insertion loss in decibels when terminated in its image impedances $= 20 \log_{10} a$ decibels.

It may be noticed that the definition of $u = a/\underline{\alpha}$, the symmetric transfer vector, is so formulated that when the network is such that under matched conditions the output voltage lags behind the input voltage, assuming lag to be represented by a negative angle, the angle α will be positive. (This definition is consistent with the common definition of propagation constant.) This is slightly confusing and is the reason why a fourpole phase lag is often represented by a positive angle.

The Quantities S_n, C_s, T_n and C_t.

These are merely short ways of writing the expressions to which they are equated.

When required, they may be evaluated by putting $u = a/\underline{\alpha}$, as follows:

$$S_n = \frac{1}{2}\left(a/\underline{\alpha} - \frac{1}{a}\overline{\backslash\alpha}\right) = \frac{1}{2}\left(a \cos \alpha + ja \sin \alpha - \frac{1}{a} \cos \alpha + j\frac{1}{a} \sin \alpha\right)$$

$$= \frac{1}{2}\left(a - \frac{1}{a}\right) \cos \alpha + j\frac{1}{2}\left(a + \frac{1}{a}\right) \sin \alpha \quad . \quad . \quad . \quad . \quad (1a)$$

$$C_s = \frac{1}{2}\left(a + \frac{1}{a}\right) \cos \alpha + j\frac{1}{2}\left(a - \frac{1}{a}\right) \sin \alpha \quad . \quad . \quad . \quad . \quad (2a)$$

$$T_n = \frac{a^2 \cos 2\alpha + ja^2 \sin 2\alpha - 1}{a^2 \cos 2\alpha + ja^2 \sin 2\alpha + 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

$$C_t = \frac{1}{T_n}, \text{ see } (3a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

Dissipationless Case. Values of S_n, C_s, T_n, and C_t when $a = 1$, so that $u = 1/\underline{\alpha}$: the fourpole has no attenuation.

Putting $a = 1$ in (1a) and (2a):

$$S_n = j \sin \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1b)$$

$$C_s = \cos \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2b)$$

$$T_n = j \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3b)$$

$$C_t = -j \cot \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4b)$$

Relations between S_n, C_s, T_n and C_t.

$$C_s^2 - S_n^2 = \tfrac{1}{4}(u+1/u)^2 - \tfrac{1}{4}(u - 1/u)^2 = 1 \quad . \quad . \quad (1c)$$

Dividing (1c) by C_s^2:

$$1 - T_n^2 = 1/C_s^2, \text{ or } T_n^2 + \frac{1}{C_s^2} = 1 \quad . \quad . \quad . \quad (2c)$$

Dividing (1c) by S_n^2:

$$C_t^2 - 1 = 1/S_n^2, \text{ or } C_t^2 - \frac{1}{S_n^2} = 1 \quad . \quad . \quad . \quad (3c)$$

Also

$$C_s + S_n = \frac{1}{2}\left(u + \frac{1}{u}\right) + \frac{1}{2}\left(u - \frac{1}{u}\right) = u \quad . \quad . \quad (4c)$$

Correlation and Nomenclature.

The quantities S_n, C_s, T_n and C_t are all functions of u which when $u = 1/\alpha$, degrade to $j \sin \alpha$, $\cos \alpha$, $j \tan \alpha$ and $-j \cot \alpha$ respectively.

Consistently two other quantities may be defined:

$$S_c = 1/C_s \text{ and } C_c = 1/S_n$$

Although for most purposes it is convenient to represent these quantities by their symbols, since they are functions of u, and in order to provide direct correlation with the complex hyperbolic functions which they replace, it is useful to give each of these functions a name. Names have therefore been chosen, derived from the trigonometrical functions. The name of each function is derived by adding an *s* to the corresponding trigonometrical function. The resulting functions will be called the Simple Vector Functions in contradistinction from the hyperbolic vector functions usually called complex hyperbolic functions.

Let $u = e^{\beta + j\alpha} = e^P$

where $P = \beta + j\alpha$ is the propagation constant of any symmetric fourpole, or the image transfer constant (see below) of any dissymmetric fourpole.

Then the following relations evidently hold:

$$S_n = \text{sins } u = \sinh P; \quad C_s = \text{coss } u = \cosh P$$
$$T_n = \text{tans } u = \tanh P; \quad C_t = \text{cots } u = \coth P$$
$$S_c = \text{secs } u = \text{sech } P; \quad C_c = \text{cosecs } u = \text{cosech } P.$$

(See also correlation and formulae in CI:15.)

It is indeed true that the pure mathematician will argue that the form of these functions does not justify their names, but this argument carries little weight against the obvious mnemonic advantage of naming them in this way.

It will be noted that S_n, C_s, etc., are all functions of u. Sometimes it is necessary to represent functions of other quantities such as $u/2$. These must be written in full, e.g. sins $u/2$, coss $u/2$, tans $u/2$, etc.

Definitions.

The *image transfer constant* of any passive fourpole has been defined (by K. S. Johnson, O. J. Zobel, and others) as half the natural logarithm of the vector ratio of the steady state volt-amperes entering the fourpole to the volt-amperes leaving the fourpole when the latter is terminated in its image impedances. The ratio is determined by dividing the value of the volt-amperes at the point nearest the transmitting end (the input) by the value of the volt-amperes at the point more remote (the output). See also correlation at end of XXIV:4.11 below.

The propagation constant of a fourpole is a special case of the image transfer constant : the image transfer constant of a symmetric fourpole is called the propagation constant.

The *image impedances* of a fourpole are those impedances which terminate the fourpole so that the impedance looking each way at input and output of the fourpole are the same.

If Z_1 and Z_2 in Fig. 1 (*a*) are the image impedances of the fourpole shown, then :

When terminals 2,2 are closed through Z_2 the impedance looking into terminals 1,1 is Z_1 : and when terminals 1,1 are closed through Z_1 the impedance looking into 2,2 is Z_2.

A fourpole of constant image impedance or a *symmetric fourpole*, is one in which the output image impedance equals the input image impedance. This type of fourpole is sometimes called an *equal-ratio fourpole*. It is realized in practice by a symmetrically disposed arrangement of impedance elements.

An *asymmetric* or *unequal-ratio fourpole* is one in which the output image impedance is unequal to the input image impedance. It is realized in practice by an unsymmetrical arrangement of impedance elements.

The *impedance ratio* of a fourpole is the ratio : input impedance divided by output impedance, and is a vector quantity.

The *root impedance ratio* is the square root of the impedance ratio.

The *power ratio* of a fourpole is the ratio : input power divided by output power, in the impedance matched condition.

The *transfer vector* of a constant impedance or symmetric fourpole is the ratio : input voltage divided by output voltage = the ratio input current divided by output current in the matched condition.

The *asymmetric transfer* vector of an asymmetric or unequal ratio fourpole is the ratio : input voltage divided by output voltage in the matched condition.

The *transfer vector* of an asymmetric or unequal ratio fourpole = the asymmetric transfer vector divided by the root impedance ratio. See definition of u under Conventions.

The *attenuation* ratio of any fourpole = the *magnitude* of the transfer vector = the square root of the power ratio of the fourpole.

The (*matched*) *phase shift* of any fourpole = the angle of the asymmetric transfer vector, which in the case of a symmetric fourpole becomes the transfer vector.

The characteristics of a fourpole are completely determined by specifying its image impedances and its transfer vector. In other words, if these quantities are known for any fourpole, the performance of the network can be determined when working between any values of generator and load impedance.

4.1. Development of Transmission Formulae. These are the formulae which relate the input and output voltages of fourpoles when terminated in any value of impedance. The sending-end impedance is also developed in terms of the fourpole parameters Z_1, Z_2 and u, and the terminating impedance Z.

The formulae are developed in terms of an asymmetric fourpole and are therefore entirely general. The case of the symmetric fourpole is covered by putting $Z_2 = Z_1$. Further, the equations are directly applicable to transmission lines and feeders by putting $u = e^{\beta+j\alpha}$, where β is the (total) line attenuation constant, and α is the (total) line phase-shift constant ; see XVI:1.3. In other words, a homogeneous transmission line is a symmetric fourpole with a transfer vector $u = e^{\beta+j\alpha}$. In this case the attenuation ratio $a = e^{\beta}$ and the symmetric phase shift α = the line attenuation constant α. A homogeneous transmission line is, of course, an example of a symmetric fourpole.

In XVI:1.3 and elsewhere the symbols β and α represent respectively the line attenuation and phase shift constants *per unit length*. In this case, if l is the length of the line, the total line attenuation constant = βl and the line phase shift constant = αl. *In the present section it would therefore have been preferable to use other symbols than β and α to denote the total line attenuation constant and total phase shift respectively*. Scarcity of symbols has, however, led to the present convention. The point is such an obvious one that no confusion should be caused.

4.11. Analysis of Asymmetric Fourpole. An asymmetric fourpole, of image impedances Z_1 and Z_2 and transfer constant u, may be replaced, for purposes of calculation, by a symmetric fourpole of image impedance Z_1 and transfer constant u, plus a fairy fourpole of image impedances Z_1 and Z_2 and transfer vector unity. The arrangement of the replacing fourpoles is shown in Fig. 1. In general the fairy fourpole has, and can have no real existence, but this in no way invalidates its usefulness as an aid to visualization of the processes involved. To those purists who may argue that the invocation of a non-physical entity is an example of fallacious argument, it may be answered, firstly, that this method gives the

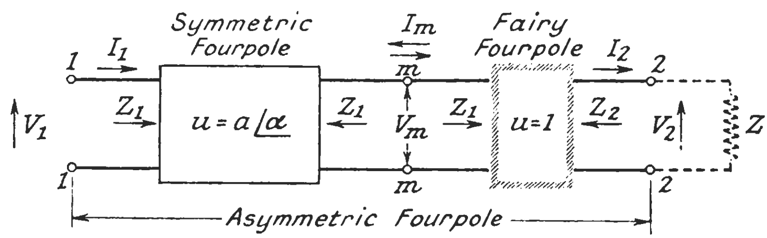


FIG. 1/XXIV:4.—Analysis of Asymmetric Fourpole.

correct results, and secondly, that the argument remains intact if the fairy fourpole is not used: the presentation then rests purely on the mathematical argument which is not so obvious when unaided.

No proof is required to show that the two networks in Fig. 1 will behave in every way the same as the asymmetric fourpole shown in Fig. 1/XXIV:3. The input image impedances are both Z_1, and the output impedances are both Z_2, while the transfer vector of the composite network is $u \times 1 = u$, which is the same as that of the original asymmetric network.

Referring to Fig. 1, when the fairy fourpole is terminated in its image impedance, $V_m/V_2 = r$ and $I_2/I_m = r$ where $r = \sqrt{Z_1/Z_2}$.

If terminals 2,2 are closed through an impedance Z_2, the output image impedance, the impedance looking into m,m, the input of the fairy fourpole is Z_1, so that the symmetric fourpole is terminated in its image impedance.

Under these conditions, if a voltage ${}_1V_1$ is applied at terminals 1,1, $V_m = {}_1V_1/u$, and the voltage appearing at terminals 2,2 is ${}_1V_2 = V_m/r = {}_1V_1/ru$.

Under the same conditions, if a current ${}_1I_1$ enters terminals 1,1, $I_m = {}_1I_1/u$, and the current flowing out at terminals 2,2 is ${}_1I_2 = rI_m = {}_1I_1 \times r/u$.

In the case of transmission through the network in the reverse direction, if terminals 1,1 are closed through an impedance Z_1, the input image impedance, the impedance at m,m looking into the output of the symmetric fourpole is Z_1, so that the fairy fourpole is terminated in its image impedance at m,m.

Under these conditions, if a voltage ${}_2V_2$ is applied at terminals 2,2, $V_m = r \times {}_2V_2$, and the voltage appearing at terminals 1,1 is ${}_2V_1 = V_m/u = {}_2V_2 \times r/u$.

Under the same conditions, if a current ${}_2I_2$ enters terminals 2,2, $I_m = {}_2I_2/r$, and the current flowing out at terminals 1,1 is ${}_2I_1 = I_m/u = {}_2I_2/ru$.

Hence $$\frac{{}_1V_1}{{}_1V_2} = \frac{{}_2I_2}{{}_2I_1} = ru \qquad (5)$$

and $$\frac{{}_2V_2}{{}_2V_1} = \frac{{}_1I_1}{{}_1I_2} = \frac{u}{r} \qquad (6)$$

or $$u = r \times \frac{{}_2V_2}{{}_2V_1} = r \times \frac{{}_1I_1}{{}_1I_2} = \frac{1}{r} \times \frac{{}_1V_1}{{}_1V_2} = \frac{1}{r} \times \frac{{}_2I_2}{{}_2I_1} \qquad (7)$$

(**Correlation.** The Image Transfer Constant is defined by K. S. Johnson in Chapter VII, section 7.52, as

$$\theta = \frac{1}{2} \log_e \frac{({}_1V_1 \times {}_1I_1)}{{}_1V_2 \times {}_1I_2} = \frac{1}{2} \log_e u^2 = \log_e u$$

$$= \log_e (a/\underline{\alpha}) = \log_e a + j\alpha$$

See also *Definitions* in XXIV:4 above.

If the Image Transfer Constant is given in the form $\beta + j\alpha$ the transfer vector $u = e^{\beta + j\alpha} = e^{\beta}/\underline{\alpha}$).

4.12. Development of Formulae from Consideration of Incident and Transmitted Voltages and Currents. The total voltages and currents in a driven fourpole are respectively constituted by the sum of the *incident* and *reflected* voltages and currents. A voltage V_1 applied at the input terminals 1,1 of a fourpole gives rise to an *incident* voltage v_1 at 1,1, which after transmission through the fourpole appears at 2,2 as $v_2 = v_1/ru$. v_1 and v_2 are called incident voltages and correspond to transmission of power through the fourpole from 1,1 to 2,2. Some of the power arriving at 2,2 is fed to the terminating impedance Z and some is *reflected*.

Corresponding to the *reflected* power there is a *reflected* voltage at 2,2 of magnitude

$$v_2' = \frac{Z - Z_2}{Z+Z_2}v_2 = \frac{Z - Z_2}{Z+Z_2} \times \frac{v_1}{ru} \qquad . \quad . \quad (8)$$

The *reflected* power, after transmission through the fourpole in the reverse direction, gives rise to a *reflected* voltage at 1,1 of magnitude

$$v_1' = \frac{r}{u}v_2' = \frac{Z - Z_2}{Z+Z_2} \times \frac{v_1}{u^2} \qquad . \quad . \quad . \quad (9)$$

It will be seen that the *incident* and *reflected* voltages are transmitted through the network as if the network were terminated in its image impedances, i.e. in accordance with equations (5) and (6). A little thought will show that this must be the case because energy advancing through the network doesn't know that the network is incorrectly terminated until it reaches the terminal, when it shows its dissatisfaction by partially coming back again.

Now consider conditions in the network under the influence of a sinusoidal applied voltage after initial transients have decayed and the steady state is established.

The total voltage at terminals 2,2 is then

$$V_2 = v_2+v_2' = v_1\left(\frac{1}{ru}+\frac{Z - Z_2}{Z+Z_2} \times \frac{1}{ru}\right) \qquad . \quad . \quad (10)$$

The total voltage at terminals 1,1 is

$$V_1 = v_1+v_1' = v_1\left(1+\frac{1}{u^2} \times \frac{Z - Z_2}{Z+Z_2}\right) \qquad . \quad . \quad (11)$$

Similarly an incident current $i_1 = v_1/Z_1$ entering terminals 1,1 gives rise to an incident current $i_2 = ri_1/u = \frac{rv_1}{uZ_1}$ at terminals 2,2, and a reflected current at 2,2 of magnitude

$$i_2'' = \frac{Z_2 - Z}{Z+Z_2}i_2 = \frac{Z_2 - Z}{Z+Z_2} \times \frac{rv_1}{uZ_1} \qquad . \quad . \quad (12)$$

The resultant reflected current at 1,1 is

$$i_1'' = \frac{i_2''}{ru} = \frac{1}{u^2} \times \frac{Z_2 - Z}{Z+Z_2} \times \frac{v_1}{Z_1} \qquad . \quad . \quad . \quad (13)$$

The total current emerging at terminals 2,2 is then

$$I_2 = i_2+i_2'' = \frac{r}{u}\left(1+\frac{Z_2 - Z}{Z+Z_2}\right)\frac{v_1}{Z_1}. \qquad . \quad . \quad (14)$$

The total current entering at terminals 1,1 is

$$I_1 = i_1+i_1'' = \left(1+\frac{1}{u^2}\frac{Z_2 - Z}{Z+Z_2}\right)\frac{v_1}{Z_1} \qquad . \quad . \quad (15)$$

The relations between V_1 and V_2, I_1 and I_2, V_1 and I_2, and V_1 and I_1 may be determined as follows. The relation between V_1 and I_1 is particularly important because it determines the sending-end or effective input impedance of the fourpole when terminated in an impedance Z.

From (10) and (11)

$$\therefore \frac{V_2}{V_1} = \frac{\frac{1}{ru} \times \left(1+\frac{Z-Z_2}{Z+Z_2}\right)}{1+\frac{1}{u^2} \times \frac{Z-Z_2}{Z+Z_2}} = \frac{\frac{2Z}{r}}{u(Z+Z_2)+\frac{1}{u}(Z-Z_2)}$$

$$\therefore V_2 = \frac{2\sqrt{\frac{Z_2}{Z_1}}V_1}{u\left(1+\frac{Z_2}{Z}\right)+\frac{1}{u}\left(1-\frac{Z_2}{Z}\right)} = \frac{2\sqrt{\frac{Z_2}{Z_1}}V_1}{u+\frac{1}{u}+\frac{Z_2}{Z}\left(u-\frac{1}{u}\right)} \quad . \quad (16)$$

$$V_2 = \frac{\sqrt{\frac{Z_2}{Z_1}}V_1}{\frac{1}{2}\left(u+\frac{1}{u}\right)+\frac{Z_2}{Z}\frac{1}{2}\left(u-\frac{1}{u}\right)} = \frac{\sqrt{\frac{Z_2}{Z_1}}V_1}{C_s+\frac{Z_2}{Z}S_n} \quad . \quad . \quad (16a)$$

Equation (16) or (16a) is the most useful form of this expression when dissipation is to be taken into account.

When there is no dissipation: i.e. when $u = 1/\underline{\alpha}$, *the expression may be simplified as follows*:

When $u = 1/\underline{\alpha}$, $S_n = j \sin \alpha$ and $C_s = \cos \alpha$

$$\therefore V_2 = \frac{\sqrt{\frac{Z_2}{Z_1}}V_1}{\cos \alpha + j\frac{Z_2}{Z} \sin \alpha} \quad . \quad . \quad . \quad (17)$$

From (14) and (15)

$$\frac{I_2}{I_1} = \frac{\frac{r}{u}\left(1+\frac{Z_2-Z}{Z+Z_2}\right)}{1+\frac{1}{u^2} \times \frac{Z_2-Z}{Z+Z_2}} = \frac{2rZ_2}{u(Z+Z_2)+\frac{1}{u}(Z_2-Z)}$$

$$\therefore I_2 = \frac{2\sqrt{\frac{Z_1}{Z_2}}I_1}{u+\frac{1}{u}+\frac{Z}{Z_2}\left(u-\frac{1}{u}\right)} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

An alternative form of (18) is

$$I_2 = \frac{\sqrt{\frac{Z_1}{Z_2}}I_1}{C_s + \frac{Z}{Z_2}S_n} \qquad . \quad . \quad . \quad . \quad (18a)$$

Equation (18) is the most useful form of the expression when dissipation is to be taken into account. When there is no dissipation and $u = 1/\underline{\alpha}$

$$I_2 = \frac{\sqrt{\frac{Z_1}{Z_2}}I_1}{\cos\alpha + j\frac{Z}{Z_2}\sin\alpha}. \qquad . \quad . \quad . \quad . \quad (19)$$

From (11) and (14)

$$\frac{I_2}{V_1} = \frac{\frac{r}{uZ_1}\left(1+\frac{Z_2 - Z}{Z+Z_2}\right)}{1+\frac{1}{u^2}\frac{Z - Z_2}{Z+Z_2}} = \frac{2r\frac{Z_2}{Z_1}}{u(Z+Z_2)+\frac{1}{u}(Z - Z_2)}$$

$$\therefore\ I_2 = \frac{2\sqrt{\frac{Z_2}{Z_1}}V_1}{Z\left(u+\frac{1}{u}\right)+Z_2\left(u - \frac{1}{u}\right)}. \qquad . \quad . \quad (20)$$

An alternative form of (20) is

$$I_2 = \frac{\sqrt{\frac{Z_2}{Z_1}}V_1}{ZC_s+Z_2S_n} \qquad . \quad . \quad . \quad . \quad (20a)$$

Equation (20) covers the case of dissipation, while the case of no dissipation is covered by:

$$I_2 = \frac{\sqrt{\frac{Z_2}{Z_1}}V_1}{Z\cos\alpha + jZ_2\sin\alpha} \qquad . \quad . \quad . \quad (21)$$

From (11) and (15) the sending-end impedance: the impedance looking into terminals 1,1 is:

$$Z_s = \frac{V_1}{I_1} = \frac{1+\frac{1}{u^2}\,\frac{Z - Z_2}{Z+Z_2}}{\frac{1}{Z_1}\left(1+\frac{1}{u^2}\times\frac{Z_2 - Z}{Z+Z_2}\right)}$$

$$= \left[\frac{u^2(Z+Z_2)+Z - Z_2}{u^2(Z+Z_2)+Z_2 - Z}\right]Z_1 . \qquad . \quad . \quad . \quad (22)$$

Equation (22) gives the sending-end impedance when there is dissipation. Equation (22) may be transformed as follows :

$$Z_s = \left[\frac{Z(u^2+1)+Z_2(u^2-1)}{Z_2(u^2+1)+Z(u^2-1)}\right]Z_1 = \left[\frac{1+\frac{Z_2}{Z}\times\frac{u^2-1}{u^2+1}}{\frac{Z_2}{Z}+\frac{u^2-1}{u^2+1}}\right]Z_1$$

$$= \left[\frac{1+\frac{Z_2}{Z}\times T_n}{\frac{Z_2}{Z}+T_n}\right]Z_1 \quad . \quad . \quad . \quad . \quad (23)$$

and when $u = 1/\alpha$, $T_n = j\tan\alpha$.
Hence, in the dissipationless case

$$Z_s = \left[\frac{1+j\frac{Z_2}{Z}\tan\alpha}{\frac{Z_2}{Z}+j\tan\alpha}\right]Z_1 \quad . \quad . \quad . \quad (24)$$

4.13. Short-Circuit and Open-Circuit Impedances of a Network. Use of " Opens and Shorts " for determining the Parameters of a Network.

Consider a fourpole with terminals 1,1 and 2,2.

S_1 = the impedance looking into terminals 1,1 with terminals 2,2 short circuited = the short-circuit impedance at 1,1.

O_1 = the impedance looking into terminals 1,1 with terminals 2,2 open circuited = the open-circuit impedance at 1,1.

S_2 = the impedance at 2,2 with 1,1 shorted.

O_2 = the impedance at 2,2 with 1,1 open circuited.

When 2,2 are shorted $Z = 0$, and when 2,2 are open $Z = \infty$. Putting the values in equation (23)

$$S_1 = Z_1T_n = Z_1\left[\frac{u^2-1}{u^2+1}\right] \quad . \quad . \quad . \quad (25)$$

With no dissipation

$$S_1 = jZ_1\tan\alpha \quad . \quad . \quad . \quad . \quad (25a)$$

Similarly,

$$O_1 = Z_1C_t = Z_1\left[\frac{u^2+1}{u^2-1}\right] \quad . \quad . \quad . \quad (26)$$

With no dissipation

$$O_1 = -jZ_1\cot\alpha \quad . \quad . \quad . \quad . \quad (26a)$$

Similarly $S_2 = Z_2T_n$, $O_2 = Z_2C_t$, etc.

Determination of Z_1, Z_2 and u, from measurements or calculations of S_1 and S_2, O_1 and O_2.

Multiplying (25) and (26) together:

$$S_1O_1 = Z_1^2T_nC_t = Z_1^2$$

$$\therefore\ Z_1 = \sqrt{S_1O_1} \quad . \quad . \quad . \quad . \quad (27)$$

Similarly

$$Z_2 = \sqrt{S_2O_2} \quad . \quad . \quad . \quad . \quad (28)$$

Dividing (25) by (26)

$$\frac{S_1}{O_1} = \frac{Z_1T_n}{Z_1C_t} = T_n^2 = \left[\frac{u^2 - 1}{u^2+1}\right]^2$$

$$\therefore\ (u^2 - 1) = (u^2+1)\sqrt{\frac{S_1}{O_1}}$$

$$\therefore\ u = \sqrt{\frac{1+\sqrt{\frac{S_1}{O_1}}}{1-\sqrt{\frac{S_1}{O_1}}} - \frac{1+\sqrt{\frac{S_1}{O_1}}}{\sqrt{1-\frac{S_1}{O_1}}}} \quad . \quad . \quad (29)$$

The above relations: (27), (28) and (29) are extremely important because they give the methods of determining by calculation of S_1, S_2, O_1 and O_2, or by measurements with an impedance bridge, the parameters Z_1, Z_2 and u, in any network, and once these are determined the performance of the network under any specified conditions can be calculated from equations (16) to (24).

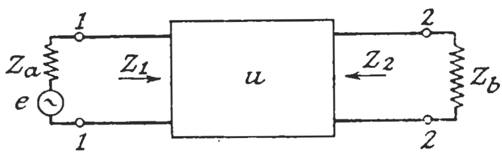


FIG. 2/XXIV:4.—Asymmetric Fourpole driven by a Generator of Internal Impedance Z_a and working into a Load of Impedance Z_b.

4.14. The Insertion Loss of a Fourpole between Impedances Z_a and Z_b. Fig. 2 shows an asymmetric fourpole inserted between a generator of internal impedance Z_a and internal e.m.f. e, and a load of impedance Z_b. The fourpole has image impedances Z_1 and Z_2, and transfer vector u.

The insertion loss of the fourpole is given by

$$L = 20 \log_{10} \left|\frac{I_0}{I_n}\right|$$

where I_0 = the current flowing through Z_b, when the generator is directly connected to Z_b

and I_n = the current flowing through Z_b with the fourpole inserted as shown in Fig. 2, the value of e remaining unchanged.

Evidently $$I_0 = \frac{e}{Z_a+Z_b}.$$

Determination of I_n.

From (23) the impedance looking into terminals 1,1 (i.e. presented towards the generator) in the circuit of Fig. 1 is

$$Z_s = \frac{1+\frac{Z_2}{Z_b}T_n}{\frac{Z_2}{Z_b}+T_n}Z_1 = \frac{Z_bC_s+Z_2S_n}{Z_2C_s+Z_bS_n}Z_1 \quad . \quad . \quad . \quad (30)$$

The voltage across terminals 1,1 is

$$E_1 = \frac{Z_s\times e}{Z_a+Z_s} = \frac{e}{1+\frac{Z_a}{Z_s}} \quad . \quad . \quad . \quad (31)$$

From (20*a*) the current through the load is

$$I_n = \frac{\sqrt{\frac{Z_2}{Z_1}}E_1}{Z_bC_s+Z_2S_n} \quad . \quad . \quad . \quad . \quad (32)$$

Hence, from (31)

$$I_n = \frac{\sqrt{\frac{Z_2}{Z_1}}e}{\left(1+\frac{Z_a}{Z_s}\right)(Z_bC_s+Z_2S_n)}$$

and from (30)

$$I_n = \frac{\sqrt{\frac{Z_2}{Z_1}}e}{Z_bC_s+Z_2S_n+\frac{Z_a}{Z_1}(Z_2C_s+Z_bS_n)}$$

$$= \frac{\sqrt{\frac{Z_2}{Z_1}}e}{\left(Z_b+\frac{Z_aZ_2}{Z_1}\right)C_s+\left(Z_2+\frac{Z_aZ_b}{Z_1}\right)S_n}$$

$$\therefore \left|\frac{I_0}{I_n}\right| = \left|\frac{\left(Z_b+\frac{Z_aZ_2}{Z_1}\right)C_s+\left(Z_2+\frac{Z_aZ_b}{Z_1}\right)S_n}{(Z_a+Z_b)\sqrt{\frac{Z_2}{Z_1}}}\right|$$

Hence, the insertion loss in decibels is

$$\text{I.L.} = 20\log_{10}\left|\frac{\left(Z_b+\dfrac{Z_aZ_2}{Z_1}\right)C_s+\left(Z_2+\dfrac{Z_aZ_b}{Z_1}\right)S_n}{(Z_a+Z_b)\sqrt{\dfrac{Z_2}{Z_1}}}\right| \quad . \quad (33)$$

If $Z_a = Z_b = R$ say, the insertion loss in decibels is

$$\text{I.L.} = 20\log_{10}\left|\frac{R\left(1+\dfrac{Z_2}{Z_1}\right)C_s+\left(Z_2+\dfrac{R^2}{Z_1}\right)S_n}{2R\sqrt{\dfrac{Z_2}{Z_1}}}\right|$$

$$= 20\log_{10}\left|\frac{\left(1+\dfrac{Z_2}{Z_1}\right)C_s+\left(\dfrac{Z_2}{R}+\dfrac{R}{Z_1}\right)S_n}{2\sqrt{\dfrac{Z_2}{Z_1}}}\right| \quad . \quad . \quad (34)$$

If $Z_a = Z_b = R$ and $Z_1 = Z_2 = Z_I$ say, the insertion loss in decibels is

$$20\log_{10}\left|C_s+\left(\frac{Z_I}{R}+\frac{R}{Z_I}\right)\frac{S_n}{2}\right| \quad . \quad . \quad . \quad (35)$$

Equation (35) is of particular interest for evaluating the insertion loss of filters working between equal impedances.

The insertion phase shift is in each case given by the angle of $\dfrac{I_0}{I_n}$. In other words, if $\dfrac{I_0}{I_n}$ is reduced to a vector of the form $U\underline{/\phi}$ $\left(\text{i.e. if } \dfrac{I_0}{I_n} = U\underline{/\phi}\right)$, then ϕ is the insertion phase shift.

It will be clear that to evaluate any of the expressions (33), (34) and (35), the values of S_n and C_s must be written in full as given in equations (1) and (2).

A word of advice should be given here. This is to the effect that although equations (33), (34) and (35) constitute comprehensive, and in the case of equation (35), easily handled formulae for calculating insertion loss, in cases where single π and T-networks are concerned, some people may find it preferable to calculate the current in the load directly by applying the method of inspection described in XXIV:2.1. If the resultant formulae are generalized (see XXI:5) they may be found simpler for substitution purposes than the equations developed immediately above.

5. Characteristics of Specific Fourpoles.

Warning. The following methods of calculating the characteristics of fourpoles are all valuable for the purpose described. In solving any particular problem, however, it is not always necessary to calculate Z_1, Z_2 and u. The Method of Inspection, or the Method of Addition may give the required answer directly and much more quickly, see XXIV:2.1 and 2.4.

5.1. Dissymmetrical T. (Solved by the Method of Inspection.) Fig. 1 shows a dissymmetrical T network working between its image impedances Z_1 and Z_2.

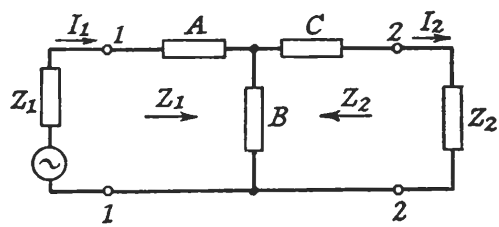


FIG. 1/XXIV:5.—Dissymmetrical T Network.

The impedance looking into terminals 1,1, with Z_1 removed and 2,2 connected to Z_2, is evidently Z_1. By inspection of the circuit

$$Z_1 = A + \frac{B(C+Z_2)}{B+C+Z_2}$$

Similarly

$$Z_2 = C + \frac{B(A+Z_1)}{B+A+Z_1}$$

Solving these two equations for Z_1 and Z_2:

$$Z_1 = \sqrt{\frac{A+B}{B+C}(AB+BC+CA)} \quad . \quad . \quad . \quad (1)$$

$$Z_2 = \sqrt{\frac{B+C}{A+B}(AB+BC+CA)} \quad . \quad . \quad . \quad (2)$$

Transfer Vector.

By inspection of Fig. 1

$$\frac{I_1}{I_2} = \frac{B+C+Z_2}{B}$$

From equation (6)/XXIV:4

$$u = r\frac{I_1}{I_2} = \frac{B+C+Z_2}{B}\sqrt{\frac{Z_1}{Z_2}} \quad . \quad . \quad . \quad (3)$$

Hence the practical procedure is to calculate Z_1 and Z_2 from equations (1) and (2), and then to use the values so obtained to calculate the value of u.

5.11. Symmetrical T. This is represented by Fig. 1 when the two series arms are equal. Assume that they are equal to A. The values of the image impedance and the transfer vector are then obtained by putting $C = A$ in equations (1), (2) and (3). This gives

$$Z_1 = Z_2 = \sqrt{A^2+2AB} = Z_I \text{ say.} \quad . \quad . \quad (4)$$

and

$$u = \frac{A+B+Z_I}{B} \quad . \quad . \quad . \quad . \quad (5)$$

5.2. Dissymmetrical π. (Solved by the Method of Inspection.) Fig. 2 shows a dissymmetrical π network working between its image impedances Z_1 and Z_2.

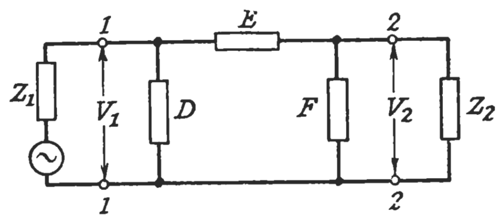


FIG. 2/XXIV:5.—Dissymmetrical π Network.

By inspection of the circuit

$$Z_1 = \frac{D\left(E+\dfrac{FZ_2}{F+Z_2}\right)}{D+E+\dfrac{FZ_2}{F+Z_2}} \quad \text{and} \quad Z_2 = \frac{F\left(E+\dfrac{DZ_1}{D+Z_1}\right)}{E+F+\dfrac{DZ_1}{D+Z_1}}$$

Solving these equations for Z_1 and Z_2:

$$Z_1 = D\sqrt{\frac{E(E+F)}{(D+E)(D+E+F)}} \quad . \quad . \quad . \quad (6)$$

$$Z_2 = F\sqrt{\frac{E(D+E)}{(E+F)(D+E+F)}} \quad . \quad . \quad . \quad (7)$$

Transfer Vector.

By inspection of Fig. 2

$$\frac{V_1}{V_2} = \frac{E+\dfrac{FZ_2}{F+Z_2}}{\dfrac{FZ_2}{F+Z_2}} = \frac{FE+FZ_2+EZ_2}{FZ_2}$$

From equation (5)/XXIV:4

$$u = \frac{1}{r}\frac{V_1}{V_2} = \sqrt{\frac{Z_2}{Z_1}}\frac{FE+FZ_2+EZ_2}{FZ_2} \quad . \quad . \quad (8)$$

$$= \sqrt{\frac{Z_2}{Z_1}}\left(1+\frac{E}{Z_2}+\frac{E}{F}\right) \quad . \quad . \quad . \quad . \quad (9)$$

Either equations (8) or (9) may be used. (8) is preferable if the impedances are given in the form $R+jX$, and (9) if they are given in the form $M/\underline{\phi}$.

5.21. Symmetrical π**.** This is represented by Fig. 2 when the two shunt arms are equal. Assume that they are equal to D. The values of the image impedance and the transfer vector are then obtained by putting $F = D$ in equations (6), (7), (8) and (9).

This gives :

$$Z_I = Z_1 = Z_2 = D\sqrt{\frac{E}{E+2D}} \quad . \quad . \quad . \quad (10)$$

and

$$u = 1+\frac{E}{Z_I}+\frac{E}{D} \quad . \quad . \quad . \quad . \quad (11)$$

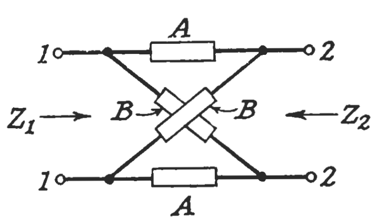


FIG. 3/XXIV:5.—Lattice Network.

5.3. Lattice. (Solved by " Opens and Shorts.") Fig. 3 shows a lattice network. Using conventions as in XXIV:4.13 :

$$S_1 = S_2 = \frac{2AB}{A+B}, \quad O_1 = O_2 = \frac{A+B}{2}$$

Hence

$$Z_1 = Z_2 = \sqrt{S_1O_1} = \sqrt{S_2O_2} = \sqrt{AB} = Z_I, \text{ say} \quad . \quad (12)$$

Also

$$T_n = \sqrt{\frac{S_1}{O_1}} = \sqrt{\frac{S_2}{O_2}} = \sqrt{\frac{4AB}{(A+B)^2}} = \frac{2\sqrt{AB}}{A+B}$$

that is

$$\frac{u^2 - 1}{u^2+1} = \frac{2\sqrt{AB}}{A+B}$$

$$u^2 = \frac{1+\dfrac{2\sqrt{AB}}{A+B}}{1-\dfrac{2\sqrt{AB}}{A+B}} = \frac{A+2\sqrt{AB}+B}{A-2\sqrt{AB}+B}$$

$$u = \pm\frac{\sqrt{A}+\sqrt{B}}{\sqrt{A}-\sqrt{B}} = \pm\frac{1+\sqrt{\dfrac{B}{A}}}{1-\sqrt{\dfrac{B}{A}}} \quad . \quad . \quad (13)$$

From (12) $B = \dfrac{Z_I^2}{A}$, $\therefore u = \dfrac{1+\dfrac{Z_I}{A}}{1-\dfrac{Z_I}{A}} = \dfrac{A+Z_I}{A-Z_I}$ . . (14)

Dissipationless Case.

This can only occur when A and B are reactances. In order that the image impedance shall be real these reactances must be opposite in sign, see equation (12). If they are $A = jX_1$ and $B = -jX_2$, then $Z_I = \sqrt{-jX_1 jX_2} = \sqrt{X_1X_2}$.

Since there is no dissipation

$$T_n = j\tan\alpha = \frac{2\sqrt{X_1X_1}}{jX_1 - jX_2} = j\frac{2\sqrt{X_1X_2}}{X_2 - X_1}$$

Hence

$$\tan\alpha = \frac{2Z_I}{\dfrac{Z_I^2}{X_1} - X_1} = \frac{2Z_IX_1}{Z_I^2 - X_1^2} = \frac{2\dfrac{X_1}{Z_I}}{1-\dfrac{X_1^2}{Z_I^2}}$$

$$\therefore \alpha = \tan^{-1}\frac{2\dfrac{X_1}{Z_I}}{1-\dfrac{X_1^2}{Z_I}}$$

Since

$$\tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x$$

$$\alpha = 2\tan^{-1}\frac{X_1}{Z_I} \quad . \quad . \quad . \quad (15)$$

The dissipationless case is important for the design of networks to insert phase shift varying with frequency in any required way. Before deciding to use a lattice network for this purpose the equivalent bridged-T structure should be examined to see that it is physically realizable, in which case it is probably a more economical structure to use as the number of reactance elements is less.

5.4. Symmetrical Bridged-T. (Solved by Matrices.) The bridged-T network is shown in Fig. 7 (*a*)/XXIV:6.

For the average engineer the following constitutes a good example of the case where the conclusions of a mathematical discussion constitute the only intelligible, and therefore the only useful part of the argument. Those who have mastered XXIV:6 will, however, find no difficulty in following the whole discussion.

From Fig. 12/XXIV:6 the A matrix of a symmetrical fourpole, in terms of its image impedance Z_I and its transfer vector u, is :

$$\| A \|_{SF} = \begin{Vmatrix} C_s & Z_I S_n \\ \dfrac{S_n}{Z_I} & C_s \end{Vmatrix} \qquad . \quad . \quad . \quad (16)$$

From equation (23) XXIV:6.71, referring to Fig. 7 (*a*) XXIV:6, the A matrix of a symmetrical bridged-T network is

$$\| A \|_{BT} = \begin{Vmatrix} \dfrac{z_1+z_2+y_4S}{z_2+y_4S} & \dfrac{S}{z_2+y_4S} \\ \dfrac{1+2z_1y_4}{z_2+y_4S} & \dfrac{z_1+z_2+y_4S}{z_2+y_4S} \end{Vmatrix}$$

where $S = Z^2+2Z_1Z_2$

Assuming $\| A \|_{SF}$ and $\| A \|_{BT}$ to represent the same fourpole, $\| A \|_{SF} = \| A \|_{BT}$, in which case corresponding terms of the two matrices are equal, so that :

$$C_s = \frac{z_1+z_2+y_4S}{z_2+y_4S} = \frac{z_4(z_1+z_2)+S}{z_2z_4+S} \qquad . \quad . \quad (17)$$

$$Z_IS_n = \frac{S}{z_2+y_4S} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$\frac{S_n}{Z_I} = \frac{1+2z_1y_4}{z_2+y_4S} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Dividing (18) by (19)

$$Z_I^2 = \frac{S}{1+2z_1y_4} = \frac{z_4S}{z_4+2z_1}$$

$$\therefore\ Z_I = \sqrt{\frac{z_4S}{z_4+2z_1}} \qquad . \quad . \quad . \quad . \quad (20)$$

Multiplying (18) by (19)

$$S_n^2 = \frac{S(1+2z_1y_4)}{(z_2+y_4S)^2}$$

$$\therefore \; S_n = \frac{\sqrt{Sz_4(z_4+2z_1)}}{z_2z_4+S} \qquad . \quad . \quad . \quad (21)$$

For many purposes it is not necessary to take the analysis further, since it is required to substitute the values of S_n and C_s directly into transmission formulae : the value of u is not generally required. The particular value of the matrix method will now be appreciated because it gives directly the values of S_n and C_s.

If, however, u is required it is very easily obtained, since from (4*c*)/XXIV:4

$$u = C_s+S_n = \frac{z_4(z_1+z_2)+S+\sqrt{Sz_4(z_4+2z_1)}}{z_2z_4+S} \qquad . \quad . \quad (22)$$

Check by Method of "Opens and Shorts".

Omitting preliminary determination of short-circuit impedances S and open-circuit impedances O, and merely stating their values :

$$S_1 = S_2 = S = \frac{z_4S}{z_4(z_1+z_2)+S}$$

$$O_1 = O_2 = O = \frac{z_4(z_1+z_2)+S}{2z_1+z_4}$$

$$Z_I = \sqrt{SO} = \sqrt{\frac{z_4S}{z_4+2z_1}} \qquad . \quad . \quad . \quad (20a)$$

also

$$\frac{S}{O} = \frac{z_4S(2z_1+z_4)}{[z_4(z_1+z_1)+S]^2} = \frac{A}{B^2}, \text{ say}$$

then

$$u = \sqrt{\frac{1+\dfrac{\sqrt{A}}{B}}{1-\dfrac{\sqrt{A}}{B}}} = \sqrt{\frac{B+\sqrt{A}}{B-\sqrt{A}}} = \frac{B+\sqrt{A}}{\sqrt{B^2-A}} \text{ (compare with equation (29)/XXIV:4)}$$

$$= \frac{z_4(z_1+z_2)+S+\sqrt{z_4S(2z_1+z_4)}}{\sqrt{[z_4(z_1+z_2)+S]^2 - z_4S(2z_1+z_4)}}$$

$$= \frac{z_4(z_1+z_2)+S+\sqrt{Sz_4(z_4+2z_1)}}{z_2z_4+S} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (22a)$$

Equation (20*a*) is the same as (20), while (22*a*) is the same as (22).

6. Circuit Synthesis : the German Method.

The following treatment is based on an article by F. Strecker and R. Feldkeller, which appeared in the *Elektrische Nachrichten-Technik* for March 1929.

During the development of the English method it has appeared that the characteristics of a fourpole can be completely specified by stating the values of any one of three sets of parameters, which will be referred to as the *normal* parameters. These are :

(1) The image impedances and transfer vector : three parameters.
(2) The short-circuit and open-circuit impedances at both input and output terminals : four parameters.
(3) The impedances of each arm of the network : number of parameters equal to the number of arms.

It has been very evident that to obtain the greatest use from these parameters it is essential to understand exactly what these parameters mean in physical terms. It is now proposed to introduce four sets of parameters without initially defining what they mean in physical terms ; means of deriving physical definitions of these parameters will, however, be given. The only purpose served by a knowledge of the physical meaning of these parameters is that the physical meaning of the parameter enables the value of the parameter to be determined in terms of any one of the three sets of parameters above. Since in later figures the values of the new parameters are tabulated, for a variety of fourpoles, in terms of certain of the above sets of parameters, and since the values of the parameters can be determined without assigning physical meanings, a knowledge of the physical meaning of these parameters is not of first importance. It is, however, rather comforting.

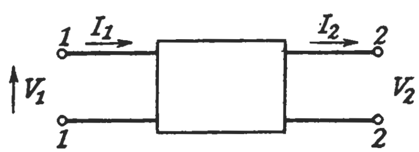


FIG. 1/XXIV:6.—General Fourpole illustrating Current and Voltage Conventions adopted for Evolving Matrices.

For the moment these parameters will be defined by the bland statement that if the values of the input and output voltages and currents V_1V_2, I_1I_2 of any fourpole, such as is shown in Fig. 1, are known, they will satisfy all the four pairs of simultaneous equations below provided suitable values are assigned to the parameters.

This statement will not be proved in general terms, but a very easy proof will present itself in all cases where the values of the parameters are known, since it is only necessary to determine the values of voltage and current at the input and output of a specific fourpole by applying the English method to any set of conditions; it will then be found that these values of voltage and current inserted in each equation, with the values of the parameters proper to the network and the equation, will satisfy the equation. See also XXIV:6.21.

The four sets of equations are:

$$V_1 = A_{11}V_2+A_{12}I_2 \quad . \quad . \quad . \quad . \quad (1a)$$
$$I_1 = A_{21}V_2+A_{22}I_2 \quad . \quad . \quad . \quad . \quad (1b)$$

$$V_1 = Z_{11}I_1+Z_{12}I_2 \quad . \quad . \quad . \quad . \quad (2a)$$
$$V_2 = Z_{21}I_1+Z_{22}I_2 \quad . \quad . \quad . \quad . \quad (2b)$$

$$I_1 = Y_{11}V_1+Y_{12}V_2 \quad . \quad . \quad . \quad . \quad (3a)$$
$$I_2 = Y_{21}V_1+Y_{22}V_2 \quad . \quad . \quad . \quad . \quad (3b)$$

$$V_1 = D_{11}V_2+D_{12}I_1 \quad . \quad . \quad . \quad . \quad (4a)$$
$$I_2 = D_{21}V_2+D_{22}I_1 \quad . \quad . \quad . \quad . \quad (4b)$$

The quantities A_{11}, A_{12}, A_{21}, A_{22}; Z_{11}, Z_{12}, Z_{21}, Z_{22}; Y_{11}, Y_{12}, Y_{21}, Y_{22}; D_{11}, D_{12}, D_{21}, D_{22}, constitute four sets of parameters, there being four parameters in each set. It will appear later that any three of the parameters in any set determine the performance of a fourpole, the fourth parameter then merely provides confirmation of some of the information already given by the other three parameters.

It is important to note the conventions which have been chosen for the sense of the voltages and currents in Fig. 1. Any other sense might have been chosen for the direction of any current or any voltage without affecting the validity of the argument which follows. The only effect of changing this convention is to change the sign of one or more of the parameters in each or any pair of equations. It is necessary to keep to this convention in order to be able to make use of existing tables of parameters without making any change of sign.

6.1. How to Determine the Physical Meaning of, and the Relations between the Parameters. These are determined by considering what happens when each pair of terminals is short circuited or open circuited in turn.

1. If terminals 2,2 are short circuited, $V_2 = O$.

Putting $V_2 = O$ in each equation in turn

$$A_{12} = \frac{V_1}{I_2}; \quad A_{22} = \frac{I_1}{I_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1c)$$

$$\frac{Z_{22}}{Z_{21}} = -\frac{I_1}{I_2} = -A_{22} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2c)$$

$$Y_{11} = \frac{I_1}{V_1}; \quad Y_{21} = \frac{I_2}{V_1} = \frac{1}{A_{12}} \quad . \quad . \quad . \quad . \quad . \quad (3c)$$

$$D_{12} = \frac{V_1}{I_1} = \frac{1}{Y_{11}} = \frac{A_{12}}{A_{22}}; \quad D_{22} = \frac{I_2}{I_1} = -\frac{Z_{21}}{Z_{22}} = \frac{1}{A_{22}} \quad . \quad (4c)$$

2. If terminals 2,2 are open circuited, $I_2 = O$.
Putting $I_2 = O$ in each equation in turn

$$A_{11} = \frac{V_1}{V_2}; \quad A_{21} = \frac{I_1}{V_2} \quad . \quad . \quad . \quad . \quad (1d)$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{A_{11}}{A_{21}}; \quad Z_{21} = \frac{V_2}{I_1} = \frac{1}{A_{21}} \quad . \quad . \quad (2d)$$

$$\frac{Y_{22}}{Y_{21}} = -\frac{V_1}{V_2} = -A_{11}. \quad . \quad . \quad . \quad . \quad . \quad (3d)$$

$$\frac{D_{22}}{D_{21}} = -\frac{V_2}{I_1} = -Z_{21} = -\frac{1}{A_{21}} \quad . \quad . \quad . \quad (4d)$$

3. If terminals 1,1 are short circuited, $V_1 = O$.
Putting $V_1 = O$ in each equation in turn

$$\frac{A_{12}}{A_{11}} = -\frac{V_2}{I_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1e)$$

$$\frac{Z_{12}}{Z_{11}} = -\frac{I_1}{I_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2e)$$

$$Y_{12} = \frac{I_2}{V_2}; \quad Y_{22} = \frac{I_2}{V_2} = -\frac{A_{11}}{A_{12}} \quad . \quad . \quad (3e)$$

(For signs of Y_{12} and Y_{22} see below.)

$$\frac{D_{12}}{D_{11}} = -\frac{V_2}{I_2} = -\frac{1}{Y_{12}} \quad . \quad . \quad . \quad . \quad (4e)$$

4. If terminals 1,1 are open circuited, $I_1 = O$.
Putting $I_1 = O$ in each equation in turn

$$\frac{A_{22}}{A_{21}} = -\frac{V_2}{I_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1f)$$

$$Z_{12} = \frac{V_1}{I_2}; \quad Z_{22} = \frac{V_2}{I_2} = -\frac{A_{22}}{A_{21}} \quad . \quad . \quad . \quad (2f)$$

$$\frac{Y_{12}}{Y_{11}} = -\frac{V_1}{V_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3f)$$

$$D_{11} = \frac{V_1}{V_2} = -\frac{Y_{12}}{Y_{11}}; \quad D_{21} = \frac{I_2}{V_2} = -\frac{A_{21}}{A_{22}}. \quad . \quad (4f)$$

It is therefore evident that the parameters are defined by short-circuit or open-circuit relations between input and/or output voltages and/or currents.

For instance, from (1*c*), $A_{12} = \frac{V_1}{I_2}$ when terminals 2,2 are shorted. A_{12} may be called the *transfer impedance* from 1,1 to 2,2. Also from (1*c*), $A_{22} = \frac{I_1}{I_2}$ with 2,2 shorted. A_{12} may be called the 2,2 *short-circuit current ratio.*

Similarly from (1*d*), A_{11} may be called the 1,1 open circuit voltage ratio, while A_{21} may be called the *transfer admittance* from 1,1 to 2,2.

There is, however, little point in attaching names to the parameters which are better defined in physical terms, by direct reference to equations (1*c*, *d*, *e*, *f*), (2*c*, *d*, *e*, *f*), etc., inserting also the proper condition appropriate to each case : i.e. 1,1 short or open, or 2,2 short or open. It will be shown in the next section that definitions in this form enable the values of the parameters to be determined in terms of any of the parameters used in the English method, which are listed at the beginning of section 6.

To determine the values of A_{12} and A_{22}, a voltage V_1 is assumed to be applied at 1,1, and the values of I_1 and I_2 are calculated with 2,2 assumed short circuited.

To determine the values of A_{11} and A_{21} a voltage V_1 is assumed to be applied at 1,1, and the values of I_1 and V_2 are calculated with 2,2 assumed open circuited.

By similar methods Y_{11}, Y_{21}, D_{12}, D_{22}, Z_{11} and Z_{21} can be determined. Fig. 9 summarizes physical characteristics.

In the case of parameters determined by shorting or opening 1,1, the initially assumed voltage or current must be respectively applied at and enter 2,2. This leads to the appearance of negative signs which do not appear from the above analysis except for indications such as are given by equation (3*e*), which shows that Y_{22} is negative (because A_{11} and A_{12} are both positive). The appearance of the minus signs is, of course, due to the conventions chosen for positive directions of voltage and current.

No difficulty need arise provided it is remembered that all parameters are positive except those listed below.

NEGATIVE PARAMETERS : $Z_{12}\, Z_{22}$; $Y_{12}\, Y_{22}$; D_{21}.

To determine the value of Y_{12} and Y_{22} ; a voltage V_2 is assumed to be applied at 2,2, and the values of I_1 and I_2 are calculated with 1,1 assumed short circuited. Since the resultant currents which flow are opposite in direction to the currents shown in Fig. 1, it is evident that

$$Y_{12} = -\frac{I_1}{V_2} \text{ and } Y_{22} = -\frac{I_2}{V_2}$$

where I_1 and I_2 are the currents consequent on the application of V_2 at 2,2.

Similarly, the other parameters which depend on 1,1 being short circuited or open circuited are determined by assuming a voltage to be applied at 2,2. The signs of the parameters are best determined by making use of the list of negative parameters above.

Any set of parameters may be expressed in terms of the parameters in any other set. For instance, the A parameters may be expressed in terms of the Z parameters as follows.

Extracting appropriate relations from the above equations :

From (4*c*)

$$A_{22} = -\frac{Z_{22}}{Z_{21}} \quad . \quad . \quad . \quad . \quad (1g)$$

From (4*d*)

$$A_{21} = \frac{1}{Z_{21}} \quad . \quad . \quad . \quad . \quad . \quad (1h)$$

From (2*d*)

$$A_{11} = A_{21}Z_{11} = \frac{Z_{11}}{Z_{21}} \quad . \quad . \quad . \quad (1j)$$

From (1*a*)

$$A_{12} = \frac{V_1}{I_2} - A_{11}\frac{V_2}{I_2}$$

Now insert simultaneous values (i.e. under the same terminating conditions) of $\frac{V_1}{I_2}$ and $\frac{V_2}{I_2}$ from (2*f*).

$$\therefore\ A_{12} = Z_{12} - A_{11}Z_{22} = Z_{12} - \frac{Z_{11}}{Z_{21}}Z_{22}$$

$$= \frac{1}{Z_{21}}(Z_{12}Z_{21} - Z_{11}Z_{22}) \quad . \quad . \quad . \quad (1k)$$

This is generally written

$$A_{12} = -\frac{|Z|}{Z_{21}} \quad . \quad . \quad . \quad . \quad (1m)$$

so introducing a new convention :

Convention.

$$\left.\begin{aligned} |A| &= A_{11}A_{22} - A_{12}A_{21} \\ |Z| &= Z_{11}Z_{22} - Z_{12}Z_{21} \\ |Y| &= Y_{11}Y_{22} - Y_{12}Y_{21} \\ |D| &= D_{11}D_{22} - D_{12}D_{21} \end{aligned}\right\} \text{These expressions are called the fourpole } \textit{determinants}.$$

This convention is not to be confused with the identical method of indicating the magnitude of an impedance : confusion is unlikely owing to the clues provided by the context. Nor is it to be confused with the symbolism $\| A \|$, $\| Z \|$, etc., which is explained later. The

	A	Z	Y	D
$\|A\|$		$-\frac{Z_{12}}{Z_{21}}$	$-\frac{Y_{12}}{Y_{21}}$	$\frac{D_{11}}{D_{22}}$
$\|A\|^{-1}$		$-\frac{Z_{21}}{Z_{12}}$	$-\frac{Y_{21}}{Y_{12}}$	$\frac{D_{22}}{D_{11}}$
$\|Z\|$	$-\frac{A_{12}}{A_{21}}$		$\|Y\|^{-1}$	$\frac{D_{12}}{D_{21}}$
$\|Y\|$	$-\frac{A_{21}}{A_{12}}$	$\|Z\|^{-1}$		$\frac{D_{21}}{D_{12}}$
$\|D\|$	$\frac{A_{11}}{A_{22}}$	$-\frac{Z_{11}}{Z_{22}}$	$-\frac{Y_{22}}{Y_{11}}$	
$\|D\|^{-1}$	$\frac{A_{22}}{A_{11}}$	$-\frac{Z_{22}}{Z_{11}}$	$-\frac{Y_{11}}{Y_{22}}$	

FIG. 2/XXIV:6.—Values of Matrix Determinants in Terms of Parameters of other Matrices.
(By courtesy of the *Electrische Nachrichten-Technik.*)

values of these determinants in terms of the other parameters are given in Fig. 2.

It will be evident that similar relations to those in (1*g*) to (1*m*) can be obtained expressing the members of any set of parameters, in terms of the members of any other set. These relations are summarized in Fig. 5, which is explained in section 6.5 below.

6.2. Determination of the Values of the New Parameters in Terms of the Normal Parameters. This will be illustrated by an example. The method is to equate each new parameter to the physical quantities which define it (e.g. in equations (1*c*, *d*, *e*, *f*), (2*c*, *d*, *e*, *f*), etc.), and then to determine the values of these physical quantities in terms of the required normal parameters by using the appropriate formulae previously established.

6.21. Determination of the *A* Parameters of a Dissymmetrical Fourpole in Terms of its Image Impedances Z_1 and Z_2 and its Transfer Vector *u*.

From (1*c*)

$$A_{12} = \frac{V_1}{I_2} \text{ with 2,2 shorted.}$$

Hence, putting $Z = O$ in equation (20*a*)/XXIV:4

$$A_{12} = \frac{V_1}{I_2} = \sqrt{Z_1 Z_2} \times S_n \quad . \quad . \quad . \quad (1n)$$

Similarly, from (1*c*) and equation (18*a*)/XXIV:4

$$A_{22} = \frac{I_1}{I_2} \text{ (2,2 shorted)} = \sqrt{\frac{Z_2}{Z_1}} C_s \quad . \quad . \quad (1p)$$

From (1*d*) and equation (16*a*)/XXIV:4

$$A_{11} = \frac{V_1}{V_2} \text{ (2,2 open)} = \sqrt{\frac{Z_1}{Z_2}} C_s \quad . \quad . \quad . \quad (1q)$$

From (1*d*) and equations (16*a*) and (23)/XXIV:4

$$A_{21} = \frac{I_1}{V_2} \text{ (2,2 open)} = \frac{I_1}{V_1} \times \frac{V_1}{V_2} = \frac{T_n}{Z_1} \times \sqrt{\frac{Z_1}{Z_2}} C_s = \frac{S_n}{\sqrt{Z_1 Z_2}} \quad . \quad (1r)$$

The *Z*, *Y* and *D* parameters can either be derived in a similar way or else by use of the formulae which express these parameters in terms of the *A* parameters.

It will be evident that if, instead of being given the values of Z_1, Z_2 and *u*, the impedance values of the arms of a specific network are given, the values of the *A*, *Z*, *Y* and *D* parameters can be determined by calculating the value of the appropriate physical counterpart of each parameter (as given by equations (1*c*, *d*, *e*, *f*), (2*c*, *d*, *e*, *f*), etc.), in terms of the impedance values of the arms of the network.

6.22. Determination of the Parameters of the Fourpole Constituted by a Simple Series Impedance. Such a fourpole is shown at (*a*) in Fig. 3. It will be appreciated that the values of the parameters are unchanged by converting the circuit to a balanced circuit by putting $\frac{1}{2}z_1$ in each leg.

Note $y_1 = \dfrac{1}{z_1}$.

From (1*c*)

$$A_{12} = z_1, \quad A_{22} = 1$$

From (1*d*)

$$A_{11} = 1, \quad A_{21} = 0$$

From (2*d*) and (2*f*)

$$Z_{11} = -Z_{12} = Z_{21} = -Z_{22} = \text{infinity}$$

From (3*c*)

$$Y_{11} = y_1, \qquad Y_{21} = y_1$$

From (3*e*)

$$Y_{12} = -y_1, \quad Y_{22} = -y_1$$

From (4*c*)

$$D_{12} = z_1, \quad D_{22} = 1$$

From (4*f*)

$$D_{11} = 1, \quad D_{21} = 0$$

Particular notice should be taken of the minus signs before the values of Y_{12} and Y_{22}. These signs are introduced because of the sense of I_1 and I_2, which are opposite in direction from that which would normally be expected from a voltage applied at 2,2, with sense according to the convention for V_2 shown in Fig. 1. It will be found that this sign agrees also with the sign obtained from equations (3*e*) and (4*e*) as follows:

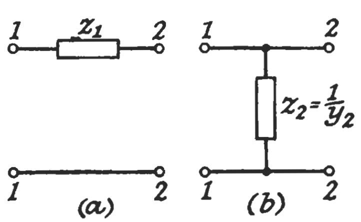


FIG. 3/XXIV:6.—Series and Shunt Impedances considered as Fourpoles.

$$Y_{22} = -\frac{A_{11}}{A_{12}} = -\frac{1}{z_1} = -y_1$$

$$Y_{12} = -\frac{D_{11}}{D_{12}} = -\frac{1}{z_1} = -y_1$$

6.23. Parameters of the Fourpole Constituted by a Simple Shunt Impedance. Such a fourpole is shown at (*b*) in Fig. 3. By the same means as above the parameters of this fourpole can be shown to be as follows:

$$A_{11} = 1\,;\quad A_{12} = 0\,;\quad A_{21} = y_2\,;\quad A_{22} = 1$$
$$Z_{11} = z_2\,;\quad Z_{12} = -z_2\,;\quad Z_{21} = z_2\,;\quad Z_{22} = -z_2$$
$$Y_{11} = -Y_{12} = Y_{21} = -Y_{22} = \text{infinity}$$
$$D_{11} = 1\,;\quad D_{12} = 0\,;\quad D_{21} = -y_2\,;\quad D_{22} = 1$$

It will be evident that y_2 has again been used to indicate $\frac{1}{z_2}$.

6.24. Parameters of the Fourpole Constituted by a Reversal or Commutation. The circuit of such a fourpole is shown at (*a*) in Fig. 4.

Applying the same method: The Y and Z parameters of this fourpole are infinite.

$$A_{11} = -1\,;\quad A_{12} = 0\,;\quad A_{21} = 0\,;\quad A_{22} = -1.$$
$$D_{11} = -1\,;\quad D_{12} = 0\,;\quad D_{21} = 0\,;\quad D_{22} = -1.$$

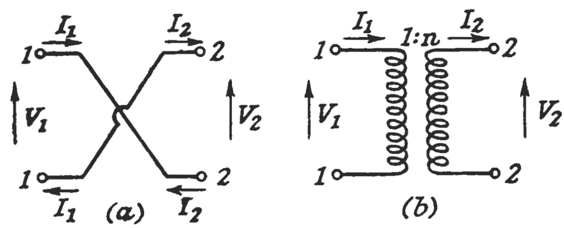


FIG. 4/XXIV:6.—Commutation and Ideal Transformer considered as Fourpoles.

6.25. Parameters of an Ideal Transformer of Turns Ratio 1 : n (and Impedance Ratio 1 : n^2). This is shown as a fourpole at (*b*) in Fig. 4.

The Y and Z parameters of this fourpole are infinite.

$$A_{11} = \frac{1}{n}\,;\quad A_{12} = 0\,;\quad A_{21} = 0\,;\quad A_{22} = n$$

$$D_{11} = \frac{1}{n}\,;\quad D_{12} = 0\,;\quad D_{21} = 0\,;\quad D_{22} = \frac{1}{n}$$

6.3. Symbolic Representation of Equations (1a) ; (1b) to (4a) ; (4b). This symbolic representation, which at first sight appears a pointless and quite unnecessary complication, is the device which gives the German method its peculiar powers by means of which simple rules of thumb may be applied easily and quickly to give the answers to what would otherwise be difficult problems.

Equations (1*a*) and (1*b*) appear below as equation (1), equations

(2*a*) and (2*b*) as equation (2), and so on. The extra equations (5) and (6) are discussed below.

$$\begin{Vmatrix} V_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{Vmatrix} \times \begin{Vmatrix} V_2 \\ I_2 \end{Vmatrix} \quad . \quad . \quad (1)$$

$$\begin{Vmatrix} V_1 \\ V_2 \end{Vmatrix} = \begin{Vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{Vmatrix} \times \begin{Vmatrix} I_1 \\ I_2 \end{Vmatrix} \quad . \quad . \quad (2)$$

$$\begin{Vmatrix} I_1 \\ I_2 \end{Vmatrix} = \begin{Vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{Vmatrix} \times \begin{Vmatrix} V_1 \\ V_2 \end{Vmatrix} \quad . \quad . \quad (3)$$

$$\begin{Vmatrix} V_1 \\ I_2 \end{Vmatrix} = \begin{Vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{Vmatrix} \times \begin{Vmatrix} V_2 \\ I_1 \end{Vmatrix} \quad . \quad . \quad (4)$$

$$\begin{Vmatrix} V_2 \\ I_2 \end{Vmatrix} = \begin{Vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{Vmatrix}^{-1} \times \begin{Vmatrix} V_1 \\ I_1 \end{Vmatrix} \quad . \quad . \quad (5)$$

$$\begin{Vmatrix} V_2 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{Vmatrix}^{-1} \times \begin{Vmatrix} V_1 \\ I_2 \end{Vmatrix} \quad . \quad . \quad (6)$$

Each of the arrangements of symbols between each pair of double vertical lines is called a matrix. Each equation therefore consists of a two-element matrix equated to a four-element matrix multiplied by a two-element matrix. An exception occurs in the case of equations (5) and (6) where, on the right-hand side, the reciprocal of a four-element matrix is multiplied by a two-element matrix. Fortunately we are not concerned with more complicated matrices.

The four-element matrix in equation (1) is called an A matrix and is designated shortly as $\| A \|$; the other parameter matrices are similarly designated so that

$$\| A \| = A \text{ Matrix} \qquad \| Z \| = Z \text{ Matrix} \qquad \| Y \| = Y \text{ Matrix}$$

$$\| D \| = D \text{ Matrix} \qquad \| A \|^{-1} = \frac{1}{\| A \|} \qquad \| D \|^{-1} = \frac{1}{\| D \|}$$

It will be seen that a four-element matrix consists of two (horizontal) rows and/or two (vertical) columns each containing two-matrix elements. In the four-element matrices above each element is equal to a parameter; more complicated elements will be met. The two-element matrices each consist of a single column with two elements.

The process of reproducing the original equations (1*a*) and (1*b*) from (1) will illustrate the laws relating to the multiplication of four-element with two-element matrices, and the conditions for the equality of matrices. For the moment it may be considered that these laws are formulated purely as rules of thumb for restoring the original equations. The whole truth is merely an extension of this statement : the same rules of thumb happen to have a more general application.

6.31. Multiplication of Four-Element and Two-Element Matrices. The rule of thumb for doing this is best stated in the form of an equation.

$$\begin{Vmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{Vmatrix} \times \begin{Vmatrix} x_1 \\ x_2 \end{Vmatrix} = \begin{Vmatrix} X_{11}x_1+X_{12}x_2 \\ X_{21}x_1+X_{22}x_2 \end{Vmatrix} \quad . \quad . \quad (7)$$

The result of multiplying a four-element matrix by a two-element matrix is therefore a two-element matrix.

Note that

$$\begin{Vmatrix} x_1 \\ x_2 \end{Vmatrix} \times \begin{Vmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{Vmatrix} = \begin{Vmatrix} X_{11}x_1 & X_{12}x_1 \\ X_{21}x_2 & X_{22}x_2 \end{Vmatrix} \quad . \quad . \quad (8)$$

The result of multiplying a two-element matrix by a four-element matrix is therefore a four-element matrix. The mathematician describes this by saying that matrices do not obey the laws of commutative algebra. If this statement makes things clearer it may be remembered; otherwise the above two equations explain the matter fully.

The multiplication of four-element matrices by four-element matrices is discussed in XXIV:6.6.

6.32. Rule for Equality of Matrices. Two matrices are equal when they contain the same number of elements and when the elements in corresponding positions are equal

$$\begin{Vmatrix} X_1 \\ X_2 \end{Vmatrix} = \begin{Vmatrix} x_1 \\ x_2 \end{Vmatrix} \text{ when } X_1 = x_1 \text{ and } X_2 = x_2 \quad . \quad (9)$$

$$\begin{Vmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{Vmatrix} = \begin{Vmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{Vmatrix} \text{ when } \left.\begin{matrix} X_{11} = x_{11} & X_{12} = x_{12} \\ X_{21} = x_{21} & X_{22} = x_{22} \end{matrix}\right\} \quad (10)$$

6.33. Resolution of Equation (1) into the Original Equations (1*a*) and (1*b*). Multiplying together the two matrices on the right-hand side :

$$\begin{Vmatrix} V_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} A_{11}V_2+A_{12}I_2 \\ A_{21}V_2+A_{22}I_2 \end{Vmatrix}$$

Since these two matrices are equal

$$V_1 = A_{11}V_2+A_{12}I_2 \quad . \quad . \quad . \quad . \quad (1a)$$

$$I_1 = A_{21}V_2+A_{22}I_2 \quad . \quad . \quad . \quad . \quad (1b)$$

These are the two original equations (1*a*) and (1*b*).

6.4. The Inverse or Reciprocal Matrices $\| A \|^{-1}$ and $\| D \|^{-1}$. It is evident that equation (5) results from dividing across in equation (1) by $\| A \|$, while equation (6) results from dividing across in equation (4) by $\| D \|$.

Although laws for dividing by matrics exist (see CI: 12), it is simpler to replace the inverse matrices by the equivalent direct matrix. This may be done very simply by solving equations (1*a*) and (1*b*) for V_2 and I_2, and by solving equations (4*a*) and (4*b*) for V_2 and I_1. This gives for the first case

$$V_2 = \frac{1}{|A|}(A_{22}V_1 - A_{12}I_1)$$

$$I_2 = \frac{1}{|A|}(-A_{21}V_1 + A_{11}I_1)$$

$$\begin{Vmatrix} V_2 \\ I_2 \end{Vmatrix} = \frac{1}{|A|}\begin{Vmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{Vmatrix} \times \begin{Vmatrix} V_1 \\ I_1 \end{Vmatrix} \quad . \quad . \quad (11)$$

Hence from (5) and (11)

$$\begin{Vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{Vmatrix}^{-1} = \frac{1}{|A|}\begin{Vmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{Vmatrix} \quad . \quad . \quad (12)$$

Similarly

$$\begin{Vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{Vmatrix}^{-1} = \frac{1}{|D|}\begin{Vmatrix} D_{22} & -D_{12} \\ -D_{21} & D_{11} \end{Vmatrix} \quad . \quad . \quad (13)$$

6.5. Relations between *A*, *Y*, *Z* and *D* Matrices. If the set of parameters corresponding to one matrix is known, the set of parameters corresponding to any other matrix can be determined by the methods described in XXIV:6.1. The result is that any matrix can be expressed in terms of the parameters of any other matrix.

Fig. 5 shows the expressions for each type of matrix in terms of its own parameters and also in terms of the parameters proper to the other matrices. The left-hand column defines the matrix to be expressed, while the headings of the other columns describe the matrix of which the parameters are used for expressing the matrices in the left-hand column.

6.6. Determination of *A* Matrix of two Fourpoles in Tandem from their Individual *A* Matrices. "Multiplication of Matrices." If the *A* matrices of two fourpoles are known it is possible to form two sets of equations similar to (1*a*) and (1*b*). If the two fourpoles are put in tandem; that is, if the output of one fourpole is connected to the input of the other, then V_2 and I_2 for the first fourpole are respectively equal to V_1' and I_1' for the second fourpole. If this relation is inserted in the two sets of equations above it is possible to derive equations defining the input

	$\Vert A\Vert$	$\Vert Z\Vert$	$\Vert Y\Vert$	$\Vert D\Vert$	Circuit Matrix
$\Vert A\Vert$	$\begin{Vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{Vmatrix}$	$\left\Vert\begin{array}{cc} Z_{11} & -\lvert Z\rvert \\ 1 & -Z_{22} \\ \hline \multicolumn{2}{c}{Z_{21}} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} -Y_{22} & 1 \\ -\lvert Y\rvert & Y_{11} \\ \hline \multicolumn{2}{c}{Y_{21}} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} \lvert D\rvert & D_{12} \\ -D_{21} & 1 \\ \hline \multicolumn{2}{c}{D_{22}} \end{array}\right\Vert$	$\begin{Vmatrix} V_1 \\ I_1 \end{Vmatrix} = \Vert A\Vert \cdot \begin{Vmatrix} V_2 \\ I_2 \end{Vmatrix}$
$\Vert A\Vert^{-1}$	$\left\Vert\begin{array}{cc} A_{22} & -A_{12} \\ -A_{21} & A_{11} \\ \hline \multicolumn{2}{c}{\lvert A\rvert} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} Z_{22} & -\lvert Z\rvert \\ 1 & -Z_{11} \\ \hline \multicolumn{2}{c}{Z_{12}} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} -Y_{11} & 1 \\ -\lvert Y\rvert & Y_{22} \\ \hline \multicolumn{2}{c}{Y_{12}} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} 1 & -D_{12} \\ D_{21} & \lvert D\rvert \\ \hline \multicolumn{2}{c}{D_{11}} \end{array}\right\Vert$	$\begin{Vmatrix} V_2 \\ I_2 \end{Vmatrix} = \Vert A\Vert^{-1} \cdot \begin{Vmatrix} V_1 \\ I_1 \end{Vmatrix}$
$\Vert Z\Vert$	$\left\Vert\begin{array}{cc} A_{11} & -\lvert A\rvert \\ 1 & -A_{22} \\ \hline \multicolumn{2}{c}{A_{21}} \end{array}\right\Vert$	$\begin{Vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{Vmatrix}$	$\left\Vert\begin{array}{cc} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \\ \hline \multicolumn{2}{c}{\lvert Y\rvert} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} -\lvert D\rvert & D_{11} \\ -D_{22} & 1 \\ \hline \multicolumn{2}{c}{D_{21}} \end{array}\right\Vert$	$\begin{Vmatrix} V_1 \\ V_2 \end{Vmatrix} = \Vert Z\Vert \cdot \begin{Vmatrix} I_1 \\ I_2 \end{Vmatrix}$
$\Vert Y\Vert$	$\left\Vert\begin{array}{cc} A_{22} & -\lvert A\rvert \\ 1 & -A_{11} \\ \hline \multicolumn{2}{c}{A_{12}} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \\ \hline \multicolumn{2}{c}{\lvert Z\rvert} \end{array}\right\Vert$	$\begin{Vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{Vmatrix}$	$\left\Vert\begin{array}{cc} 1 & -D_{11} \\ D_{22} & -\lvert D\rvert \\ \hline \multicolumn{2}{c}{D_{12}} \end{array}\right\Vert$	$\begin{Vmatrix} I_1 \\ I_2 \end{Vmatrix} = \Vert Y\Vert \cdot \begin{Vmatrix} V_1 \\ V_2 \end{Vmatrix}$
$\Vert D\Vert$	$\left\Vert\begin{array}{cc} \lvert A\rvert & A_{12} \\ -A_{21} & 1 \\ \hline \multicolumn{2}{c}{A_{22}} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} Z_{12} & \lvert Z\rvert \\ 1 & -Z_{21} \\ \hline \multicolumn{2}{c}{Z_{22}} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} -Y_{12} & 1 \\ \lvert Y\rvert & Y_{21} \\ \hline \multicolumn{2}{c}{Y_{11}} \end{array}\right\Vert$	$\begin{Vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{Vmatrix}$	$\begin{Vmatrix} V_1 \\ I_2 \end{Vmatrix} = \Vert D\Vert \cdot \begin{Vmatrix} V_2 \\ I_1 \end{Vmatrix}$
$\Vert D\Vert^{-1}$	$\left\Vert\begin{array}{cc} 1 & -A_{12} \\ A_{21} & \lvert A\rvert \\ \hline \multicolumn{2}{c}{A_{11}} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} Z_{21} & \lvert Z\rvert \\ 1 & -Z_{12} \\ \hline \multicolumn{2}{c}{Z_{11}} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} -Y_{21} & 1 \\ \lvert Y\rvert & Y_{12} \\ \hline \multicolumn{2}{c}{Y_{22}} \end{array}\right\Vert$	$\left\Vert\begin{array}{cc} D_{22} & -D_{12} \\ -D_{21} & D_{11} \\ \hline \multicolumn{2}{c}{\lvert D\rvert} \end{array}\right\Vert$	$\begin{Vmatrix} V_2 \\ I_1 \end{Vmatrix} = \Vert D\Vert^{-1} \cdot \begin{Vmatrix} V_1 \\ I_2 \end{Vmatrix}$

FIG. 5/XXIV:6.—Relations between Matrices.

(By courtesy of the *Electrische Nachrichten-Technik*)

voltages and currents to the tandem combination in terms of the output voltages and currents of the tandem combination. If the parameters of the driving fourpole are $A_{11}A_{12}A_{21}A_{22}$, and those of the driven fourpole are $A'_{11}A'_{12}A'_{21}A'_{22}$, these equations are :

$$V_1 = (A_{11}A'_{11}+A_{12}A'_{21})V'_2+(A_{11}A'_{12}+A_{12}A'_{22})I'_2$$
$$I = (A_{21}A'_{11}+A_{22}A'_{21})V'_2+(A_{21}A'_{12}+A_{22}A'_{22})I'_2$$

or in matrix form :

$$\begin{Vmatrix} V_1 \\ I_1 \end{Vmatrix} = \begin{Vmatrix} A_{11}A'_{11}+A_{12}A'_{21} & A_{11}A'_{12}+A_{12}A'_{22} \\ A_{21}A'_{11}+A_{22}A'_{21} & A_{21}A'_{12}+A_{22}A'_{22} \end{Vmatrix} \times \begin{Vmatrix} V'_2 \\ I'_2 \end{Vmatrix} \quad . \quad (14)$$

It now appears that the A matrix of the tandem combination has been derived. From equation (14) and consideration of the two original A and A' matrices it is possible to formulate a law or rule of thumb for deriving the matrix of the tandem combination of two fourpoles. This is as follows :

Write down side by side the two individual matrices of the component fourpoles in the same order as the fourpoles : the matrix of the left-hand fourpole on the left and the matrix of the right-hand fourpole on the right. Insert a multiplication sign between them and " multiply " the matrices together in accordance with the formula

$$\begin{Vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{Vmatrix} \times \begin{Vmatrix} A'_{11} & A'_{12} \\ A'_{21} & A'_{22} \end{Vmatrix} = \begin{Vmatrix} A_{11}A'_{11}+A_{12}A'_{21} & A_{11}A'_{12}+A_{12}A'_{22} \\ A_{21}A'_{11}+A_{22}A'_{21} & A_{21}A'_{12}+A_{22}A'_{22} \end{Vmatrix} \quad . \quad (15)$$

The resultant matrix is the matrix of the tandem combination.

It will be appreciated that the use of the term " multiplication " to describe the operation is arbitrary but convenient.

Example 1.

Determine the A matrix of the L fourpole obtained by combining the two elementary fourpoles shown in Fig. 3. Given that the A matrix of the left-hand (series element) fourpole is $\begin{Vmatrix} 1 & z_1 \\ 0 & 1 \end{Vmatrix}$ (see XXIV:6.22) and the A matrix of the right-hand (shunt element) fourpole is $\begin{Vmatrix} 1 & 0 \\ y_2 & 1 \end{Vmatrix}$ (see XXIV:6.23).

Multiplying the two matrices in proper order the required matrix is given by :

$$\begin{Vmatrix} 1 & z_1 \\ 0 & 1 \end{Vmatrix} \times \begin{Vmatrix} 1 & 0 \\ y_2 & 1 \end{Vmatrix} = \begin{Vmatrix} 1+z_1y_2 & z_1 \\ y_2 & 1 \end{Vmatrix} \quad . \quad . \quad (16)$$

In passing it may be noted that the determinants of the component fourpoles, and of the resultant fourpole are all equal to unity. *The determinant of the A matrix of a passive fourpole is always equal to unity.* This forms a useful check on the correctness of any matrix operation.

Example 2.

Find the A matrix of a T network.

This can be done by adding a series arm z_3 on the right of the network of Example 1. The resultant matrix is evidently given by

$$\begin{Vmatrix} 1+z_1y_2 & z_1 \\ y_2 & 1 \end{Vmatrix} \times \begin{Vmatrix} 1 & z_3 \\ 0 & 1 \end{Vmatrix} = \begin{Vmatrix} 1+z_1y_2 & z_3+z_1y_2z_3+z_1 \\ y_2 & y_2z_3+1 \end{Vmatrix} \quad . \quad (17)$$

It may not be evident from the above, but it is a fact, that for the purpose of finding the equivalent matrix of two fourpoles in tandem, A matrices must be used : the A matrices of the individual fourpoles are multiplied together to give the A matrix of the combination. If any other matrix of the combination is required it can then be found from the appropriate formulae in Fig. 4.

6.7. Determination of Y Matrix of Two Fourpoles in Parallel from their Individual Y Matrices. Addition of Matrices. Y matrices are used for combining fourpoles in parallel.

Fig. 6 shows two fourpoles with their inputs and outputs in parallel. It is evident that the total input current is the sum of the individual input currents, while the output current is the sum of the output currents. The input voltages of the individual fourpoles and of the composite fourpole are the same, as are also the output voltages. It follows that equations similar to (3*a*) and (3*b*) for the composite fourpole can be derived by adding the equations of the form of (3*a*) and (3*b*) constituted with the individual parameters of each fourpole. Reference to equation (3) shows that the matrix derived from the resultant equations is equal to the matrix formed by adding together corresponding elements of the matrices of the component fourpoles. Hence follows the rule for addition of matrices and the rule for the combination of fourpoles in parallel.

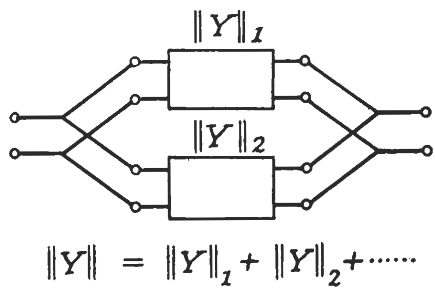


FIG. 6/XXIV:6.—Combination of Fourpoles in Parallel-Parallel by use of Y Matrices.

Matrices are added together in accordance with the formula

$$\left\| \begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix} \right\| + \left\| \begin{matrix} Y'_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} \end{matrix} \right\| = \left\| \begin{matrix} Y_{11}+Y'_{11} & Y_{12}+Y'_{12} \\ Y_{21}+Y'_{21} & Y_{22}+Y'_{22} \end{matrix} \right\| . \quad (18)$$

Fourpoles are combined in parallel-parallel (i.e. with inputs in parallel and outputs in parallel) by adding the Y matrices.

This device is useful for determining the matrices of complex fourpoles which would otherwise be difficult to evaluate. An example of this is given immediately below.

6.71. Determination of Y Matrix of Bridged-T Fourpole. A bridged-T fourpole is shown at (*a*) in Fig. 7, and its resolution into a T fourpole and a single-series element fourpole is shown at (*b*).

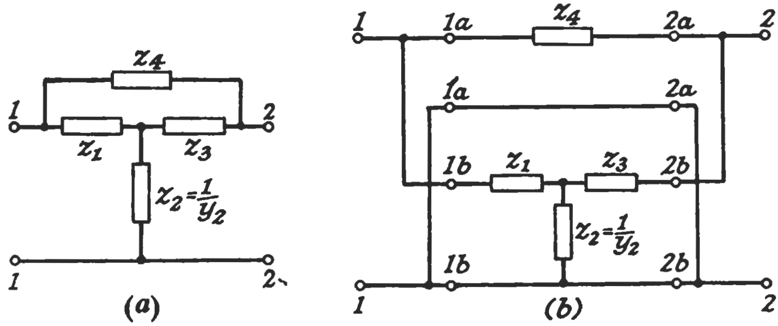


FIG. 7/XXIV:6.—Derivation of Matrix of Bridged-T Fourpole by representing it as Two More Simple Fourpoles in Parallel-Parallel Connection.

The Y matrix of the composite fourpole is obtained by adding the Y matrices of the two component fourpoles in accordance with (18).

The Y matrix of the single-series element is obtained by assembling its Y parameters which are derived in XXIV:6.22, and is

$$\| Y \|_s = \begin{Vmatrix} y_4 & -y_4 \\ y_4 & -y_4 \end{Vmatrix} \quad . \quad . \quad . \quad (19)$$

Note that in the above, $y_4 = 1/z_4$; also, below $y_1 = 1/z_1$; $y_2 = 1/z_2$; $y_3 = 1/z_3$.

The Y matrix of the **T** fourpole is obtained from its A matrix, given in (17)/XXIV:6.6, by using the appropriate transformation from Fig. 5.

Hence

$$\| Y \|_T = \frac{1}{A_{12}} \begin{Vmatrix} A_{22} & -|A| \\ 1 & -A_{11} \end{Vmatrix} = \frac{1}{z_3 + z_1 y_2 z_3 + z_1} \begin{Vmatrix} 1 + y_2 z_3 & -1 \\ 1 & -(1 + z_1 y_2) \end{Vmatrix}$$

$$= \frac{1}{z_1 z_2 + z_2 z_3 + z_1 z_3} \begin{Vmatrix} z_2 + z_3 & -z_2 \\ z_2 & -(z_1 + z_2) \end{Vmatrix} = \frac{1}{S} \begin{Vmatrix} z_2 + z_3 & -z_2 \\ z_2 & -(z_1 + z_2) \end{Vmatrix} \quad (20)$$

where $S = z_1 z_2 + z_2 z_3 + z_1 z_3$.

Hence, from (19) and (20), the Y matrix of the bridged **T** is:

$$\| Y \|_{BT} = \| Y_S \| + \| Y \|_T = \begin{Vmatrix} \frac{1}{S}(z_1 + z_3 + y_4 S) & -\frac{1}{S}(z_2 + y_4 S) \\ \frac{1}{S}(z_2 + y_4 S) & -\frac{1}{S}(z_1 + z_2 + y_4 S) \end{Vmatrix} \quad (21)$$

The A matrix of the bridged **T** is then obtained from the Y matrix in (21) by using the appropriate transformation from Fig. 4:

$$A \|_{BT} = \frac{1}{Y_{21}} \begin{Vmatrix} -Y_{22} & 1 \\ -|Y| & Y_{11} \end{Vmatrix} = \begin{Vmatrix} \dfrac{z_1 + z_2 + y_4 S}{z_2 + y_4 S} & \dfrac{S}{z_2 + y_4 S} \\ \dfrac{1 + (z_1 + z_3) y_4}{z_2 + y_4 S} & \dfrac{z_2 + z_3 + y_4 S}{z_2 + y_4 S} \end{Vmatrix} . \quad (22)$$

As a check, note that from (22) $|A|_{BT} = 1$.

In practice, bridged-T fourpoles are usually symmetrical, in which case $z_3 = z_1$. Putting $z_3 = z_1$ in (22) :

$$\| A \|_{BT} = \begin{Vmatrix} \dfrac{z_1+z_2+y_4S}{z_2+y_4S} & \dfrac{S}{z_2+y_4S} \\ \dfrac{1+2z_1y_4}{z_2y_4S} & \dfrac{z_1+z_2+y_4S}{z_1+y_4S} \end{Vmatrix} \quad . \quad . \quad (23)$$

where $S = z_1^2+2z_1z_2$.

6.8. Use of Y, Z and D Matrices and Inverse D Matrix.

Z and *D* and inverse *D* matrices are added by exactly the same rule that is given for *Y* matrices in equation (18).

The application of these matrices is illustrated in Fig. 8.

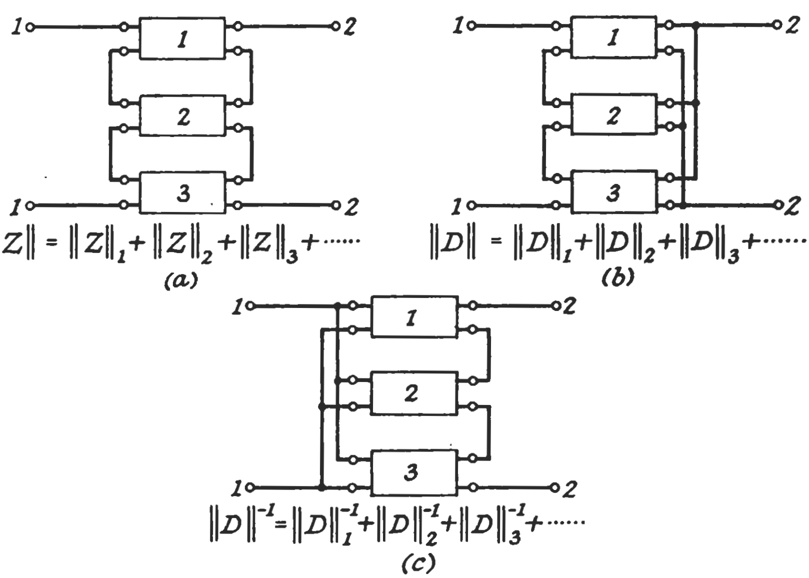


FIG. 8/XXIV:6.—Combination of Fourpoles : In Series-Series. By use of *Z* Matrices : in Series-Parallel by use of *D* Matrices : in Parallel Series by use of Inverse *D* Matrices.

Y matrices are used for combining fourpoles with their inputs in parallel and their outputs in parallel.

Z matrices are used for combining fourpoles with their inputs in series and their outputs in series, as shown at (*a*).

D matrices are used for combining fourpoles with their inputs in series and their outputs in parallel, as shown at (*b*).

D^{-1} matrices are used for combining fourpoles with their inputs in parallel and their outputs in series, as shown at (*c*).

$$\|A\| = \left\| \begin{matrix} \frac{V_1}{V_2}(2,2,0,C.) & \frac{V_1}{I_2}(2,2,S,C) \\ \frac{I_1}{V_2}(2,2,0,C.) & \frac{I_1}{I_2}(2,2,S,C) \end{matrix} \right\|$$

$$\|Z\| = \left\| \begin{matrix} \frac{V_1}{I_1}(2,2,0,C.) & -\frac{V_1}{I_2}(1,1,0,C.) \\ \frac{V_2}{I_1}(2,2,0,C.) & -\frac{V_2}{I_2}(1,1,0,C.) \end{matrix} \right\|$$

$$\|Y\| = \left\| \begin{matrix} \frac{I_1}{V_1}(2,2,S,C.) & -\frac{I_1}{V_2}(1,1,S,C.) \\ \frac{I_2}{V_1}(2,2,S,C.) & -\frac{I_2}{V_2}(1,1,S,C.) \end{matrix} \right\|$$

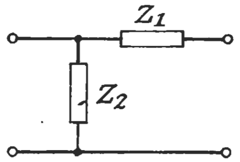
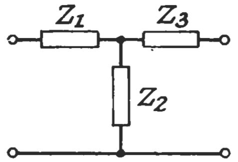
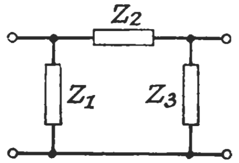
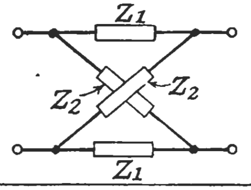
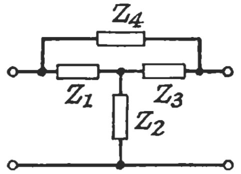
$$\|D\| = \left\| \begin{matrix} \frac{V_1}{V_2}(1,1,0,C.) & \frac{V_1}{I_1}(2,2,S,C.) \\ -\frac{I_2}{V_2}(1,1,0,C.) & \frac{I_2}{I_1}(2,2,S,C.) \end{matrix} \right\|$$

FIG. 9/XXIV:6.—Expression of Matrix Parameters in Fundamental Physical Terms.

The method in each case is to add the matrices of the component fourpoles. The resultant matrix is then, of course, of the same kind as the component matrices. If any other kind of matrix is required, it must be derived in the appropriate manner.

The method of using D^{-1} matrices is to express them in terms of the parameters of the D matrix of the fourpole by using the relation :

$$\|D\|^{-1} = \left\| \begin{matrix} \frac{D_{22}}{|D|} & -\frac{D_{12}}{|D|} \\ -\frac{D_{21}}{|D|} & \frac{D_{11}}{|D|} \end{matrix} \right\| . \quad . \quad . \quad (24)$$

Fourpole	$\|A\|$	$\|A\|^{-1}$	$\|Z\|$	$\|Y\|$	$\|D\|$	$\|D\|^{-1}$
	$\begin{Vmatrix} 1 & Z_1 \\ \frac{1}{Z_2} & 1+\frac{Z_1}{Z_2} \end{Vmatrix}$	$\begin{Vmatrix} 1+\frac{Z_1}{Z_2} & -Z_1 \\ -\frac{1}{Z_2} & 1 \end{Vmatrix}$	$\begin{Vmatrix} Z_2 & -Z_2 \\ Z_2 & -(Z_1+Z_2) \end{Vmatrix}$	$\begin{Vmatrix} \frac{1}{Z_1}+\frac{1}{Z_2} & -\frac{1}{Z_1} \\ \frac{1}{Z_1} & -\frac{1}{Z_1} \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_2}{Z_1+Z_2} & \frac{Z_1 Z_2}{Z_1+Z_2} \\ \frac{-1}{Z_1+Z_2} & \frac{Z_2}{Z_1+Z_2} \end{Vmatrix}$	$\begin{Vmatrix} 1 & -Z_1 \\ \frac{1}{Z_2} & 1 \end{Vmatrix}$
	$\begin{Vmatrix} 1+\frac{Z_1}{Z_2} & \frac{S}{Z_2} \\ \frac{1}{Z_2} & 1+\frac{Z_3}{Z_2} \end{Vmatrix}$	$\begin{Vmatrix} 1+\frac{Z_3}{Z_2} & -\frac{S}{Z_2} \\ -\frac{1}{Z_2} & 1+\frac{Z_1}{Z_2} \end{Vmatrix}$	$\begin{Vmatrix} Z_1+Z_2 & -Z_2 \\ Z_2 & -(Z_2+Z_3) \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_2+Z_3}{S} & -\frac{Z_2}{S} \\ \frac{Z_2}{S} & -\frac{Z_1+Z_2}{S} \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_2}{Z_2+Z_3} & \frac{S}{Z_2+Z_3} \\ -\frac{1}{Z_2+Z_3} & \frac{Z_2}{Z_2+Z_3} \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_2}{Z_1+Z_2} & \frac{-S}{Z_1+Z_2} \\ \frac{1}{Z_1+Z_2} & \frac{Z_2}{Z_1+Z_2} \end{Vmatrix}$
	$\begin{Vmatrix} 1+\frac{Z_2}{Z_3} & Z_2 \\ \frac{Z_1+Z_2+Z_3}{Z_1 Z_3} & 1+\frac{Z_2}{Z_1} \end{Vmatrix}$	$\begin{Vmatrix} 1+\frac{Z_2}{Z_1} & -Z_2 \\ -\frac{Z_1+Z_2+Z_3}{Z_1 Z_3} & 1+\frac{Z_2}{Z_3} \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3} & -\frac{Z_1 Z_3}{Z_1+Z_2+Z_3} \\ \frac{Z_1 Z_3}{Z_1+Z_2+Z_3} & -\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3} \end{Vmatrix}$	$\begin{Vmatrix} \frac{1}{Z_1}+\frac{1}{Z_2} & -\frac{1}{Z_2} \\ \frac{1}{Z_2} & -\left(\frac{1}{Z_2}+\frac{1}{Z_3}\right) \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_1}{Z_1+Z_2} & \frac{Z_1 Z_2}{Z_1+Z_2} \\ -\frac{Z_1+Z_2+Z_3}{Z_3(Z_1+Z_2)} & \frac{Z_1}{Z_1+Z_2} \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_3}{Z_2+Z_3} & -\frac{Z_2 Z_3}{Z_2+Z_3} \\ \frac{Z_1+Z_2+Z_3}{Z_1(Z_2+Z_3)} & \frac{Z_3}{Z_2+Z_3} \end{Vmatrix}$
	$\begin{Vmatrix} \frac{Z_1+Z_2}{Z_2-Z_1} & \frac{2Z_1 Z_2}{Z_2-Z_1} \\ \frac{2}{Z_2-Z_1} & \frac{Z_1+Z_2}{Z_2-Z_1} \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_1+Z_2}{Z_2-Z_1} & -\frac{2Z_1 Z_2}{Z_2-Z_1} \\ -\frac{2}{Z_2-Z_1} & \frac{Z_1+Z_2}{Z_2-Z_1} \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_1+Z_2}{2} & \frac{Z_1-Z_2}{2} \\ \frac{Z_2-Z_1}{2} & -\frac{Z_1+Z_2}{2} \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_1+Z_2}{2Z_1 Z_2} & \frac{Z_1-Z_2}{2Z_1 Z_2} \\ \frac{Z_2-Z_1}{2Z_1 Z_2} & -\frac{Z_1+Z_2}{2Z_1 Z_2} \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_2-Z_1}{Z_2+Z_1} & \frac{2Z_1 Z_2}{Z_1+Z_2} \\ -\frac{2}{Z_1+Z_2} & \frac{Z_2-Z_1}{Z_2+Z_1} \end{Vmatrix}$	$\begin{Vmatrix} \frac{Z_2-Z_1}{Z_2+Z_1} & -\frac{2Z_1 Z_2}{Z_1+Z_2} \\ \frac{2}{Z_1+Z_2} & \frac{Z_2-Z_1}{Z_2+Z_1} \end{Vmatrix}$
	$\begin{Vmatrix} S+(Z_1+Z_2)Z_4 & SZ_4 \\ Z_1+Z_3+Z_4 & S+(Z_2+Z_3)Z_4 \end{Vmatrix}$ $S+Z_2 Z_4$	$\begin{Vmatrix} S+(Z_2+Z_3)Z_4 & -SZ_4 \\ -(Z_1+Z_3+Z_4) & S+(Z_1+Z_2)Z_4 \end{Vmatrix}$ $S+Z_2 Z_4$	$\begin{Vmatrix} S+(Z_1+Z_2)Z_4 & -(S+Z_2 Z_4) \\ S+Z_2 Z_4 & -[S+(Z_2+Z_3)Z_4] \end{Vmatrix}$ $Z_1+Z_3+Z_4$	$\begin{Vmatrix} S+(Z_2+Z_3)Z_4 & -(S+Z_2 Z_4) \\ S+Z_2 Z_4 & -[S+(Z_1+Z_2)Z_4] \end{Vmatrix}$ SZ_4	$\begin{Vmatrix} S+Z_2 Z_4 & SZ_4 \\ -(Z_1+Z_3+Z_4) & S+Z_2 Z_4 \end{Vmatrix}$ $S+(Z_2+Z_3)Z_4$	$\begin{Vmatrix} S+Z_2 Z_4 & -SZ_4 \\ Z_1+Z_3+Z_4 & S+Z_2 Z_4 \end{Vmatrix}$ $S+(Z_1+Z_2)Z_4$

$$S = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$$

In the case of the Bridged T Fourpole all terms are divided by the denominator shown below each matrix.

FIG. 11/XXIV:6.—Matrices of Passive Fourpoles.

(The "A" Determinant of all these Fourpoles is Unity.)

By courtesy of B.B.C. and *Electrische Nachrichten-Technik*. Checked by W. H. Ward.)

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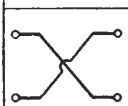
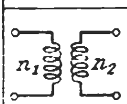
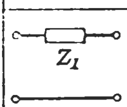
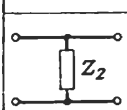
The matrix so formed is then used exactly as any of the other matrices, remembering, of course, that the result of combining a number of inverse matrices is an inverse matrix.

No one appears to have found much use for the inverse A matrix.

6.9. Matrix Tables. It will be clear to the simplest mind that the use of logarithms would not have achieved a very wide popularity if before doing a calculation it was necessary to work out the logarithms of the numbers it was required to multiply together. Such a procedure would be many times more long and difficult than doing the multiplication by the longest and slowest uncontracted method known. The same reasoning applies to matrices. The value of matrices depends on the existence of proper matrix tables. The proper use of these tables avoids most of the working described above.

Fig. 9 shows the values of the parameters of each matrix in terms of the currents and voltages which result from applying voltages to the pair of terminals which are *not* specified as open circuited, or short circuited ; see XXIV:6.1.

	$\Vert A\Vert$	$\Vert A\Vert^{-1}$	$\Vert Z\Vert$	$\Vert Y\Vert$	$\Vert D\Vert$	$\Vert D\Vert^{-1}$
	$\begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}$	$\begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}$	$\begin{Vmatrix} \infty & -\infty \\ \infty & -\infty \end{Vmatrix}$	$\begin{Vmatrix} \infty & -\infty \\ \infty & -\infty \end{Vmatrix}$	$\begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}$	$\begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix}$
	$\begin{Vmatrix} -1 & 0 \\ 0 & -1 \end{Vmatrix}$	$\begin{Vmatrix} -1 & 0 \\ 0 & -1 \end{Vmatrix}$	$\begin{Vmatrix} \infty & \infty \\ -\infty & -\infty \end{Vmatrix}$	$\begin{Vmatrix} \infty & \infty \\ -\infty & -\infty \end{Vmatrix}$	$\begin{Vmatrix} -1 & 0 \\ 0 & -1 \end{Vmatrix}$	$\begin{Vmatrix} -1 & 0 \\ 0 & -1 \end{Vmatrix}$
	$\begin{Vmatrix} \frac{n_1}{n_2} & 0 \\ 0 & \frac{n_2}{n_1} \end{Vmatrix}$	$\begin{Vmatrix} \frac{n_2}{n_1} & 0 \\ 0 & \frac{n_1}{n_2} \end{Vmatrix}$	$\begin{Vmatrix} \infty & -\infty \\ \infty & -\infty \end{Vmatrix}$	$\begin{Vmatrix} \infty & -\infty \\ \infty & -\infty \end{Vmatrix}$	$\begin{Vmatrix} \frac{n_1}{n_2} & 0 \\ 0 & \frac{n_1}{n_2} \end{Vmatrix}$	$\begin{Vmatrix} \frac{n_2}{n_1} & 0 \\ 0 & \frac{n_2}{n_1} \end{Vmatrix}$
	$\begin{Vmatrix} 1 & Z_1 \\ 0 & 1 \end{Vmatrix}$	$\begin{Vmatrix} 1 & -Z_1 \\ 0 & 1 \end{Vmatrix}$	$\begin{Vmatrix} \infty & -\infty \\ \infty & -\infty \end{Vmatrix}$	$\begin{Vmatrix} \frac{1}{Z_1} & -\frac{1}{Z_1} \\ \frac{1}{Z_1} & -\frac{1}{Z_1} \end{Vmatrix}$	$\begin{Vmatrix} 1 & Z_1 \\ 0 & 1 \end{Vmatrix}$	$\begin{Vmatrix} 1 & -Z_1 \\ 0 & 1 \end{Vmatrix}$
	$\begin{Vmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{Vmatrix}$	$\begin{Vmatrix} 1 & 0 \\ -\frac{1}{Z_2} & 1 \end{Vmatrix}$	$\begin{Vmatrix} Z_2 & -Z_2 \\ Z_2 & -Z_2 \end{Vmatrix}$	$\begin{Vmatrix} \infty & \infty \\ \infty & \infty \end{Vmatrix}$	$\begin{Vmatrix} 1 & 0 \\ -\frac{1}{Z_2} & 1 \end{Vmatrix}$	$\begin{Vmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{Vmatrix}$

(The 'A' Determinant of all these Fourpoles is Unity)

FIG. 10/XXIV:6.—Matrices of Passive Fourpoles.

(By courtesy of the B.B.C. and *Electrische Nachrichten-Technik*.)

Figs. 10, 11 and 12 gives the matrices of the more common types of fourpole, including the important case of the general dissymmetrical fourpole, and the even more important case of the transmission line. It will be seen that the symmetrical fourpole matrices are derived from those of the dissymmetrical fourpole by putting $Z_1 = Z_2 = Z_I$, while the transmission line is of course a symmetrical fourpole. For clarity its matrices have, however, been shown separately by putting $Z_I = Z_0$, which is the conventional symbol for the characteristic impedance of a transmission line. It is hardly necessary to point out that the image impedances of a homogeneous transmission line are equal to its characteristic impedance.

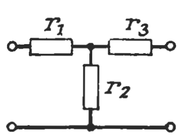
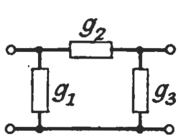
		$\|A\|$	$\|Z\|$	$\|Y\|$	$\|D\|$
	r_1	$\frac{A_{11}-1}{A_{21}}$	$Z_{11}-Z_{21}$	$\frac{Y_{22}+Y_{21}}{\lvert Y\rvert}$	$\frac{-\lvert D\rvert+D_{22}}{D_{21}}$
	r_2	$\frac{1}{A_{21}}$	Z_{21}	$-\frac{Y_{21}}{\lvert Y\rvert}$	$-\frac{D_{22}}{D_{21}}$
	r_3	$\frac{A_{22}-1}{A_{21}}$	$-Z_{22}-Z_{21}$	$\frac{-Y_{11}+Y_{21}}{\lvert Y\rvert}$	$\frac{D_{22}-1}{D_{21}}$
	g_1	$\frac{A_{22}-1}{A_{12}}$	$\frac{Z_{22}+Z_{21}}{\lvert Z\rvert}$	$Y_{11}-Y_{21}$	$\frac{1-D_{22}}{D_{12}}$
	g_2	$\frac{1}{A_{12}}$	$-\frac{Z_{21}}{\lvert Z\rvert}$	Y_{21}	$\frac{D_{22}}{D_{12}}$
	g_3	$\frac{A_{11}-1}{A_{12}}$	$\frac{-Z_{11}+Z_{21}}{\lvert Z\rvert}$	$-Y_{22}-Y_{21}$	$\frac{\lvert D\rvert-D_{22}}{D_{12}}$

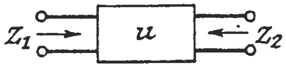
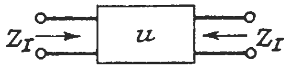
FIG. 13/XXIV:6.—Arms of T and π Fourpoles in Terms of Matrix Parameters.

(By courtesy of the *Electrische Nachrichten-Technik*.)

The matrices of the dissipationless transmission line are derived from the general case of a transmission line by substituting the values of S_n, C_s, T_n and C_t from equations (1*b*), (2*b*), (3*b*) and (4*b*)/XXIV:4.

The matrices of the half-wave and full-wave dissipationless transmission line are derived from the general dissipationless case by inserting the values of the corresponding trigonometrical functions of 180° and 360°.

Fig. 13 shows the values of the arms of T and π fourpoles in terms of the matrix parameters of the fourpoles. Evidently the table can be extended for any other fourpole by solving the equations derived

Network	$\|A\|$	$\|A\|^{-1}$	$\|Z\|$	$\|Y\|$	$\|D\|$	$\|D\|^{-1}$
Dissymmetrical Fourpole  *Image Imped: Z_1 and Z_2* *Transfer Vector: u*	$\begin{matrix} \sqrt{\frac{Z_1}{Z_2}}C_s & \sqrt{Z_1Z_2}S_n \\ \frac{S_n}{\sqrt{Z_1Z_2}} & \sqrt{\frac{Z_2}{Z_1}}C_s \end{matrix}$	$\begin{matrix} \sqrt{\frac{Z_2}{Z_1}}C_s & -\sqrt{Z_1Z_2}S_n \\ -\frac{S_n}{\sqrt{Z_1Z_2}} & \sqrt{\frac{Z_1}{Z_2}}C_s \end{matrix}$	$\begin{matrix} Z_1C_t & -\frac{\sqrt{Z_1Z_2}}{S_n} \\ \frac{\sqrt{Z_1Z_2}}{S_n} & Z_2C_t \end{matrix}$	$\begin{matrix} \frac{C_t}{Z_1} & -\frac{1}{\sqrt{Z_1Z_2}S_n} \\ \frac{1}{\sqrt{Z_1Z_2}S_n} & -\frac{C_t}{Z_2} \end{matrix}$	$\begin{matrix} \frac{1}{C_S}\sqrt{\frac{Z_1}{Z_2}} & Z_1T_n \\ -\frac{T_n}{Z_2} & \frac{1}{C_S}\sqrt{\frac{Z_1}{Z_2}} \end{matrix}$	$\begin{matrix} \frac{1}{C_S}\sqrt{\frac{Z_2}{Z_1}} & -Z_2T_n \\ \frac{T_n}{Z_1} & \frac{1}{C_S}\sqrt{\frac{Z_2}{Z_1}} \end{matrix}$
Symmetrical Fourpole  *Image Impedance: Z_I* *Transfer Vector: u*	$\begin{matrix} C_s & Z_IS_n \\ \frac{S_n}{Z_I} & C_s \end{matrix}$	$\begin{matrix} C_s & -Z_IS_n \\ -\frac{S_n}{Z_I} & C_s \end{matrix}$	$\begin{matrix} Z_IC_t & -\frac{Z_I}{S_n} \\ \frac{Z_I}{S_n} & Z_IC_t \end{matrix}$	$\begin{matrix} \frac{C_t}{Z_I} & -\frac{1}{Z_IS_n} \\ \frac{1}{Z_IS_n} & -\frac{C_t}{Z_I} \end{matrix}$	$\begin{matrix} \frac{1}{C_S} & Z_IT_n \\ -\frac{T_n}{Z_I} & \frac{1}{C_S} \end{matrix}$	$\begin{matrix} \frac{1}{C_S} & -Z_IT_n \\ \frac{T_n}{Z_I} & \frac{1}{C_S} \end{matrix}$
Transmission Line *Char. Impedance: Z_0* $u=e^{\beta+j\alpha}$	$\begin{matrix} C_s & Z_0S_n \\ \frac{S_n}{Z_0} & C_s \end{matrix}$	$\begin{matrix} C_s & -Z_0S_n \\ -\frac{S_n}{Z_0} & C_s \end{matrix}$	$\begin{matrix} Z_0C_t & -\frac{Z_0}{S_n} \\ \frac{Z_0}{S_n} & Z_0C_t \end{matrix}$	$\begin{matrix} \frac{C_t}{Z_0} & -\frac{1}{Z_0S_n} \\ \frac{1}{Z_0S_n} & -\frac{C_t}{Z_0} \end{matrix}$	$\begin{matrix} \frac{1}{C_S} & Z_0T_n \\ -\frac{T_n}{Z_0} & \frac{1}{C_S} \end{matrix}$	$\begin{matrix} \frac{1}{C_S} & -Z_0T_n \\ \frac{T_n}{Z_0} & \frac{1}{C_S} \end{matrix}$
Dissipationless Transmission Line. *Char. Impedance: Z_0* $u=e^{j\alpha}=1\angle\alpha$	$\begin{matrix} \cos\alpha & jZ_0\sin\alpha \\ \frac{j\sin\alpha}{Z_0} & \cos\alpha \end{matrix}$	$\begin{matrix} \cos\alpha & -jZ_0\sin\alpha \\ -\frac{j\sin\alpha}{Z_0} & \cos\alpha \end{matrix}$	$\begin{matrix} -jZ_0\cot\alpha & \frac{jZ_0}{\sin\alpha} \\ \frac{-jZ_0}{\sin\alpha} & -jZ_0\cot\alpha \end{matrix}$	$\begin{matrix} \frac{-j\cot\alpha}{Z_0} & \frac{j}{Z_0\sin\alpha} \\ \frac{-j}{Z_0\sin\alpha} & \frac{j\cot\alpha}{Z_0} \end{matrix}$	$\begin{matrix} \frac{1}{\cos\alpha} & jZ_0\tan\alpha \\ \frac{-j\tan\alpha}{Z_0} & \frac{1}{\cos\alpha} \end{matrix}$	$\begin{matrix} \frac{1}{\cos\alpha} & -jZ_0\tan\alpha \\ \frac{j\tan\alpha}{Z_0} & \frac{1}{\cos\alpha} \end{matrix}$
Half-Wave ($\frac{\lambda}{2}$) Dissipationless Transmission Line. *Char. Impedance: Z_0* $u=e^{j\pi}=1\angle 180°$	$\begin{matrix} -1 & 0 \\ 0 & -1 \end{matrix}$	$\begin{matrix} -1 & 0 \\ 0 & -1 \end{matrix}$	$\begin{matrix} \infty & \infty \\ \infty & \infty \end{matrix}$ (signs of infinities omitted)	$\begin{matrix} \infty & \infty \\ \infty & \infty \end{matrix}$	$\begin{matrix} -1 & 0 \\ 0 & -1 \end{matrix}$	$\begin{matrix} -1 & 0 \\ 0 & -1 \end{matrix}$
Full-Wave (λ) Dissipationless Transmission Line. *Char. Impedance: Z_0* $u=e^{j2\pi}=1\angle 360°$	$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$	$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$	$\begin{matrix} \infty & \infty \\ \infty & \infty \end{matrix}$	$\begin{matrix} \infty & \infty \\ \infty & \infty \end{matrix}$	$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$	$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$

(The 'A' Determinant of all these Fourpoles is Unity)

FIG. 12/XXIV:6.—Matrices of Passive Fourpoles.

(By courtesy of B.B.C. and *Electrische Nachrichten-Technik.* Checked by W. H. Ward.)

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from Figs. 10, 11 and 12, or similar matrix tables for other fourpoles, by equating the expressions for the parameters of any matrix (in terms of its arms) respectively to the parameters of the matrix and solving the resultant equations simultaneously.

6.10. Application of Matrices. Matrices should never be used where simpler methods of obtaining the required answer exist. For instance, while in XXIV:5.4 an example is given of the use of matrices to find the transfer vector and image impedances of a bridged-T fourpole, usually one of the other methods given in XXIV:5 will be found more useful.

There are, however, many cases where matrices save not only a lot of calculation but also a lot of thinking, which should certainly recommend them in certain quarters. Excellent examples of the latter case are given in CIII:3 and 4, to which reference should be made.

The matrix method has the value, however, that it gives directly the quantities C_s and S_n for any fourpole. And as these quantities are generally required for substitution in transmission formulae, it is not always necessary to derive the value of the transfer vector which can, however, be derived very simply by the matrix method. When calculating the insertion loss of a fourpole or a filter, for instance, the value of the transfer vector is not required, but only the values of C_s and S_n.

6.10.1. Equivalent Fourpoles. As has already been indicated, one of the most useful applications of matrices is for the determination of the relations between the arms of equivalent fourpoles. Two fourpoles are equivalent when they have the same image impedances and the same transfer vector. They are also equivalent when they have the same open-circuit impedances and the same short-circuit impedances : this is really a corollary of the last statement. It will now be evident that two fourpoles are equivalent when they have the same A matrices, and therefore the same Z matrices, the same Y matrices and the same D matrices.

Hence, the relations between the arms of any two fourpoles which are to be equivalent can be established by equating their matrices (of the same kind), each matrix being expressed in terms of the arms of the fourpole to which it applies. It will be found that relations between the arms of any two fourpoles can be found regardless of their form, and this might give rise to the hope that any fourpole has an equivalent fourpole of every other possible form. In practice this is not the case since, while it is possible to calculate the arms of any fourpole in terms of the arms of the fourpole to which it is equivalent, these arms are not always physically realizable : for their realization they may require the introduction of elements of negative-impedance : negative-resistance, inductance and capacity.

For any two equivalent fourpoles F_1 and F_2, it is possible to express the relations between their arms in two ways. The arms of F_1 can be expressed in terms of the arms of F_2, or the arms of F_2 can be expressed in terms of the arms of F_1. Evidently where the relations are simple it is immaterial which method is used for purposes of record, since the other can easily be derived. Where the relations are not simple both methods must be used. In any particular problem the particular set of relations used evidently depends on which fourpole has been solved and for which a solution is required : the most frequent use of tables of equivalent fourpoles occurs when a solution of some particular requirement has been obtained in terms of one type of fourpole, and it is required to use another type of fourpole because it is cheaper, or because the configuration of elements in it is more easily realized physically.

The paucity of tables of equivalent fourpoles can be offset, until such tables are published, by deriving the relations of equivalence by means of matrices. It is to be hoped that now that matrices have become vulgarized an article supplying the missing information will shortly appear.

Example 1.

*Dissymmetrical-*T *Fourpole Equivalent to General Dissymmetrical Fourpole.*

The process of determining the conditions of equivalence consists merely in determining the values of the arms of the dissymmetrical fourpole in terms of the image impedances and transfer vector of the general dissymmetrical fourpole. This is done by equating any matrix of the one fourpole to the (same kind of) matrix of the other fourpole. For this purpose the kind of matrix which gives the simplest equations should, of course, be chosen. In the present case the Z matrix evidently gives the simplest equations. Equating the Z matrix of the dissymmetrical-T network in Fig. 11 to the Z matrix of the general dissymmetrical fourpole in Fig. 12 and writing x_1, x_2, x_3 instead of z_1, z_2, z_3 for the unknown parameters :

$$\begin{Vmatrix} x_1+x_2 & -x_2 \\ x_2 & -(x_2+x_3) \end{Vmatrix} = \begin{Vmatrix} Z_1C_t & -\dfrac{\sqrt{Z_1Z_2}}{S_n} \\ \dfrac{\sqrt{Z_1Z_2}}{S_n} & -Z_2C_t \end{Vmatrix}. \quad . \quad (25)$$

Hence,

$$x_1+x_2 = Z_1C_t \quad . \quad . \quad . \quad . \quad . \quad (25a)$$

$$x_2 = \frac{\sqrt{Z_1Z_2}}{S_n} \quad . \quad . \quad . \quad . \quad . \quad (25b)$$

$$x_2+x_3 = Z_2C_t \quad . \quad . \quad . \quad . \quad . \quad (25c)$$

$$\therefore\ x_1 = Z_1C_t - \frac{\sqrt{Z_1Z_2}}{S_n} \quad . \quad . \quad . \quad (26)$$

$$x_3 = Z_2C_t - \frac{\sqrt{Z_1Z_2}}{S_n} \quad . \quad . \quad . \quad (27)$$

x_2 is then given by equation (25*b*).

These relations were used in determining the values of the unequal-ratio T-attenuator shown in Fig. 2/VII:17. In this figure it should be noted that r is the power ratio of the network so that it is equal to the square of the transfer vector, which in that case is a numeric, i.e. it is non-complex.

The values of the arms of π attenuator shown in Fig. 2/VII:17 were obtained in a similar way.

Example 2.

Express the arms of a lattice fourpole, supposed unknown, in terms of the arms of its equivalent **T** *fourpole, supposed known.*

The arms of the unknown (lattice) fourpole will be called x instead of z, while those of the **T** network will be called z as usual.

Equating the Z matrices of the two networks (and putting $z_3 = z_1$ in the Z matrix of the **T** to provide the matrix of a symmetrical **T**), see Fig. 11 :

$$\left\|\begin{matrix} \dfrac{x_1+x_2}{2} & \dfrac{x_1-x_2}{2} \\ \dfrac{x_2-x_1}{2} & -\dfrac{x_1+x_2}{2} \end{matrix}\right\| = \left\|\begin{matrix} z_1+z_2 & -z_2 \\ z_2 & -(z_1+z_2) \end{matrix}\right\| \quad . \quad . \quad (28)$$

$$\therefore\ x_2+x_1 = 2z_1+2z_2 . \quad . \quad . \quad . \quad (28a)$$

$$x_2 - x_1 = 2z_2 \quad . \quad . \quad . \quad . \quad (28b)$$

The arms of the lattice fourpole are therefore

$$x_2 = z_1+2z_2 \quad . \quad . \quad . \quad . \quad (29)$$

$$x_1 = z_1 \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Equations (29) and (30) can easily be transformed to give the arms of the **T** in terms of the lattice :

$$x_1 = z_1$$

$$x_2 = \tfrac{1}{2}(z_2 - z_1)$$

where x_1 and x_2 are the arms of the **T** and z_1 and z_2 the arms of the lattice.

Example 3.

*Express the arms of a bridged-***T** *fourpole, supposed unknown, in terms of its equivalent lattice fourpole, supposed known.* Using x for the unknown arms as before, and equating the Z matrices :

$$\begin{Vmatrix} \frac{S+(x_1+x_2)x_4}{2x_1+x_4} & -\frac{S+x_2x_4}{2x_1+x_4} \\ \frac{S+x_2x_4}{2x_1+x_4} & -\frac{S+(x_1+x_2)x_4}{2x_1+x_4} \end{Vmatrix} = \begin{Vmatrix} \frac{z_1+z_2}{2} & \frac{z_1-z_2}{2} \\ \frac{z_2-z_1}{2} & -\frac{z_1+z_2}{2} \end{Vmatrix} \quad . \quad (31)$$

where $S = x_1^2+2x_1x_2$ and x_3 has been made equal to x_1 since a symmetrical structure is evidently required (because the lattice is a symmetrical structure).

It is to be noticed that (31) only yields two independent equations for the determination of three unknowns : x_1, x_2 and x_4. This means that one of the arms of the bridged **T** can be chosen arbitrarily, so that there are an infinite number of ideal bridged-**T** networks which are the equivalent of any given lattice structure. (On the contrary, there is only one lattice structure which is the equivalent of any given symmetrical bridged-**T** structure.) In order to obtain a practical bridged-**T** equivalent of a lattice structure it is therefore necessary to introduce some other condition. A condition which is commonly introduced is $x_4 = z_I^2/x_2$, where Z_I is the image impedance of the bridged **T**, and therefore of the lattice also. But the image impedance of a lattice is $\sqrt{z_1z_2}$: hence $x_4 = \frac{z_1z_2}{x_2}$. Inserting this condition in the equations derived from (31) by equating corresponding matrix terms (i.e. parameters) :

$$\frac{x_1^2+2x_1x_2+(x_1+x_2)\frac{z_1z_2}{x_2}}{2x_1+\frac{z_1z_2}{x_2}} = \frac{1}{2}z_1+\frac{1}{2}z_2 \quad . \quad . \quad (31a)$$

$$\frac{x_1^2+2x_1x_2+x_2\frac{z_1z_2}{x_2}}{2x_1+\frac{z_1z_1}{x_2}} = \frac{1}{2}z_2-\frac{1}{2}z_1 \quad . \quad . \quad . \quad (31b)$$

From (31*a*)

$$\frac{x_1^2x_2+2x_1x_2^2+z_1z_2x_1+z_1z_2x_2}{2x_1x_2+z_1z_2} = \frac{1}{2}z_1+\frac{1}{2}z_2 \quad . \quad . \quad (32)$$

From (31*b*)

$$\frac{x_1^2x_2+2x_1x_2^2+z_1z_2x_2}{2x_1x_2+z_1z_2} = \frac{1}{2}z_2 - \frac{1}{2}z_1. \qquad (33)$$

Subtracting (33) from (32)

$$\therefore \quad \frac{z_1z_2x_1}{2x_1x_2+z_1z_2} = z_1$$

Cancelling z_1 across :

$$\therefore \quad z_2x_1 = 2x_1x_2+z_1z_2. \qquad (34)$$

$$\therefore \quad x_2 = \frac{z_2x_1 - z_1z_2}{2x_1}. \qquad (35)$$

Note also from (34) that

$$2x_1x_2^2+z_1z_2x_2 = z_2x_1x_2 \qquad (36)$$

Substituting (36) in (33)

$$\therefore \quad \frac{x_1^2x_2+z_2x_1x_2}{2x_1x_2+z_1z_2} = \frac{1}{2}z_2 - \frac{1}{2}z_1$$

$$\therefore \quad x_1^2x_2+z_2x_1x_2 = z_2x_1x_2 - z_1x_1x_2+\tfrac{1}{2}z_1z_2(z_2 - z_1)$$

$$\therefore \quad x_1^2x_2+z_1x_1x_2 - \tfrac{1}{2}z_1z_2(z_2 - z_1) = 0 \qquad (37)$$

Substituting (35) in (37)

$$\therefore \quad \tfrac{1}{2}x_1(z_2x_1 - z_1z_2)+\tfrac{1}{2}z_1(z_2x_1 - z_1z_2) - \tfrac{1}{2}z_1z_2(z_2 - z_1) = 0$$

$$\therefore \quad z_2x_1^2 - z_1z_2x_1+z_1z_2x_1 - z_1^2z_2 - z_1z_2^2+z_1^2z_2 = 0$$

Cancelling similar terms of opposite signs

$$\therefore \quad x_1 = \sqrt{z_1z_2} \qquad (38)$$

Substituting (38) in (35)

$$x_2 = \frac{z_2\sqrt{z_1z_2} - z_1z_2}{2\sqrt{z_1z_2}} = \frac{1}{2}(z_2 - \sqrt{z_1z_2}) \qquad (39)$$

Hence

$$x_4 = \frac{Z_I^2}{x_2} = \frac{2z_1z_2}{z_2 - \sqrt{z_1z_2}} \qquad (40)$$

Example 4.

Establish the equivalence of fourpoles (f) to (k) in Fig. 4/XXI:4 and show that the insertion losses of these fourpoles are the same as those of fourpoles (d) and (e) in the same figure.

The proofs will not be given in full, but the lines along which the proofs may be established will be indicated. The choice of these lines of proof is a good example of the kind of opportunism which is so useful in avoiding unnecessary labour in all network calculations. Starting with (*f*), the bridged-**T** fourpole: from equation (38) the image impedance $Z_1 = x_1 = R$. Hence, if fourpoles (*g*) to (*k*) are equivalent to (*f*), their image impedances will be R and their transfer vectors will be the same as that of (*f*); hence their insertion losses will be the same as that of (*f*).

The equivalence of the lattice structure at (*h*) to the bridged-**T** structure at (*f*) may be established by showing that the values of the impedances of the lattice structure, when substituted in the formulae for the equivalent bridge structure shown against the lattice structure No. 15 in Fig. 16/XXIV:6, give rise to the same impedances as the corresponding arms of the bridged-**T** structure at (*h*). It will be seen that the values of the arms in the equivalent bridged-**T** structure are obtained from equations (38), (39) and (40) above.

The equivalence of the **T** structure at (*k*) to the lattice structure at (*h*), and hence to the bridged-**T** structure at (*f*), may be established by showing that their Z matrices are equal when the values of the parameters are expressed in terms of the impedances of the arms as shown at (*h*) and (*k*).

The equivalence of the π structure at (*j*) to the **T** structure at (*k*) may be established by showing that their A matrices are equal when the values of the parameters are expressed in terms of the impedances of the arms of each structure as shown at (*j*) and (*k*).

The input impedances (i.e. at terminals 1,1) of the structures at (*d*) and (*e*) may be shown to be equal to R by the method of inspection. The insertion losses of these structures may then be shown to be equal to the insertion loss of the **T** structure at (*k*) by calculating the received currents for the circuit configurations at (*d*), (*e*) and (*k*), assuming, of course, the same e.m.f. e in each case.

It will, of course, be appreciated that the method, or the matrix chosen in each case is that which gives rise to the minimum amount of analytical manipulation. This will be obvious in the case of the matrices, because the matrices chosen are those which have the simplest expressions for their parameters when expressed in terms of the arms of each of the types of structure in question.

Figs. 14, 15, 16 and 17 show a number of equivalent fourpoles. Each of the one or two networks in the middle and right-hand column is equivalent to the network shown in the left-hand column when its arms have the values shown in terms of the arms of the left-hand network.

The fourpoles equivalent to the transmission line or symmetric fourpole shown at (1) in Fig. 14 will be recognized as the equivalent-**T** and π for a transmission line, except that the series arms of the **T** and the shunt arms of the π are not expressed in their usual and most simple form. The peculiar form adopted has been chosen in order to give direct relation to the arms of the equivalent fourpoles of the dissymmetrical fourpole shown at (2) in Fig. 14.

The series arms of the **T** corresponding to the symmetric fourpole may be converted to a form *corresponding* to the usual form as follows :

$$Z_0\left[C_t - \frac{1}{S_n}\right] = Z_0\left[\frac{u^2+1}{u^2-1} - \frac{2}{u - \frac{1}{u}}\right] = Z_0\left[\frac{u^2+1}{u^2-1} - \frac{2u}{u^2-1}\right]$$

$$= Z_0\frac{(u-1)^2}{u^2-1} = Z_0\frac{u-1}{u+1} = Z_0 \operatorname{tans}\sqrt{u} \quad . \quad (41)$$

Similarly, the shunt arms of the π corresponding to the transmission line or symmetric fourpole are equal to $Z_0 \operatorname{cots}\sqrt{u}$.

The equivalent fourpoles shown against fourpoles (3) to (8) are merely a way of tabulating the image impedances and transfer vectors of these fourpoles ; they are none the less useful.

The equivalents of fourpoles (9) to (14) in Fig. 15 are straight equivalents in which the values of the arms are shown in terms of the arms of the fourpole in the left-hand column.

Fourpole (15) in Fig. 16 is an example of a constant resistance fourpole.

A *constant resistance* fourpole is a fourpole in which the image impedance(s) is (are) equal to a resistance (the same resistance) at all frequencies.

The lattice equivalent of the bridged-**T** structure (16) is an unrestricted and entirely general equivalent of the bridged-**T** structure shown.

(17), (18) and (19) show the same coupled circuit : two inductances L_1 and L_2 with a mutual coupling M. Six alternative equivalents of this structure are shown, each of which will be found useful for different purposes. As an example of the application of these equivalents, (20) shows how a double-parallel-tuned mutual coupling can be shown to be equivalent to a mid-shunt terminated section of a prototype band-pass filter plus an ideal transformer. (21) shows how a series-parallel-tuned mutual coupling can be shown to be equivalent to a half-section of prototype band-pass filter plus an ideal transformer. See XXV:5 and CIX.

An *ideal transformer* is a transformer with infinite inductances, unity coupling factor k, and the required or indicated impedance ratio, i.e. ratio of L_1 to L_2.

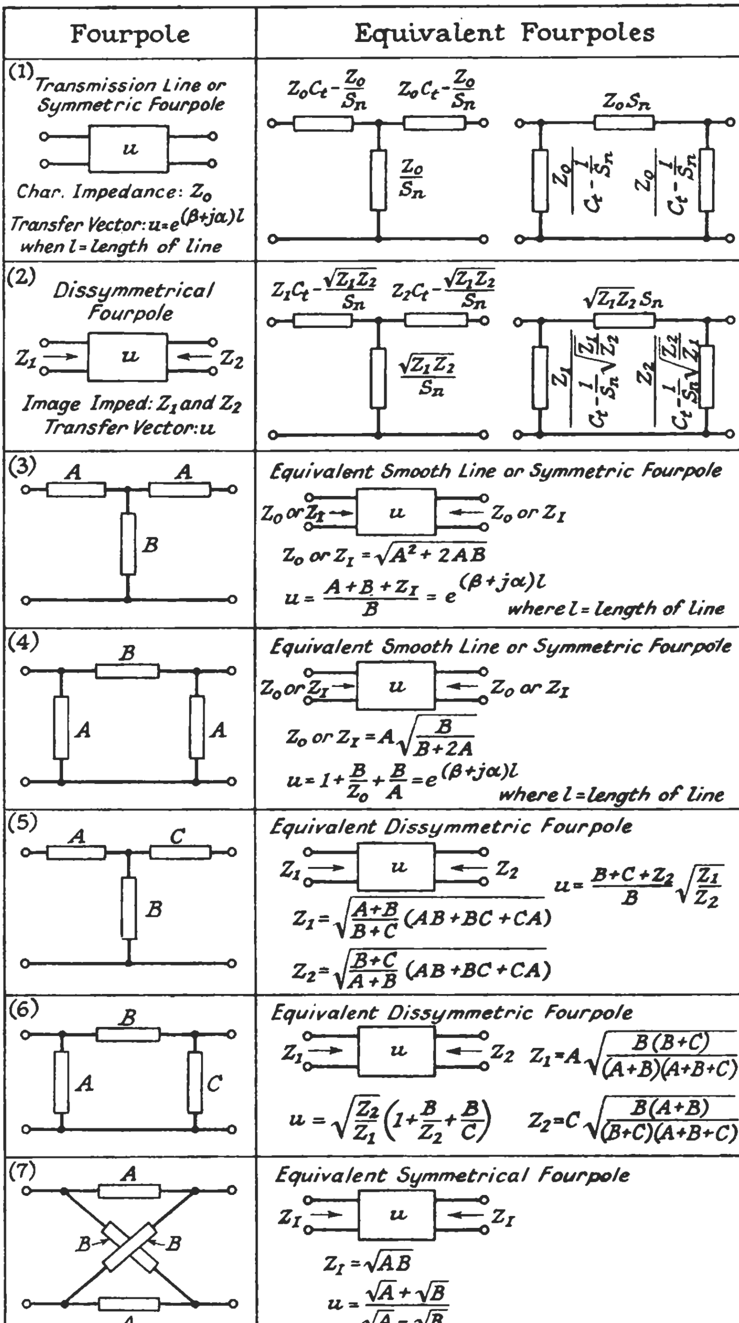


FIG. 14/XXIV:6.—Equivalent Fourpoles.

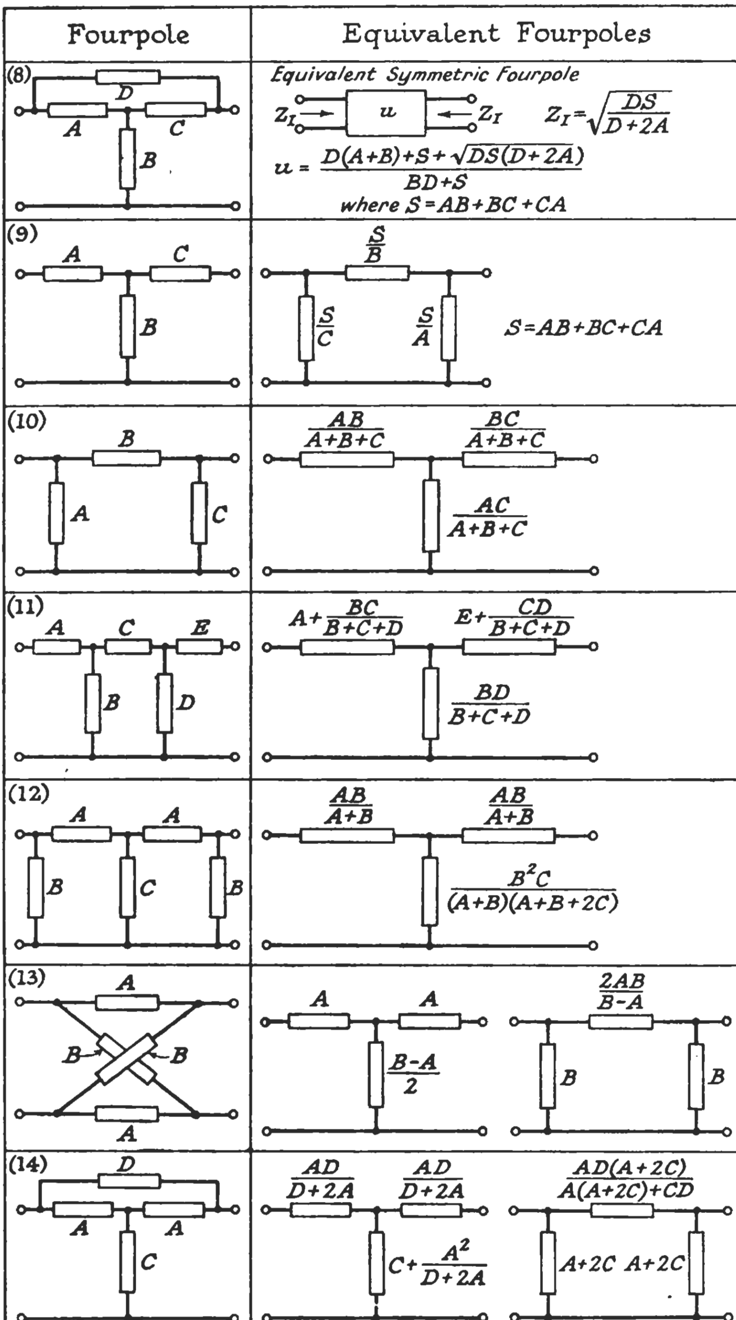


FIG. 15/XXIV:6.—Equivalent Fourpoles (*continued*).

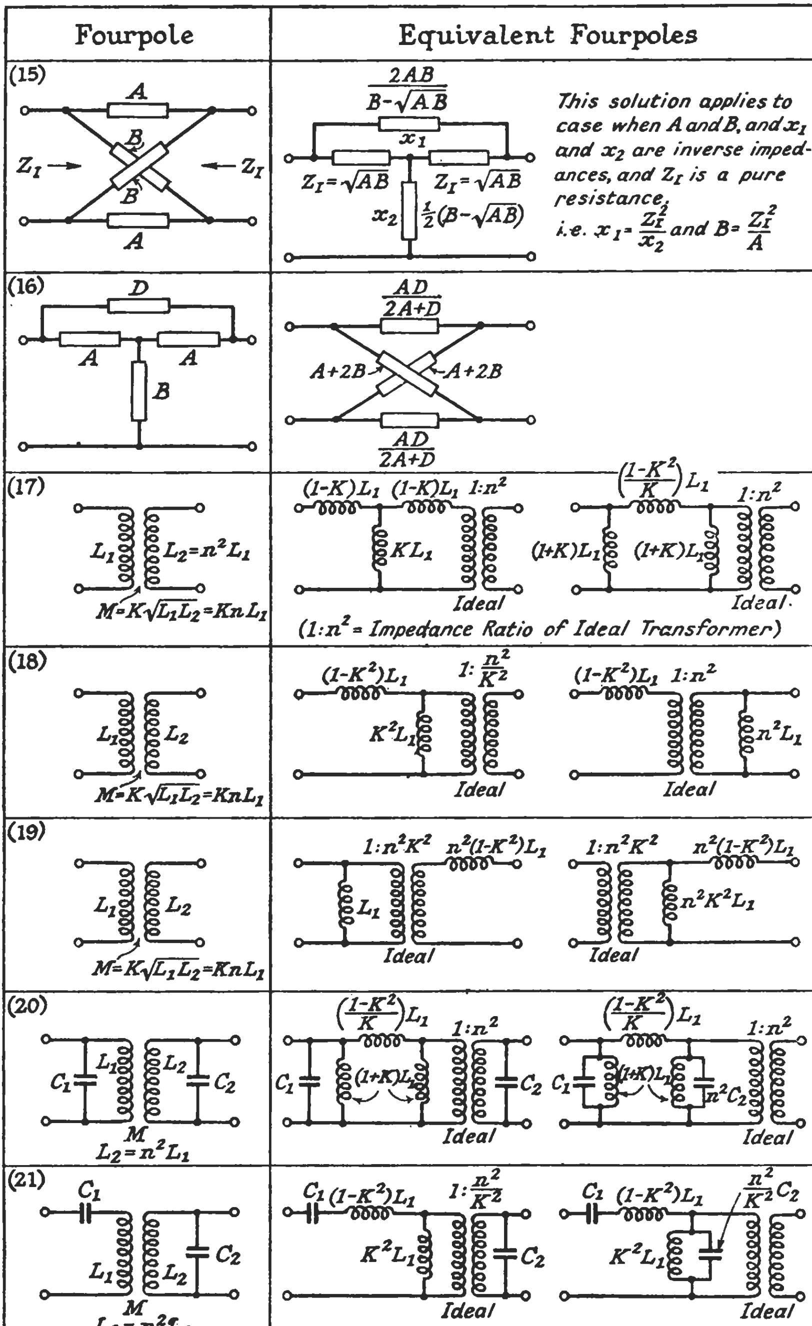


FIG. 16/XXIV:6.—Equivalent Fourpoles (*continued*).

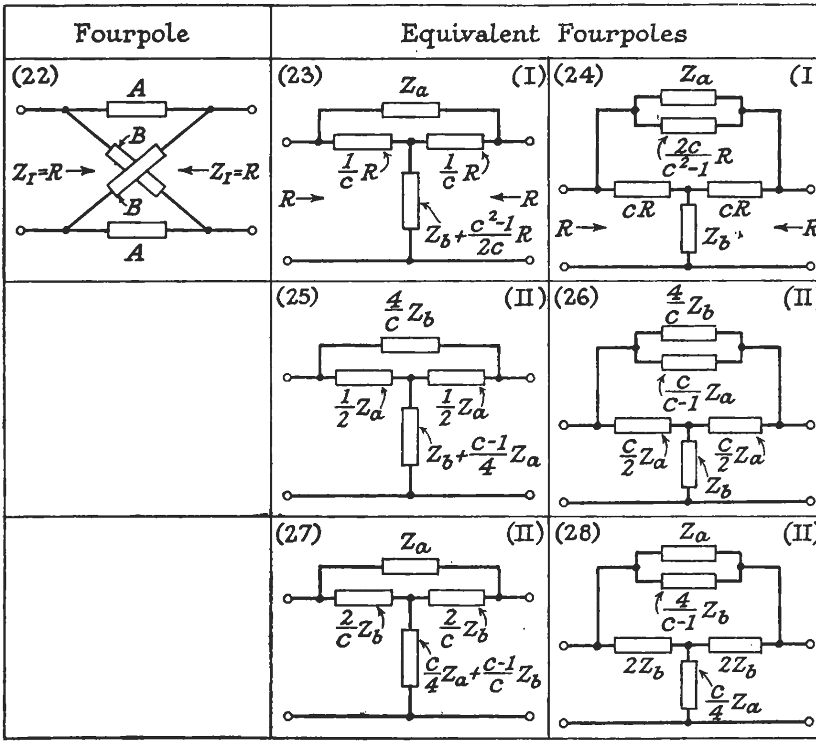


FIG. 17/XXIV:6.—Equivalent Constant Resistance Fourpoles.
(By courtesy of the *Bell System Technical Journal*.)

Fig. 17 shows six constant-resistance bridged-T structures which are the equivalents of the lattice structure shown, when the conditions below hold. These equivalents are taken from a paper by O. J. Zobel, " Distortion Correction in Electrical Circuits with Constant Resistance Recurrent Networks ", *Bell System Technical Journal*, July 1928, p. 438. They have not all been checked, but anyone requiring to use them should have little difficulty in establishing their validity by applying the methods developed above.

The bridged-T structures can be classified into two types: Type I at (23) and (24), and Type II at (25), (26), (27) and (28). The essential difference between the two types is that the series arms of Type I are always resistive so that the fourpoles must have dissipation, while Type II *may* consist entirely of reactances in which case the magnitude of the transfer vector will be unity. Such fourpoles are used for introducing required amounts of phase shift, for instance, for reducing phase distortion.

It will be noted that the bridged-T fourpoles contain a factor c; this is an arbitrary parameter which can have any value, greater than or equal to unity, without changing the image impedance or the transfer vector of the structure.

Conditions for the equivalence of the bridged-T's and lattice structure are given below. The relations for the Type II structure are taken from the above paper by Zobel and have not been checked. The relations given by Zobel for the Type I structure appear to be wrong and the correct relations have been substituted.

Zobel states that the transfer vector of the Type I bridged-T structures is

$$u = \frac{1+(c+1)Z_a/2R}{1+(c-1)Z_a/2R} \quad . \quad . \quad . \quad (42)$$

while the transfer vector for the Type II structures is:

$$u = \frac{1+Z_a/2R+c(Z_a/2R)^2}{1-Z_a/2R+c(Z_a/2R)^2} \quad . \quad . \quad . \quad (43)$$

The general conditions for the equivalence of the bridged-T structures and the lattice are that all the structures are constant-resistance structures of image impedance R, in which case $R^2 = AB$, and also that $Z_aZ_b = R^2$; in other words, Z_a and Z_b are inverse networks inverted about R.

Conditions for Bridged-T Fourpoles to be Equivalent to Lattice.

Type I Bridged T. $$Z_a = \frac{1}{\dfrac{1}{2A} - \dfrac{c}{2R}} \quad . \quad . \quad . \quad (44)$$

Type II Bridged T. $$Z_a = \frac{2}{c}(B \pm \sqrt{B^2 - cR^2}) \quad . \quad (45)$$

In each case $Z_b = \dfrac{R^2}{Z_a}$

Conditions for Lattice to be Equivalent to Bridged-T Fourpoles.

To Type I Bridged T. $$A = \frac{1}{\dfrac{2}{Z_a} + \dfrac{c}{R}} \quad . \quad . \quad (46)$$

To Type II Bridged T. $$A = \frac{1}{\dfrac{1}{Z_a} + \dfrac{c}{4Z_b}} \quad . \quad . \quad (47)$$

In each case $B = \dfrac{R^2}{A}$.

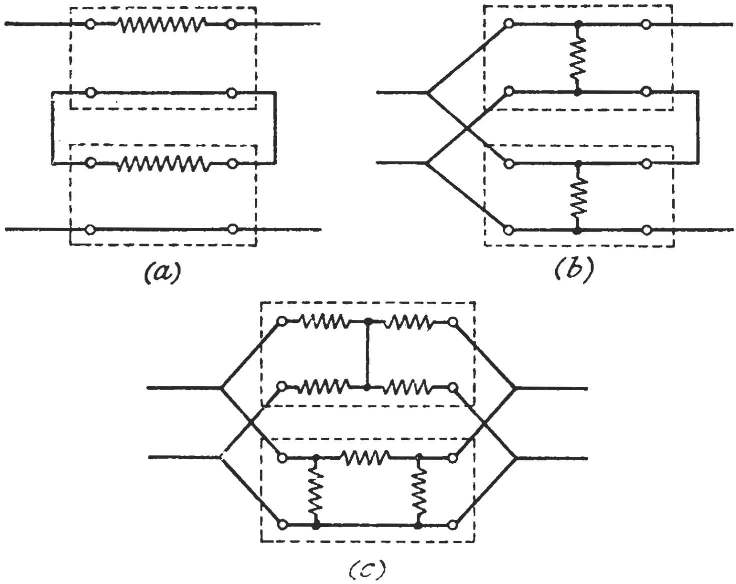


FIG. 1/XXIV:7.—Forbidden Fourpole Combinations.

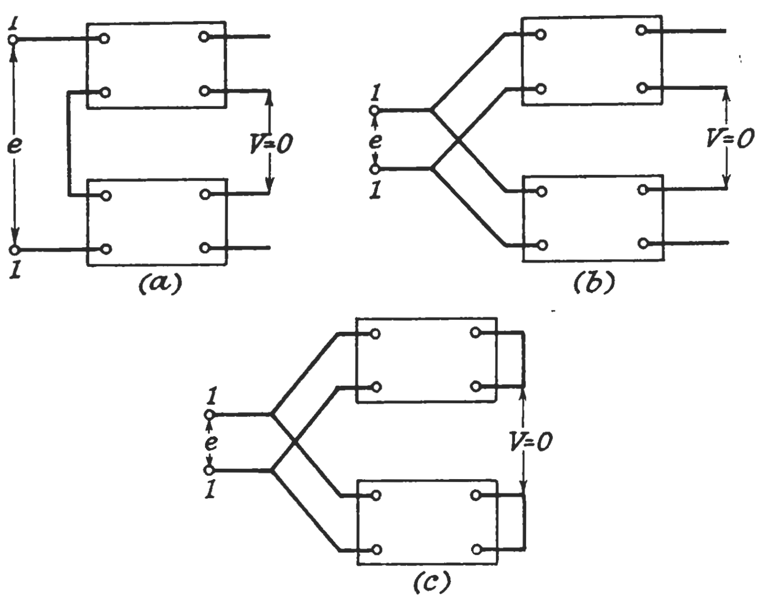


FIG. 2/XXIV:7.—Validity Tests for Matrix Combinations of Fourpoles. (*a*) Series-Series. (*b*) Parallel-Series. (*c*) Parallel-Parallel.

7. Failure of Matrix Combination of Fourpoles.

In practice, the permissible combinations of fourpoles by the matrix methods described above are severely limited.

It is evident, for instance, that the matrices of the combinations shown in Fig. 1 are not equal to the matrices obtained by performing the prescribed operations on the component matrices. By the appropriate use of transformers, approaching sufficiently closely to ideal transformers, inserted each side of the component networks where necessary, networks could, of course, be constructed from the component fourpoles which would have matrices equal to the matrices obtained by combination of the component matrices in the way described. This is, however, not always a practical proposition and it is required to establish some criterion for determining when it is permissible to apply the methods described for obtaining the composite matrix of an assembly of fourpoles. Unfortunately, existing criteria are not infallible.

Fig. 2 shows the classical criteria to be applied to determine whether the combined matrix is represented by the sum of the appropriate matrices of the component fourpoles. These are due to O. Brune, *E.N.T.*, Vol. 9, No. 6, p. 234, 1932, and a proof of their validity is given by E. A. Guillemin in *Communication Networks*, p. 148.

In each case an e.m.f. is applied at terminals 1,1, and if the voltage V between terminals, which it is required to connect together to complete the combination, is zero, the combination is permissible. The outputs of the component fourpoles are shorted or open circuited as indicated. Applying the criterion in Fig. 2 (*a*) to the case of Fig. 1 (*a*) it will be seen that the criterion fails : it pronounces the combination to be permissible when actually it is forbidden. These criteria must therefore be supplemented by a certain amount of intelligent discrimination.

CHAPTER XXV

FILTERS

WHILE filter theory has many complex ramifications, the practical design of ordinary filters suitable for a great many purposes is extremely simple. The discussion below describes, in the earlier sections, easy methods of design and of predicting the performance of basic filter types. Refinements are presented in the later sections, which normally need only be referred to when occasion arises.

1. Descriptions and Definitions.

A *fourpole* (American : Transducer) is any network of reactances and resistances with two (input) terminals to which power is supplied, and two (output) terminals which supply power to an impedance load after transmission through the network.

Filters are combinations of reactances constituting fourpoles which have the property of passing certain bands of frequencies and rejecting other bands of frequencies. In other words, filters present zero attenuation to certain bands of frequencies, the pass bands, and finite attenuation, of amount varying with frequency, to other bands of frequencies. The cut-off frequencies of a filter are the frequencies which define the points of demarcation between the pass bands and attenuated or rejected bands.

A *low-pass filter* is a filter passing all frequencies from zero up to its cut-off frequency and attenuating all frequencies above its cut-off frequency.

A *high-pass filter* is a filter passing all frequencies from infinity down to its cut-off frequency and attenuating all frequencies below its cut-off frequency.

A *band-pass filter* is a filter passing all frequencies between two cut-off frequencies f_1 and f_2 and attenuating all frequencies below f_1 and above f_2. In general, a band-pass filter has more than one pass band, but most practical band-pass filters have only one pass band, and it is simplest to define a band-pass filter in terms of the case which is most useful in practice.

A *band-rejection filter* is a filter rejecting or attenuating all frequencies between two cut-off frequencies f_1 and f_2 and passing all frequencies below f_1 and above f_2. In general, a band-rejection filter has more than one rejection band, but most practical band-rejection filters have only one rejection band, and the definition has therefore been given again in terms of the practical case.

A number of structures having band-pass characteristics are described in VII:14, while a band-rejection filter is described in XVIII:5.22.14.

Within limits, any structure of loss-free reactances can have such values assigned to its elements that, when terminated by appropriate impedances, it will pass certain bands of frequencies without attenuation and will attenuate other bands.

Inside the pass bands the image impedances of a filter structure are pure resistances, of magnitude varying with frequency. Outside the pass bands, that is in the attenuating bands, the image impedances are pure reactances of magnitude varying with frequency. At the cut-off frequencies the image impedances are either zero or infinity. It should be noticed that zero frequency is not a cut-off frequency.

In low-pass filters the design is determined in terms of the image impedances at zero frequency. The design of filters to pass a single band of frequencies is determined in terms of the image impedances at the geometric mid-band frequency. If f_1 and f_2 are the cut-off frequencies of a pass band, then the geometric mid-band frequency is $f_m = \sqrt{f_1 f_2}$. The design of filters to reject a single band of frequencies is determined in terms of the image impedances at zero and at infinite frequency, which are equal to one another. The design of high-pass filters is determined in terms of the image impedances at infinite frequency.

When a filter is terminated in its image impedances at any frequency inside its pass band, it has no loss at that frequency; when terminated in any other value of resistance it has loss. When terminated at each end in a physical resistance equal to its geometric mid-band image impedance a filter has no loss at its geometric mid-band frequency; it has loss at all other frequencies inside the pass band which rises to a maximum (usually 1 or 2 db.) at the cut-off frequencies.

In the case of filters with one cut-off frequency, or in the case of filters with image impedances, both of which rise to infinity, or both of which fall to zero at the cut-off frequencies, the transition loss at the ends of the pass band can be reduced at the expense of an increase in the middle of the band, by terminating the filters in resistances which are greater or less than the mid-band image impedance: greater for impedances which rise towards cut-off, and less for impedances which fall towards cut-off. It is at this point that art and experience have to step in and aid science. For most purposes, however, entirely satisfactory results can be obtained by terminating such structures in impedances equal to their mid-band image impedances.

In the discussion which follows the image impedances will be called Z_1 and Z_2 in accordance with the list of conventions below. The image impedances at the geometric mid-band frequency will be termed R_1 and R_2. The ratio between R_1 and R_2 will be termed the impedance ratio of the structure. In most cases $Z_1 = Z_2$ and $R_1 = R_2$, so that the impedance ratio of the structure is unity.

Conventions

Z_1 = Input image impedance of filter structure.
Z_2 = Output image impedance of filter structure.
$Z_I = Z_1 = Z_2$ when $Z_1 = Z_2$ = image impedance of symmetric filter structure.
R_1 = Input geometric mid-band image impedance.
R_2 = Output geometric mid-band image impedance.
$R = R_1 = R_2$ when $R_1 = R_2$ = mid-band image impedance of symmetric filter structure.
f_c = Cut-off frequency of low-pass or high-pass filter.
f_1 = Lower cut-off frequency of band pass or band-rejection filter.
f_2 = Upper cut-off frequency of band-pass or band-rejection filter.
f_∞ = frequency of maximum attenuation in m-derived filters.
$a = \frac{f_\infty}{f_c}$ in m-derived low-pass filters
$= \frac{f_c}{f_\infty}$ in m-derived high-pass filters.
Z_a = reactance of full series arm in ladder-type filter.
Z_b = reactance of full shunt arm in ladder-type filter.
Z_{sh} = mid-shunt image impedance.
Z_{se} = mid-series image impedance.

2. Ladder-Type Filters.

The most common filters in general use are constituted by a half-section or one or more sections of the infinite structure shown in Fig. 1 (*a*).

Mid-series Terminated Section. This is shown in Fig. 1 (*b*). It may be regarded as being obtained by cutting out the section of the infinite ladder structure at (*a*) in the way indicated by the section lines at S_1 and S_2. It consists of two series arms each equal to half a full series arm, and one shunt arm equal to a full shunt arm of the general ladder structure.

Mid-shunt Terminated Section. This is shown in Fig. 1 (*c*). It may be regarded as being obtained by cutting out a section of the infinite ladder structure by section lines drawn through the shunt arms. It consists of two shunt arms each having twice the reactance of a normal full shunt arm, and a series arm between them equal to a full series arm of the ladder structure.

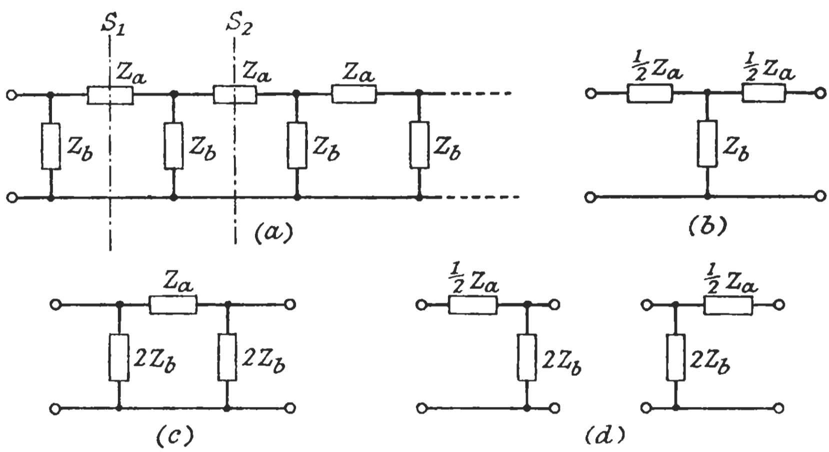


FIG. 1/XXV:2.—(*a*) Ladder Filter. (*b*) Mid-Series Terminated Section. (*c*) Mid-Shunt Terminated Section. (*d*) Half Section.

Half-section. In Fig. 1 (*d*) are shown two half-sections each consisting of one series arm of reactance equal to half a full series arm, and one shunt arm of reactance equal to twice the reactance of a full shunt arm.

The factors which determine whether a mid-shunt or a mid-series termination is to be used are :

1. The comparative ease of construction of the series and shunt arms.
2. The behaviour of the filter impedance outside the pass band. Since the mid-series and mid-shunt image impedances of prototype filters are mutually inverse impedances (inverted about the mid-band image impedance $R = R_1 = R_2$; i.e. $Z_{sh}Z_{se} = R^2$) they vary with frequency in opposite ways. For instance, in the case of a prototype band-pass filter the mid-shunt image impedances vary from infinity at the cut-

off frequencies f_1 and f_2 to zero at zero frequency and infinity respectively, while the mid-series image impedances vary from zero at the cut-off frequencies f_1 and f_2 to infinity at zero frequency and infinity respectively.

3. The behaviour of the image impedances inside the pass band. For instance, in the case of a prototype band-pass filter the mid-shunt image impedance varies from R at the geometric mid-band frequency to infinity at the cut-off frequencies f_1 and f_2, while the mid-series image impedance varies from R at the geometric mid-band frequency to zero at the cut-off frequencies f_1 and f_2.

Usually for prototype low-pass filters mid-shunt termination is used. For prototype high-pass filters the mid-series termination is generally used. This choice is guided largely by the cheapness of condensers as compared with inductances, but sometimes, in the case of low-pass filters, by the requirement that the filter must present a low impedance at frequencies above cut-off. If a high-pass filter is to present a low impedance at frequencies below cut-off it must, of course, be mid-shunt terminated.

3. Design of Low-Pass Filters.

There are three types of ladder-type low-pass filters : the prototype or constant k type, the series derived m type and the shunt derived m type.

Prototype Filter. The prototype contains inductances only in its series arms and condensers only in its shunt arms. The values of the inductances and condensers in the full arms are given in Fig. 1, in terms of R, the image impedance at zero frequency and the cut-off frequency f_c. Normally R is made equal to the impedance terminating the filter, while f_c is evidently determined by the band of frequencies it is required to pass.

A mid-shunt terminated section consists of a π section with an inductance equal to the full series inductance ($L = R/\pi f_c$) in the series arm, and condensers equal to half the full shunt arm condenser in the shunt arms (i.e. equal to $1/2\pi f_c R$). Any number of such sections may be connected in tandem. In this case the two condensers at the junction points of each section may of course be replaced by one condenser equal to a full shunt arm.

A mid-series terminated section consists of a T section with a condenser equal to the full shunt condenser ($C = 1/\pi f_c R$) in the shunt arm, and inductances equal to half the full series arm inductance in the series arms (i.e. equal to $R/2\pi f_c$). Any number of such sections may be connected in tandem. In this case the two inductances at the junction points of each section should be constituted by a single inductance equal to the full series arm.

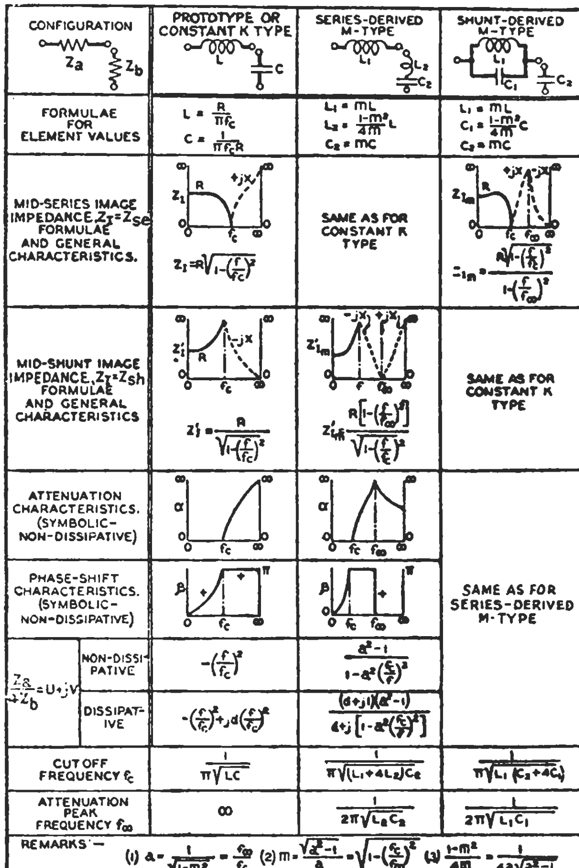


Fig. 1/XXV:3.—Characteristics and Design Formulae for Low-Pass Filters.
(By courtesy of Messrs. D. Van Nostrand & Co. Inc., New York, U.S.A.)

Fig. 1 shows :

1. The configuration of the full series and shunt arms.
2. The values of the elements constituting the full series and full shunt arms.
3. The formula for the mid-series image impedance and the way in which this impedance varies with frequency, both inside and outside the pass range. Inside the pass range it is seen

to be a resistance, as already indicated, and outside the pass range it is a positive reactance.

4. The formula for and the form of the mid-shunt image impedance.
5. The form of the attenuation-frequency characteristic of the filter when terminated in its image impedances. This is not to scale but is very useful in seeing whether the attenuation rises continuously with departure from cut-off, or whether it rises to a maximum and then falls away, etc.
6. The form of the phase shift-frequency characteristic of the filter when terminated in its image impedances. This again is not to scale, but is useful in determining the sign of the phase shift and the total variation over the pass range.
7. The ratio of the series arm impedance to four times the shunt arm impedance at any frequency f: expressed in terms of the cut-off frequency and the frequency f. This is a useful ratio for determining the insertion loss of a filter using the technique described by Shea ; in the discussion below, use is made of a quantity equal to the simple ratio : series arm impedance divided by shunt arm impedance.
8. The cut-off frequency expressed in terms of the values of the elements of the full arms.
9. The frequency of (theoretically) infinite attenuation.

m-Type Filters. Fig. 1 shows the same information for the two m-type low-pass filters.

The values of the elements of the m-type filters are specified in terms of the values of the elements of the prototype filter. It will be seen that the value of $L_1 = mL = mR/\pi f_c$, while the values of the other elements are obtained by multiplying the appropriate elements of the prototype filter by the appropriate fraction, which, it will be noticed, always contains the quantity m.

The value of m. In low pass filters the value of m is given by

$$m = \sqrt{1 - \frac{1}{a^2}} \text{ where } a = \frac{f_\infty}{f_c}$$

and f_∞ = the frequency of infinite attenuation.

The normal use of m-type sections is in conjunction with prototype sections, in order to obtain a more rapid rise of attenuation than is possible with prototype sections alone. For this purpose the frequency of infinite attenuation is chosen with reference to Fig. 2, having regard to the required attenuation at all frequencies above cut-off, and the attenuation introduced by other sections being used. The cut-off frequency of the *m*-type sections is made the same as that of the prototype low-pass filter with which they are associated. A good combination of sections is represented by values of $a = f_\infty/f_c$, equal, respectively, to 1·1, 1·25, and 2·5. Such a combination of sections with one prototype section will result in an attenuation frequency characteristic which rises to infinity at a frequency 10% higher than cut-off and never falls below about 50 db. at higher frequencies.

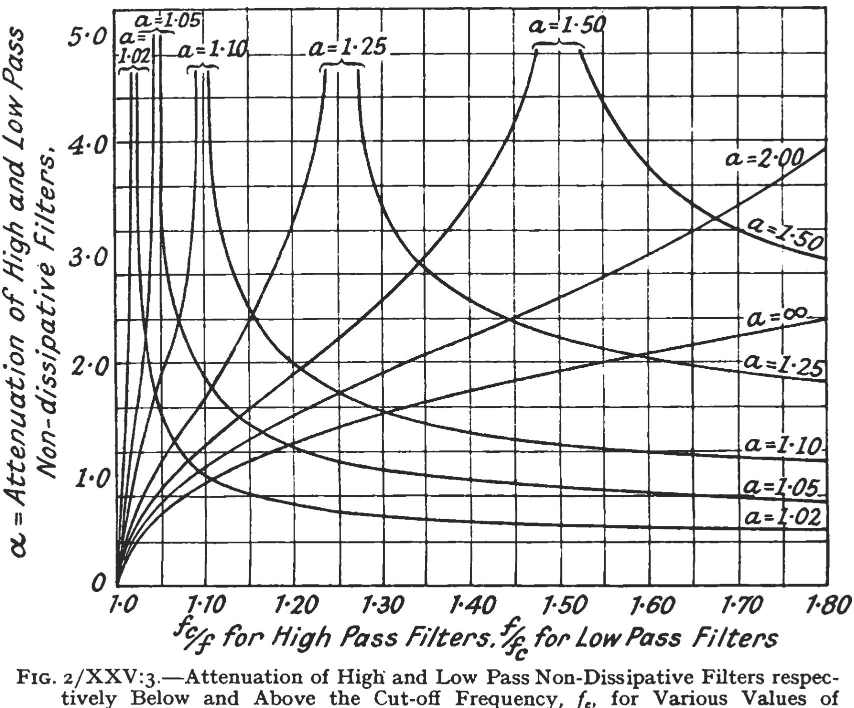


FIG. 2/XXV:3.—Attenuation of High and Low Pass Non-Dissipative Filters respectively Below and Above the Cut-off Frequency, f_c, for Various Values of $a = f_\infty/f_c$.

(By courtesy of Messrs. D. Van Nostrand & Co. Inc., New York, U.S.A.)

Fig. 2 shows the attenuation frequency characteristics per single section of *m*-type filter : the curve marked $a = \infty$ being the curve for the prototype filter. The vertical scale is attenuation in Nepers and can be converted to decibels by the relation 1 Neper = 8·686 decibels. The horizontal scale is frequency, f in terms of f/f_c.

When filter sections are properly connected together in tandem, see below, the total attenuation ratio is the sum of the attenuations read from Fig. 2. Evidently if a half-section is used or more than one section of the same kind the attenuation on Fig. 2 must be multiplied by the appropriate factor.

3.1. Rules for connecting *m*-Type and Prototype Sections in Tandem. Adjacent image impedances must be equal, and all sections must be designed with the same cut-off frequencies.

The mid-series image impedance of a series derived *m*-type section is the same as the mid-series image impedance of a prototype section : these impedances may therefore be adjacent.

The mid-shunt image impedance of a shunt-derived *m*-type section is the same as the mid-shunt image impedance of a prototype section : these impedances may therefore be adjacent.

Series-derived *m*-type sections may be connected together at mid-series termination only, and shunt-derived at mid-shunt, unless the value of *a* is the same for both adjacent sections.

Except when terminating a filter a series-derived *m*-type section must always be terminated at mid-series, while a shunt-derived section must always be terminated at mid-shunt.

3.2. Use of *m*-Type Sections to Provide a Filter Terminal Impedance which is more constant (with Frequency) than a Prototype Section. Fig. 3 shows the ratio of Z_{sh}/R for series-derived *m*-type sections where Z_{sh} is the mid-shunt image impedance. The same curves define the ratio R/Z_{se} for shunt-derived *m*-type sections, where Z_{se} is the mid-series image impedance. The scale of frequency is in terms of f/f_c, and R is the geometric mid-band image impedance.

Each curve corresponds to a different value of $a = f_\infty/f_c$. It will be seen that values of a between 1·1 and 1·25 provide image impedances which vary less over the pass band than do other values of a. The curve corresponding to $a = \infty$, which is the curve of the image impedance of the prototype filter, falls away very rapidly towards cut-off, so that a very large advantage is to be obtained by using an *m*-type section to terminate a filter.

3.3. Example of Composite Filter. Fig. 4 shows the way in which a composite filter would be built up consisting of one section for each of the values of a : 1·1, 1·25, 1·5 and infinity. The infinity section is of course a prototype section.

It will be noted that the *m*-type sections are all shunt-derived. The reason for this is that it is a sound principle always to choose a circuit configuration with condensers across the coils : if this is

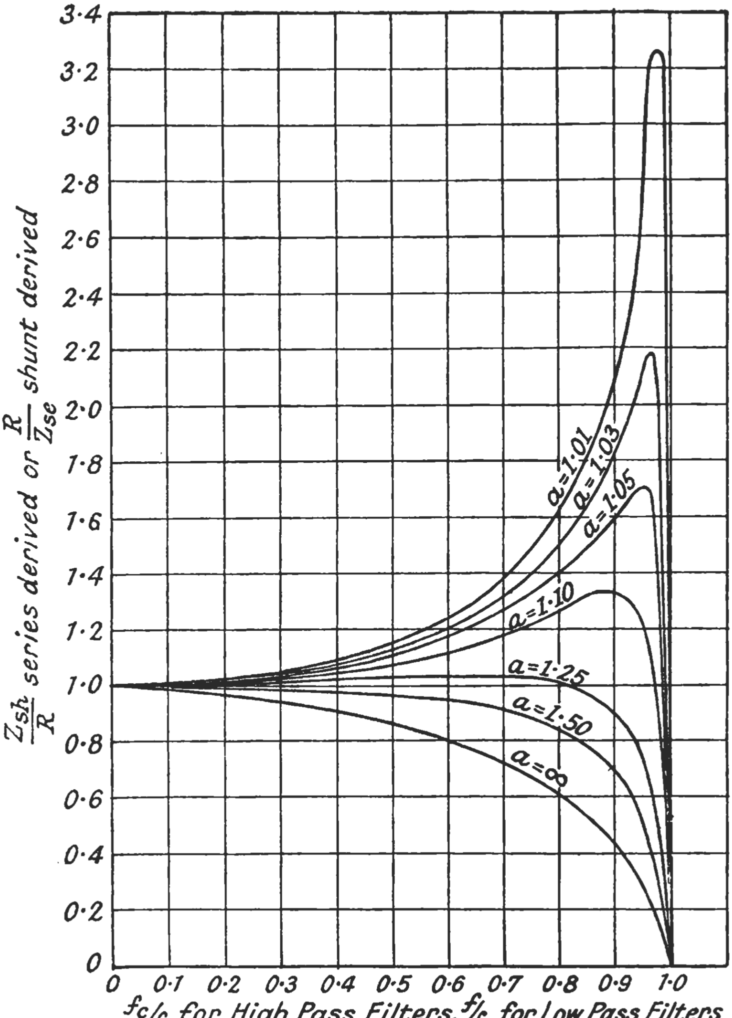


FIG. 3/XXV:3.—Variation of the Mid-Series and Mid-Shunt Image Impedance in the Transmission Band, for Various Values of *a*.

(By courtesy of Messrs. D. Van Nostrand & Co. Inc., New York, U.S.A.)

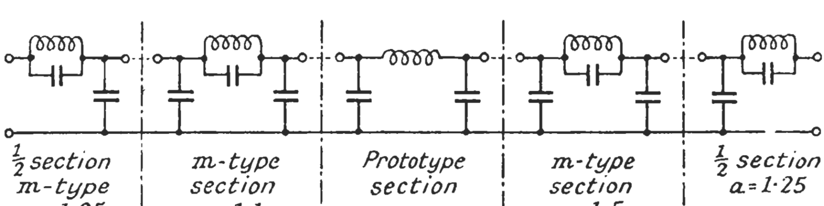


FIG. 4/XXV:3.—Analysis of Low-Pass Filter consisting of One Section each with a = 1·1, 1·25, 1·5 and ∞.

done, any self-capacity across the coils can be compensated for by reducing the value of the parallel condenser.

It should be noted also that the image impedances of the composite filter are those of mid-series terminated *m*-type shunt-derived sections with $a = 1{\cdot}25$. Reference to Fig. 3 shows that the impedance does not depart from R by more than 20%, up to 93% of the cut-off frequency. This constitutes a large improvement on the prototype filter. Where matching is required up to a higher percentage of the cut-off frequency, end sections with a rather less than 1·25 may be used. This will improve the match near the cut-off frequency at the expense of frequencies farther away.

It must not be thought, because a comparatively complicated filter has been chosen as an example, that simpler types of filter are useless. The case is very much the reverse, and it is comparatively seldom that such a complicated filter is justified. Many filters consist of prototype sections only.

4. Design of High-Pass Filters.

As for low-pass filters, there are three types of high-pass filter : the prototype or constant k type, the series derived m type and the shunt derived m type.

Fig. 1 shows for high-pass filters the same information that Fig. 1/XXV:3 shows for low-pass filters.

Prototype Filter. The prototype contains condensers only in its series arms and inductances only in its shunt arms. The values of the condensers and inductances in the full arms are given in Fig. 1 in terms of R, the image impedance at infinite frequency and the cut-off frequency f_c.

Mid-shunt and mid-series sections and half-sections are built up in an analogous way to that described with reference to low-pass filters. Any number of sections may be connected in tandem.

The Value of *m*. In high-pass filters the value of m is given by :

$$m = \sqrt{1 - \frac{1}{a^2}}$$

where
$$a = \frac{f_c}{f_\infty}.$$

m-Type sections can be combined with prototype sections according to the laws given for low-pass filters. Fig. 2/XXV:3 shows also the attenuation characteristics of high-pass filter sections : the scale of frequency is now in terms of f_c/f.

Fig. 3/XXV:3, using the scale of frequency f_c/f, shows the

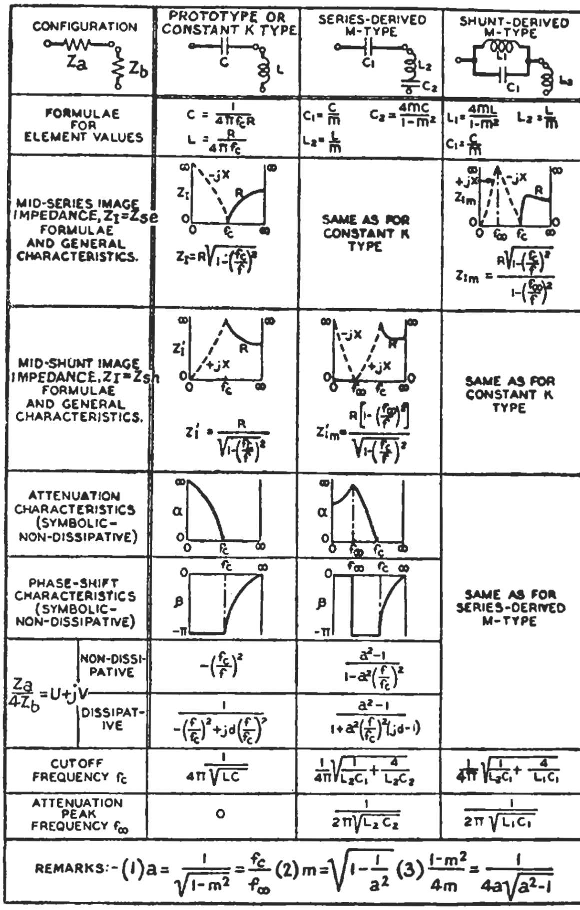


FIG. 1/XXV:4.—Characteristics and Design Formulae for High-Pass Filters.
(By courtesy of Messrs. D. Van Nostrand & Co. Inc., New York, U.S.A.)

variation of the image impedance with frequency, as in the case of low-pass sections. The curve marked $a = \infty$ is again the curve for the prototype section.

A composite high-pass filter is built up, using the same rules that apply to low-pass filters.

5. Design of Band-Pass Filters.

The variety of band-pass filters is greater than that of low-pass and high-pass filters. There are first of all the band-pass prototype and *m*-type filters shown in Fig. 1/XXV:5, and in addition a number

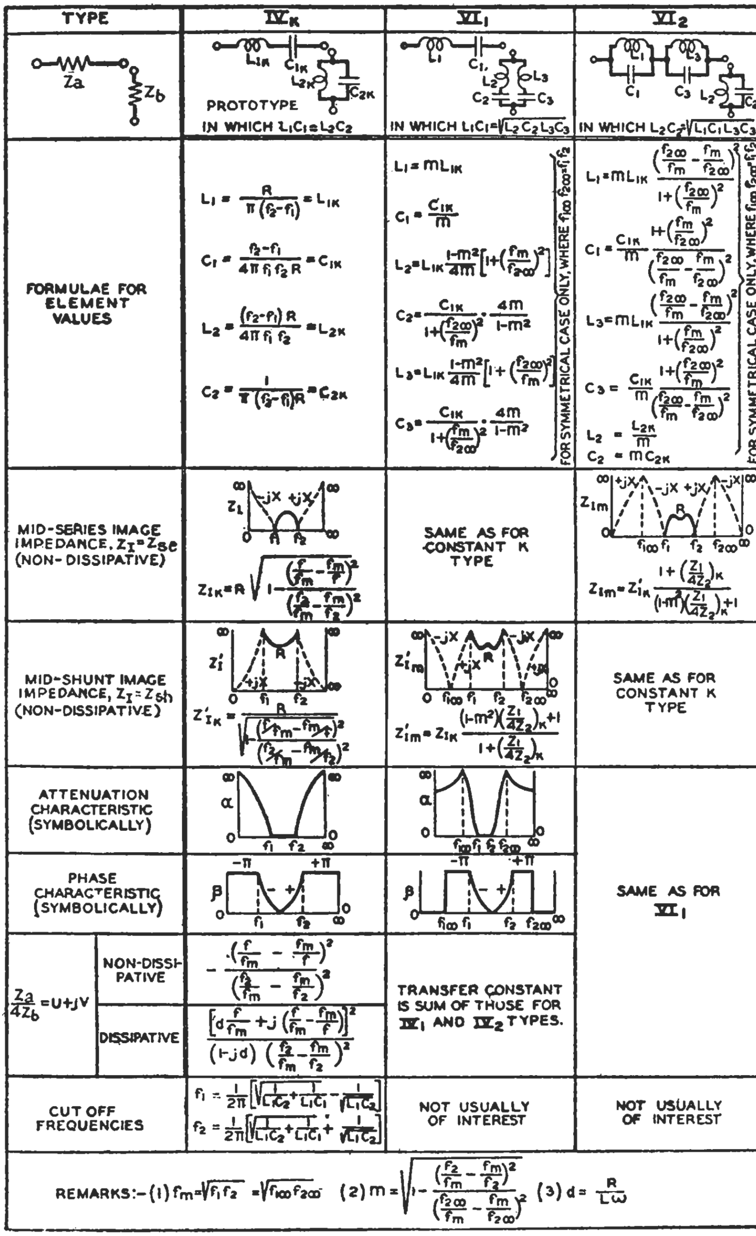


FIG. 1/XXV:5.—Characteristics and Design Formulae for Band-Pass Filters:
(By courtesy of Messrs. D. Van Nostrand & Co. Inc., New York, U.S.A.)

of other types which may be called Dissymmetric-Loss Sections because the attenuation frequency characteristic, when plotted against a logarithmic scale of frequency, is unsymmetrical about the geometric mid-band frequency f_m; the loss of those in Fig. 1 is symmetrical.

Filters are classified by the number of elements contained in one series arm plus one shunt arm and by an arbitrary order. Four-element filters are labelled IV_1, IV_2, IV_3, etc.; five-element filters are labelled V_1, V_2, V_3, etc. Prototype filters are given the suffix *k*.

In the design of band-pass filters, apart from the extra complication introduced by the rather more complex formulae defining the values of the elements, no problems are met which have not been encountered already.

Figs. 1, 2, 3 and 4 show for band-pass filters the same information that Fig. 1/XXV:3 gives for low-pass filters. (Remember to derive the mid-series and mid-shunt terminations correctly. This is done exactly as before: a mid-shunt terminated section is a π network with a full series arm and two shunt arms each twice the impedance of the full shunt arm: a mid-series terminated section is a T network with a full shunt arm and two series arms each half the impedance of a full series arm.)

Prototype and *m*-Type Filters. The three filters on Fig. 1 are respectively: IV_k, the prototype band-pass filter; VI_1, the series derived *m*-type band-pass filter; VI_2, the shunt derived *m*-type band-pass filter. These may be combined according to the rules laid down in XXV:3.1 and 3.2.

As a very rough guide to the forms of the attenuation and impedance characteristics of *narrow* band filters, Figs. 2 and 3/XXV:3 may be used with changed frequency scales as follows:

For frequencies above f_m the numbers on the frequency scales define the ratio $\frac{f - f_m}{f_2 - f_m}$.

For frequencies below f_m the numbers on the frequency scales define the ratio $\frac{f_m - f}{f_m - f_1}$.

Value of *m*. The value of *m* for band-pass *m*-type filters is given at the bottom of Fig. 1.

It will be seen that all the remaining filters, which are shown on Figs. 2, 3 and 4, have the property that either the mid-shunt image impedance, or else the mid-series image impedance is the same as that of the prototype (IV_k) filter. This means that a section or a half-section of any of these types may be joined to a prototype

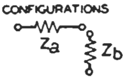
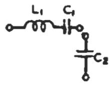
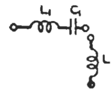
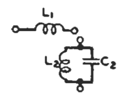
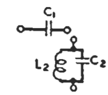
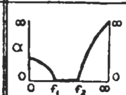
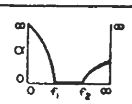
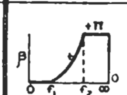
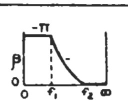
TYPE		III_1	III_2	III_3	III_4
CONFIGURATIONS 					
ELEMENT VALUES		$L_1 = \frac{R}{\pi(f_2 - f_1)}$ $C_1 = \frac{f_2 - f_1}{4\pi f_1 f_2 \cdot R}$ $C_2 = \frac{1}{\pi(f_1 + f_2)R}$	$L_1 = \frac{f_1 R}{\pi f_2 (f_2 - f_1)}$ $C_1 = \frac{f_2 - f_1}{4\pi f_1 f_2 R}$ $L_2 = \frac{(f_1 + f_2) R}{4\pi f_1 f_2}$	$L_1 = \frac{R}{\pi(f_1 + f_2)}$ $L_2 = \frac{(f_2 - f_1) R}{4\pi f_1^2}$ $C_2 = \frac{1}{\pi(f_2 - f_1) R}$	$C_1 = \frac{(f_1 + f_2)}{4\pi f_1 f_2 R}$ $L_2 = \frac{(f_2 - f_1) R}{4\pi f_1 f_2}$ $C_2 = \frac{f_1}{\pi f_2 (f_2 - f_1) R}$
MID - SERIES IMAGE IMPEDANCE $Z_I = Z_{se}$		SAME AS FOR CONSTANT K (SEE IV_K)	SAME AS FOR CONSTANT K (SEE IV_K)	NOT USUALLY OF INTEREST	NOT USUALLY OF INTEREST
MID - SHUNT IMAGE IMPEDANCE $Z_I = Z_{sh}$		NOT USUALLY OF INTEREST	NOT USUALLY OF INTEREST	SAME AS FOR CONSTANT K (SEE IV_K)	SAME AS FOR CONSTANT K (SEE IV_K)
ATTENUATION CHARACTERISTIC (SYMBOLICALLY)				SAME AS FOR III_1	SAME AS FOR III_2
PHASE SHIFT CHARACTERISTIC (SYMBOLICALLY)					
$\frac{Z_a}{4Z_b} = U + jV$	NON-DISSIPATIVE	$\dfrac{\frac{f_1}{f_2} - \left(\frac{f}{f_m}\right)^2}{\frac{f_2}{f_1} - \frac{f_1}{f_2}}$	$\dfrac{-\left(\frac{f}{f_m}\right)^2 \frac{f_1}{f_2} - 1}{\left(\frac{f}{f_m}\right)^2 \left[\frac{f_2}{f_1} - \frac{f_1}{f_2}\right]}$	SAME AS FOR III_1	SAME AS FOR III_2
	DISSIPATIVE	$\dfrac{\frac{f_1}{f_2} - \left(\frac{f}{f_m}\right)^2 + jd\left(\frac{f}{f_m}\right)^2}{\frac{f_2}{f_1} - \frac{f_1}{f_2}}$	$\dfrac{d + j\left[1 - \left(\frac{f_m}{f}\right)^2 \left(\frac{f_2}{f_1}\right)\right]}{\left[d + j1\right]\left[\left(\frac{f_2}{f_1}\right)^2 - 1\right]}$		
UPPER CUTOFF FREQUENCY		$f_2 = \frac{1}{2\pi}\sqrt{\frac{C_2 + 4C_1}{L_1 C_1 C_2}}$	$f_2 = \frac{1}{2\pi\sqrt{L_1 C_1}}$	$f_2 = \frac{1}{2\pi}\sqrt{\frac{L_1 + 4L_2}{L_1 L_2 C_2}}$	$f_2 = \frac{1}{2\pi\sqrt{L_2 C_2}}$
LOWER CUTOFF FREQUENCY		$f_1 = \frac{1}{2\pi\sqrt{L_1 C_1}}$	$f_1 = \frac{1}{2\pi\sqrt{C_1(L_1 + 4L_2)}}$	$f_1 = \frac{1}{2\pi\sqrt{L_2 C_2}}$	$f_1 = \frac{1}{2\pi\sqrt{L_2(C_2 + 4C_1)}}$
REMARKS		(1) $f_m = \sqrt{f_1 f_2}$	(2) $d = \frac{R}{L\omega}$		

FIG. 2/XXV:5.—Characteristics and Design Formulae for Band-Pass Filters.

Note : (1) The values of *L* and *C* given for III_1 and III_2 hold only for Mid-Series Terminations and the Values of *L* and *C* given for III_3 and III_4 hold only for Mid-Shunt Terminations ; (2) the Image Impedance Characteristics of any of these Filter Sections may be obtained from " Mutual Inductance in Wave Filters with an Introduction on Filter Design ", by K. S. Johnson and T. E. Shea, pages 9–12, *B.S.T.J.*, January 1925.

(By courtesy of Messrs. D. Van Nostrand & Co. Inc., New York, U.S.A.)

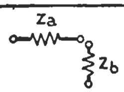
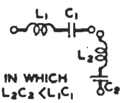
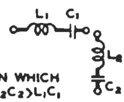
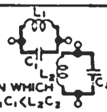
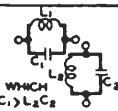
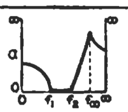
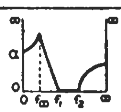
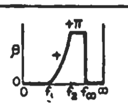
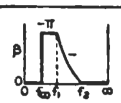
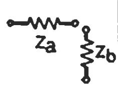
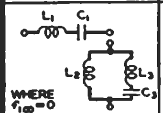
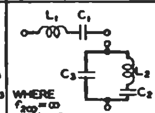
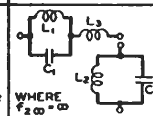
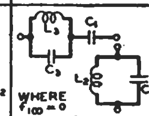
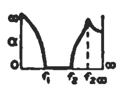
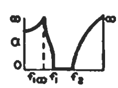
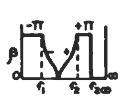
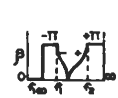
TYPE		IV_1	IV_2	IV_3	IV_4
		 IN WHICH $L_2C_2 < L_1C_1$	 IN WHICH $L_2C_2 > L_1C_1$	 IN WHICH $L_1C_1 < L_2C_2$	 IN WHICH $L_1C_1 > L_2C_2$
FORMULAE FOR ELEMENT VALUES		$L_1 = m_1 L_{1K}$ $C_1 = \frac{C_{1K}}{m_2}$ $L_2 = \frac{(1-m_1^2)}{4m_1} L_{1K}$ $C_2 = \frac{4m_2}{1-m_2^2} C_{1K}$ WHERE $m_2 = \frac{f_1}{f_2} m_1$ $m_1 = \sqrt{\frac{1-\left(\frac{f_2}{f_{2\infty}}\right)^2}{1-\left(\frac{f_1}{f_{2\infty}}\right)^2}}$	SAME AS FOR IV_1 EXCEPT: $m_1 = \frac{f_1}{f_2} m_2$ $m_2 = \sqrt{\frac{1-\left(\frac{f_{1\infty}}{f_1}\right)^2}{1-\left(\frac{f_{1\infty}}{f_2}\right)^2}}$	$L_1 = \frac{4m_2}{1-m_2^2} L_{2K}$ $C_1 = \frac{1-m_1^2}{4m_1} C_{2K}$ $L_2 = \frac{L_{2K}}{m_2}$ $C_2 = m_1 C_{2K}$ FOR VALUES OF m_1 AND m_2 SEE IV_1	SAME AS FOR IV_3 FOR VALUES OF m_1 AND m_2 SEE IV_2
MID-SERIES IMAGE IMPEDANCE $Z_I = Z_{se}$		SAME AS FOR CONSTANT K (SEE IV_K)	SAME AS FOR CONSTANT K (SEE IV_K)	NOT USUALLY OF INTEREST	NOT USUALLY OF INTEREST
MID-SHUNT IMAGE IMPEDANCE $Z_I = Z_{sh}$		NOT USUALLY OF INTEREST	NOT USUALLY OF INTEREST	SAME AS FOR CONSTANT K SEE IV_K	SAME AS FOR CONSTANT K SEE IV_K
ATTENUATION CHARACTERISTIC (SYMBOLICALLY)				SAME AS FOR IV_1	SAME AS FOR IV_2
PHASE CHARACTERISTIC (SYMBOLICALLY)					
$\frac{Z_a}{4Z_b} = U + jV$	NON-DISSIPATIVE	$\frac{\left[1-\left(\frac{f_\infty}{f_2}\right)^2\right]\left[1-\left(\frac{f_1}{f}\right)^2\right]}{\left[1-\left(\frac{f_1}{f_2}\right)^2\right]\left[\left(\frac{f_\infty}{f}\right)^2-1\right]}$	SAME AS FOR IV_4	SAME AS FOR IV_1	$\frac{\left[1-\left(\frac{f_\infty}{f_1}\right)^2\right]\left[1-\left(\frac{f_2}{f}\right)^2\right]}{\left[1-\left(\frac{f_2}{f_1}\right)^2\right]\left[\left(\frac{f_\infty}{f}\right)^2-1\right]}$
	DISSIPATIVE	$\frac{\left(\frac{f_1}{f_\infty}\right)^2\left[\left(\frac{f_\infty}{f_2}\right)^2-1\right]\left[d\left(\frac{f}{f_1}\right)^2+j\left\{\left(\frac{f}{f_1}\right)^2-1\right\}\right]}{\left[1-\left(\frac{f_1}{f_2}\right)^2\right]\left[d\left(\frac{f}{f_\infty}\right)^2+j\left\{\left(\frac{f}{f_\infty}\right)^2-1\right\}\right]}$			$\frac{\left[\left(\frac{f_1}{f_\infty}\right)^2-1\right]\left[d\left(\frac{f}{f_2}\right)^2+j\left\{\left(\frac{f}{f_2}\right)^2-1\right\}\right]}{\left[1-\left(\frac{f_1}{f_2}\right)^2\right]\left[d\left(\frac{f}{f_\infty}\right)^2+j\left\{\left(\frac{f}{f_\infty}\right)^2-1\right\}\right]}$
LOWER CUTOFF FREQUENCY		$f_1 = \frac{1}{2\pi\sqrt{L_1C_1}}$	$f_1 = \frac{1}{2\pi}\sqrt{\frac{C_2+4C_1}{C_1C_2(L_1+4L_2)}}$	$f_1 = \frac{1}{2\pi\sqrt{L_2C_2}}$	$f_1 = \frac{1}{2\pi}\sqrt{\frac{L_1+4L_2}{L_1L_2(C_2+4C_1)}}$
UPPER CUTOFF FREQUENCY		$f_2 = \frac{1}{2\pi}\sqrt{\frac{C_2+4C_1}{C_1C_2(L_1+4L_2)}}$	$f_2 = \frac{1}{2\pi\sqrt{L_1C_1}}$	$f_2 = \frac{1}{2\pi}\sqrt{\frac{L_1+4L_2}{L_1L_2(C_2+4C_1)}}$	$f_2 = \frac{1}{2\pi\sqrt{L_2C_2}}$

FIG. 3/XXV:5.—Characteristics and Design Formulae for Band-Pass Filters.
(By courtesy of Messrs. D. Van Nostrand & Co. Inc., New York, U.S.A.)

TYPE	V_1	V_2	V_3	V_4
				
FORMULAE FOR ELEMENT VALUES	$L_1=m_1L_{1K}$ $C_1=C_{1K}$ $L_2=aL_{1K}$ $C_3=\frac{h}{a}C_{1K}$ $L_3=\frac{(1-m_1^2)}{4h}L_{1K}$	$L_1=L_{1K}$ $C_1=\frac{C_{1K}}{m_2}$ $L_2=\frac{d}{g}L_{1K}$ $C_2=\frac{4g}{1-m_2^2}C_{1K}$ $C_3=\frac{C_{1K}}{d}$	$L_1=\frac{4g}{1-m_2^2}L_{2K}$ $C_1=\frac{d}{g}C_{2K}$ $L_3=\frac{L_{2K}}{d}$ $L_2=\frac{L_{2K}}{m_2}$ $C_2=C_{2K}$	$C_1=aC_{2K}$ $L_3=\frac{h}{a}L_{2K}$ $C_3=\frac{(1-m_1^2)}{4h}C_{2K}$ $L_2=L_{2K}$ $C_2=m_1C_{2K}$
MID-SERIES IMAGE IMPEDANCE $Z_I=Z_{se}$	SAME AS FOR CONSTANT K (SEE IV_K)	SAME AS FOR CONSTANT K (SEE IV_K)	NOT USUALLY OF INTEREST	NOT USUALLY OF INTEREST
MID-SHUNT IMAGE IMPEDANCE $Z_I=Z_{sh}$	NOT USUALLY OF INTEREST	NOT USUALLY OF INTEREST	SAME AS FOR CONSTANT K SEE IV_K	SAME AS FOR CONSTANT K SEE IV_K
ATTENUATION CHARACTERISTIC (SYMBOLICALLY)			SAME AS FOR V_2	SAME AS FOR V_1
PHASE CHARACTERISTIC (SYMBOLICALLY)				
ATTENUATION AND PHASE CONSTANT	SAME AS FOR III_2+IV_1	SAME AS FOR III_1+IV_2	SAME AS FOR III_1+IV_2	SAME AS FOR III_2+IV_1
CUT OFF FREQUENCIES	NOT USUALLY OF INTEREST	NOT USUALLY OF INTEREST	NOT USUALLY OF INTEREST	NOT USUALLY OF INTEREST

REMARKS

(1) $h=\sqrt{\left(1-\frac{f_1^2}{f_{2\infty}^2}\right)\left(1-\frac{f_2^2}{f_{2\infty}^2}\right)}$

(2) $m_1=\frac{f_1f_2}{f_{2\infty}^2}+h$

(3) $a=\frac{(1-m_1^2)f_{2\infty}^2}{4f_1f_2}$

(4) $g=\sqrt{\left(1-\frac{f_{1\infty}^2}{f_1^2}\right)\left(1-\frac{f_{1\infty}^2}{f_2^2}\right)}$

(5) $m_2=g+\frac{f_{1\infty}^2}{f_1f_2}$

(6) $d=\frac{(1-m_2^2)f_1f_2}{4f_{1\infty}^2}$

(7) SEE IV_K FOR L_{1K} ETC.

FIG. 4/XXV:5.—Characteristics and Design Formulae for Band-Pass Filters.
(By courtesy of Messrs. D. Van Nostrand & Co. Inc., New York, U.S.A.)

filter at the appropriate termination : i.e. mid-series or mid-shunt. Also, of course, any filters which have the same mid-shunt image impedance may be joined together at mid-shunt (termination), while any filters which have the same mid-series image impedance may be joined together at mid-series.

Provided the design instructions given on Figs. 2, 3, and 4 are followed, little difficulty should be experienced in designing any of these sections, or in combining them to give any required overall attenuation characteristic. The following comments may, however, be of assistance.

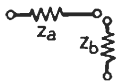
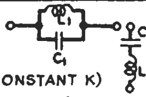
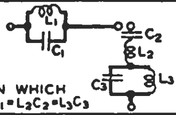
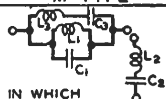
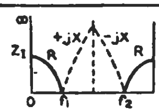
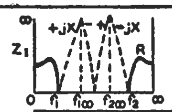
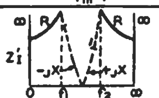
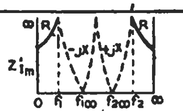
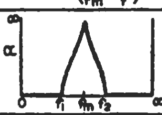
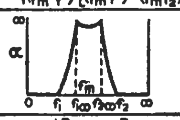
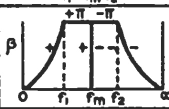
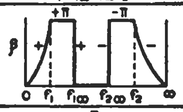
Type III Filter Sections. These are shown in Fig. 2. These sections are normally of little use because one attenuation band introduces only a small amount of attenuation. Their performance is therefore that of high-pass or low-pass filters with some attenuation at one end of the pass band. A prototype high-pass or low-pass filter is simpler and cheaper and usually gives a performance nearer the performance which is required. A use for one of these sections, the III_3, is sometimes found in wide-band amplifiers where the input capacity of a valve is made to constitute part of the filter. These filters have the advantage over low-pass filters used for this purpose in that the value of condenser in the shunt arm is larger for a given upper cut-off frequency than in the case of a low-pass filter. This means that it is possible to pass a higher band of frequencies with a given valve input capacity.

Type IV Filter Sections. These are shown in Fig. 3. These sections also suffer from the disadvantage that the attenuation on one side is low, being substantially the same as that on the low attenuation side of the type III filter sections. They are, however, of value for adding to a prototype section or sections to steepen the attenuation characteristic on one side of the pass band when this is required.

It should be noted that two values of m are used in calculating the values of the elements of these sections : m_1 and m_2 ; the values of these are given in Fig. 3 in the sections giving the formulae for the element values.

Type V Filter Sections. These are shown in Fig. 4. These sections give an attenuation characteristic similar to the prototype section on one side of the pass band and a steeper rise of attenuation on the other side where a frequency of infinite attenuation occurs.

It should be noted that six constants are used in determining the values of the elements : the values of these constants are given at the bottom of Fig. 4.

TYPE	PROTOTYPE	SERIES-DERIVED M-TYPE	SHUNT-DERIVED M-TYPE
	 (CONSTANT K) IN WHICH $L_1C_1 = L_2C_2$	 IN WHICH $L_1C_1 = L_2C_2 = L_3C_3$	 IN WHICH $L_1C_1 = L_2C_2 = L_3C_3$
FORMULAE FOR ELEMENT VALUES	$L_1 = \frac{(f_2-f_1)R}{\pi f_1 f_2} = L_{1K}$ $C_1 = \frac{1}{4\pi(f_2-f_1)R} = C_{1K}$ $L_2 = \frac{R}{4\pi(f_2-f_1)} = L_{2K}$ $C_2 = \frac{f_2-f_1}{\pi f_1 f_2 R} = C_{2K}$	$L_1 = mL_{1K}$ $C_1 = \frac{C_{1K}}{m}$ $L_2 = \frac{L_{2K}}{m}$ $C_2 = mC_{2K}$ $L_3 = \frac{1-m^2}{4m}L_{1K}$ $C_3 = \frac{4m}{1-m^2}C_{1K}$	$L_1 = mL_{1K}$ $C_1 = \frac{C_{1K}}{m}$ $L_2 = \frac{L_{2K}}{m}$ $C_2 = mC_{2K}$ $L_3 = \frac{4m}{1-m^2}L_{2K}$ $C_3 = \frac{1-m^2}{4m}C_{2K}$
MID-SERIES IMAGE IMPEDANCE $Z_I = Z_{se}$	 $Z_I = R\sqrt{1-\frac{\left(\frac{f_2}{f_m}-\frac{f_m}{f_2}\right)^2}{\left(\frac{f}{f_m}-\frac{f_m}{f}\right)^2}}$	SAME AS FOR CONSTANT K TYPE	 $Z_{Im} = \frac{R\sqrt{\left(\frac{f}{f_m}-\frac{f_m}{f}\right)^2\left[\left(\frac{f}{f_m}-\frac{f_m}{f}\right)^2-\left(\frac{f_2}{f_m}-\frac{f_m}{f_2}\right)^2\right]}}{\left(\frac{f}{f_m}-\frac{f_m}{f}\right)^2-\left(\frac{f_{2\infty}}{f_m}-\frac{f_m}{f_{2\infty}}\right)^2}$
MID-SHUNT IMAGE IMPEDANCE $Z_I = Z_{sh}$	 $Z'_I = \frac{R}{\sqrt{1-\frac{\left(\frac{f_2}{f_m}-\frac{f_m}{f_2}\right)^2}{\left(\frac{f}{f_m}-\frac{f_m}{f}\right)^2}}}$	 $Z'_{Im} = \frac{R\left[\left(\frac{f}{f_m}-\frac{f_m}{f}\right)^2-\left(\frac{f_{2\infty}}{f_m}-\frac{f_m}{f_{2\infty}}\right)^2\right]}{\sqrt{\left(\frac{f}{f_m}-\frac{f_m}{f}\right)^2\left[\left(\frac{f}{f_m}-\frac{f_m}{f}\right)^2-\left(\frac{f_2}{f_m}-\frac{f_m}{f_2}\right)^2\right]}}$	SAME AS FOR CONSTANT K TYPE
ATTENUATION CHARACTERISTICS (SYMBOLICALLY)			SAME AS FOR MID-SERIES M-TYPE
PHASE SHIFT CHARACTERISTICS (SYMBOLICALLY)			
VALUE OF $\frac{Z_a}{4Z_b} = u + jv$ — NON-DISSIPATIVE	$-\frac{\left(\frac{f_2}{f_m}-\frac{f_m}{f_2}\right)^2}{\left(\frac{f}{f_m}-\frac{f_m}{f}\right)^2} = \left(\frac{Z_1}{4Z_2}\right)_K$	$\frac{m^2\left(\frac{Z_1}{4Z_2}\right)_K}{(1-m^2)\left(\frac{Z_1}{4Z_2}\right)_K+1}$	SAME AS FOR SERIES-DERIVED M-TYPE
VALUE OF $\frac{Z_a}{4Z_b} = u + jv$ — DISSIPATIVE	$\frac{(1-jd)\left(\frac{f_2}{f_m}-\frac{f_m}{f_2}\right)^2}{\left[d\frac{f}{f_m}+j\left(\frac{f}{f_m}-\frac{f_m}{f}\right)\right]^2} = \left(\frac{Z_1}{4Z_2}\right)'_K$	$\frac{m^2\left(\frac{Z_1}{4Z_2}\right)'_K}{(1-m^2)\left(\frac{Z_1}{4Z_2}\right)'_K+1}$	
CUTOFF FREQUENCIES NOTE – USE PLUS (+) FOR f_2 USE MINUS (−) FOR f_1	$\frac{\sqrt{L_1C_2+16L_1C_1}\pm\sqrt{L_1C_2}}{8\pi L_1C_1}$	$\frac{\sqrt{16L_1C_1+C_3(L_1+4L_3)}\pm\sqrt{C_3(L_1+4L_3)}}{8\pi L_1C_1}$	$\frac{\sqrt{16L_1C_1+L_1(C_2+4C_3)}\pm\sqrt{L_1(C_2+4C_3)}}{8\pi L_1C_1}$

REMARKS:—(1) $f_m = \sqrt{f_1 f_2} = \sqrt{f_{1\infty} f_{2\infty}}$ (2) $m = \sqrt{1-\frac{\left(\frac{f_{2\infty}}{f_m}-\frac{f_m}{f_{2\infty}}\right)^2}{\left(\frac{f_2}{f_m}-\frac{f_m}{f_2}\right)^2}}$ (3) $d = \frac{R}{\omega L}$

FIG. 1/XXV:6.—Characteristics and Design Formulae for Band-Elimination Filters.
(By courtesy of Messrs. D. Van Nostrand & Co. Inc., New York, U.S.A.)

6. Design of Band-Elimination or Band-Rejection Filters.

Of these only three types are important : the prototype and the series- and shunt-derived *m*-type.

In this case the *m* type should not be considered as accessories to improve the performance of the prototype but as an improved section which, when used by itself, affords an attenuation characteristic which gives more effective rejection over a band of frequencies than does the prototype. A glance at the attenuation characteristics of the two types of filter shown in Fig. 1 will make this clear. It is important to note that the band over which the *m*-type sections give high rejection is the band contained between the two frequencies of infinite attenuation and not the band between the cut-off frequencies.

The value of *m* is given at the bottom of Fig. 1, which gives all the standard information about the prototype and the two *m*-type sections.

Although the *m*-type sections give a better performance than does the prototype, the prototype itself gives such a large improvement over the performance of a single resonant rejector or acceptor circuit that it is almost true to say that the *m*-type section is never required. It does, however, offer a way out when very high discrimination is required.

7. Type of Filter to Use.

The type of filter to use is the simplest that will do the required job. If a low-pass filter is required it should first be established that a simple shunt condenser or a series inductance is inadequate. Then a single section, or even a half-section, of the prototype filter should be considered. If this is inadequate, try more sections. The decision as to whether to add more prototype sections or to add *m*-type sections is governed by whether it is necessary to add attenuation close above cut-off or remote from cut-off. If attenuation is needed near cut-off, *m*-type sections are required ; if remote from cut-off, prototype sections are generally required unless it is necessary to increase the attenuation in a limited frequency band only.

Similar arguments apply to high-pass filters.

In the case of band-pass filters the prototype filter is the first to consider because it is one of the simplest to design and adjust. If slow rising attenuation is required on one side of the pass band, and only a small amount of attenuation on the other, a type III filter may be used.

If rapidly rising attenuation is required both sides of the pass band, then one or more prototype sections may be combined into a composite filter with the required number and type of *m*-type sections.

If rapidly rising attenuation is required on one side of the pass band only, with a high final value of attenuation on the other, a prototype section may be used in combination with a type IV section. This will give higher values of attenuation than a single section of type V which will give the same form of characteristic, but will be harder to design, and probably harder to adjust.

8. Adjustment of Filters.

Prototype low-pass and high-pass filters normally require no adjustment because the effective values of the elements, when in circuit, do not normally differ sufficiently from their measured values before insertion in circuit, to affect appreciably either the cut-off frequency or the filter impedance.

Prototype band-pass filters and band-rejection filters should be adjusted with their elements in situ (i.e. mounted in their final position in the apparatus), so that the resonant frequencies of the shunt and series arms occur at the geometric mid-band frequency f_m.

m-type sections should be adjusted in situ so that the arms determining the frequency of infinite attenuation resonate at the frequency of infinite attenuation.

All other sections should be adjusted so that all arms resonate at the theoretical frequency determined from the element values. For instance, the shunt arm of a III_4 section resonates at the resonant frequency of L_2 and C_2, which is

$$f_r = \frac{1}{2\pi\sqrt{L_2C_2}} = \frac{1}{2\pi\sqrt{\dfrac{(f_2-f_1)R}{4\pi f_1 f_2}\cdot\dfrac{f_1}{\pi f_2(f_2-f_1)R}}} = f_2$$

Adjustments should be made by adjustment of the capacities in the circuit. This is, firstly, because the variations are largely due to stray capacities, and, secondly, because adjustment of capacity is most easily done. For this reason *the condensers in all critical circuits should be made slightly smaller than the calculated values and finally adjusted by the addition of small trimmer condensers.*

9. Calculation of Image Impedances and Transfer Vectors of Filter Sections.

Each mid-shunt terminated section constitutes a symmetrical π fourpole so that its image impedance is given by equation

(10)/XXIV:5.21, while its transfer vector is given by (11)/XXIV:5.21.

Each mid-series terminated section constitutes a symmetrical T fourpole so that its image impedance is given by equation (4)/XXIV:5.11, while its transfer vector is given by (5)/XXIV:5.11.

A half-section has one image impedance equal to the mid-series image impedance of the filter and one image impedance equal to the mid-shunt image impedance of the filter type in question. The transfer vector of a half section is equal to the square root of the transfer vector of a full section, whether mid-shunt or mid-series terminated.

As a corollary the transfer vector of a mid-shunt terminated section is the same as the transfer vector of a mid-series terminated section of the same type of filter. As this is a rather important point it is demonstrated below. The working should be followed in detail because it illustrates the conventions to be observed in substituting in the equations of XXIV:5 referred to immediately above.

Consider the case of a ladder-type filter as shown in Fig. 1 (a)/XXV:2. The mid-series terminated section then consists of a T fourpole with $\frac{Z_a}{2}$ in each series arm and Z_b in the shunt arm.

The mid-shunt terminated section consists of a π fourpole with $2Z_b$ in each shunt arm and Z_a in the series arm.

From equations (4) and (5)/XXIV:5, the transfer vector of the mid-series terminated section is:

$$u = 1+\frac{A}{B}+\frac{Z_I}{B} = 1+\frac{Z_a}{2Z_b}+\frac{1}{Z_b}\sqrt{\frac{Z_a^2}{4}+Z_aZ_b}$$

$$\left(\text{since } A = \frac{Z_a}{2} \text{ and } B = Z_b\right)$$

$$= 1+\frac{Z_a}{2Z_b}+\sqrt{\frac{Z_a^2}{4Z_b^2}+\frac{Z_a}{Z_b}} \quad . \quad . \quad . \quad . \quad (1)$$

From equations (10) and (11)/XXIV:5 the transfer vector of the mid-shunt terminated section is:

$$u = 1+\frac{E}{D}+\frac{E}{Z_I} = 1+\frac{Z_a}{2Z_b}+\frac{Z_a}{2Z_b\sqrt{\frac{Z_a}{Z_a+4Z_b}}}$$

$$(\text{since } E = Z_a \text{ and } D = 2Z_b)$$

$$= 1+\frac{Z_a}{2Z_b}+\sqrt{\frac{Z_a^2}{4Z_b^2}+\frac{Z_a}{Z_b}} \quad . \quad . \quad . \quad . \quad (1a)$$

Similarly, if Z_{se} is the mid-series image impedance and Z_{sh} is the mid-shunt image impedance,

$$Z_{se} = \sqrt{A^2+2AB} = \sqrt{\frac{Z_a^2}{4}+Z_aZ_b} = Z_a\sqrt{\frac{1}{4}+\frac{Z_b}{Z_a}} \quad . \quad (2)$$

$$Z_{sh} = D\sqrt{\frac{E}{E+2D}} = 2Z_b\sqrt{\frac{Z_a}{Z_a+4Z_b}} = \frac{Z_b}{\sqrt{\frac{1}{4}+\frac{Z_b}{Z_a}}} \quad . \quad (3)$$

For the calculation of transfer vectors and image impedances respectively: use equations (1), (2) and (3) above when it is required to substitute full series and full shunt elements: when it is required to substitute actual elements in a **T** or π section, use equations (4) and (5), (10) and (11) from XXIV:5. When using the full elements for the determination of the transfer vector the method of calculation given under XXV:9.1 below will, however, be found very much simpler than full substitution in equation (1).

9.1. Values of $\frac{Z_a}{Z_b}$ in Pass Bands and Attenuating Bands.

Equation (1) may be transformed as follows:

Let $\frac{Z_a}{Z_b} = w$, then

$$\begin{aligned} u &= 1+0{\cdot}25w+\sqrt{0{\cdot}25w^2+w}+0{\cdot}25w \\ &= (\sqrt{1+0{\cdot}25w}+\sqrt{0{\cdot}25w})^2 \\ &= \frac{(\sqrt{1+0{\cdot}25w}+\sqrt{0{\cdot}25w})^2(\sqrt{1+0{\cdot}25w}-\sqrt{0{\cdot}25w})}{\sqrt{1+0{\cdot}25w}-\sqrt{0{\cdot}25w}} \\ &= \frac{\sqrt{1+0{\cdot}25w}+\sqrt{0{\cdot}25w}}{\sqrt{1+0{\cdot}25w}-\sqrt{0{\cdot}25w}} \end{aligned}$$

$$\therefore\ u = a/\underline{\alpha} = \frac{\sqrt{4+w}+\sqrt{w}}{\sqrt{4+w}-\sqrt{w}} \quad . \quad . \quad . \quad (4)$$

In the pass range $a = 1$. It follows that in the pass range w lies between 0 and -4. This is because $\frac{x+y}{x-y}$ is only equal to unity when y is in quadrature with a: $\sqrt{w}$ can only be in quadrature with $\sqrt{4+w}$ when w is negative and $\sqrt{4+w}$ is real, that is when w is between 0 and -4.

The above statement is general and unrestricted. It is, however, not advisable to deduce from this that filter cut-off frequencies always occur when $w = 0$ or -4, since pass bands defined in this way are sometimes contiguous: the value of w, for instance, may travel from -4 to zero and back to -4, in which case the zero value of w does not correspond to a practical cut-off frequency.

The statement that cut-off frequencies occur when the image impedances are zero or infinity is subject to no such limitation and is the criterion to use for filter design.

Note that from (4), when $w = 0$, $\alpha = 0$, and when $w = -4$, $u = 1\underline{/\alpha} = -1$, so that $\alpha = \pm 180°$. When w is finite and positive, both numerator and denominator are real and positive, so that $\alpha = 0$, independent of the magnitude of w. When w is negative and greater than -4, both numerator and denominator are imaginary, the numerator being a positive imaginary and the denominator a negative imaginary so that $\alpha = \pm 180°$.

9.11. Value of u in Pass Bands and Attenuating Bands. The numerical evaluation of equation (1) may be simplified by the following process.

Put $\dfrac{Z_a}{Z_b} = w$ and solve equation (1) for w in terms of u.

Then $u - 1 - \dfrac{1}{2}w = \sqrt{\dfrac{w^2}{4} + w}$.

Squaring both sides

$$u^2 + 1 + \frac{w^2}{4} - 2u - uw + w = \frac{w^2}{4} + w$$

$$\therefore \ w = u - 2 + \frac{1}{u} \quad . \quad . \quad . \quad . \quad (5)$$

When w is negative and greater than -4. From (4)

$$u = a\underline{/180°} \text{ or } \overline{a\backslash 180°} = -a$$

Hence, from (5)

$$w = -a - 2 - \frac{1}{a} \quad . \quad . \quad . \quad (5a)$$

When w lies between 0 *and* -4. From (4)

$$u = 1\underline{/\alpha}$$

Hence, from (5)

$$w = 1\underline{/\alpha} + \overline{1\backslash\alpha} - 2$$

$$= 2\cos\alpha - 2$$

$$\therefore \ \alpha = \pm\cos^{-1}\left(1 + \frac{w}{2}\right) \quad . \quad . \quad . \quad (5b)$$

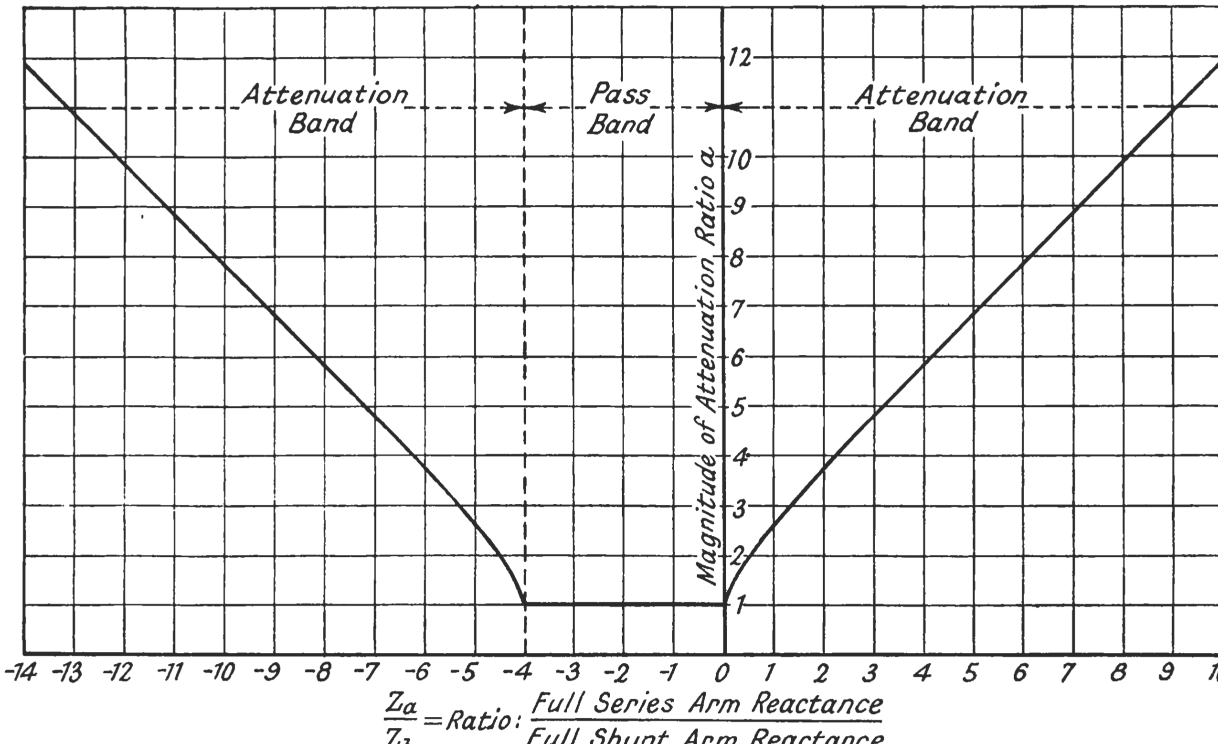


FIG. 1/XXV:9.—Magnitude of Attenuation Ratio α in Attenuation Bands and Pass Band.

When w is positive. From (4)

$$u = a/\underline{0} = a$$

Hence, from (5)

$$w = a + \frac{1}{a} - 2 \quad . \quad . \quad . \quad . \quad (5c)$$

The sign of α is determined from the signs of Z_a and Z_b. If Z_a is positive and Z_b is negative, α is positive, indicating lag. With the reverse signs α is negative indicating lead. These conventions are reversed with regard to normal conventions because u = input voltage/output voltage of fourpole and *not* output voltage/input voltage.

If Z_a and Z_b are both positive α is zero; if Z_a and Z_b are both negative α is zero.

Equations (5*a*) and (5*c*) are plotted in Fig. 1, which shows the value of the attenuation ratio a for values of $w = \frac{Z_a}{Z_b}$ from -14 to $+10$.

Outside this range the magnitude of a may be obtained with an error decreasing as w and a increase, and which is never greater than within 1%, by subtracting 2 from the magnitude of w when w is negative and by adding 2 to the magnitude of w when w is positive.

Equation (5*b*) is plotted in Fig. 2 and shows the value of the phase-shift α for values of w from 0 to -4.

Figs. 1 and 2 enable the values of a and α to be obtained at any frequency at which the reactances of the full series and full shunt arms of the filter are determined. This gives the value of u for a mid-series or mid-shunt terminated section of a ladder-type filter. The transfer vector of a half-section of such a filter is equal to the square root of the transfer vector for the full section.

It should be realized that a full section of a ladder filter is a symmetrical structure (although its half-section is unsymmetrical) and that this method evidently does not apply to sections which are unsymmetrical.

The method of deriving the design formulae for an unsymmetrical-**T** section is given in XXV:11, but the method of determining the loss and phase-shift of such a structure is not given. The best method is to calculate the insertion loss directly by finding the received current in the terminating load with and without the filter inserted. For this, the method of inspection or the method of addition may be used. This, of course, does not give the value of u, but does give the insertion loss which is generally the quantity of most interest. If u is required it may be found from the insertion loss of the structure when working between its image impedances.

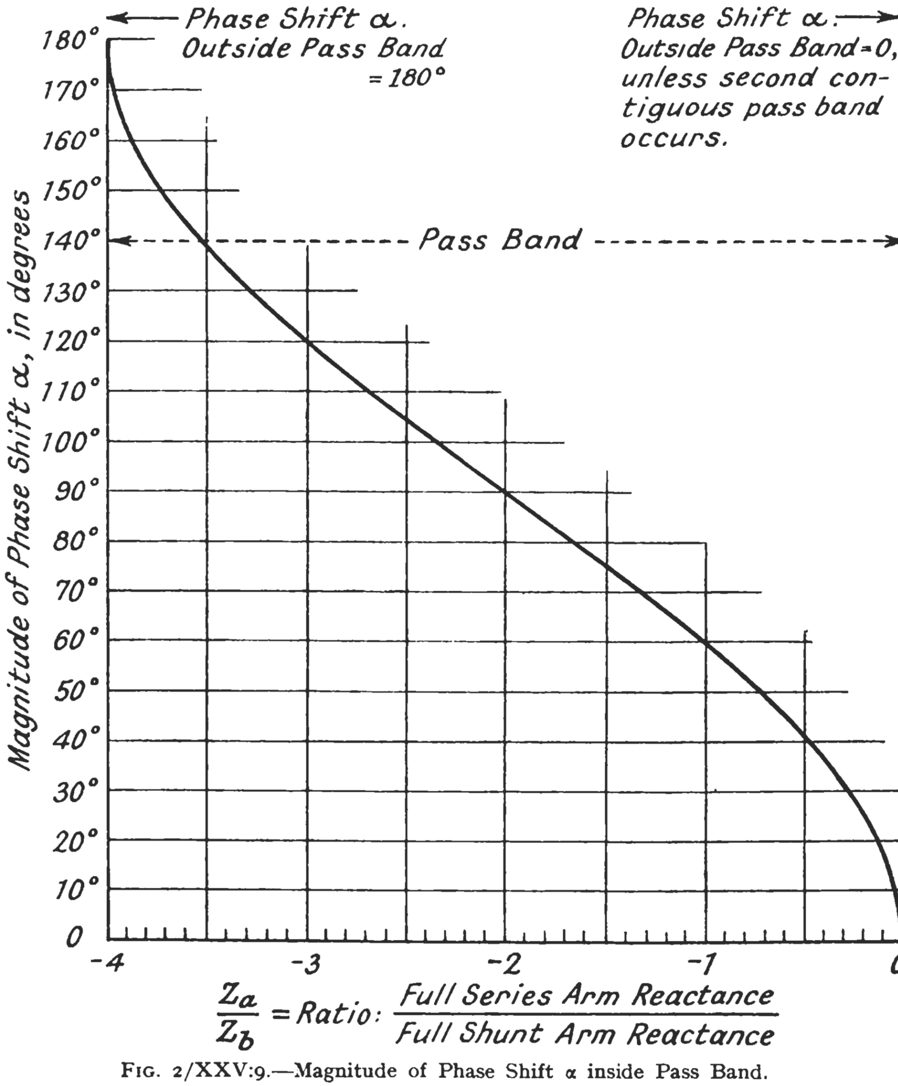


FIG. 2/XXV:9.—Magnitude of Phase Shift α inside Pass Band.

A final point which is normally of small practical importance relates to the conventions adopted with regard to phase shift which apparently lead to anomalies when a multiband filter is considered. These disappear when it is remembered that the addition of 2π to an angle does not change the magnitude of any of its trigonometrical functions. The true phase shift in a composite network at any frequency is obtained by adding up the phase changes in all frequency ranges below that frequency. If, therefore, it *appears* that the phase shift at the high frequency end of an attenuation band is, for instance, $+\pi$, while the phase shift at the beginning of the contiguous pass band on the high-frequency side is $-\pi$, the difficulty is resolved when it is realized that a pass band, extending from π to 2π, when examined only in its own frequency range, behaves exactly like a band extending from $-\pi$ to 0. When the true phase shift at the cut-off frequency in question is determined, it may be found that it is in fact greater than π by any integral multiple of 2π.

9.2. Lattice Filter Sections. A lattice fourpole, as shown in Fig. 3/XXIV:5, may be used as a filter section. The method of design of such a section, to have given cut-off frequencies and a given mid-band image impedance, is given in XXV:11.2.

Once the section has been designed, its image impedance is given by equation (12)/XXIV:5 and the value of its transfer vector by equation (13)/XXIV:5.

From examination of equation (13)/XXIV:5 it is evident that pass bands occupy the frequency ranges in which $w = \frac{B}{A}$ is negative, regardless of its magnitude. Attenuating bands occupy the frequency ranges in which w is positive. If, for instance, A is constituted by a condenser and B by any combination of inductances and capacities, a cut-off frequency will occur at every frequency at which the reactance of B crosses the axis of frequency and so changes sign.

Since the value of u in equation (13)/XXIV:5 was obtained by extracting a square root, there is an ambiguity in the sign of u. This may be expressed by saying that this equation defines the magnitude of the phase shift but not the sign. Assuming, as before, that lag is represented by a positive angle and lead by a negative angle, the sign of the phase shift is given by the following rules:

In the pass bands

$\alpha = +2 \tan^{-1} \sqrt{w}$, when A is positive and B is negative

$\alpha = -2 \tan^{-1} \sqrt{w}$, when A is negative and B is positive.

This convention refers to the configuration shown in Fig. 3/XXIV:5.

In the attenuating bands

$\alpha = 0$, when A is less than B

$\alpha = \pi$, when A is greater than B.

Apparent phase-shift anomalies at the junction of pass bands and attenuating bands may be resolved as in the case of ladder-type structures.

The value of a in the attenuating bands is evidently given by

$$a = \frac{1 + \sqrt{w}}{1 - \sqrt{w}}$$

where w is, of course, a positive real quantity.

10. Insertion Loss of a Filter.

Since a composite filter is made up from an assembly of sections which are impedance matched at their common terminals, the total transfer vector of a composite filter is equal to the product of the transfer vectors of the component sections. The whole filter can then be treated as a single fourpole of input image impedance equal to the input image impedance of the first section, and of output impedance equal to the output image impedance of the last section.

The insertion loss at any frequency, whether inside or outside the pass band, is then given by equation (33)/XXIV:4. Since, however, the filter is usually terminated in the same impedance at both ends, the most commonly applicable formula is equation (34)/XXIV:4, which is repeated below.

The insertion loss is given by:

$$L = 20 \log_{10} \left| \frac{R\left(1+\frac{Z_2}{Z_1}\right)C_s + \left(Z_2 + \frac{R^2}{Z_1}\right)S_n}{2\sqrt{\frac{Z_1}{Z_2}}} \right| \text{ decibels}$$

where Z_1 = the input image impedance of the first section determined as above.

Z_2 = the output image impedance of the last section determined as above.

R = the impedance of driving source and terminating impedance.

$$C_s = \tfrac{1}{2}\left(u + \frac{1}{u}\right).$$

$$S_n = \tfrac{1}{2}\left(u - \frac{1}{u}\right).$$

$u = u_1 u_2 u_3 \ldots$ etc., where u_1 and u_2, etc., are the transfer vectors of the component sections, and are determined as below.

The vertical lines indicate that the magnitude of the complex quantity between them is to be taken.

The insertion loss is usually the loss which is effective in exercising discrimination in the circuit : it is the loss which determines the relative efficiency of reproduction of each frequency.

For single sections it is probably preferable to calculate the insertion loss by calculating the current in the terminating impedance by the method of inspection, see XXIV:2.1, or the method of addition, see XXIV:2.4. The insertion loss is then given by

$$L = 20 \log_{10} \left| \frac{I_0}{I_n} \right| \text{ decibels}$$

where I_0 = the current in the terminating impedance with the driving source directly connected to the terminating impedance.

and I_n = the current in the terminating impedance with the filter in circuit.

Insertion Loss in Prototype H.P. and L.P. Filters. O. S. Meixell (*R.C.A. Rev.*, Vol. 5, p. 337) has shown that the insertion loss and phase shift of prototype low-pass and high-pass filters, working between their image impedances respectively at zero frequency and infinite frequency, can be expressed at any frequency f in terms of r, where $r = f/f_c$ for low-pass filters and f_c/f for high-pass filters. The formulae below are valid for both mid-shunt and mid-series terminations.

Insertion Loss and Phase Shift of One Section of Prototype L.P. or H.P. Filter between Impedances $R = \sqrt{\frac{L}{C}}$, where L and C are the values of the full arms, series or shunt as the case may be.

$$\text{Insertion Loss } L = 10 \log_{10} (1+r^6) \text{ db.}$$

$$\text{Insertion Phase Shift } A = \tan^{-1} \pm \left[\frac{r(2 - r^2)}{1 - 2r^2}\right].$$

The plus sign in the phase-shift formula is taken for high-pass filters, indicating lead, while the negative sign is taken for low-pass filters, indicating lag.

Insertion Loss and Phase Shift of Two Sections of Prototype L.P. or H.P. Filter between Impedances $R = \sqrt{\frac{L}{C}}$, where L and C are the values of the full arms, series or shunt as the case may be.

$$\text{Insertion Loss} = 10 \log_{10} [1+4r^6(1 - 2r^2)^2]$$

$$\text{Insertion Phase Shift} = \tan^{-1} \pm \left[\frac{2r(2 - r^2)(1 - 2r^2)}{2(1 - 2r^2)^2 - 1}\right].$$

The sign in the phase-shift formula is determined as in the single-section case.

10.1. Approximate Determinations of Filter Performance. For many purposes an approximate determination of the performance of a filter is adequate. Two types of approximation can be made, both of which disregard the transition loss at the terminals of the filter.

The first approximation is that $L = 20 \log_{10} u$, where $u = u_1 u_2 u_3$, etc. This is obviously inaccurate, but is useful in assessing the probable number of sections which will be required.

The other approximation is of use particularly when designing low-pass filters for smoothing purposes when the terminating impedances are often rather indeterminate. It can best be illustrated by means of an example.

Fig. 1(*a*) shows two sections of mid-shunt terminated prototype low-pass filter designed for a (zero frequency) image impedance of

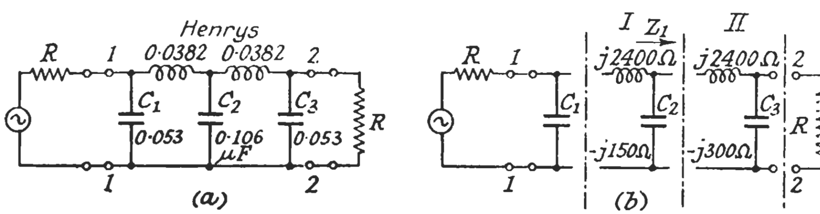


FIG. 1/XXV:10.—Illustrating Approximate Method of calculating Filter Attenuation.

600 ohms and a cut-off frequency of 5,000 c/s. It can also be regarded as a mid-series terminated section with a half-section at each end building the filter out to mid-shunt ; this is an incidental point.

The inductances are each 0·0382 henrys, the terminating condensers are each 0·053 μF, while the full shunt arm in the centre is 0·106 μF.

To make a rough estimate of the ratio of the output voltage V_2, appearing at 2,2, to the input voltage V_1 effective across 1,1 at 10 kc/s, the procedure is as follows.

At 10 kc/s the reactance of each inductance is $6{,}280 \times 10 \times 0{\cdot}0382 = 2{,}400\ \Omega$ and the reactance of the full shunt condenser is $\dfrac{159}{0{\cdot}106 \times 10} = 150\ \Omega$, while that of the terminating condensers is 300 Ω.

The filter can therefore be split up into two potentiometers, as in Fig. 1(*b*). Assuming that the impedances Z_1 and R are respectively high compared to the impedance of C_2 and C_3 the ratio between V_1 and V_2 is the product of the ratios of the two potentiometers I and II, i.e.

$$\frac{V_2}{V_1} = \frac{-150}{2{,}400 - 150} \cdot \frac{-300}{2{,}400 - 300} = \frac{150}{2{,}250} \cdot \frac{300}{2{,}100} = \frac{1}{105}$$

This ratio may provide all the information that is required. Alternatively the ratio may be converted to the decibel equivalent in the normal manner, assuming equal impedances at 1,1, and 2,2, which again is of course an unwarrantable assumption.

The same procedure may be used to determine a rough indication of the voltage suppression at any other frequency.

11. Derivation of Design Formulae for Filters.

This refers to the formulae giving the values of the elements of each filter section in terms of the mid-band image impedances and the cut-off frequencies. The procedure below can be applied to any structure of reactances.

This procedure consists in the following steps:

1. Find the image impedances of the structure in terms of the reactances of the structure.
2. By equating the resultant expressions for the image impedances to zero and infinity, derive expressions for the cut-off frequencies in terms of the reactances of the circuit elements in the structure.
3. Eliminate all the reactances except one between the several equations obtained, and derive an expression for one image impedance in terms of the remaining circuit element, the cut-off frequencies, and frequency. By making the frequency equal to the geometric mid-band frequency (the geometric mean of the cut-off frequencies) obtain an expression for the mid-band image impedance in terms of the remaining circuit element and the cut-off frequencies.
4. Invert the equation last obtained to give an explicit expression for the remaining circuit element in terms of the mid-band image impedance and the cut-off frequencies.
5. Repeat for the remaining circuit elements.

In practice the expressions obtained for the image impedances give directly the value of the square of the image impedances and are most conveniently left in this form until the end of the analysis. Also, the analysis is most conveniently conducted in terms of the angular frequency instead of frequency, but these modifications are evidently unimportant.

The procedure is illustrated below for the case of a dissymmetrical-T fourpole, using inductances in the series arms and a capacity in the shunt arm. See also Bib. F7:20.

11.1. Dissymmetrical-T Fourpole Designed as a Filter Section. The particular type of dissymmetrical T chosen for this example is one in which the series arms are constituted by inductances L_1 and L_2, and the shunt arm is constituted by a condenser C, as shown in Fig. 1.

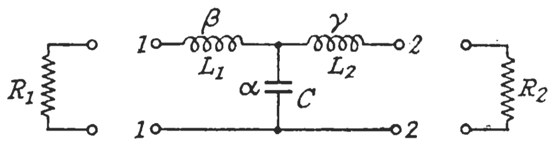


FIG. 1/XXV:11.—Dissymmetrical-T Fourpole between its Mid-Band Image Impedances.

Conventions

Z_1 = image impedance at terminals 1,1, Fig. 1.

Z_2 = image impedance at terminals 2,2, Fig. 1.

SC_1 = short-circuit impedance looking into terminals 1,1 (i.e. with terminals 2,2, shorted).

SC_2 = short-circuit impedance looking into terminals 2,2.

OC_1 = open-circuit impedance looking into terminals 1,1.

OC_2 = open-circuit impedance looking into terminals 2,2.

R_1 and R_2 = values of Z_1 and Z_2 at geometric mid-band frequency.

α = the impedance of condenser C.

β = the impedance of inductance L_1.

γ = the impedance of inductance L_2.

$$a = -\alpha\omega = \frac{j}{C}.$$

$$b = \frac{\beta}{\omega} = jL_1.$$

$$mb = \frac{\gamma}{\omega} = jL_2,$$ where m is any numeric greater than unity.

f_1 = lower cut-off frequency.

f_2 = upper cut-off frequency.

f_3 = cut-off frequency in symmetrical case (not of interest).

$\omega_1, \omega_2, \omega_3 = 2\pi f_1, 2\pi f_2, 2\pi f_3$ respectively.

$$\phi = \frac{f_2}{f_1}.$$

Then

$$SC_1 = \frac{\alpha\beta+\beta\gamma+\alpha\gamma}{\alpha+\gamma} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$OC_1 = \alpha+\beta \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$SC_2 = \frac{\alpha\beta+\beta\gamma+\alpha\gamma}{\alpha+\beta} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$OC_2 = \alpha+\gamma \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Determination of Image Impedances.

$$Z_1^2 = SC_1OC_2 = \frac{(\alpha+\beta)(\alpha\beta+\beta\gamma+\alpha\gamma)}{\alpha+\gamma} \quad . \quad . \quad (5)$$

$$Z_2^2 = SC_2OC_2 = \frac{(\alpha+\gamma)(\alpha\beta+\beta\gamma+\alpha\gamma)}{\alpha+\beta} \quad . \quad . \quad (6)$$

Also

$$\frac{Z_1}{Z_2} = \frac{\alpha+\beta}{\alpha+\gamma}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Equations (1) to (7) are quite general for all **T** structures. For the structure shown in Fig. 1 the following substitutions are now made :

$$\alpha = \frac{1}{jC\omega} = -\frac{j}{C\omega} = -\frac{a}{\omega} \quad . \quad . \quad . \quad (8)$$

$$\beta = jL_1\omega = b\omega \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\gamma = jL_2\omega = mb\omega \; . \quad . \quad . \quad . \quad . \quad (10)$$

Then equations (5), (6) and (7) become

$$Z_1^2 = \frac{(b\omega^2 - a)(mb^2\omega^2 - ab.\overline{1+m})}{mb\omega^2 - a} \quad . \quad . \quad (5a)$$

$$Z_2^2 = \frac{(mb\omega^2 - a)(mb^2\omega^2 - ab.\overline{1+m})}{b\omega^2 - a} \quad . \quad . \quad (6a)$$

$$\frac{Z_1}{Z_2} = \frac{b\omega - \frac{a}{\omega}}{mb\omega - \frac{a}{\omega}} = \frac{b\omega^2 - a}{mb\omega^2 - a} \quad . \quad . \quad . \quad (7a)$$

Determination of Cut-off Frequencies. The cut-off frequencies of a structure exist at the frequencies at which the image impedances Z_1 and Z_2 are respectively 0,0 ; 0, ∞ ; ∞,0 ; or ∞, ∞.

Three of these conditions can be satisfied by equating appropriate functions of ω from equations (5*a*) and (6*a*) to zero. There are, therefore, three cut-off frequencies to be considered, corresponding to ω_1, ω_2 and ω_3, where:

$$\omega_1^2 = \frac{a}{b} \left(\text{hence } b = \frac{a}{\omega_1^2}, \text{ etc.}\right) \quad . \quad . \quad (11)$$

$$\omega_2^2 = \frac{a}{b} \cdot \frac{1+m}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\omega_3^2 = \frac{a}{mb} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Since m is greater than unity, these cut-off frequencies are arranged as in Fig. 2.

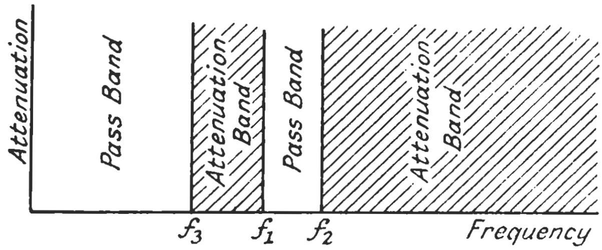


FIG. 2/XXV:11.—Arrangement of Cut-off Frequencies.

This structure can therefore be designed as a filter section *either*

(1) To pass a band from 0 to f_3.

or (2) To pass a band from f_1 to f_2.

or (3) To pass a band from 0 to f_3 and from f_1 to f_2 *only* in the particular case where $f_3 = f_1$.

It can be shown that cases (1) and (3) lead to the same design in which $Z_1 = Z_2$. In both these cases there is only a single pass band giving rise to a mid-series terminated section of the normal prototype low-pass filter.

If the structure is designed to pass a band from f_1 to f_2 (i.e. case 2), the pass band from 0 to f_3 does not exist.

Case (2) is evidently the case under discussion, since the dissymmetrical case is being investigated. This case enables the structure to be used for impedance matching because the image impedances of the structure are unequal.

Values of L_1, L_2 and C in terms of R_1.

From equations (11) and (12)

$$\frac{\omega_2^2}{\omega_1^2} = \frac{1+m}{m} \quad . \quad . \quad . \quad . \quad (14)$$

$$\therefore m = \frac{\omega_1^2}{\omega_2^2 - \omega_1^2} = \frac{1}{\phi^2 - 1} \text{ and } 1+m = \frac{\omega_2^2}{\omega_2^2 - \omega_1^2} \quad . \quad (15)$$

Value of C. Eliminate b and m, by substituting their values from (11) and (15) in (5a). Also replace ω^2 by $\omega_1\omega_2$ so that $Z_1 = R_1$. Whence

$$R_1^2 = \frac{\left[\dfrac{a\omega_1\omega_2}{\omega_1^2} - a\right]\left[\dfrac{\omega_1^3\omega_2}{\omega_2^2 - \omega_1^2} \times \dfrac{a^2}{\omega_1^4} - \dfrac{a^2}{\omega_1^2} \times \dfrac{\omega_2^2}{\omega_2^2 - \omega_1^2}\right]}{\dfrac{\omega_1^3\omega_2}{\omega_2^2 - \omega_1^2} \times \dfrac{a}{\omega_1^2} - a}$$

Multiply above and below by $\omega_2^2 - \omega_1^2$.

$$\therefore\ R_1^2 = \frac{a^2\left[\dfrac{\omega_2}{\omega_1} - 1\right]\left[\dfrac{\omega_2}{\omega_1} - \dfrac{\omega_2^2}{\omega_1^2}\right]}{\omega_1\omega_2 - \omega_2^2 + \omega_1^2}$$

Multiply above and below by $-\dfrac{\omega_1^3}{\omega_2}$ and replace a by $\dfrac{j}{C}$

$$\therefore\ R_1^2 = \frac{(\omega_2 - \omega_1)^2}{-C^2\omega_1^4\left(\dfrac{\omega_2}{\omega_1} - \dfrac{\omega_1}{\omega_2} - 1\right)}$$

$$\therefore\ C = \frac{\dfrac{f_2}{f_1} - 1}{2\pi f_1 R_1\sqrt{1 + \dfrac{f_1}{f_2} - \dfrac{f_2}{f_1}}} = \frac{\phi - 1}{2\pi f_1 R_1}\sqrt{\frac{\phi}{1 + \phi - \phi^2}} \quad . \quad (16)$$

where $\phi = \dfrac{f_2}{f_1}$.

Value of L_1. Eliminate a and m by substituting their values from (11) and (15) in (5a). As before, replace ω^2 by $\omega_1\omega_2$ so that

$$R_1^2 = \frac{(b\omega_1\omega_2 - b\omega_1^2)\left[\dfrac{\omega_1^3\omega_2 b^2}{\omega_2^2 - \omega_1^2} - \dfrac{\omega_2^2\omega_1^2 b^2}{\omega_2^2 - \omega_1^2}\right]}{\dfrac{\omega_1^3\omega_2 b}{\omega_2^2 - \omega_1^2} - b\omega_1^2}$$

Replacing b by jL_1

$$R_1^2 = \frac{L_1^2(\omega_2 - \omega_1)(\omega_2 - \omega_1)}{1 - \dfrac{\omega_2}{\omega_1} + \dfrac{\omega_1}{\omega_2}}$$

$$\therefore\ L_1 = \frac{R_1\sqrt{1 - \dfrac{f_2}{f_1} + \dfrac{f_1}{f_2}}}{2\pi(f_2 - f_1)} = \frac{R_1}{2\pi f_1(\phi - 1)}\sqrt{\frac{1 + \phi - \phi^2}{\phi}} \quad . \quad (17)$$

Value of L_2. $L_2 = mL_1$; hence from (15) and (17)

$$L_2 = \frac{L_1}{\phi^2 - 1} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Impedance Transformation Ratio in terms of $\frac{f_2}{f_1}$.

Insert in (7*a*) the values of b and m from (11) and (15), and replace ω^2 by $\omega_1\omega_2$.

$$\therefore \frac{R_1}{R_2} = \frac{a\frac{\omega_1\omega_2}{\omega_1^2} - a}{\frac{\omega_1^2}{\omega_2^2 - \omega_1^2} \times \frac{a\omega_1\omega_2}{\omega_1^2} - a}$$

$$= \frac{f_2^2\frac{f_2}{f_1} - f_1^2\frac{f_2}{f_1} - f_2^2 + f_1^2}{f_1^2\frac{f_2}{f_1} - f_2^2 + f_1^2}$$

$$= \frac{\frac{f_2^3}{f_1^3} - \frac{f_2}{f_1} - \frac{f_2^2}{f_1^2} + 1}{\frac{f_2}{f_1} - \frac{f_2^2}{f_1^2} + 1}$$

$$= \frac{(\phi^2 - 1)(\phi - 1)}{1 + \phi - \phi^2} \text{ where } \phi = \frac{f_2}{f_1} \quad . \quad (19)$$

Equations (16), (17), (18) and (19) supply all the information that is necessary for the design of the filter in terms of f_1, f_2 and R_1. As a matter of interest, the values of the circuit elements may be found in terms of R_2 by substituting the value of R_1 from (19) into (16) and (17).

Hence

$$C = \frac{(\phi - 1)(1 + \phi - \phi^2)}{2\pi f_1 R_2(\phi^2 - 1)(\phi - 1)}\sqrt{\frac{\phi}{1 + \phi - \phi^2}}$$

$$= \frac{\sqrt{\phi(1 + \phi - \phi^2)}}{2\pi f_1 R_2(\phi^2 - 1)} \quad . \quad . \quad . \quad . \quad . \quad (16a)$$

$$L_1 = \frac{(\phi^2 - 1)(\phi - 1)R_2}{2\pi f_1(\phi - 1)(1 + \phi - \phi^2)}\sqrt{\frac{1 + \phi - \phi^2}{\phi}}$$

$$= \frac{(\phi^2 - 1)R_2}{2\pi f_1\sqrt{\phi(1 + \phi - \phi^2)}} \quad . \quad . \quad . \quad . \quad . \quad (17a)$$

$$L_2 = \frac{L_1}{\phi^2 + 1} = \frac{R_2}{2\pi f_1\sqrt{\phi(1 + \phi - \phi^2)}} \quad . \quad . \quad . \quad (18a)$$

Summary.

$$C = \frac{\phi - 1}{2\pi f_1 R_1}\sqrt{\frac{\phi}{1+\phi-\phi^2}} = \frac{\sqrt{\phi(1+\phi-\phi^2)}}{2\pi f_1 R_2(\phi^2-1)} \quad . \quad . \quad . \quad (16b)$$

$$L_1 = \frac{R_1}{2\pi f_1(\phi - 1)}\sqrt{\frac{1+\phi-\phi^2}{\phi}} = \frac{(\phi^2-1)R_2}{2\pi f_1\sqrt{\phi(1+\phi-\phi^2)}} \quad . \quad . \quad (17b)$$

$$L_2 = \frac{R_1}{2\pi f_1(\phi - 1)(\phi^2-1)}\sqrt{\frac{1+\phi-\phi^2}{\phi}} = \frac{R_2}{2\pi f_1\sqrt{\phi(1+\phi-\phi^2)}} \quad (18b)$$

$$\frac{R_2}{R_1} = \frac{1+\phi-\phi^2}{(\phi^2-1)(\phi-1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19b)$$

R_1 and R_2 are in ohms, L_1 and L_2 in henrys, C in farads, and f_1 in cycles per second.

It is to be noted that as ϕ $\left(\text{i.e. } \frac{f_2}{f_1}\right)$ is increased from unity the value of the ratio $\frac{R_2}{R_1}$ falls, reaching unity when $\frac{f_2}{f_1} = \sqrt{2}$. For this reason the above equations cannot be used for values of ϕ outside the limits 1 and $\sqrt{2}$.

ϕ is determined by the step-up ratio and vice versa. In other words, if the frequency band is determined (in location and width) the step-up ratio of the structure is determined. Alternatively, if the step-up ratio is determined, the ratio of the cut-off frequencies is determined. It is therefore possible to design a filter either from the required cut-off frequencies, or from the impedance step-up ratio and the location of the pass band in the frequency spectrum. An example of the last case will now be considered.

Example. Design a dissymmetrical T of this type to match between 20 ohms and 100 ohms for a carrier frequency of 1 Mc/s. The carrier frequency should be located at the geometric mid-band so that $\sqrt{f_1 f_2} = 10^6$. The value of ϕ cannot be obtained from (19) because this equation cannot readily be converted to give ϕ as an explicit function of $\frac{R_2}{R_1}$. It is therefore necessary to plot the relation of equation (19). This is done in Fig. 3 from which it is seen that, when $R_2/R_1 = 5$, $\phi = \frac{f_2}{f_1} = 1{\cdot}25$.

Hence $f_2 = 1{\cdot}25 f_1$, and $f_1 f_2 = 1{\cdot}25 f_1^2 = 10^{12}$, so that $f_1 = 895{,}000$. Also $R_1 = 20$ and $R_2 = 100$.

$$\sqrt{\phi(1+\phi-\phi^2)} = 0{\cdot}928 \qquad \phi^2 - 1 = 0{\cdot}56$$

ϕ	ϕ^2	ϕ^2-1	$\phi-1$	$(\phi^2-1)(\phi-1)$	$1+\phi-\phi^2$	$\frac{R_2}{R_1}$	ϕ	ϕ^2	ϕ^2-1	$\phi-1$	$(\phi^2-1)(\phi-1)$	$1+\phi-\phi^2$	$\frac{R_2}{R_1}$
1	1	0	0	0	1	∞	1·04	1·082	0·082	0·04	0·00328	0·958	293
1·1	1·21	0·21	0·1	0·021	0·89	42·4	1·06	1·124	0·124	0·06	0·00744	0·936	126
1·2	1·44	0·44	0·2	0·088	0·76	8·53	1·08	1·166	0·166	0·08	0·01328	0·914	68·8
1·3	1·69	0·69	0·3	0·207	0·61	2·95	1·14	1·3	0·3	0·14	0·042	0·84	20
1·4	1·96	0·96	0·4	0·384	0·44	1·145	1·16	1·346	0·346	0·16	0·0553	0·814	14·7
$\sqrt{2}$	2	1	0·414	0·414	0·414	1	1·18	1·392	0·392	0·18	0·0705	0·788	11·2
1·01	1·02	0·02	0·01	0·0002	0·99	4900	1·24	1·538	0·538	0·24	0·129	0·702	5·45
1·02	1·04	0·04	0·02	0·0008	0·98	1223	1·26	1·588	0·588	0·26	0·153	0·672	4·4

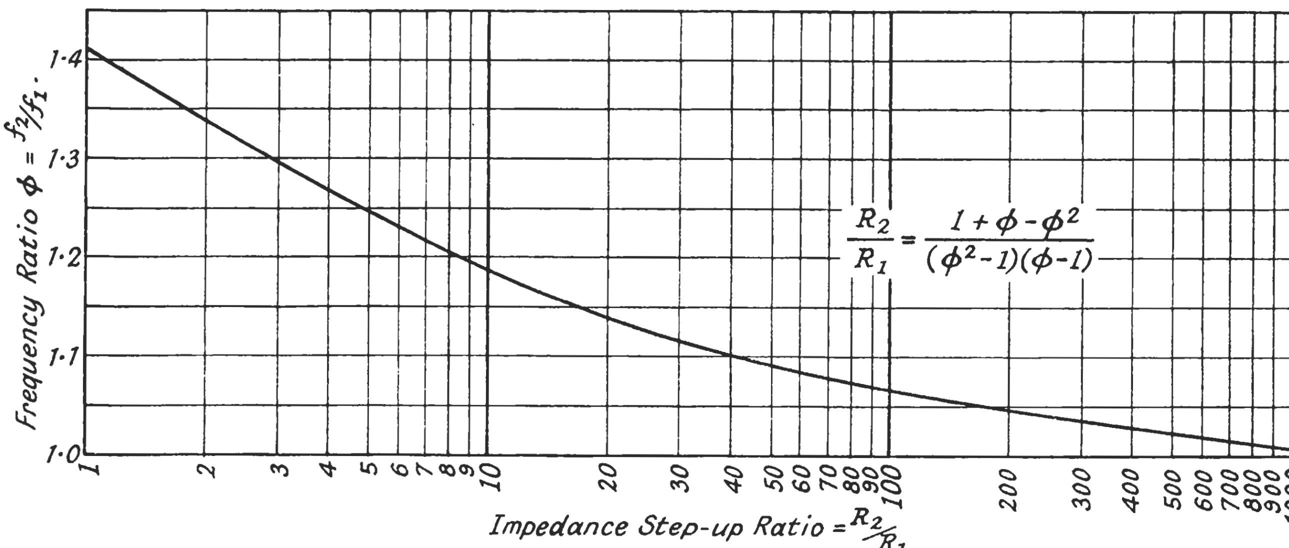


FIG. 3/XXV:11.—Relation between $\phi = \frac{f_2}{f_1}$ and $\frac{R_2}{R_1}$.

Then evaluating in terms of R_2:

$$C = \frac{0 \cdot 928}{2\pi \times 895{,}000 \times 100 \times 0 \cdot 56} \text{ Farads} = 0 \cdot 00295\ \mu\text{F}$$

$$L_1 = \frac{0 \cdot 56 \times 100}{2\pi \times 895{,}000 \times 0 \cdot 928} \text{ Henrys} = 10 \cdot 75\ \mu\text{H}$$

$$L_2 = \frac{100}{2\pi \times 895{,}000 \times 0 \cdot 928} \text{ Henrys} = 19 \cdot 15\ \mu\text{H}.$$

As a check, calculate the impedance at 1,1, Fig. 1, at 1 Mc/s when 2,2 are closed by 100 ohms. This should evidently be 20 ohms.

By inspection

$$Z = jL_1\omega + \frac{\dfrac{1}{jC\omega}(R_2 + jL_2\omega)}{R_2 + j\left(L_2\omega - \dfrac{1}{C\omega}\right)}$$

$$= jL_1\omega + \frac{\dfrac{L_2}{C} - j\dfrac{R_2}{C\omega}}{R_2 + j\left(L_2\omega - \dfrac{1}{C\omega}\right)}$$

$$= j67 \cdot 5 + \frac{6{,}490 - j5{,}390}{100 + j66 \cdot 1}$$

And from equation (2)/V:10

$$Z = j67 \cdot 5 + \frac{649{,}000 - 356{,}000}{100^2 + 66 \cdot 1^2} - j\frac{539{,}000 + 429{,}000}{100^2 + 66 \cdot 1^2}$$

$$= j67 \cdot 5 + 20 \cdot 4 - j67 \cdot 3.$$
$$= 20 \cdot 4 + j0 \cdot 2$$

which, by slide rule, is a good approximation to the correct answer.

Determination of Voltage across the Condenser C. The voltage across the condenser C depends on Z_c, the impedance level at this point.

This may best be determined by enunciating a series of semi-axioms which the reader should have no difficulty in proving.

(1) The limiting condition of two conjugate impedances, as their reactive components are reduced, is represented by two resistive impedances. A conjugate impedance match therefore embraces the case of a resistive impedance match.

(2) Any dissipationless linear passive system which is impedance matched on a conjugate basis at any one point of section

is matched on a conjugate basis at all other possible points of section.

(3) The equivalent parallel resistances of two conjugate impedances are equal.

(4) The only contribution to impedance matching which can be made by a shunt reactance between two impedances is the cancellation of the resultant or residual parallel reactance.

Corollary. Z_c is equivalent to half the equivalent parallel resistance of the impedance looking either way from the bridge point chosen with the circuit cut either side of condenser C. Whether condenser C is removed or not is evidently immaterial.

Hence

$$Z_c = R_1\left(1+\frac{L_1^2\omega^2}{R_1^2}\right) = R_1+\frac{L_1^2\omega^2}{R_1}$$

$$= R_1\left[1+\frac{(1+\phi+\phi^2)\omega_1\omega_2}{4\pi^2 f_1^2(\phi-1)^2\phi}\right]$$

$$= R_1\left[1+\frac{(1+\phi+\phi^2)}{(\phi-1)^2}\right]$$

$$= R_1\left[\frac{1+\phi+\phi^2+\phi^2-2\phi+1}{(\phi-1)^2}\right]$$

$$= R_1\left[\frac{2-\phi+2\phi^2}{(\phi-1)^2}\right] \quad . \quad . \quad . \quad . \quad (20)$$

Similarly

$$Z_c = R_2\left[\frac{2+\phi-\phi^2}{1+\phi+\phi^2}\right] \quad . \quad . \quad . \quad (20a)$$

If the transmitted power is P watts, the R.M.S. voltage across condenser C is equal to $\sqrt{PZ_c}$.

It follows from the above analysis that an alternative way of designing this type of network as a band-pass filter is to design two **L** networks as described in XVI:8.4, one to work from R_1 to Z_c and one to work from Z_c to R_2. The value of Z_c is obtained from equation (20) or (20*a*) and the value of ϕ is obtained by entering the impedance ratio in Fig. 3. If the value of ω used in designing the **L** networks is equal to the driving or carrier frequency, this method of design will locate the carrier or driving frequency at the geometric mid-band frequency of the structure and will provide exact impedance matching at that frequency.

The above methods of design of an inductance capacity **T** network are the correct ones to use to obtain band-pass characteristics and may be used for matching aerial circuits. Their advantages in this connection, from a band-pass point of view are, however, only important in long-wave circuits and television circuits. Three methods of designing a **T** section of this type have therefore been described.

(1) The band-pass method above which provides the optimum pass band when the section is inserted between unequal ratio impedances.

(2) The compromise method described in XVI:8.6, which provides the minimum value of the shunt condenser for the maximum permissible voltage across the condenser, and matches two unequal impedances. This method does not produce a band-pass structure, but the fall away in side-band frequencies is tolerable at medium and short waves, except when television is involved.

(3) The method described in CIII:4.1, which designs a low-pass filter with required phase shift at a given frequency, inserted between equal impedances, impedance matched at the given frequency.

It will be evident that the π network shown in Fig. 4/VII:14 may also be reduced to two **L** networks which step down to a common *low* impedance; the relation between ϕ and R_2/R_1 is given by Fig. 3 for this network also.

11.2. Lattice Fourpole Designed as a Filter Section. If a symmetrical reactive fourpole is realizable at all, it is realizable as a lattice structure. It is therefore the most general form of symmetrical fourpole, and characteristics can be realized with a lattice network which cannot be obtained with a **T** or π structure.

On the other hand, if a lattice structure has a real physical **T** or π structure as an equivalent, it is generally but not always, preferable to use the ladder (**T** or π) equivalent.

In the lattice each component reactance occurs twice, whereas in the **T** or π structures one set of reactances occurs once only.

In the lattice structure, because it is essentially a bridge, the permissible tolerances for a given degree of simulation to the ideal performance are less than in the ladder structure.

The lattice structure has, however, an importance additional to its generality, which is that its transfer vector is a function of the ratio of the impedances of its arms while its image impedance is a function of the product of these impedances. See equations (12) and (13)/XXV:3. The result is that the lattice offers the possibility of designing a structure which is capable of meeting the requirements on transfer vector and image impedance independently. The transfer vector determines the phase shift in the pass band and the attenuation in the non-pass band.

It is usually required to make the image impedance approximately constant in the pass band, and to make the phase shift approximately proportional to frequency in the pass band. Alternatively, the network may be designed to give approximately constant image impedance in the pass band while affording a choice of attenuation characteristics in the non-pass band. The means of doing this are indicated in *Communication Networks*, by E. A. Guillemin. As such methods belong to the specialized art of filter design they are not discussed here. They are evidently valuable since they enable the form of the elements of the lattice to be chosen in order to give a required performance.

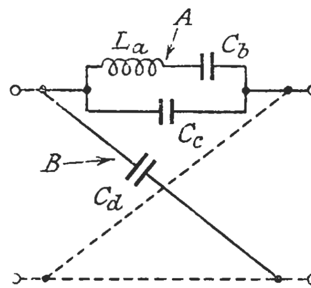


FIG. 4/XXV:11. Lattice Filter Structure.

What has been given in XXV:11.1 is the means of determining the values of the elements of a structure of chosen form in order to locate the cut-off frequencies at specified values.

A lattice structure of chosen form may be dealt with in exactly the same way.

Example. Determine the design formulae for the lattice structure shown in Fig. 4.

Conventions.

Z_I = image impedance
R = geometric midband image impedance

$$a = jL_a \quad \therefore \; jL_a\omega = a\omega$$

$$b = \frac{j}{C_b} \quad \therefore \; \frac{1}{jC_b\omega} = -\frac{b}{\omega}$$

$$c = \frac{j}{C_c}$$

$$d = \frac{j}{C_d}$$

f_1, f_2 = cut-off frequencies
$\omega_1, \omega_2 = 2\pi f_1, 2\pi f_2$
A and B = the impedance of the arms as indicated.

Then $$Z_I^2 = AB = \frac{\dfrac{cd}{\omega^2}\left(a\omega - \dfrac{b}{\omega}\right)}{-\dfrac{c}{\omega}+a\omega+\dfrac{b}{\omega}}$$

$$\therefore\ \omega_1^2 = \frac{b}{a}$$

and $$\omega_2^2 = \frac{b+c}{a}$$

There are therefore only three equations to determine four unknowns : one may therefore be chosen arbitrarily. The arbitrarily chosen element is best expressed as some constant times one of the other impedances. Let the value of this constant be h.

The values of the elements of inductance and capacity may then be found in terms of R and the cut-off frequencies as before. The resultant formulae will contain the constant h. The constant h may then be chosen to give a required performance, or may be given a value convenient to the circuit used for realizing the impedance A.

If, for instance, this impedance is realized by means of a crystal, the capacity C_c may be fixed, or may bear a constant ratio to one of the other elements. If this is the case, this condition should be used in defining the value of the fourth element before developing the equations above to obtain the required design formulae.

CHAPTER CI

FORMULAE

Greek Alphabet.

Letters.	Names.	Letters.	Names.
A α	Alpha	N ν	Nu
B β	Beta	Ξ ξ	Xi
Γ γ	Gamma	O o	Omicron
Δ δ	Delta	Π π	Pi
E ε	Epsilon	P ρ	Rho
Z ζ	Zeta	Σ σ ς	Sigma
H η	Eta	T τ	Tau
Θ θ	Theta	Υ υ	Upsilon
I ι	Iota	Φ ϕ	Phi
K κ	Kappa	X χ	Chi
Λ λ	Lambda	Ψ ψ	Psi
M μ	Mu	Ω ω	Omega

1. Conventions and Definitions.

An *integral number* or an *integer* is *any* whole number such as 1, 2, 5, 57, 1,024 (as distinct from numbers containing fractions or decimals such as 1·5, $14\frac{3}{7}$, 1,000·6, etc.).

Ordinates are distances on a graph measured parallel to the vertical axis: the axis of ordinates.

Abscissae are distances on a graph measured parallel to the horizontal axis: the axis of abscissae.

$$e = 1+\frac{1}{1}+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\ \ldots = 2{\cdot}7182813\ \ldots$$

$\pi = 3{\cdot}14159265\ \ldots$

$-$ = minus = (1) the sign of a negative quantity.

(2) an operator which, when written as a prefix to a vector magnitude, rotates that vector magnitude 180° in a counter-clockwise direction with regard to any vector magnitude written without such a prefix.

$\underline{/A} = 1\underline{/A}$

$j = \underline{/90^\circ} = e^{j90^\circ}$ = an operator which, when written as a prefix to a vector magnitude, rotates that vector magnitude 90°

in a counter-clockwise direction with regard to any vector written without such a prefix.

$$-j = \overline{\backslash 90^\circ} = e^{-j90^\circ}.$$

$$j^2 = j \times j = \underline{/90^\circ} \times \underline{/90^\circ} = e^{j90^\circ} \times e^{j90^\circ} = \underline{/180^\circ}$$
$$= e^{j180^\circ} = -1.$$

Hence $j = \sqrt{-1}.$

$$e^{jA} = \underline{/A}$$
$$e^{-jA} = \overline{/-A} = \overline{\backslash A}$$
$$-e^{jA} = -\underline{/A} = \underline{/A+180^\circ}$$

$=$ means : equals
$\neq$,, : is not equal to
$\simeq$,, : is approximately equal to
$>$,, : is greater than
$\ngtr$,, : is not greater than
$<$,, : is less than
$\nless$,, : is not less than
∞ ,, : infinity
$\propto$,, : is proportional to
Σ ,, : the sum of
$//$,, : in parallel with (electrically)
$\equiv$,, : is equivalent to (but not equal to)
. (full stop) means : multiplied by
$\sqrt{\ }$ means : the square root of

$$a^x \times a^y = a^{x+y}; \quad a^x \div a^y = a^{x-y}; \quad a^{-x} = \frac{1}{a^x}$$

$$a^{\frac{1}{x}} = \sqrt[x]{a}; \quad a^{\frac{x}{y}} = \sqrt[y]{a^x} = (\sqrt[y]{a})^x$$

2. Trigonometrical Formulae.

(1)

$$\sin(-A) = -\sin A \qquad \cos(-A) = \cos A$$
$$\tan(-A) = -\tan A$$

$$\sin(90^\circ+A) = \cos A \qquad \cos(90^\circ+A) = -\sin A$$
$$\tan(90^\circ+A) = -\cot A$$

$$\sin(90^\circ-A) = \cos A \qquad \cos(90^\circ-A) = \sin A$$
$$\tan(90^\circ-A) = \cot A$$

$$\sin(180^\circ+A) = -\sin A \qquad \cos(180^\circ+A) = -\cos A$$
$$\tan(180^\circ+A) = \tan A$$

$$\sin(180^\circ-A) = \sin A \qquad \cos(180^\circ-A) = -\cos A$$
$$\tan(180^\circ-A) = -\tan A$$

(2) $$\sin^2 A + \cos^2 A = 1 \qquad \sec^2 A - \tan^2 A = 1 \qquad \operatorname{cosec}^2 A - \cot^2 A = 1$$

(3)
$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}} \qquad \cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$
$$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$
$$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} \qquad \cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$$
$$\tan A = \frac{2\tan\frac{1}{2}A}{1-\tan^2\frac{1}{2}A}$$
$$\sin 2A = 2\sin A\cos A \qquad \cos 2A = \cos^2 A - \sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$
$$\sin 3A = 3\sin A - 4\sin^3 A \qquad \cos 3A = 4\cos^3 A - 3\cos A$$
$$\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$$
$$\cos^2 A = \tfrac{1}{2}+\tfrac{1}{2}\cos 2A \qquad \sin^2 A = \tfrac{1}{2}-\tfrac{1}{2}\cos 2A$$

(4)
$$\sin(A+B) = \sin A\cos B + \cos A\sin B;$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B;$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$$
$$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$$

(5)
$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$
$$\sin A - \sin B = 2\cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$
$$\cos A + \cos B = 2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$
$$\cos A - \cos B = -2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$

(6)
$$\sin A\cos B = \tfrac{1}{2}\sin(A+B) + \tfrac{1}{2}\sin(A-B)$$
$$\cos A\sin B = \tfrac{1}{2}\sin(A+B) - \tfrac{1}{2}\sin(A-B)$$
$$\cos A\cos B = \tfrac{1}{2}\cos(A+B) + \tfrac{1}{2}\cos(A-B)$$
$$\sin A\sin B = \tfrac{1}{2}\cos(A-B) - \tfrac{1}{2}\cos(A+B)$$

$$(7)\begin{cases} A \sin \omega t + B \cos \omega t = \sqrt{A^2+B^2} \sin \left(\omega t + \tan^{-1} \dfrac{B}{A}\right) \\ A \sin \omega t + B \sin (\omega t + \phi) \\ = \sqrt{A^2+B^2+2AB \cos \phi} \sin \left[\omega t + \tan^{-1} \dfrac{B \sin \phi}{A + B \cos \phi}\right] \end{cases}$$

$$(8)\begin{cases} \sin A = \dfrac{1}{2j}(e^{jA} - e^{-jA}) = \dfrac{1}{2j}(\underline{/A} - \overline{\backslash A}) \\ \cos A = \tfrac{1}{2}(e^{jA} + e^{-jA}) = \tfrac{1}{2}(\underline{/A} + \overline{\backslash A}) \\ \cos A + j \sin A = e^{jA} = \underline{/A} \\ e^{A+jB} = e^A e^{jB} = e^A \cos B + j e^A \sin B \end{cases}$$

2.1. Formulae Relating Inverse Trigonometrical Functions.

$$(1)\begin{cases} \sin^{-1} x = \operatorname{cosec}^{-1} \dfrac{1}{x} = \cos^{-1} \sqrt{1 - x^2} = \tan^{-1} \dfrac{x}{\sqrt{1-x^2}} \\ = \sec^{-1} \dfrac{1}{\sqrt{1-x^2}} = \cot^{-1} \dfrac{\sqrt{1-x^2}}{x} = \tfrac{1}{2} \cos^{-1} (1 - 2x^2) \\ \tan^{-1} x = \tfrac{1}{2} \cos^{-1} \left(\dfrac{1-x^2}{1+x^2}\right) = \sin^{-1} \dfrac{x}{\sqrt{1+x^2}} = \cos^{-1} \dfrac{1}{\sqrt{1+x^2}} \\ = \cot^{-1} \dfrac{1}{x} = \sec^{-1} \sqrt{1+x^2} = \operatorname{cosec}^{-1} \dfrac{\sqrt{1+x^2}}{x} \\ \cos^{-1} x = \sin^{-1} \sqrt{1-x^2} = \tan^{-1} \dfrac{\sqrt{1-x^2}}{x} = \cot^{-1} \dfrac{x}{\sqrt{1-x^2}} \\ = \sec^{-1} \dfrac{1}{x} = \operatorname{cosec}^{-1} \dfrac{1}{\sqrt{1-x^2}} \end{cases}$$

$$(2)\begin{cases} \sin^{-1} x + \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}] \\ = \cos^{-1} [\sqrt{(1-x^2)(1-y^2)} - xy] \\ \sin^{-1} x - \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} - y\sqrt{1-x^2}] \\ = \cos^{-1} [\sqrt{(1-x^2)(1-y^2)} + xy] \\ \cos^{-1} x + \cos^{-1} y = \cos^{-1} [xy - \sqrt{(1-x^2)(1-y^2)}] \\ = \sin^{-1} [y\sqrt{1-x^2} + x\sqrt{1-y^2}] \\ \cos^{-1} x - \cos^{-1} y = \cos^{-1} [xy + \sqrt{(1-x^2)(1-y^2)}] \\ = \sin^{-1} [y\sqrt{1-x^2} - x\sqrt{1-y^2}] \\ \tan^{-1} x - \tan^{-1} y = \tan^{-1} \dfrac{x-y}{1+xy}; \\ \tan^{-1} x + \tan^{-1} y = \tan^{-1} \dfrac{x+y}{1+xy} \end{cases}$$

3. Algebraical Formulae.

Binomial Theorem.

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 \ldots$$

$$\ldots + \frac{n(n-1)(n-2)\ldots(n-r+2)}{(r-1)!}a^{n-r+1}b^{r-1} \ldots \quad (1)$$

where r is the number of the term.

Factorial Number.

$$m! = m(m-1)(m-2)\ldots 5.4.3.2.1 \quad (2)$$

e.g.

$$1! = 1$$
$$2! = 2.1 = 2$$
$$3! = 3.2.1 = 6$$
$$4! = 4.3.2.1 = 24$$

Solution of quadratic equation $ax^2+bx+c=0$ is:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad (3)$$

In the Arithmetic Series of n terms: $a, a+d, a+2d, a+3d \ldots l$. Last term $l = a+(n-1)d$; Sum of n terms $= \frac{n}{2}(a+l)$

$$= \frac{n}{2}[2a+(n-1)d]. \quad (4)$$

In the Geometric Series of n terms: $a, ar, ar^2, ar^3 \ldots l$. Last term $l = ar^n - 1$;

$$\text{Sum of } n \text{ terms} = \frac{rl-a}{r-1} = \frac{a(r^n-1)}{r-1} \quad (5)$$

Permutations: the number of possible arrangements of n things taken p at a time is:

$$\frac{n!}{(n-p)!} \quad (6)$$

Combinations: the number of combinations of n things taken p at a time is:

$$\frac{n!}{p!(n-p)!} \quad (7)$$

Determinants.

A *Determinant* is a grouping of letters to define the value of a given expression : very seldom useful, but sometimes very useful.

e.g. $$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - b_1a_2$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$\begin{aligned} &= a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 \\ &= a_1A_1 + a_2A_2 + a_3A_3 \\ &= b_1B_1 + b_2B_2 + b_3B_3 \\ &= c_1C_1 + c_2C_2 + c_3C_3 \end{aligned}$$

$$\left.\begin{matrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{matrix}\right\} \text{are called the cofactors of} \left\{\begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix}\right. \text{ respectively.}$$

Evidently $A_1 = \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$, $A_2 = \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}$, $A_3 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$, etc.

Notice that

$$b_1A_1 + b_2A_2 + b_3A_3 = \begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & b_2 & c_2 \\ b_3 & b_3 & c_3 \end{vmatrix} = 0. \quad \text{Two columns the same.}$$

Also

$$a_1A_2 + b_1B_2 + c_1C_2 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0. \quad \text{Two rows the same.}$$

Any determinant with two rows or two columns the same = 0.

The sign ∇ is sometimes used to mean " determinant ". Interchanging columns changes sign of ∇.

$$\nabla\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\begin{vmatrix} b_1 & a_1 & c_1 \\ b_2 & a_2 & c_2 \\ b_3 & a_3 & c_3 \end{vmatrix} = +\begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix} = -\begin{vmatrix} c_1 & b_1 & a_1 \\ c_2 & b_2 & a_2 \\ c_3 & b_3 & a_3 \end{vmatrix}$$

Columns may be changed into rows without altering value of ∇ if done according to rule :

$$\nabla\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_3 & a_2 & a_1 \\ b_3 & b_2 & b_1 \\ c_3 & c_2 & c_1 \end{vmatrix} = \begin{vmatrix} b_3 & b_2 & b_1 \\ c_3 & c_2 & c_1 \\ a_3 & a_2 & a_1 \end{vmatrix} = -\begin{vmatrix} b_3 & b_2 & b_1 \\ a_3 & a_2 & a_1 \\ c_3 & c_2 & c_1 \end{vmatrix}$$

$$\nabla \begin{vmatrix} K & a_1 & a_2 & a_3 \\ K & b_1 & b_2 & b_3 \\ K & c_1 & c_2 & c_3 \end{vmatrix} = K \times \nabla \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\nabla \begin{vmatrix} a_1+x_1 & b_1 & c_1 \\ a_2+x_2 & b_2 & c_2 \\ a_3+x_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x_1 & b_1 & c_1 \\ x_2 & b_2 & c_2 \\ x_3 & b_3 & c_3 \end{vmatrix}$$

Solution of Simultaneous Equations.

To solve equations

$$\begin{aligned} a_1x+b_1y+c_1z &= d_1 \qquad (\times A_1) \\ a_2x+b_2y+c_2z &= d_2 \qquad (\times A_2) \\ a_3x+b_3y+c_3z &= d_3 \qquad (\times A_3) \end{aligned}$$

To find x, multiply as indicated, by cofactors

$$A_1, A_2 \text{ and } A_3 \text{ of } \nabla \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\therefore\ x(A_1a_1+A_2a_2+A_2a_3)+y(A_1b_1+A_2b_2+A_3b_3) \\ +z(A_1c_1+A_2c_2+A_3c_3) = A_1d_1+A_2d_2+A_3d_3$$

$$\therefore\ x \times \nabla + y \times 0 + z \times 0 = d_1A_1+d_2A_2+d_3A_3$$

$$\therefore\ x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \div \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Similarly, multiplying by B_1, B_2 and B_3:

$$y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \div \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Finally, multiplying by c_1, c_2 and c_3:

$$z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} \div \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

4. Logarithms.

The common logarithm of a number is the power to which 10 *must be raised to equal that number ; the base of common logarithms is* 10. Other bases can be used, but the only other base in common use is $e = 2{\cdot}71828 \ldots$ Logarithms with this base are called Napierian logarithms.

The logarithm L of any number N to base a is written

$$L = \log_a N \qquad . \quad . \quad . \quad . \qquad (1)$$

in which case, by the definition above

$$a^L = N \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Just as L is the logarithm of N, so N is called the antilogarithm (antilog) of L.

If m and n are respectively the logarithms of M and N to base 10, $10^m = M$ and $10^n = N$, so that the product $MN = 10^m 10^n = 10^{m+n}$

$$= \text{the antilogarithm of } m+n \text{ to base } 10$$

$$= \text{antilog}_{10}\,(m+n) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Similarly

$$\frac{M}{N} = \text{antilog}_{10}\,(m-n) \quad . \quad . \quad . \quad (3a)$$

Also

$$N^p = (10^L)^p = 10^{Lp} = \text{antilog}_{10}\,pL \quad . \quad . \quad (4)$$

Similarly

$$N^{\frac{1}{p}} = (10^L)^{\frac{1}{p}} = 10^{\frac{L}{p}} = \text{antilog}_{10}\,\frac{L}{p}. \quad . \quad . \quad (5)$$

$$\log_{10} x = \log_{10} e \log_e x = 0{\cdot}4343 \log_e x \quad . \quad . \quad (6)$$

$$\log_e x = \frac{\log_{10} x}{\log_{10} e} = \frac{\log_{10} x}{0{\cdot}4343} \quad . \quad . \quad . \quad . \quad (7)$$

5. Series.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \cdots \quad . \quad . \quad (1)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots \quad . \quad . \quad (2)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots \quad . \quad . \quad (3)$$

$$f(x) = f(a) + \frac{x-a}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots \quad (4)$$

$$f(a+x) = f(a) + \frac{x}{1!}f'(a) + \frac{x^2}{2!}f''(a) + \frac{x^3}{3!}f'''(a) + \cdots \quad . \quad (5)$$

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots \quad . \quad (6)$$

6. Complex Algebra.

If an impedance $Z = P\underline{/\phi} = A + jb$, P is called the argument or magnitude and ϕ the angle, while A is called the real or resistive component and B is called the imaginary or reactive component.

$$P\underline{/\phi} = P\cos\phi + jP\sin\phi = A + jB, \text{ say} \quad . \quad . \quad (1)$$

$$A+jB = \sqrt{A^2+B^2}\,\underline{/\tan^{-1}\frac{B}{A}} = P\underline{/\theta}, \text{ say} \quad . \quad . \quad (2)$$

$$A_1+jB_1+A_2+jB_2+ \ . \ . \ . = A_1+A_2+j(B_1+B_2)$$

$$P\underline{/\phi}+Q\underline{/\theta} = P\cos\phi+Q\cos\theta+j(P\sin\phi+Q\sin\theta) \quad . \quad (3)$$

$$(A+jB)(C+jD) = AC - BD+j(AD+BC) \quad . \quad . \quad (4)$$

$$P\underline{/\phi} \times Q\underline{/\theta} = PQ\underline{/\phi+\theta} \qquad \frac{P\underline{/\phi}}{Q\underline{/\theta}} = \frac{P}{Q}\underline{/\phi-\theta} \quad . \quad (5)$$

$$\frac{A+jB}{C+jD} = \sqrt{\frac{A^2+B^2}{C^2+D^2}}\,\underline{/\tan^{-1}\frac{B}{A} - \tan^{-1}\frac{D}{C}} \quad . \quad (6)$$

$$= \sqrt{\frac{A^2+B^2}{C^2+D^2}}\,\underline{/\tan^{-1}\frac{BC-AD}{AC+BD}} \quad . \quad . \quad (7)$$

$$\frac{1}{P\underline{/\phi}} = \frac{1}{P}\overline{\backslash\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\frac{1}{A+jB} = \frac{A}{A^2+B^2} - \frac{jB}{A^2+B^2} \quad . \quad . \quad . \quad (9)$$

$$\log_e(A+jB) = \tfrac{1}{2}\log_e(A^2+B^2)+j\left[\tan^{-1}\frac{B}{A}\right]$$

$$\log_e(-A+jB) = \tfrac{1}{2}\log_e(A^2+B^2)+j\left[\tan^{-1}\left(-\frac{B}{A}\right)\pm\pi\right]$$

The above are particular examples of the general solutions which are :

$$\log_e(A+jB) = \tfrac{1}{2}\log_e(A^2+B^2)+j\left(\tan^{-1}\frac{B}{A}\pm 2\pi n\right)$$

$$\log_e(-A+jB) = \tfrac{1}{2}\log_e(A^2+B^2)+j\left[\tan^{-1}\left(-\frac{B}{A}\right)\pm(2n+1)\pi\right] \quad . \quad (10)$$

$$\left.\begin{array}{l} e^{j\pi} = -1 \qquad e^{j\frac{\pi}{2}} = \underline{/90^\circ} \qquad e^{j\theta} = \underline{/\theta} \qquad e^{-j\theta} = \overline{\backslash\theta} \\ \log_e A\underline{/\theta} = \log_e A+j(\theta+2\pi n) \\ \log_e(+1) = \pm j2\pi n \text{ or zero, } \log_e(-1) = \pm j\pi(2n+1) \\ \log_e(+j) = \pm j\pi(2n+\tfrac{1}{2}), \qquad \log_e(-j) = \pm j\pi(2n+)\tfrac{3}{2} \end{array}\right\} \quad . \quad (11)$$

In (10) and (11) n is any integer, and one solution is always obtained by putting $n = 0$.

$$\left.\begin{aligned} &\text{If } R_s+jX_s = R_p//jX_p \\ &R_p = R_s\left(1+\frac{X_s^2}{R_s^2}\right) \qquad X_p = X_s\left(1+\frac{R_s^2}{X_s^2}\right) \\ &R_s = \frac{R_p}{1+\dfrac{R_p^2}{X_p^2}} \qquad X_s = \frac{X_p}{1+\dfrac{X_p^2}{R_p^2}} \end{aligned}\right\} \quad . \quad (12)$$

Transformations from Scalar Representation to Mixed Representation in terms of sin *ωt*.

Scalar Representation	*Equivalent Mixed Representation*
$\sin(\omega t+90°) = \cos \omega t$	$j \sin \omega t$
$\sin(\omega t - 90°) = -\cos \omega t$	$-j \sin \omega t$
$\cos(\omega t+90°)$	$-\sin \omega t$
$\cos(\omega t - 90°)$	$\sin \omega t$
$\sin(\omega t+\theta)$	$\underline{/\theta} \sin \omega t$
$\sin(\omega t - \theta)$	$\overline{\backslash\theta} \sin \omega t$
$\cos(\omega t+\theta) = \sin(\omega t+\theta+90°)$	$\underline{/90°+\theta} \sin \omega t$
$\cos(\omega t - \theta)$	$\underline{/90° - \theta} \sin \omega t$

. (13)

7. Electrical Formulae.

Ohms Law :

$$I \text{ (amps.)} = \frac{E \text{ (volts)}}{R \text{ (ohms)}}; \quad E \text{ (volts)} = IR \text{ (amps.-ohms)} \quad . \quad (1)$$

Power due to a voltage E volts driving a current I amps. through a resistance R ohms is

$$P = EI = \frac{E^2}{R} = I^2R \text{ watts} \quad . \quad . \quad . \quad (2)$$

Resistances in series $R = r_1+r_2+r_3+r_4$, etc. . . . (3)

Resistances in parallel $R = \dfrac{1}{\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}+\ldots}$ . . . (4)

For two resistances only $R = \dfrac{r_1 r_2}{r_1+r_2}$ . . . . . . (5)

Condensers in parallel $C = c_1+c_2+c_3+$, etc. . . (6)

Condenser in series $C = \dfrac{1}{\dfrac{1}{C_1}+\dfrac{1}{C_2}+\dfrac{1}{C_3}\ \ldots}$ . . . (7)

Two condensers in series $C = \dfrac{C_1C_2}{C_1+C_2}$ . . . . (8)

Charge in a Condenser, etc.

C = capacity of condenser in Farads.
Q = quantity of electricity stored in condenser in coulombs.
W_c = energy stored in a condenser in watt seconds = joules.
E = voltage across condenser.

$$Q = EC \quad E = \frac{Q}{C} \quad W_c = \tfrac{1}{2}E^2C. \qquad (9)$$

Current (amps.) flowing into a condenser = $C \times$ rate of change of volts across condenser in volts per second.

Current through an inductance and energy stored in magnetic field of inductance.

L = inductance in Henrys.
I = current through inductance in amperes.
E = voltage across inductance.
W_L = energy stored in magnetic field of inductance in joules.

Rate of change of current through inductance in amperes per second $= \dfrac{E}{L}$.
$E = L \times$ rate of change of current through inductance
$W_L = \tfrac{1}{2}LI^2$. (10)

In a circuit containing Inductance L, Capacity C and Resistance R:

The resonant frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

The natural frequency $f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$

The Decay Factor per second $= e^{-R/2L}$

The Decrement $= \dfrac{2fL}{R}$; see VI:5 . . (11)

For Time Constants, see IV:7.

Impedance of Two-terminal Networks.

Circuit Element	*Impedance*
L (Henrys)	$Z=jL\omega$
C (Farads)	$Z=\frac{1}{jC\omega}=-j\frac{1}{C\omega}$
$R//C$ (ohms, Farads)	$Z=\frac{R}{1+jRC\omega}=\frac{R}{1+R^2C^2\omega^2}-j\frac{R^2C\omega}{1+R^2C^2\omega^2}$
$R//L$ (ohms, Henrys)	$Z=\frac{jRL\omega}{R+jL\omega}=\frac{RL^2\omega^2}{R^2+L^2\omega^2}+j\frac{R^2L\omega}{R^2+L^2\omega^2}$
R, L and C in series	$Z=R+j\left(L\omega-\frac{1}{C\omega}\right)$
Inductance L (of resistance R) $//C$ (Parallel resonant circuit)	$Z=\frac{R+j\omega[L-C(R^2+L^2\omega^2)]}{(1-LC\omega^2)^2+R^2C^2\omega^2}$ See VI:2 and XXIV:2.23.

(12)

R.M.S. current and voltage $=\frac{1}{\sqrt{2}}\times$ peak current or voltage (when of sinusoidal wave form).

$$I_{\text{R.M.S.}}=\sqrt{I_1^2+I_2^2+I_3^2+\text{ etc.}}\quad E_{\text{R.M.S.}}=\sqrt{E_1^2+E_2^2+E_3^2} \quad . \quad . \quad . \quad . \quad (13)$$

where I_1, I_2, I_3, etc., E_1, E_2, E_3, etc., are the R.M.S. values of the component currents or voltages.

Power factor of an impedance $R+jX=P\underline{/\phi}$ is

$$\cos\phi=\frac{R}{\sqrt{R^2+X^2}} . \quad . \quad . \quad . \quad . \quad (14)$$

$$\text{Power}=E_{\text{R.M.S.}}\times I_{\text{R.M.S.}}\cos\phi=\tfrac{1}{2}\hat{e}\hat{\imath}\cos\phi \quad . \quad . \quad (15)$$

Number of decibels difference between two powers P_1 and P_2 is

$$N=10\log_{10}\frac{P_1}{P_2} \quad . \quad . \quad . \quad . \quad (16)$$

N is positive if $P_1>P_2$ and is negative if $P_1<P_2$.

Transformer : Turns Ratio $=\frac{T_1}{T_2}$ Voltage Ratio $=\frac{E_1}{E_2}$,

Current Ratio $=\frac{I_1}{I_2}$, Impedance Ratio $=\frac{R_1}{R_2}$, Inductance Ratio $=\frac{L_1}{L_2}$

$$\frac{E_1}{E_2}=\frac{I_2}{I_1}=\sqrt{\frac{R_1}{R_2}}=\frac{T_1}{T_2}=\sqrt{\frac{L_1}{L_2}} \quad . \quad . \quad . \quad (17)$$

Coupled Coils of inductance L_1 and L_2 and coupling factor k.

Mutual inductance $M = k\sqrt{L_1L_2}$

Impedance looking into L_1 with

(*a*) L_2 short circuited (*b*) L_2 closed through impedance Z

$$= jL_1\omega(1 - k^2) \qquad = L_1j\omega + \frac{M^2\omega^2}{jL_2\omega + Z}$$

E.M.F. induced in L_2 by current I_1 through $L_1 = jI_1M\omega$

Inductance of L_1 and L_2 in series $= L_1+L_2\pm 2M$

$$\text{Inductance of } L_1 \text{ and } L_2 \text{ in parallel} = \frac{L_1L_2 - M^2}{L_1+L_2\pm 2M}$$

$$= \tfrac{1}{2}(L\pm M) \text{ if } L_1 = L_2 = L$$

Measurement of coupling factor k.

If $L_a = L_1+L_2+2M$ and $L_b = L_1+L_2 - 2M$

$$k = \frac{L_a+L_b}{4\sqrt{L_1L_2}} \qquad \ldots (18)$$

8. Class C Amplifier.

Conventions.

E = H.T. volts.

E_b = working grid bias.

E_c = grid-bias volts necessary to take valve to cut-off with H.T. volts E.

$\hat{e}_g$ = grid peak volts.

$\hat{e}_a$ = anode peak volts.

i_m = R.M.S. value of make-up current.

$\hat{i}_p$ = peak anode current.

$\hat{i}_f$ = peak fundamental frequency component of current.

$\hat{i}_2, \hat{i}_3, \hat{i}_4$, etc. = peak values of harmonic frequency components of current.

$\bar{i}$ = mean anode current (anode feed).

$$f = \frac{\hat{i}_f}{\hat{i}_g} \quad g = \frac{\bar{i}}{\hat{i}_p} \quad u = \frac{\hat{i}_f}{\bar{i}} = \frac{f}{g} \quad \text{(See Fig. 1/X:22)}$$

$$f_2 = \frac{\hat{i}_2}{\hat{i}_p}, \quad f_3 = \frac{\hat{i}_3}{\hat{i}_p}, \quad f_4 = \frac{\hat{i}_4}{\hat{i}_p}, \text{ etc.}$$

P_0 = R.F. power output.

P_{in} = D.C. power input to anode circuit.

$$h = \frac{\hat{e}_a}{E}.$$

Z_c = impedance facing anode.
Z_L = load line impedance.
η = anode efficiency.
θ = angle of current flow.
μ = amplification factor of valve.

Formulae.

$$Z_L = \frac{\hat{e}_a}{\dot{i}_p} \qquad (1)$$

$$Z_c = \frac{\hat{e}_a}{\hat{i}_f} = \frac{Z_L}{f} \qquad (2)$$

$$\hat{i}_f = \frac{\hat{e}_a}{Z_c} = f\,\dot{i}_p \qquad (3)$$

$$P_0 = \frac{\hat{e}_a^2}{2Z_c} = \tfrac{1}{2}\hat{i}_f^2 Z_c = \tfrac{1}{2}f^2\dot{i}_p^2 Z_c = \tfrac{1}{2}f\,\dot{i}_p^2 Z_L = \tfrac{1}{2}\hat{e}_a\hat{i}_f = \tfrac{1}{2}\hat{e}_a f\,\dot{i}_p \qquad (4)$$

$$P_{in} = E\bar{i} \qquad (5)$$

$$\eta = \frac{P_0}{P_{in}} = \frac{1}{2}\frac{\hat{e}_a\hat{i}_f}{E\bar{i}} = \frac{1}{2}u\frac{\hat{e}_a}{E} \qquad (6)$$

Also $\hat{i}_2 = f_2\dot{i}_p$, $\hat{i}_3 = f_3\dot{i}_p$, $\hat{i}_4 = f_4\dot{i}_p$, etc. (7)

$$\dot{i}_m = \dot{i}_p\sqrt{\tfrac{1}{2}(f^2+f_2^2+f_3^2+f_4^2+\ldots)} \qquad (8)$$

$$\theta = \cos^{-1}\frac{E_b - E_c}{\hat{e}_g - \frac{\hat{e}_a}{\mu}} \qquad (9)$$

Class B Amplifier. In a class B amplifier $\theta = 180°$, $f = 0{\cdot}5$, $g = \frac{1}{\pi} = 0{\cdot}318$, and $u = \frac{\pi}{2} = 1{\cdot}57$. All the class C relations hold in the case of a class B Amplifier, but certain of them can be simplified as follows :

$$\dot{i}_p = \pi\bar{i}, \quad \hat{i}_f = 0{\cdot}5\dot{i}_p = 1{\cdot}57\bar{i} \qquad (10)$$

$$Z_c = 2Z_L \qquad (11)$$

$$P_0 = \tfrac{1}{8}\dot{i}_p^2 Z_c = \tfrac{1}{4}\dot{i}_p^2 Z_L = \tfrac{1}{4}\hat{e}_a\dot{i}_p \qquad (12)$$

$$\eta = \frac{\pi}{4}\frac{\hat{e}_a}{E} = 0{\cdot}785\frac{\hat{e}_a}{E} = 0{\cdot}785h. \qquad (13)$$

Efficiency of Linear Class B Amplifier for 100% Modulated Wave. When no modulation is applied

$$\eta_0 = 0{\cdot}785 \times \tfrac{1}{2}\hat{h} = 0{\cdot}3925\hat{h} \qquad (14)$$

where $\hat{h}$ is the maximum value of $\frac{\hat{e}_a}{E}$, i.e. the value at 100% peak modulation.

9. Modulation.

9.1. Power in Modulated Carrier Wave.

$$P = P_0\left(1+\frac{m^2}{2}\right) \quad . \quad . \quad . \quad . \quad (1)$$

where P_0 = power in unmodulated wave
m = modulation depth.

9.2. Heising Formulae for Determination of Percentage Modulation from Aerial Currents.

I_0 = observed R.M.S. aerial current without modulation.
I_m = observed R.M.S. aerial current with percentage modulation m due to a sinusoidal modulating wave.

$$m = \sqrt{2\left[\frac{I_m^2}{I_0^2} - 1\right]} \quad . \quad . \quad . \quad (2)$$

$$I_m = I_0\sqrt{1+\frac{m^2}{2}} \quad . \quad . \quad . \quad . \quad (3)$$

10. Transmission Line and Network Formulae.

Conventions.

E_s = sending end voltage.
E_x = voltage at any point distant x from the sending end.
E_r = receiving-end voltage.
I_s = sending-end current.
I_x = current at any point distant x from the sending end.
I_r = receiving-end current.
l = length of transmission line.
α' = phase-shift constant of transmission line per unit length.
$\alpha = \alpha' l$ = phase-shift constant of transmission line = phase shift constant of fourpole, = the angle of u, see below.
β' = attenuation constant of transmission line per unit length.
$\beta = \beta' l$ = attenuation constant of transmission line = attenuation constant of fourpole.
p = the propagation constant per unit length = $\beta' + j\alpha'$.
P = the propagation constant = pl = the image transfer constant in the case of fourpoles.
u = transfer vector of transmission line or fourpole, see XXIV:4.
a = the attenuation ratio of the network = $|u|$.
Z_0 = characteristic impedance of transmission line.
Z_1 = input image impedance of fourpole = Z_0 in the case of a transmission line.

Z_2 = output image impedance of fourpole = Z_0 in the case of a transmission line.

$r = \sqrt{\dfrac{Z_1}{Z_2}}$

Z_I = image impedance of a symmetric fourpole = $Z_1 = Z_2$.

$S_n = \frac{1}{2}(u - 1/u)$.

$C_s = \frac{1}{2}(u + 1/u)$.

$T_n = \dfrac{u^2 - 1}{u^2 + 1}$

$C_t = \dfrac{u^2 + 1}{u^2 - 1}$

Z_r = receiving-end impedance = impedance terminating output of line or fourpole.

Z_s = sending-end impedance = impedance looking into input of line or fourpole.

Formulae.

$$u = S_n + C_s = e^P = e^{\beta + j\alpha} = e^\beta \times e^{j\alpha} = e^\beta \underline{/\alpha} = a\underline{/\alpha} \quad . \quad (1)$$

$$a = e^\beta \qquad e^{j\alpha} = \underline{/\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Lossless ($a = 1, \beta = 0$) *Transmission Line Open Circuited at Receiving End.*

$$E_x = E_s \frac{\cos(\alpha' l - \alpha' x)}{\cos \alpha' l}; \quad E_r = \frac{E_s}{\cos \alpha' l} = \frac{-jZ_0 I_s}{\sin \alpha' l} \quad . \quad (3)$$

$$I_x = I_s \frac{\sin(\alpha' l - \alpha' x)}{\sin \alpha' l}; \quad I_r = 0 \quad . \quad . \quad . \quad . \quad (4)$$

$$Z_s = -jZ_0 \cot \alpha' l = \frac{E_s}{I_s} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Lossless Dissymmetrical Fourpole Open Circuited at Receiving End.

$$E_r = \frac{E_s}{r \cos \alpha} = \frac{-jZ_1 I_s}{r \sin \alpha}; \quad I_r = 0 \quad . \quad . \quad (6)$$

$$Z_s = -jZ_1 \cot \alpha = \frac{E_s}{I_s} \quad . \quad . \quad . \quad . \quad (7)$$

Lossless Transmission Line Short Circuited at Receiving End.

$$E_x = E_s \frac{\sin(\alpha' l - \alpha' x)}{\sin \alpha' l}; \quad E_r = 0 \quad . \quad . \quad . \quad . \quad (8)$$

$$I_x = I_s \frac{\cos(\alpha' l - \alpha' x)}{\cos \alpha' l}; \quad I_r = \frac{I_s}{\cos \alpha' l} = \frac{-jE_s}{Z_0 \sin \alpha' l} \quad . \quad (9)$$

$$Z_s = jZ_0 \tan \alpha' l = \frac{E_s}{I_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Lossless Dissymmetrical Fourpole, Short Circuited at Receiving End

$$E_r = 0\,; \quad I_r = \frac{rI_s}{\cos\alpha} = \frac{-jrE_s}{Z_1 \sin\alpha} \quad . \quad . \quad (11)$$

$$Z_s = jZ_1 \tan\alpha = \frac{E_s}{I_s} \quad . \quad . \quad . \quad . \quad (12)$$

Lossless Transmission Line Terminated in an Impedance Z_r at Receiving End.

$$E_x = E_s \frac{(Z_0+Z_r)\overline{\backslash \alpha' x} + (Z_r - Z_0)\overline{\backslash 2\alpha' l - \alpha' x}}{Z_0+Z_r+(Z_r - Z_0)\overline{\backslash 2\alpha' l}} . \quad . \quad (13)$$

$$I_x = I_s \frac{(Z_0+Z_r)\overline{\backslash \alpha' x} - (Z_r - Z_0)\overline{\backslash 2\alpha' l - \alpha' x}}{Z_0+Z_r-(Z_r - Z_0)\overline{\backslash 2\alpha' l}} \quad . \quad (14)$$

$$E_r = E_s \frac{2Z_r}{(Z_0+Z_r)\underline{/\alpha' l}+(Z_r - Z_0)\overline{\backslash \alpha' l}}$$

$$= \frac{E_s}{\cos\alpha' l + j\dfrac{Z_0}{Z_r}\sin\alpha' l} . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$= \frac{I_s Z_0}{\dfrac{Z_0}{Z_r}\cos\alpha' l + j\sin\alpha} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$I_r = I_s \frac{2Z_0}{(Z_0+Z_r)\underline{/\alpha' l}+(Z_0 - Z_r)\overline{\backslash \alpha' l}}$$

$$= \frac{I_s}{\cos\alpha' l + j\dfrac{Z_r}{Z_0}\sin\alpha' l} . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$= \frac{E_s}{Z_r \cos\alpha' l + jZ_0 \sin\alpha' l} \quad . \quad . \quad . \quad . \quad (18)$$

$$Z_s = Z_0 \frac{Z_r + jZ_0 \tan\alpha' l}{Z_0 + jZ_r \tan\alpha' l} = \frac{E_s}{I_s} \quad . \quad . \quad . \quad . \quad (19)$$

Dissipationless Quarter-Wave Network.

Receiving End Open.

$$E_r = E_s \times \text{infinity} = -jZ_0 I_s \quad . \quad . \quad . \quad (20)$$

$$I_r = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$Z_s = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Receiving End Shorted.

$$E_r = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$I_r = I_s \times \text{infinity} = \frac{-jE_s}{Z_0} \quad . \quad . \quad . \quad (24)$$

$$Z_s = \text{infinity} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Receiving End Terminated in Z_r.

$$E_r = -j\frac{Z_r}{Z_0} \times E_s = -jZ_0I_s. \quad . \quad . \quad (26)$$

$$I_r = -j\frac{Z_0}{Z_r} \times I_s = -j\frac{E_s}{Z_0} \quad . \quad . \quad . \quad (27)$$

$$Z_s = \frac{Z_0^2}{Z_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

General Dissymmetrical Fourpole with Dissipation.

This embraces all the above cases in section 10.

Remember that $r = \sqrt{\frac{Z_1}{Z_2}}$.

$$E_r = \frac{\frac{1}{r}E_s}{C_s + \frac{Z_2}{Z_r}S_n} \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$= \frac{\frac{1}{r}I_sZ_1}{\frac{Z_2}{Z_r}C_s + S_n} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

$$I_r = \frac{rI_s}{C_s + \frac{Z_r}{Z_0}S_n} \quad . \quad . \quad . \quad . \quad . \quad (31)$$

$$= \frac{\frac{1}{r}E_s}{Z_rC_s + Z_2S_n} \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$Z_s = Z_1\frac{Z_r + Z_2T_n}{Z_2 + Z_rT_n} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

The lossless (dissipationless) case of a dissymmetrical fourpole can be derived by making substitutions from (34) below.

When $\beta = 0$:

$$\left.\begin{aligned} S_n &= j \sin \alpha \\ C_s &= \cos \alpha \\ T_n &= j \tan \alpha \\ C_t &= -j \cot \alpha \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (34)$$

Relations between the Simple Vector Functions S_n, C_s, T_n, C_t and Hyperbolic Functions. **See CI : 15.**

Remember that $u = e^P$, and $\sqrt{u} = e^{P/2}$ . . . (35)

$$\left.\begin{aligned} S_n &= \text{sins } u = \sinh P & C_s &= \text{coss } u = \cosh P \\ T_n &= \text{tans } u = \tanh P & C_t &= \text{cots } u = \coth P \\ S_c &= \text{secs } u = \text{sech } P & C_c &= \text{cosecs } u = \text{cosech } P \end{aligned}\right\}. \quad (36)$$

It follows that any relations between hyperbolic functions of P are satisfied by the corresponding simple vector functions of u. Hence :

$$\left.\begin{aligned} C_s^2 - S_n^2 = 1, \quad 1 - T_n^2 = 1/C_s^2 \\ C_t^2 - 1 = 1/S_n^2 \end{aligned}\right\} \quad . \quad . \quad (37)$$

$$\sinh P/2 = \text{sins } \sqrt{u}, \ \cosh P/2 = \text{coss } \sqrt{u}, \text{ etc.} \quad . \quad . \quad (38)$$

11. Reflection Effects.

Reflection Loss is difference in decibels between power in actual load $|L|\underline{/\theta}$ and power in ideal load matched to generator impedance $|G|\underline{/\theta}$; it is given by :

$$\text{R.L.} = 10 \log_{10}\left[\frac{|L|^2+|G|^2+2|LG|\cos(\phi-\theta)}{4|LG|} \times \frac{\cos\phi}{\cos\theta}\right] \text{db.} \quad . \quad (1)$$

When $\phi = \theta = 0$ so that G and L represent pure resistances :

$$\text{R.L.} = 20 \log_{10} \frac{1+r}{2\sqrt{r}} \text{ db. where } r = \frac{L}{G} \text{ or } \frac{G}{L}. \quad . \quad (1a)$$

Transition Loss is difference in decibels between power in load $|L|\underline{/\theta}$ and power in load having impedance equal to conjugate of generator impedance. It is given by :

$$\text{T.L.} = 10 \log_{10}\left[\frac{|L|^2+|G|^2+2|LG|\cos(\phi-\theta)}{4|LG|\cos\phi\cos\theta}\right] \text{db.} \quad . \quad (2)$$

Insertion Loss = the power loss in decibels consequent on the insertion of a fourpole, of image impedances Z_1 and Z_2 and transfer vector u, between a generator of impedance G and a load of impedance L. It is given by :

$$\text{I.L.} = 20 \log_{10}\left[\frac{(LZ_1+GZ_2)C_s+(Z_1Z_2+LG)S_n}{(L+G)Z_1}\right]r \text{ db.} \quad . \quad (3)$$

where $r = \sqrt{\frac{Z_1}{Z_2}}$, $G = |G|\underline{/\phi}$, $L = |L|\underline{/\theta}$.

Standing Waves in Transmission Line without Loss.

E_0 and I_0 = voltage and current in a line terminated in its characteristic impedance Z_0, for any power P supplied to the sending-end.

E_n and I_n = voltage and current for the same power, at nodes of voltage and current respectively, when the line is terminated in an impedance other than its characteristic impedance Z_0.

E_a and I_a = voltage and current at antinodes of voltage and current respectively under the last condition.

Z_{va} = impedance at voltage antinode.

Z_{vn} = impedance at voltage node.

r = standing wave ratio.

Voltage antinodes and current nodes occur at the same place ; and voltage nodes and current antinodes occur at the same place.

$$\frac{E_0^2}{Z_0} = I_0^2Z_0 = \frac{E_n^2}{Z_{vn}} = I_a^2Z_{vn} = \frac{E_a^2}{Z_{va}} = I_n^2Z_{va} \quad . \quad . \quad (4)$$

$$\sqrt{r} = \frac{E_0}{E_n} = \frac{I_n}{I_0} = \frac{E_a}{E_0} = \frac{I_0}{I_a} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$r = \frac{E_a}{E_n} = \frac{I_a}{I_n} = \frac{Z_{va}}{Z_0} = \frac{Z_0}{Z_{vn}} = \sqrt{\frac{Z_{va}}{Z_{vn}}}. \quad . \quad . \quad (6)$$

12. Matrices.

Multiplication of Matrices :

$$\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} \times \begin{Vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{Vmatrix} = \begin{Vmatrix} a_{11}A_{11}+a_{12}A_{21} & a_{11}A_{12}+a_{12}A_{22} \\ a_{21}A_{11}+a_{22}A_{21} & a_{21}A_{12}+a_{22}A_{22} \end{Vmatrix}$$

Division of Matrices :

$$\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} \div \begin{Vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{Vmatrix} = \frac{\begin{Vmatrix} A_{12}a_{21} - A_{22}a_{11} & A_{12}a_{22} - A_{22}a_{12} \\ A_{21}a_{11} - A_{11}a_{21} & A_{21}a_{12} - A_{11}a_{22} \end{Vmatrix}}{A_{12}A_{21} - A_{11}A_{22}}$$

13. Wire Resistance.

ρ = specific resistance of material of conductor in ohms/cm.^{3}.
l = length of wire in centimetres.
A = area of cross-section in square centimetres.

$$\text{Resistance } R = \rho\frac{l}{A}$$

14. Johnson Noise: The e.m.f. of Thermal Agitation.

In any resistance R at temperature $T°$ C. the open circuit noise voltage appearing at the terminals of the resistance in any frequency range from f_1 to f_2 c/s is:

$$N = 7{\cdot}4 \times 10^{-6}\sqrt{(273+T)(f_2 - f_1)R} \text{ microvolts.}$$

15. Correlation of Simple Vectors with Hyperbolic Functions, and Related Formulae.

It will probably have been appreciated by now that anyone who has mastered the use of simple vectors has also mastered the use of hyperbolic functions in all applications relating to transmission problems. This is because, on the one hand, any transmission formulae which have been derived in terms of simple vectors, can be converted into hyperbolic functions by means of equations (5) below, and on the other, any formulae which are expressed in terms of hyperbolic functions can be expressed in terms of simple vectors by means of equations (5) below. Since the majority of adept users of hyperbolic functions have to go through analogous, or even more complicated, processes in developing or interpreting expressions in terms of hyperbolic functions, the user of simple vectors is now as well equipped as the user of hyperbolic functions.

It may, however, be claimed that the habitual user of hyperbolic functions is at an advantage compared with the user of simple vectors in that there exists a tabulated system of relations between hyperbolic functions, and between hyperbolic functions and other functions, to which he can refer when he wants to transform an expression into a form which he hopes will be simpler. The user of simple vectors will not normally need such an aid to thinking. In order to bolster up his morale, however, there are given below the more common relations in terms of hyperbolic functions and the equivalent relations in terms of simple vectors. Often the two sets of relations are equated. A few special relations between simple vectors and hyperbolic functions are also given.

Conventions.

u = transfer vector = $a\underline{/\alpha}$ $\quad$ P = propagation constant = $\beta+j\alpha$

a = attenuation ratio $\quad$ β = attenuation constant

α = phase shift constant

e = base of Napierean logarithms — 2·718 . . .

$$\left.\begin{aligned} u &= e^P,\ \frac{1}{u} = e^{-P} \\ a\underline{/\alpha} &= e^{\beta+j\alpha} \\ a &= e^{\beta} \\ \underline{/\alpha} &= e^{j\alpha} = 1\underline{|\alpha} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\left.\begin{aligned} S_n &= \text{sins}\ u = \tfrac{1}{2}\left(u - \frac{1}{u}\right) = \tfrac{1}{2}(e^P - e^{-P}) = \sinh P \\ C_n &= \text{coss}\ u = \tfrac{1}{2}\left(u+\frac{1}{u}\right) = \tfrac{1}{2}(e^P+e^{-P}) = \cosh P \\ T_n &= \text{tans}\ u = \frac{u^2 - 1}{u^2 + 1} = \frac{e^{2P} - 1}{e^{2P} + 1} = \tanh P \\ \text{sins}\ a\underline{/\alpha} &= \sinh(\beta+j\alpha) \qquad \text{coss}\ a\underline{/\alpha} = \cosh(\beta+ja) \end{aligned}\right\} \quad . \quad (2)$$

$$\left.\begin{aligned} \text{coss}^2\ u - \text{sins}^2\ u &= 1 \\ \text{tans}^2\ u+\text{secs}^2\ u &= 1 \\ \text{cots}^2\ u - \text{cosec}^2\ u &= 1 \\ \text{coss}\ u = \sqrt{1+\text{sins}^2\ u} &= \sqrt{\frac{1}{1 - \text{tans}^2\ u}} \\ \text{sins}\ u = \sqrt{\text{coss}^2\ u - 1} &= \sqrt{\frac{1}{\text{cots}^2\ u - 1}} \\ \text{coss}\ u+\text{sins}\ u &= u \end{aligned}\right\} \quad . \quad . \quad (3)$$

$$\left.\begin{aligned} \sinh \beta\pm j\alpha &= \sinh\beta\cos\alpha\pm j\cosh\beta\sin\alpha = \pm j\sin(\alpha\mp j\beta) \\ \text{sins}\ a\underline{/\alpha} &= \text{sins}\ a\cos\alpha+j\ \text{coss}\ a\sin\alpha = j\sin(\alpha - j\beta) \\ &= \tfrac{1}{2}\left(a - \frac{1}{a}\right)\cos\alpha+j\tfrac{1}{2}\left(a+\frac{1}{a}\right)\sin\alpha \\ \text{sins}\ a\overline{\backslash\alpha} &= \text{sins}\ a\cos\alpha - j\ \text{coss}\ a\sin\alpha = -j\sin(\alpha+j\beta) \\ &= \tfrac{1}{2}\left(a - \frac{1}{a}\right)\cos\alpha - j\tfrac{1}{2}\left(a+\frac{1}{a}\right)\sin\alpha \\ \cosh \beta\pm j\alpha &= \cosh\beta\cos\alpha\pm j\sinh\beta\sin\alpha = \cos(\alpha\mp j\beta) \\ \text{coss}\ a\underline{/\alpha} &= \text{coss}\ a\cos\alpha+j\ \text{sins}\ a\sin\alpha = \cos(\alpha - j\beta) \\ &= \tfrac{1}{2}\left(a+\frac{1}{a}\right)\cos\alpha+j\tfrac{1}{2}\left(a - \frac{1}{a}\right)\sin\alpha \\ \text{coss}\ a\overline{\backslash\alpha} &= \text{coss}\ a\cos\alpha - j\ \text{sins}\ a\sin\alpha = \cos(\alpha+j\beta) \end{aligned}\right\} \quad . \quad (4)$$

$$= \tfrac{1}{2}\left(a+\frac{1}{a}\right)\cos\alpha - j\tfrac{1}{2}\left(a-\frac{1}{a}\right)\sin\alpha$$
$$\text{tans}\, a\underline{/\alpha} = \frac{\text{tans}\, a + j\tan\alpha}{1 + j\,\text{tans}\, a\tan\alpha} = \frac{\tanh\beta + j\tan\alpha}{1 + j\tanh\beta\tan\alpha}$$
$$\text{tans}\, a\overline{\backslash\alpha} = \frac{\text{tans}\, a - j\tan\alpha}{1 - j\,\text{tans}\, a\tan\alpha} = \frac{\sinh 2\beta - j\sin 2\alpha}{\cosh 2\beta + \cos 2\alpha}$$
$$\text{sins}\,\underline{/\beta} = \sinh j\beta = j\sin\beta$$
$$\text{coss}\,\underline{/\beta} = \cosh j\beta = \cos\beta$$
$$\text{tans}\,\underline{/\beta} = \tanh j\beta = j\tan\beta$$

(4) *contd.*

$$\sin\beta = -j\tfrac{1}{2}(e^{j\beta} - e^{-j\beta}) = -j\tfrac{1}{2}(\underline{/\beta} - \overline{\backslash\beta})$$
$$= -j\sinh j\beta = -j\,\text{sins}\underline{/\beta}$$
$$\cos\beta = \tfrac{1}{2}(e^{j\beta} + e^{-j\beta}) = \tfrac{1}{2}(\underline{/\beta} + \overline{\backslash\beta}) = \cosh j\beta = \text{coss}\underline{/\beta}$$
$$\tan\beta = -j\frac{e^{j2\beta} - 1}{e^{j2\beta} + 1} = -j\frac{\underline{/2\beta} - 1}{\underline{/2\beta} + 1}$$
$$= -j\tanh j\beta = -j\tan\underline{/\beta}$$
$$\sin(\beta \pm j\alpha) = \sin\beta\cosh\alpha \pm j\cos\beta\sinh\alpha$$
$$= \sin\beta\,\text{coss}\, e^{\alpha} \pm j\cos\beta\,\text{sins}\, e^{\alpha}$$
$$= \pm j\sinh(\alpha \mp j\beta) = j\,\text{sins}\,(e^{\alpha - j\beta}) \text{ or } -j\,\text{sins}\,(e^{\alpha + j\beta})$$
$$\cos(\beta \pm j\alpha) = \cos\beta\cosh\alpha \mp j\sin\beta\sinh\alpha$$
$$= \cos\beta\,\text{coss}\, e^{\alpha} \mp j\sin\beta\,\text{sins}\, e^{\alpha}$$
$$= \cosh(\alpha \mp j\beta) = \text{coss}\,(e^{\alpha - j\beta}) \text{ or } \text{coss}\,(e^{\alpha + j\beta})$$
$$\tan(\beta \pm j\alpha) = \frac{\sin 2\beta \pm j\sinh 2\alpha}{\cosh 2\beta + \cosh 2\alpha} = \frac{\sin 2\beta \mp j\,\text{sins}\, e^{2\alpha}}{\cos 2\beta + \text{coss}\, e^{2\alpha}}$$

(5)

If n = any integer.

$$\text{sins}\underline{/n\pi} = \sinh jn\pi = 0$$
$$\text{coss}\underline{/n\pi} = \cosh jn\pi = \cos n\pi = (-1)^n$$
$$\text{tans}\underline{/n\pi} = \tanh jn\pi = 0$$
$$\text{sins}\, a\underline{/\tfrac{\pi}{2}} = \sinh\left(\beta + j\frac{\pi}{2}\right) = j\cosh\beta = j\,\text{coss}\, a$$
$$\text{sins}\frac{\underline{/\tfrac{\pi}{2}}}{a} = \sinh\left(-\beta + j\frac{\pi}{2}\right) = j\cosh\beta = j\,\text{coss}\, a$$
$$\text{sins}\, a\overline{\backslash\tfrac{\pi}{2}} = \sinh\left(\beta - j\frac{\pi}{2}\right) = -j\cosh\beta = -j\,\text{coss}\, a$$

(6)

$$
\left.
\begin{aligned}
\text{sins}\; a\underline{/n\pi} &= \sinh(\beta + jn\pi) = (-1)^n \sinh\beta = (-1)^n\, \text{sins}\; a \\
\text{sins}\; a\underline{/2\pi} &= \sinh(\beta + j2\pi) = \sinh\beta = \text{sins}\; a \\
\text{coss}\; a\underline{\Big/\tfrac{\pi}{2}} &= \cosh\left(\beta + j\frac{\pi}{2}\right) = j\sinh\beta = j\,\text{sins}\; a \\
\text{coss}\; a\overline{\Big\backslash\tfrac{\pi}{2}} &= \cosh\left(\beta - j\frac{\pi}{2}\right) = -j\sinh\beta = -j\,\text{sins}\; a \\
\text{coss}\, \frac{\underline{/\frac{\pi}{2}}}{a} &= \cosh\left(-\beta + j\frac{\pi}{2}\right) = -j\sinh\beta = -j\,\text{sins}\; a \\
\text{coss}\; a\underline{/n\pi} &= \cosh(\beta + jn\pi) = (-1)^n \cosh\beta = (-1)^n\, \text{coss}\; a \\
\text{coss}\; a\underline{/2\pi} &= \cosh(\beta + j2\pi) = \cosh\beta = \text{coss}\; a \\
\text{tans}\; a\underline{\Big/\tfrac{\pi}{2}} &= \tanh\left(\beta + j\frac{\pi}{2}\right) = \coth\beta = \text{cots}\; a \\
\text{tans}\; a\overline{\Big\backslash\tfrac{\pi}{2}} &= \tanh\left(\beta - j\frac{\pi}{2}\right) = \coth\beta = \text{cots}\; a \\
\text{tans}\; a\underline{/n\pi} &= \tanh(\beta + jn\pi) = \tanh\beta = \text{tans}\; a \\
\text{sins}\; \underline{\Big/(2n+1)\tfrac{\pi}{2}} &= \sinh j(2n+1)\frac{\pi}{2} = j\sin(2n+1)\frac{\pi}{2} = \pm j^{*} \\
\text{coss}\; \underline{\Big/(2n+1)\tfrac{\pi}{2}} &= \cosh j(2n+1)\frac{\pi}{2} = 0
\end{aligned}
\right\} \quad (6)
$$

$$\log_e \text{sins}^{-1} x = \sinh^{-1} x = \log_e\left(x + \sqrt{x^2+1}\right)$$

$$\log_e \text{coss}^{-1} x = \cosh^{-1} x = \log_e\left(x + \sqrt{x^2-1}\right)$$

$$\log_e \text{tans}^{-1} x = \tanh^{-1} x = \tfrac{1}{2}[\log_e(1+x) - \log_e(1-x)]$$

$$\text{sins}^{-1} x = x + \sqrt{x^2+1} \qquad \text{sins}^{-1} jx = jx + \sqrt{1-x^2}$$

$$\text{coss}^{-1} x = x + \sqrt{x^2-1} \qquad \text{coss}^{-1} jx = jx + j\sqrt{1+x^2}$$

$$\text{tans}^{-1} x = \sqrt{\frac{1+x}{1-x}} \qquad \text{tans}^{-1} jx = \sqrt{\frac{1+jx}{1-jx}}$$

* $+j$ when $n = 0, 2, 4, 6$, etc.
$-j$ when $n = 1, 3, 5, 7$, etc.

CHAPTER CII

TUNING BY CLASSICAL METHODS DESCRIBED IN VII:14.4 ; AND UNTUNABLE CIRCUITS

1. Values to which Condensers in Double Parallel Tuned Mutual Coupling are adjusted by Classical Methods of Tuning.

THESE are determined in CII:1.2 after the preliminary discussion in CII:1.1.

1.1. Resonance Condition in a Circuit containing an Inductance in series with a Parallel Combination of a Resistance and a Condenser. The circuit to be considered is shown in Fig. 1. The resonance condition is defined for any frequency,

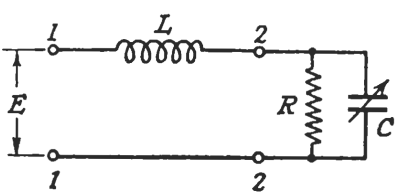


FIG. 1/CII:1.—Circuit tuned to Minimum Impedance at 1,1, by Variation of Capacity C.

any values having been assigned to two out of the three elements, by the value of the third element which gives maximum current at terminals 1,1 when an e.m.f. is applied across 1,1 as shown. Resonance corresponds therefore to maximum current through L, and therefore through R, since when the current through L is a maximum the voltage across R is a maximum.

1.11. Case 1. Values assigned to R and C ; and L adjusted to maximum current through L and R. In this case the impedance at 1,1 is

$$Z = jL\omega - j\frac{R^2C}{1+R^2C^2\omega^2} + \frac{R}{1+R^2C^2\omega^2} \quad . \quad . \quad (1)$$

If L is varied the impedance is evidently a minimum when the total reactance is zero, i.e. when

$$L\omega = \frac{R^2C\omega}{1+R^2C^2\omega^2} \quad \text{or} \quad L = \frac{R^2C}{1+R^2C^2\omega^2}$$

1.12. Case 2. Values assigned to L and R; and C adjusted to maximum current through L and R, i.e. to minimum impedance at 1,1.

To determine the value of C. This can most simply be determined as follows.

Let $a+jb$ be the impedance presented at terminals 2,2 by R and C in parallel.

Then
$$a = \frac{R}{1+R^2C^2\omega^2} \quad . \quad . \quad . \quad . \quad (2)$$

and
$$b = -\frac{R^2C\omega}{1+R^2C^2\omega^2}. \quad . \quad . \quad . \quad (2a)$$

Eliminating ω between (2) and (2a):
From (1)

$$a+aR^2C^2\omega^2 = R$$

$$\therefore \omega^2 = \frac{R-a}{aR^2C^2} \text{ and } \omega = \sqrt{\frac{R-a}{aR^2C^2}}$$

Substituting these values of ω^2 and ω in (2)

$$\therefore b = -\frac{R^2C\sqrt{\dfrac{R-a}{aR^2C^2}}}{1+\dfrac{R^2C^2(R-a)}{aR^2C^2}} = -\frac{R\sqrt{a(R-a)}}{a+R-a}$$

$$\therefore b = -\sqrt{aR-a^2} \quad . \quad . \quad . \quad (3)$$

or
$$a^2+b^2-aR = 0. \quad . \quad . \quad . \quad (4)$$

If b is plotted as a function of a on a plane of resistance and reactance, the resultant curve is a semi-circle of radius $\frac{1}{2}R$ with its

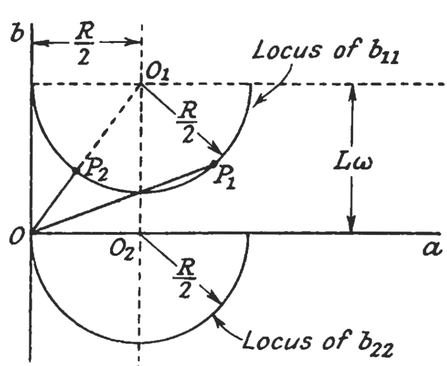


FIG. 2/CII:1.—Plot of Resistance a against Resistance b.

centre at the point $a = \frac{R}{2}$, $b = 0$; see Fig. 2. This semicircle is evidently a plot of the reactance appearing at 2,2 against the resistance appearing at 2,2, when R and ω are fixed and C is varied from 0 to infinity (or when C is fixed and ω is varied from 0 to infinity, but this case is not here of interest).

The reactance b observed at 2,2 may conveniently be called b_{22}, i.e. $b = b_{22}$.

Taking the case where ω is fixed, the value of $L\omega$ does not change as C is varied and b_{11}, the total reactance in circuit appearing at 1,1, is given by the value $b_{22}+L\omega$, that is,

$$b_{11} = b_{22}+L\omega = -\sqrt{aR - a^2}+L\omega \quad . \quad . \quad (5)$$

If b_{11} is plotted as a function of a, the resultant curve is simply the curve defined by equation (4) displaced in the positive direction by an amount $L\omega$. In other words, it is a semicircle of radius $R/2$ with centre at the point $a = \frac{R}{2}$, $b = L\omega$.

The two semicircles drawn on the ab plane constituting the loci of b_{11} and b_{22} are shown in Fig. 2, b_{22} being drawn with centre O_1 and b_{11} with centre O_2.

If a_1 and b_1 are particular corresponding values of a and b_{11}, when plotted on the ab plane, the point P_1 so defined must lie somewhere on the locus of b_{11}. Further, since $OP_1 = \sqrt{a_1^2+b_1^2}$, OP_1 is the magnitude of the corresponding impedance. Hence the length of any line drawn from O to the curve b_{11} gives the magnitude of the impedance at terminals 1,1 corresponding to some value of C. The shortest line drawn from O to the circle b_{11} is evidently the line which, when projected, passes through O_1. This is the line OP_2, and OP_2 is the minimum value of impedance which can be obtained with a given resistance R and inductance L.

The corresponding value of C is now obtained as follows: first determine the value of a at the point P_2.

The line OP_2 is defined by the equation

$$b = \frac{2L\omega}{R}.a \quad . \quad . \quad . \quad . \quad (6)$$

But from equation (5)

$$b = \sqrt{aR - a^2}+L\omega$$

$$\therefore \frac{2L\omega}{R}.a - L\omega = \sqrt{aR - a^2}$$

Putting $L\omega = X$ and squaring

$$\therefore\ \frac{4X^2a^2}{R^2}+X^2-\frac{4X^2a}{R}=aR-a^2$$

$$\therefore\ \left(1+\frac{4X^2}{R^2}\right)a^2-\left(R+\frac{4X^2}{R}\right)a+X^2=0$$

$$\therefore\ (R^2+4X^2)a^2-R(R^2+4X^2)a+R^2X^2=0$$

(The solution of equations (5) and (6) as simultaneous equations corresponds to the classical method for finding the co-ordinates of the point of intersection of two curves.)

$$\therefore\ a=a_{11}$$

$$=\frac{R(R^2+4X^2)\pm\sqrt{R^2(R^4+8R^2X^2+16X^4)-4R^2X^2(R^2+4X^2)}}{2(R^2+4X^2)}$$

$$=\frac{R(R^2+4X^2)\pm R\sqrt{R^4+8R^2X^2+16X^4-4R^2X^2-16X^4}}{2(R^2+4X^2)}$$

$$=\frac{R(R^2+4X^2)\pm R\sqrt{R^4+4R^2X^2}}{2(R^2+4X^2)}$$

$$=\frac{1}{2}\left[R-\frac{R^2}{\sqrt{R^2+4X^2}}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The minus sign is chosen in front of the root since the smallest value of a is required ; this is the value corresponding to the point of intersection of the line OO_1 with the circle b_{11}, at P_2, i.e. to OP_2.

But a, which is the resistance component of the impedance, and is the same at 1,1 as at 2,2, is given by :

$$a_{11}=\frac{R}{1+R^2C^2\omega^2}$$

Hence
$$C=\frac{1}{R\omega}\sqrt{\frac{R}{a_{11}}-1} \quad . \quad . \quad . \quad . \quad (8)$$

Substituting in (8) the value of a from (7)

$$C=\frac{1}{R\omega}\sqrt{\frac{2}{1-\frac{R}{\sqrt{R^2+4L^2\omega^2}}}-1} \quad . \quad . \quad (9)$$

It may be noted also that from (6) and (7)

$$b_{11}=L\omega\left[1-\frac{R}{\sqrt{R^2+4L^2\omega^2}}\right] \quad . \quad . \quad . \quad (10)$$

since b_{11} is the value of b when a is defined by equation (7).

The above method of solution was suggested by Grp. Capt. Wilson.

1.2. Example of a Practical Circuit to which the above Analysis applies: The Double Parallel Tuned Mutual Coupling. Fig. 3 shows a case where a condenser assumes the value determined by equation (9). In practice the secondary circuit is tuned by adjusting the coupling to a minimum and then varying C_2 until maximum voltage is observed across R or maximum current through L_2.

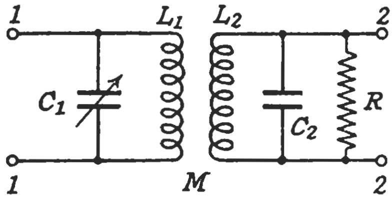


FIG. 3/CII:1.—Double-Parallel Tuned Coupling Circuit.

It will be clear that the value of C_2 is then defined by equation (9) where L is replaced by L_2 and the value of R is the resistance terminating the secondary circuit in Fig. 3.

In practice the circuit of Fig. 3 may be connected in the anode circuit of a valve: terminals 1,1 would then constitute the anode load while the terminals 2,2 might be connected to the grid of the following valve.

The value to which C_1 is adjusted by classical methods of tuning may then be determined as follows.

It is assumed that the condenser C_2 has been adjusted by tuning the secondary to resonance, that is, to maximum current through L_2 (an e.m.f. being applied at 1,1), with the mutual inductance M negligibly small compared to L_2. In this case, as stated, the value of C_2 is given by equation (9), while the total series impedance effective in the secondary circuit, viewed from the midpoint of L_2, assumed broken for the impedance observation, is then $Z_s = a_{11}+jb_{11}$ where a_{11} and b_{11} have the values given in (7) and (10).

The impedance transferred to L_1 is therefore

$$\frac{M^2\omega^2}{Z_s} = \frac{M^2\omega^2}{a_{11}+jb_{11}}$$

and the impedance looking into L_1 is

$$A+jB = jL_1\omega + \frac{M^2\omega^2}{a_{11}+jb_{11}} = \frac{M^2\omega^2 a_{11}}{a_{11}^2+b_{11}^2} + j\left[L\omega - \frac{M^2\omega^2 b_{11}}{a_{11}^2+b_{11}^2}\right] \quad . \quad (11)$$

Condenser C_1 is normally tuned to maximum impedance looking into terminals 1,1 (minimum anode feed) or to maximum circulating current through L_1. These two conditions prescribe exactly the same value for C_1, since the condition of maximum anode impedance gives maximum volts across L_1. This condition also corresponds to that of zero angle impedance presented at 1,1, as is proved incidentally in the remainder of the discussion :

If $R_p//X_p = A + jB$

$$R_p = A\left(1+\frac{B^2}{A^2}\right) \text{ and } X_p = B\left(1+\frac{A^2}{B^2}\right) \text{ (inductive)}$$

The value of C_1 to give maximum impedance must evidently have a reactance equal to X_p, since this makes the total parallel reactance infinity, without changing the value of R_p, and so makes the impedance at 1,1 a maximum.

$$\therefore \frac{1}{C_1\omega} = B\left(1+\frac{A^2}{B^2}\right) \qquad . \quad . \quad . \quad (12)$$

where the values of A and B are defined by equation (11).

2. "Untunable" Circuits.

Consider the circuit of Fig. 1 in which X_1 and X_2 are reactances of opposite sign and X_2 is variable. (If X_1 is variable the circuit is easily tunable under the practical case when terminals 1,1 are connected to the anode of a valve, since the value of X_1, which gives maximum impedance at terminals 1,1 also gives a pure resistance looking into terminals 1,1.) *If terminals 1,1 are connected to the anode of a valve and an attempt is made to tune the circuit by tuning to minimum anode current* (maximum impedance looking into terminals 1,1), the circuit is not adjusted to a proper condition of impedance match because the impedance looking into terminals 1,1 has a reactive component. The energy transfer from the valve to the load R is therefore not a maximum.

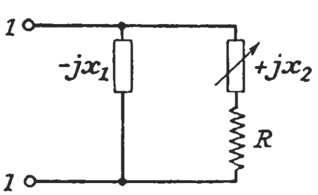


FIG. 1/CII:2.—Basic "Untunable" Circuit.

Proof. The admittance looking into terminals 1,1 is

$$\frac{j}{X_1}+\frac{1}{R+jX_2} = jX_1 - \frac{jX_2}{R^2+X_2^2}+\frac{R}{R^2+X_2^2}$$

$$= jX_1 - j\frac{g^2X_2}{1+g^2X_2^2}+\frac{g}{1+g^2X_2^2} \qquad . \quad . \quad (1)$$

where $$g = \frac{1}{R}$$

This equation is of the same form as equation (1)/CII:1, as can be seen by replacing $L\omega$ by X_1, R by g, and $C\omega$ by X_2.

The impedance looking into terminals 1,1 is a maximum when the admittance is a minimum. By reference to equation (9)/CII:1 it can be seen that this occurs when

$$X_2 = \frac{1}{g}\sqrt{\frac{2}{1 - \frac{g}{\sqrt{g^2+4X_1^2}}} - 1} \quad . \quad . \quad . \quad (2)$$

In this condition, by analogy with equations (8) and (9)/CII:1, the admittance looking into terminals 1,1 is

$$g_{11}+jy_{11} = \frac{g}{1+g^2X_2^2}+jX_1\left[1 - \frac{g}{\sqrt{g^2+4X_2^2}}\right]$$

And the impedance looking into terminals 1,1 is

$$Z_{11} = \frac{1}{g_{11}+jy_{11}} = \frac{g_{11}}{g_{11}^2+y_{11}^2} - j\frac{y_{11}}{g_{11}^2+y_{11}^2}$$

Since g_{11} and y_{11} are both finite the impedance Z_{11} is complex : it has a reactive component.

If, on the other hand, with terminals 1,1 connected to the anode of a valve of internal impedance R_0, X_2 is varied to give maximum current through R, X_2 will evidently assume a value equal to $\frac{X_1}{1+\frac{X_1^2}{R_0^2}}$, which is the value required to neutralize the series reactance presented by R_0 and X_1 in parallel. But in order to provide optimum impedance matching it should assume the value satisfying the relation $X_1 = X_2\left(1+\frac{R^2}{X_2^2}\right)$, that is, such that X_1 neutralizes the parallel reactance presented by X_2 and R in series. In other words, the required value for X_2 is $\frac{X_1+\sqrt{X_1^2 - 4R^2}}{2}$, the plus sign being taken before the root to accord with practical relative magnitudes of resistance and reactance.

Hence in circuits similar to Fig. 1, X_2 should not be made a variable for purposes of tuning by normal indications. It is of course permissible to adjust X_2 and then tune in the normal way by adjusting X_1, but this is not the case in point. Examples of " untunable " circuits are shown in Fig. 2.

These circuits cannot be tuned in the conventional way by varying the element shown as variable. The circuit at (*b*) is the so-called capacity tap circuit. If it is required to tune this circuit it should be tuned by a variable capacity in the position shown dotted and not by varying the upper of the two series condensers.

An "untunable" circuit may be tuned by special methods. An R.F. bridge may be used either to adjust the variable element to

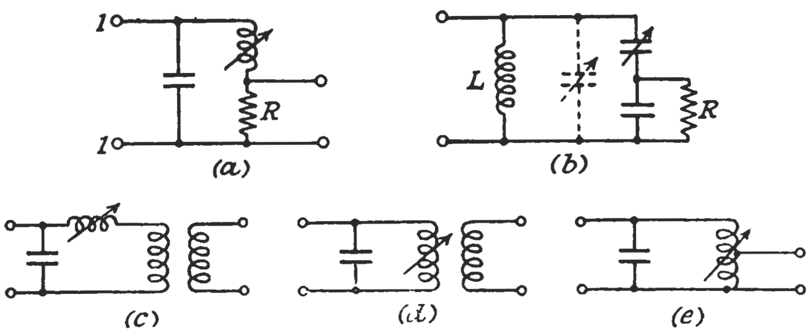


FIG. 2/CII:2.—" Untunable " Circuits.

a known required value, or by connecting the bridge to terminals 1,1 the variable element may be adjusted until a pure resistance is observed at terminals 1,1. Alternatively, if no bridge is available the variable element may be adjusted until the ratio between the current through the load *R* divided by the voltage applied across terminals 1,1 is a maximum.

CHAPTER CIII

SOLUTION OF NETWORK PROBLEMS RELATED TO IMPEDANCE MATCHING IN LONG-WAVE AERIAL CIRCUITS

1. Design of Double Parallel Tuned Mutual Coupling as a Quarter-Wave Network.

THIS network is shown in Fig. 1 (*a*) and is equivalent to an ideal transformer plus a mid-shunt terminated section of a Type III_3 filter. See Bibliography F4:20 and XXV.

The circuit of the latter only is shown in Fig. 1 (*b*).

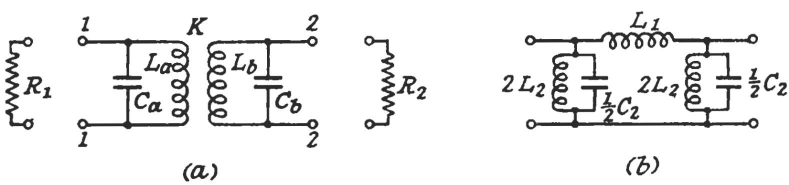


FIG. 1/CIII:1.—(*a*) Double Parallel Tuned Mutual Coupling. (*b*) Equivalent Circuit Less Ideal Transformer.

The values of the elements are given in terms of the cut-off frequencies f_1 and f_2 and the mid-band image impedance R as follows :

$$L_1 = \frac{R}{\pi(f_1+f_2)}, \quad L_2 = \frac{(f_2 - f_1)R}{4\pi f_1^2}, \quad C_2 = \frac{1}{\pi(f_2 - f_1)R}$$

This network has 90° phase shift (when terminated in its image impedance at the frequency concerned) when the effective reactance of each shunt arm is equal in magnitude and opposite in sign to the reactance of the series arm, that is when :

$$\frac{1}{j\frac{1}{2}C_2\omega + \frac{1}{j2L_2\omega}} = -jL_1\omega$$

$$C_2 j\omega + \frac{1}{jL_2\omega} = -\frac{2}{jL_1\omega}$$

$$C_2 - \frac{1}{L_2\omega^2} - \frac{2}{L_1\omega^2} = 0$$

$$\omega^2 = \frac{1}{L_2C_2} + \frac{2}{L_1C_2}$$

$$\therefore f_{90°} = \frac{1}{2\pi}\sqrt{\frac{1}{L_2C_2}+\frac{2}{L_1C_2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{4\pi^2f_1^2(f_2-f_1)}{f_2-f_1}+2\pi^2(f_2^2-f_1^2)}$$

$$= \sqrt{f_1^2+\tfrac{1}{2}f_2^2-f_1^2} = \sqrt{\tfrac{1}{2}(f_1+f_2^2)}$$

$$= \sqrt{f_1f_2}\sqrt{\frac{1}{2}\left(\frac{f_1}{f_2}+\frac{f_2}{f_1}\right)} = f_m\sqrt{\frac{1}{2}\left(\frac{f_1}{f_2}+\frac{f_2}{f_1}\right)}$$

The mid-shunt image impedance at $f_{90°}$ is

$$R_{90°} = \frac{R}{\sqrt{1-\dfrac{\dfrac{f}{f_m}-\dfrac{f_m}{f}}{\dfrac{f_2}{f_m}-\dfrac{f_m}{f_2}}}}$$

where $f_m = \sqrt{f_1f_2}$ and $f = f_{90°}$.

R is the mid-band image impedance, and for band widths less than 20% the deviation of $R_{90°}$ from R can be neglected, when the circuit is lined up on a bridge as described below. For greater band widths the deviation should be allowed for, when the circuit is designed to work between impedances R_1 and R_2, by inserting in the formulae on Fig. 1/VII:14, instead of R_1 and R_2, the quantities $\frac{R}{R_{90°}}R_1$ and $\frac{R}{R_{90°}}R_2$.

In practice, after the circuit has been designed and the values of L_a and L_b and the coupling adjusted to their correct values, the circuit is finally adjusted as follows :

(1) A short is applied at terminals 1,1 and a bridge is connected to 2,2. The condenser C_b is then adjusted until the circuit goes to infinite impedance at 2,2 at the carrier frequency.

(2) C_a is adjusted to give infinite impedance at 1,1 with 2,2 shorted.

(3) A resistance equal to R_2 is connected to 2,2 and the impedance looking into 1,1 is measured. If this is greater than R_1 the coupling is increased ; if it is less than R_1 it is reduced.

(4) (1), (2) and (3) are repeated until the required impedance at the carrier frequency is observed at 1,1.

2. Design of π Network as a Quarter-Wave Network.

The circuit of this is shown in Fig. 1, in which α, β and γ are respectively the impedances of L, C_2 and C_1.

Two cases arise : the unity impedance ratio case and the unequal ratio case.

In the unity impedance ratio case, where $R_1 = R_2 = R$ say, the phase shift constant is 90° and the characteristic impedance is equal to R when $L\omega = \frac{1}{C_1\omega} = \frac{1}{C_2\omega} = R$. The circuit then corresponds to a mid-shunt terminated section of prototype low-pass filter operating at a frequency equal to $\frac{1}{\sqrt{2}}$ times its cut-off frequency and terminated in $\sqrt{2}$ times its characteristic impedance.

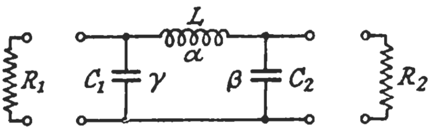


FIG. 1/CIII:2.—π Network.

In the unequal ratio case the circuit must be designed in accordance with the formulae given in Fig. 4/VII:14. It then becomes a band-pass filter and the phase-shift constant is equal to 90° at the low cut-off frequency, i.e.

$$f_{90^\circ} = f_1$$

This means that in practice the unequal ratio network cannot be used to give exactly 90° phase shift ; since if the carrier frequency is located at f_1 the lower sideband will be attenuated. The most satisfactory method of treatment therefore appears to be to design all such networks in accordance with Fig. 4/VII:14, but to locate the carrier frequency at $f_m = \sqrt{f_1 f_2}$.

This means that the phase shift contributed by each network will be greater than 90°, since the phase shift of the unequal ratio structure varies from 90° to 180° throughout its pass band. At some other point in the circuit phase shift must then be added to build up the phase shift of the circuit to the required multiple of 90°. Exceptional cases may of course arise where the departure of the overall phase shift from an integral multiple of 90° is not serious ; in some cases it may be beneficial ; for instance, in restoring the symmetry of an asymmetric impedance curve. Droitwich was an example of such a case and it was not necessary to phase compensate the π networks there.

The value of f_{90° in the unequal ratio structure may be derived as follows. If α, β and γ are respectively the impedances of L, C_1 and C_2, it may be shown that the network has a phase shift of 90° when $\alpha = -\beta$, or $\gamma = -\alpha$. See Bibliography F4:20.

Referring to Fig. 4/VII:14, where $R_1 > R_2$, the design formulae for L and C_2 are

$$L = \frac{R_1\sqrt{\phi(1+\phi-\phi^2)}}{2\pi f_1(\phi^2-1)} \qquad C_2 = \frac{\phi^2-1}{2\pi f_1 R_1\sqrt{\phi(1+\phi-\phi^2)}}$$

Hence 90° phase shift will occur when $\alpha = jL\omega$ is equal to

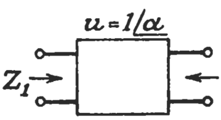
$$-\beta = j\frac{1}{C\omega}.$$

$$\therefore\ \omega^2_{90^\circ} = \frac{1}{LC_2} = 4\pi^2 f_1^2$$

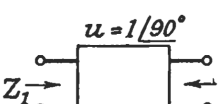
$$\therefore\ f_{90^\circ} = f_1$$

3. Method of Building Up Quarter-Wave Networks from Component Networks.

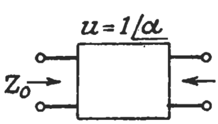
While any number of component networks may be used the present discussion considers the use of two only.



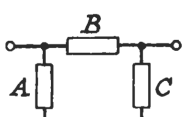
$$\|A_1\| = \begin{Vmatrix} \sqrt{\frac{Z_1}{Z_2}}\cos\alpha & j\sqrt{Z_1Z_2}\sin\alpha \\ \frac{j}{\sqrt{Z_1Z_2}}\sin\alpha & \sqrt{\frac{Z_2}{Z_1}}\cos\alpha \end{Vmatrix} \quad (a)$$



$$\|A_{90^\circ}\| = \begin{Vmatrix} 0 & j\sqrt{Z_1Z_2} \\ \frac{j}{\sqrt{Z_1Z_2}} & 0 \end{Vmatrix} \quad (b)$$



$$\|A_\alpha\| = \begin{Vmatrix} \cos\alpha & jZ_0\sin\alpha \\ \frac{j\sin\alpha}{Z_0} & \cos\alpha \end{Vmatrix} \quad (c)$$



$$\|A_\pi\| = \begin{Vmatrix} 1+\frac{B}{C} & B \\ \frac{1}{A}+\frac{B}{CA}+\frac{1}{C} & 1+\frac{B}{A} \end{Vmatrix} \quad (d)$$

FIG. 1/CIII:3.—Matrices of Networks Shown.

The component networks may consist of two networks of lumped capacity and inductance or a length of feeder and a lumped network (the term "lumped" is merely used to distinguish the ordinary inductances and capacities constituting circuit elements from the distributed constants of a transmission line or feeder).

The present problem is to determine the values of the elements of networks to be added in tandem with a line of a given angle (phase shift constant) in order to provide a composite network with a 90° phase shift constant and required image impedances.

Conventions.

Fig. 1 gives the A matrices of four structures:

(*a*) An inequality impedance ratio lossless fourpole with image impedances Z_1 and Z_2 and phase shift constant α. This matrix is called $\| A_1 \|$.

(*b*) As (*a*) in which $\alpha = 90°$. Matrix $= \| A_{90°} \|$.

(*c*) An equality impedance ratio fourpole which may represent a lossless transmission line or feeder of characteristic impedance Z_0 and phase-shift constant α. Matrix $= \| A_\alpha \|$.

(*d*) A π network. Matrix $= \| A_\pi \|$:

Z_0 = characteristic impedance of feeder or line.
α = total phase shift constant of feeder or line.
A, B and C = Impedances of arms of π network.
Z_1 and Z_2 = Image impedances of composite network: Z_1 looking into line; Z_2 looking into π network.

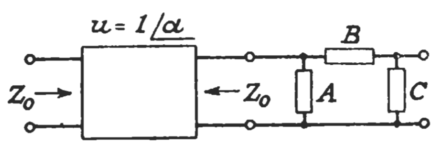


FIG. 2/CIII:3.—Combination of Transmission Line and π Network.

Fig. 2 shows the combination of a length of feeder represented by an equal ratio fourpole and a π network which in general is required to constitute an unequal ratio fourpole with a phase-shift constant equal to 90°. Its matrix must therefore be equal to $\| A_{90°} \|$ as defined by Fig. 1 (*a*), hence, having regard to the order in which the two networks in which Fig. 2 appear,

$$\| A_\alpha \| \times \| A_\pi \| = \| A_{90°} \| \text{ or } A_\pi = \frac{\| A_{90°} \|}{\| A_\alpha \|}$$

Writing this operation in full

$$\begin{Vmatrix} \cos\alpha & jZ_0\sin\alpha \\ \dfrac{\sin\alpha}{Z_0} & \cos\alpha \end{Vmatrix} \times \begin{Vmatrix} 1+\dfrac{B}{C} & B \\ \dfrac{1}{A}+\dfrac{B}{CA}+\dfrac{1}{C} & 1+\dfrac{B}{A} \end{Vmatrix} = \begin{Vmatrix} 0 & j\sqrt{Z_1Z_2} \\ \dfrac{j}{\sqrt{Z_1Z_2}} & 0 \end{Vmatrix}$$

Divide each side by $\| A_\alpha \|$:

(The formula for the division of matrices is given in CI:12; the fact that the next step is correct can however be proved by multiplying across by $\| A_\alpha \|$ in the line below when the line above appears.)

$$\therefore \begin{Vmatrix} 1+\dfrac{B}{C} & B \\ \dfrac{1}{A}+\dfrac{B}{CA}+\dfrac{1}{C} & 1+\dfrac{B}{A} \end{Vmatrix} = \begin{Vmatrix} \dfrac{Z_0}{\sqrt{Z_1Z_2}}\sin\alpha & j\sqrt{Z_1Z_2}\cos\alpha \\ \dfrac{j}{\sqrt{Z_1Z_2}}\cos\alpha & \dfrac{\sqrt{Z_1Z_2}}{Z_0}\sin\alpha \end{Vmatrix}$$

The condition of equality between matrices demands that all corresponding terms shall be equal. Hence:

$$\therefore\ 1+\frac{B}{C} = \frac{Z_0}{\sqrt{Z_1Z_2}}\sin\alpha \quad . \quad . \quad . \quad (1)$$

$$B = j\sqrt{Z_1Z_2}\cos\alpha \quad . \quad . \quad . \quad (2)$$

$$\frac{1}{A}+\frac{B}{CA}+\frac{1}{C} = \frac{j}{\sqrt{Z_1Z_2}}\cos\alpha \quad . \quad . \quad . \quad (3)$$

$$1+\frac{B}{A} = \frac{\sqrt{Z_1Z_2}}{Z_0}\sin\alpha \quad . \quad . \quad . \quad (4)$$

Equations (1), (2), (3) and (4) provide three independent equations and one dependent equation. In other words, any three of these equations may be solved simultaneously, the fourth equation will then provide confirmation of one or more of the results.

3.1. Given $Z_1 = Z_2 = Z_3$ and α to find A, B and C.

From (2), $B = jZ_0\cos\alpha$ . . . . (5)

From (1), $C = \dfrac{B}{\sin\alpha - 1} = \dfrac{jZ_0\cos\alpha}{\sin\alpha - 1}$ . . . . . (6)

From (4), $A = \dfrac{B}{\sin\alpha - 1} = \dfrac{jZ_0\cos\alpha}{\sin\alpha - 1}$ . . . . . (7)

Note that B is positive and therefore inductive, while A and C are negative and therefore capacitative, since $\sin\alpha < 1$.

3.2. Given $Z_1 = Z_0$, C = infinity, and the value of Z_2, to find A B and α. The solution is given in equations (8), (11) and (12), which are derived as follows:

From (1) $\sin\alpha = \sqrt{\frac{Z_2}{Z_0}}$ . . . . (8)

$\therefore \cos\alpha = \sqrt{1 - \frac{Z_2}{Z_0}}$ . . . . . . (9)

Substituting (8) and (9) in (4) and (2) respectively,

$\therefore 1 + \frac{B}{A} = \frac{Z_2}{Z_0}$ . . . . (10)

and $B = j\sqrt{Z_0 Z_2\left(1 - \frac{Z_2}{Z_0}\right)} = j\sqrt{Z_0 Z_2 - Z^2}$ . . 11)

From (10) $A = \dfrac{B}{\frac{Z_2}{Z_0} - 1} = \dfrac{jZ_0\sqrt{Z_0 Z_2 - Z_2^2}}{Z_2 - Z_0}$ . . (12)

From (11) it is evident that Z_2 must be less than Z_0, and that B is positive and therefore inductive. From (12), since $Z_2 < Z_0$ it is evident that A is negative and therefore capacitative.

3.3. Given $Z_1 = Z_0$, A = infinity, and the value of Z_2, to find B, C and α. The solution is given in equations (13), (16) and (17), which are derived as follows:

From (4) $\sin\alpha = \sqrt{\frac{Z_0}{Z_2}}$ . . . . (13)

$\therefore \cos\alpha = \sqrt{1 - \frac{Z_0}{Z_2}}$ . . . . (14)

Substituting (14) and (15) in (1) and (2) respectively,

$\therefore 1 + \frac{B}{C} = \frac{Z_0}{Z_2}$ . . . . . (15)

and $B = j\sqrt{Z_0 Z_2\left(1 - \frac{Z_0}{Z_2}\right)} = j\sqrt{Z_0 Z_2 - Z_0^2}$ . . (16)

From (15) $C = \dfrac{B}{\frac{Z_0}{Z_2} - 1} = \dfrac{jZ_2\sqrt{Z_0 Z_2 - Z_0^2}}{Z_0 - Z_2}$ . . . (17)

From (16) it is evident that Z_2 must be greater than Z_0 and that B is positive and therefore inductive. From (17), since $Z_2 > Z_0$, it is evident that C is negative and therefore capacitative.

4. Design of Lossless Symmetrical π Network to have Required Characteristic Impedance and Phase-Shift Constant.

To obtain the design constants of this network it is only necessary to equate $\| A_\alpha \|$ to $\| A_\pi \|$; see Fig. 1/CIII:3.

Whence
$$1+\frac{B}{C} = \cos\alpha \qquad (1)$$
$$B = jZ_0 \sin\alpha \qquad (2)$$
$$\frac{1}{A}+\frac{B}{CA}+\frac{1}{C} = \frac{j\sin\alpha}{Z_0} \qquad (3)$$
$$1+\frac{B}{A} = \cos\alpha \qquad (4)$$

From (1) and (4) $A = C = \dfrac{B}{\cos\alpha - 1} = \dfrac{jZ_0\sin\alpha}{\cos\alpha - 1}$ (5)

The value of B is given by (2).

Equation (5) may be transformed as follows:

$$A = C = \frac{-jZ_0\sqrt{1-\cos^2\alpha}}{1-\cos\alpha} = -jZ_0\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$$
$$= -jZ_0\sqrt{\frac{1+2\cos^2\frac{\alpha}{2}-1}{1-2\cos^2\frac{\alpha}{2}+1}} = -jZ_0\sqrt{\frac{2\cos^2\frac{\alpha}{2}}{2-2\cos^2\frac{\alpha}{2}}}$$
$$= -jZ_0\cot\frac{\alpha}{2} \qquad (6)$$

4.1. Lossless Symmetrical T Network. In the same way the arms of a lossless symmetrical-T network to have a phase shift α and a characteristic impedance Z_0 are given by:

$$\text{Series arms} = jZ_0\tan\frac{\alpha}{2};$$
$$\text{Shunt arm} = -j\frac{Z_0}{\sin\alpha}$$

In the case of both T and π networks the phase shift with the above values of the arms constitutes a lag. If the signs of the reactances are reversed, a leading phase shift of the same amount is obtained.

The above formulae can also be derived from the equivalent **T** and π networks shown at the top of Fig. 14/XXIV:6, making use of the relation :

$$C_t - \frac{1}{S_n} = \text{tans}\,\sqrt{u} = \text{tans}\,\sqrt{1/\alpha} = j\tan\frac{\alpha}{2}$$

CHAPTER CIV

MEASUREMENT OF PHASE SHIFT ON CATHODE-RAY OSCILLOGRAPH

Two sinusoidal waves of equal amplitude:

$$y = A \sin \omega t \quad \ldots \quad (1)$$

and

$$x = A \sin (\omega t + \phi) \quad \ldots \quad (2)$$

are applied to the plates of a cathode-ray oscillograph so that y causes vertical deflections and x causes horizontal deflections.

By eliminating ωt between (1) and (2) the form of the figure on the cathode-ray oscillograph is obtained:

From (1) $\sin \omega t = \dfrac{y}{A}$ and $\cos \omega t = \sqrt{1 - y^2/A^2}$

From (2)

$$\begin{aligned} x &= A \sin \omega t \cos \phi + A \cos \omega t \sin \phi \\ &= y \cos \phi + A \sin \phi \sqrt{1 - y^2/A^2} \\ &= y \cos \phi + \sqrt{A^2 - y^2} \sin \phi. \end{aligned}$$

$\therefore\ x - y \cos \phi = \sqrt{A^2 - y^2} \sin \phi$, and squaring both sides

$$x^2 - 2xy \cos \phi + y^2 \cos^2 \phi = A^2 \sin^2 \phi - y^2 \sin^2 \phi$$

$$\therefore\ x^2 - 2xy \cos \phi + y^2 = A^2 \sin^2 \phi \quad \ldots \quad (3)$$

This is the equation of an ellipse, the major and minor axes of which always lie respectively and alternatively on the lines $y = x$ and $y = -x$; the centre of the ellipse must then be the origin.

It is to be noted that since $\cos \phi = \cos - \phi$, the same form of ellipse is obtained, regardless of which pair of plates carries the leading voltage.

Determination of the Axes of the Ellipse. This is based on the axiomatic fact that any straight line which cuts the ellipse at right angles must be an axis.

Put $\dfrac{dy}{dx} = p$.

Differentiating (3) with regard to x:

$$2x - 2xp \cos \phi - 2y \cos \phi + 2yp = 0$$

$$\therefore\ (2y - 2x \cos \phi)p = 2y \cos \phi - 2x$$

$$\therefore\ p = \frac{y \cos \phi - x}{y - x \cos \phi} \quad \ldots \quad (4)$$

When $y = x, \quad p = -1 \qquad \left(\because\ p = \dfrac{\cos\phi - 1}{1 - \cos\phi}\right)$

That is, the slope of the ellipse is normal to the straight line $y = x$. Similarly when $y = -x$, $p = +1$: the slope of the ellipse is also normal to $y = -x$.

Hence $y = x$ and $y = -x$ are the two axes of the ellipse, they will be referred to as the positive and negative axes.

Referring to equation (3),

If $\phi = 0$, $\cos\phi = 1$, $\sin\phi = 0$, and equation becomes:

$$x^2 - 2xy + y^2 = 0 \text{ or } x - y = 0 \quad . \quad . \quad . \quad (5)$$

that is $y = x$; the ellipse degenerates into a straight line at 45° to each of the axes of co-ordinates, and length $= 2\sqrt{2}A$.

If $\phi = 90°$, $\cos\phi = 0$, $\sin\phi = 0$, and the equation becomes

$$x^2 + y^2 = A^2 \quad . \quad . \quad . \quad . \quad (6)$$

which is a circle of radius A.

In between these two conditions the ellipse under consideration always has its longer or major axis along the line $y = x$. When ϕ is greater than 90° the ellipse has its longer or major axis along the line $y = -x$.

Determination of Phase Shift of Ellipse in Terms of *a* and *b*, the Lengths of the Positive and Negative Axes of the Ellipse, and *A*, the Amplitude of each Input Wave. Referring to Fig. 5/XX:10, *a* the positive axis of the ellipse lies along the line $y = x$,

$$a = 2\sqrt{x^2 + y^2} = 2\sqrt{2} \times x = 2\sqrt{2} \times y$$

$$\therefore\ x = y = \frac{a}{2\sqrt{2}}$$

Substituting these values in (3)

$$\therefore\ \frac{a^2}{8} - \frac{a^2}{4}\cos\phi + \frac{a^2}{8} = A^2 \sin^2\phi$$

$$\therefore\ a^2(1 - \cos\phi) = 4A^2 \sin^2\phi$$

$$\therefore\ a = \frac{2A\sin\phi}{\sqrt{1 - \cos\phi}} \text{ or } \frac{a}{A} = \frac{2\sin\phi}{\sqrt{1 - \cos\phi}}$$

Similarly

$$b = \frac{2A\sin\phi}{\sqrt{1 + \cos\phi}} \text{ or } \frac{b}{A} = \frac{2\sin\phi}{\sqrt{1 + \cos\phi}}$$

On Fig. 6/XX:10 the value of ϕ is plotted as a function of $\frac{a}{A}$ and $\frac{b}{A}$. Hence the phase shift between two waves can be obtained by measuring the lengths of the major axis a or the minor axis b and the amplitude A of either wave acting alone.

Alternative Determination of ϕ. The set-up and conventions are as above.

Draw the major and minor axes of the ellipse. Join the intersection of the axis with the ellipse occurring in the positive quadrant to the two intersections of the other axis with the ellipse. The angle between the two lines so drawn is equal to ϕ.

The method previously described is, however, preferred on the grounds of accuracy.

CHAPTER CV

CONDITIONS IN A MODULATED AMPLIFIER FACED WITH A LOAD IMPEDANCE VARYING WITH FREQUENCY

THE symmetrical case only is considered, that is the case in which the load impedance at each sideband frequency is the same; see XVI:12. All load impedances considered are those which face one valve in the modulated amplifier. All currents are the currents flowing through one valve in the modulated amplifier and the anode peak volts are the volts developed by that valve. All impedances, load lines, etc., refer to the same valve.

Conventions.

R_c = Load impedance at the carrier frequency.

$Z_s/\underline{\theta}$ = Load impedance at each sideband frequency. θ is positive for one sideband and negative for the other.

m = Percentage current modulation in anode circuit.

m_v = Percentage voltage modulation in anode circuit.

i_f = Instantaneous current in anode circuit.

$\hat{i}_f$ = Peak amplitude of carrier current in anode circuit (varying with envelope amplitude).

$\hat{i}_c$ = Peak amplitude of carrier current when unmodulated.

$\hat{i}_s$ = Peak amplitude of one sideband current (i.e. of each sideband current).

i_p = Peak anode current in valve (varying with envelope amplitude).

i = Anode current as measured on a quick-acting ammeter responsive at audio frequency (varying with envelope amplitude).

$f = \frac{\hat{i}_f}{i_p}$ assumed constant but really determined by angle of current flow. (Recent investigations have shown that f is substantially constant over the modulation cycle so this assumption is justified.)

$u = \frac{\hat{i}_f}{i}$ assumed constant but really determined by angle of current flow. (Recent investigations have shown that u is very nearly constant over the modulation cycle.)

e_a = Anode volts.

$\hat{e}_a$ = Peak anode volts (peak amplitude of carrier frequency volts varying with modulation).

$Z_c = \frac{\hat{e}_a}{\hat{\imath}_f}$ = Discontinuous instantaneous circuit impedance facing anode of valve defined only at instants at which $\hat{e}_a$ and $\hat{\imath}_f$ occur.

$Z_L = fZ_c$ = Impedance corresponding to slope of load line of valve. (Discontinuous and varying with time.)

R_a = Impedance corresponding to slope of asymptote of anode-current/anode-voltage characteristics of valve = effective internal impedance of hard-driven valve.

E_0 = Value of H.T. volts effective on modulated amplifier.

E_m = Modulating voltage effective on anodes of modulated amplifier.

$E = E_0+E_m$.

I_0 = Anode feed of modulated amplifier due to E_0 and grid drive in carrier condition.

I_m = Varying current supplied by modulator to modulated amplifier.

$\hat{I}_m$ = Peak value of I_m.

$I = I_0+I_m$ = Value of envelope of anode feed at any time $=\bar{\imath}$.

f_m = Modulating frequency.

$v = 2\pi f_m$.

f_0 = Carrier frequency.

$c = 2\pi f_0$.

Suppose that, as the result of an applied modulating frequency $f_m = \frac{v}{2\pi}$, a modulated current flows in the anode load defined by

$$i_f = \hat{\imath}_c[1+m \sin vt] \sin ct \qquad . \quad . \quad . \quad (1)$$

where m is the percentage current modulation and $c = 2\pi f_0$ where f_0 is the carrier frequency.

$$\text{Then } i_f = \hat{\imath}_c\left[\sin ct+\frac{m}{2}\cos(c-v)t - \frac{m}{2}\cos(c+v)t\right]$$

$$= \hat{\imath}_c \sin ct+\hat{\imath}_s \cos(c-v)t - \hat{\imath}_s \cos(c+v)t \qquad . \quad . \quad (2)$$

where $\hat{\imath}_s= \frac{m}{2}\hat{\imath}_c$ = the sideband current amplitude.

The anode volts are obtained as the sum of the voltage due to $\hat{\imath}_c$ flowing through R_c and each sideband current $\hat{\imath}_s$ flowing through $Z_s\underline{/\theta}$:

$$e_a = R_c\hat{i}_c \sin ct + Z_s\hat{i}_s \cos[(c-v)t-\theta] - Z_s\,\hat{i}_s \cos[(c+v)t+\theta]$$

$$= R_c\hat{i}_c\left[1+2\frac{Z_s\hat{i}_s}{R_c\hat{i}_c}\sin(vt+\theta)\right]\sin ct$$

Putting $\hat{i}_s = \frac{m}{2}\hat{i}_c$

$$e_a = R_c\hat{i}_c\left[1+\frac{Z_s}{R_c}m\sin(vt+\theta)\right]\sin ct \quad . \quad . \quad (3)$$

The peak anode volts occur when $\sin ct = 1$.

Hence

$$\hat{e}_a = R_c\hat{i}_c\left[1+\frac{Z_s}{R_c}m\sin(vt+\theta)\right] \quad . \quad . \quad . \quad (4)$$

Similarly, from (1), the peak fundamental frequency load current is given by

$$\hat{i}_f = \hat{i}_c(1+m\sin vt) \quad . \quad . \quad . \quad . \quad (5)$$

Hence the (discontinuous) instantaneous circuit impedance as seen by the valve, defined only at the times of occurrence of $\hat{e}_a$ and $\hat{i}_f$, is given by

$$Z_c = \frac{\hat{e}_a}{\hat{i}_f} = \frac{R_c+Z_sm\sin(vt+\theta)}{1+m\sin vt} \quad . \quad . \quad . \quad (6)$$

Therefore

$$Z_L = fZ_c = f\frac{R_c+Z_sm\sin(vt+\theta)}{1+m\sin vt} \quad . \quad . \quad (7)$$

Now, $$E = \left(\frac{R_a+Z_L}{Z_L}\right)\hat{e}_a = \left(1+\frac{R_a}{Z_L}\right)\hat{e}_a \quad . \quad . \quad (8)$$

where E is the instantaneous value of the total H.T. effective on the anode (i.e. $E = E_0+E_m$). Hence from (4), (7) and (8)

$$E = \left[1+\frac{R_a(1+m\sin vt)}{f[R_c+Z_sm\sin(vt+\theta)]}\right]\hat{i}_c[R_c+Z_sm\sin(vt+\theta)]$$

$$= \hat{i}_c\left[R_c+Z_sm\sin(vt+\theta)+\frac{R_a}{f}+\frac{mR_a}{f}\sin vt\right] \quad . \quad . \quad . \quad (9)$$

E therefore consists of a constant component:

$$E_0 = \hat{i}_c\left(R_c+\frac{R_a}{f}\right) \quad . \quad . \quad . \quad . \quad (10)$$

which corresponds to the steady H.T., and a varying component

$$E_m = \hat{i}_c\left[Z_sm\sin(vt+\theta)+\frac{mR_a}{f}\sin vt\right] \quad . \quad . \quad (11)$$

which corresponds to the modulating voltage.

Similarly the instantaneous value of the envelope of the anode feed is

$$I = \frac{\hat{i}_f}{u} = \frac{\hat{i}_c}{u}(1+m \sin vt) \quad . \quad . \quad . \quad (12)$$

This consists of a constant component

$$I_0 = \frac{\hat{i}_c}{u} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and a varying component

$$I_m = \frac{\hat{i}_c}{u} m \sin vt \quad . \quad . \quad . \quad . \quad (14)$$

The D.C. resistance of the modulated amplifier is therefore

$$Z_{\text{D.C.}} = \frac{E_0}{I_0} = u\left(R_c + \frac{R_a}{f}\right) \quad . \quad . \quad . \quad (15)$$

The A.C. impedance of the modulated amplifier as seen by the modulator is:

$$Z_{\text{A.C.}} = \frac{E_m}{I_m} = \frac{u\left[Z_s m \sin (vt+\theta) + \frac{R_a m}{f} \sin vt\right]}{m \sin vt}$$

$$= u\left(Z_s \underline{/\theta} + \frac{R_a}{f}\right) \quad . \quad . \quad . \quad . \quad . \quad (16)$$

A.F. to Sideband Conversion Efficiency. The audio-frequency power input to the modulator = the real part of

$$\tfrac{1}{2}\hat{I}_m Z_{\text{A.C.}} = \frac{\frac{1}{2}\hat{i}_c^2}{u^2} m^2 u\left(Z_s \underline{/\theta} + \frac{1}{f}R_a\right)$$

$$= \frac{m^2 \hat{i}_c^2}{2u}\left(Z_s \cos \theta + \frac{1}{f}R_a\right). \quad . \quad . \quad (17)$$

The sideband power $= \frac{1}{2}\hat{i}_1^2 Z_s \cos\theta + \frac{1}{2}\hat{i}_2^2 Z_s \cos\theta$

where $\hat{i}_1 = \hat{i}_2 = \hat{i}_s = \frac{m}{2}\hat{i}_c$

Hence sideband power $= \frac{m^2}{4}\hat{i}_c^2 Z_s \cos\theta \quad . \quad . \quad . \quad . \quad . \quad (18)$

The efficiency of conversion from A.F. to sideband is therefore:

$$\eta_{\text{A.C.}} = \frac{\frac{u}{2} Z_s \cos\theta}{Z_s \cos\theta + \frac{1}{f}R_a} \quad . \quad . \quad . \quad (19)$$

D.C. to Carrier Conversion Efficiency. The D.C. power input to the transmitter

$$= \bar{i}^2 Z_{\text{D.C.}} = \bar{i}^2\left(R_c + \frac{1}{f}R_a\right)u \quad . \quad . \quad . \quad (20)$$

$$\text{The carrier power} = \tfrac{1}{2}\hat{i}_c^2 R_c \quad . \quad . \quad . \quad (21)$$

$$(\hat{i}_f = \hat{i}_c \text{ when } m = 0)$$

The D.C. to Carrier Conversion Efficiency is therefore:

$$\eta_{\text{D.C.}} = \tfrac{1}{2}\frac{\hat{i}_c^2}{\bar{i}^2} \times \frac{R_c}{\left(R_c + \frac{1}{f}R_a\right)u}$$

(Note that, since in the carrier condition $\hat{i}_f = \hat{i}_c$, $u = \frac{\hat{i}_f}{\bar{i}} = \frac{\hat{i}_c}{\bar{i}}$).

$$= \tfrac{1}{2}u\frac{\hat{e}_a}{E_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where E_0 is the H.T. volts and u is the current utilization. Equation (22) serves as a check on the accuracy of the whole calculation since this is the classical expression for efficiency. For comparison with (19) note that:

$$\eta_{\text{D.C.}} = \frac{u}{2} \times \frac{R_c}{R_c + \frac{R_a}{f}} \quad . \quad . \quad . \quad . \quad (23)$$

Equation (19) is therefore of the same form as equation (23) which is known to be correct.

Relation between Current Modulation and Voltage Modulation. From (1) and (3), when the current modulation is m, the voltage modulation is

$$\frac{Z_s}{R_c}m$$

Conversely, when the voltage modulation is m, the current modulation is $\frac{R_c}{Z_s}m$. Hence for 100% voltage modulation the current modulation is $\frac{R_c}{Z_s}$. In the case of a circuit presenting a higher impedance at the sideband frequencies than at the carrier frequency, since the voltage modulation can never exceed 100% the current modulation can never exceed R_c/Z_s. In the case of a circuit presenting a lower impedance at the sideband frequencies than at the carrier frequency, the current modulation can never exceed 100% (since otherwise the valves would have to pass reverse current) and the voltage modulation can never exceed Z_s/R_c.

Power Modulation or Mean Modulation. In a normal circuit having the same (resistive) impedance at carrier and sideband frequencies, the power delivered at the carrier frequency is $P_c = e_0 i_0$, where e_0 and i_0 are the R.M.S. values of voltage and current in the unmodulated condition. If the current modulation is m the voltage modulation is m. The power in the sideband frequencies is then :

$$P_s = \tfrac{1}{2}\, m e_0 m i_0$$

Therefore $$\frac{P_s}{P_c} = \frac{m^2}{2}, \text{ so that } m = \sqrt{\frac{2P_s}{P_c}}. \qquad (24)$$

In a circuit in which the voltage modulation m_v is different from the current modulation m and determined by the relation $m_v = \dfrac{Z_s m}{R}$, it is convenient to define the mean modulation or the power modulation by a relation similar to equation (24). In this case the mean (or power) modulation is :

$$\bar{m} = \sqrt{\frac{2P_s}{P_c}} = \sqrt{\frac{2 \times \frac{1}{2} m_v e_0 m i_0 \cos\theta}{e_0 i_0}} = \sqrt{m_v m \cos\theta}\ . \qquad (25)$$

$\bar{m}$ then defines both the current modulation and the voltage modulation at any later point in the circuit (if such a point exists) where the circuit is restored to constant resistance. Inserting in (25) the relations $m_v = \dfrac{Z_s}{R_c}$ and $m = \dfrac{R_c}{Z_s} m_v$

$$\bar{m} = m\sqrt{\frac{Z_s \cos\theta}{R_c}} = m_v\sqrt{\frac{R_c \cos\theta}{Z_s}} \qquad (26)$$

Modulation Conditions for Rising and Falling Anode Impedance Characteristics.

With a Falling Impedance Characteristic the value of m, the percentage current modulation, cannot exceed 100%. Putting $m = 1$ in (26) the *maximum* value of *power modulation* obtainable with a falling characteristic (Z_s less than R_c) is

$$\bar{m} = \sqrt{\frac{Z_s \cos\theta}{R_c}} \qquad (27)$$

The *maximum* permissible value of *voltage modulation,* i.e. to give 100% current modulation and the power modulation defined by (27), is $\dfrac{Z_s}{R_c}$.

With a Rising Impedance Characteristic the value of m_v, the percentage voltage modulation, cannot exceed 100%. Putting

$m_v = 1$ in (26) the ***maximum*** value of ***power modulation*** obtainable with a rising characteristic (Z_s greater than R_c) is

$$\bar{m} = \sqrt{\frac{R_c \cos\theta}{Z_s}} \quad . \quad . \quad . \quad . \quad (28)$$

With 100% voltage modulation, which gives the power modulation defined by (28), the current modulation is $\frac{R_c}{Z_s}$.

Modulation of the Radiated Field. Neglecting the effect of variation of the aerial polar diagram with frequency, the percentage modulation of the radiated field is equal to $\bar{m}$.

Modulations at Points of Impedance Inversion. At points situated an odd integral number of quarter wavelengths from the anodes, the magnitudes of voltage and current modulations are interchanged, i.e. as compared with their magnitudes at the anodes. This means, for instance, that with a symmetrical falling characteristic at the anodes, the maximum permissible value of voltage modulation observed a quarter wavelength from the anodes, is unity.

In interpreting results of measurements of modulation, it is therefore essential that proper account is taken of the impedance characteristics at the point of measurement, and also of the type of modulation being measured: current or voltage.

CHAPTER CVI

CALIBRATION OF AERIAL POWER DISTRIBUTION DIAGRAMS IN *ED* PER ROOT KILOWATT

1. AERIAL power distribution diagrams, plotted on constant solid-angle-to-area ratio charts, such as Figs. 2 to 10/XVI:7, to show the directivity of aerials, are provided with a calibration enabling the value of field strength (at one kilometre distance measured along the radius vector) to be directly determined in millivolts per metre per kilowatt radiated, from the arbitrary contour markings of the chart, for any direction of transmission. This quantity is usually referred to as *ED* per root kilowatt. Attenuation due to dissipation is assumed to be zero.

The charts are also marked with a factor K by which the contour markings on the chart must be multiplied to convert to power density in watts per solid radian.

Referring, for instance, to Fig. 4/XVI:7, $ED/\sqrt{kW} = 458\sqrt{\frac{P_c}{100}}$ where P_c is the chart contour line. So that at the 100 point $ED/\sqrt{kW} = 458$, on the 50-contour line $ED/\sqrt{kW} = 325$, and on the 25-contour line $ED/\sqrt{kW} = 229$ millivolts per metre × kilometres per kilowatt radiated. At one kilometre the field strengths for one kilowatt radiated are then respectively 458, 325 and 229 millivolts per metre.

The above follows from the formula (derived below):

$ED = 100\sqrt{\frac{KP_c}{26\cdot 5}}$ millivolts per metre × kilometres per root kilowatt radiated.

The value of K is given by $K = \frac{1{,}000n}{\int_0^A P_c da}$

where n, A and a are quantities having the meanings assigned under the conventions given immediately below.

The graphical method of determining the value of $\int_0^A P_c da$ and the proof of the formula for *ED* per root kilowatt follow.

2. Conventions.

P_c = arbitrary marking of power density contours on constant solid-angle-to-area ratio : power density diagrams.

$P_r = KP_c$ = power density at each contour in watts per solid radian per kilowatt radiated.

K = a constant peculiar to each antenna for converting contour markings into watts per solid radian.

n = number of square inches of chart per solid radian. ($n = 20{\cdot}4$ for the charts of Figs. 2 to 10/XVI:7 when enlarged so that the base line is 14·13 inches, and is of course proportional to the square of this base line.)

$P'' = \frac{1}{n}P_r = \frac{K}{n}P_c$ = power density at each contour line in watts per square inch of chart per kilowatt radiated.

ω = solid angle.

a = area of chart in square inches, equivalent to solid angle ω, $= n\omega$.

A = area of chart in square inches, corresponding to 4π radians $= 4\pi n$.

a_1, a_2, a_3, etc. = area, in square inches, contained between adjacent contour lines.

$\overline{P}_{c_1}$, $\overline{P}_{c_2}$, $\overline{P}_{c_3}$, etc. = mean arbitrary energy density marked on chart respectively in regions a_1, a_2, a_3, etc., e.g. $P_{c_1} = \frac{1}{2}(100+90)$, $P_{c_2} = \frac{1}{2}(90+80)$, etc.

W = total power radiated by antenna in watts.

A_{z_1} = area in square inches of chart representing power density in one half-hemisphere extending from horizon level to zenith.

A_{z_2} = corresponding area in other half-hemisphere extending from horizon to zenith.

A_{a_1} = area of chart representing power density in one half-hemisphere extending from horizon level to antipodes.

A_{a_2} = corresponding area in other half-hemisphere extending from horizon to antipodes.

W_{cm} = power in watts traversing each square centimetre of space normal to direction of propagation.

D = distance in kilometres measured from the transmitting aerial along the radius vector to any point.

E = field strength in millivolts per metre.

ED_{100} = value of ED along radius vector directed towards point 100 on contour chart.

E_s and E_v = field strength respectively in c.g.s. electrostatic units per centimetre and in volts per centimetre.

H_m = electromagnetic component of field in c.g.s. electromagnetic units.

ε = specific inductive capacity of space = 1 in Gaussian units.

μ = permeability of space = 1 in Gaussian units.

c = velocity of light in centimetres per second.

δ = directivity, see CVI:4.6.

3. Introduction.

Power density diagrams are each a plot to an arbitrary scale (such that the maximum power density is represented by the figure 100) of the contour lines of power density, on a surface in which the lines of longitude and latitude have been so drawn that equal solid angles are represented by equal areas.

An integration of power density over the area of the chart in square inches therefore gives to some arbitrary scale the power radiated into the hemisphere represented by the chart.

If this integration is performed for each of the four half-hemispheres constituting the sphere surrounding the aerial, and the total arbitrary power figure so found is multiplied by an arbitrary constant K' and the product equated to the known power in watts radiated from the antenna, it is evident that the value of K' is determined. If the integration has been carried out as a summation of an infinite series of products, of dimensions equal to square inches multiplied by chart contour markings P_c, then $K'P_c$ is the energy density in watts per square inch. To convert to watts per solid radian it is necessary to multiply $K'P_c$ by n (the number of square inches of chart per solid radian). In the analysis below it has been found convenient to derive directly the quantity $nK' = K$.

Once this last has been obtained the various values of ED follow immediately.

4. Proofs.

4.1. Total Radiated Power.

P_c = arbitrary marking of power density contours on charts of Figs. 2 to 10/XVI:7.

$P_r = KP_c$ = power density at each contour in watts per solid radian per kilowatt radiated.

K = a constant peculiar to each antenna for converting contour markings into watts per solid radian.

n = number of square inches of chart per solid radian. ($n = 20{\cdot}4$ when base line of power density diagram is 14·13 inches.)

$P'' = \frac{1}{n}P_r = \frac{K}{n}P_c$ = power density at each contour line in watt per square inch of chart per kilowatt radiated.

ω = solid angle.

a = area of chart in square inches, equivalent to solid angle ω, $= n\omega$.

A = area of chart in square inches, corresponding to 4π radians $= 4\pi n$.

W = total power radiated by antenna in watts.

Then evidently $$W = \int_0^{4\pi} P_r d\omega = \int_0^A P''da = \frac{K}{n}\int_0^A P_c da \quad . \quad (1)$$

4.2. Graphical Determination of $\int_0^A P_c da$.

a_1, a_2, a_3, etc. = area, in square inches, contained between adjacent contour lines.

$\bar{P}_{c_1}$, $\bar{P}_{c_2}$, $\bar{P}_{c_3}$, etc. = mean arbitrary energy density marked on chart respectively in regions a_1, a_2, a_3, etc., e.g. $\bar{P}_{c_1} = \frac{1}{2}(100+90)$, $\bar{P}_{c_2} = \frac{1}{2}(90+80)$, etc.

A_{z_1} = area in square inches of chart representing power density in one half-hemisphere extending from horizon level to zenith.

A_{z_2} = corresponding area in other half-hemisphere extending from horizon to zenith.

A_{a_1} = area of chart representing power density in one half-hemisphere extending from horizon level to antipodes.

A_{a_2} = corresponding area on other half-hemisphere extending from horizon to antipodes.

If each of the areas a_1, a_2, a_3, . . . etc., contained between adjacent contour lines is measured in square inches and multiplied respectively by the mean arbitrary contour markings on the charts of Figs. 2 to 10/XVI:7, a series of quantities are obtained of the form $a_1\bar{P}_{c_1}$, $a_2\bar{P}_{c_2}$, $a_3\bar{P}_{c_3}$, etc.

Evidently the sum of all these quantities for a single chart represents the integrated arbitrary power density per square inch over one half-hemisphere.

Call this quantity for the chart representing, for instance, one hemisphere above the horizon:

$$\sum_{0}^{A_{z_1}} a\bar{P}_c = a_1\bar{P}_{c_1} + a_2\bar{P}_{c_2} + a_3\bar{P}_{c_3} + \ldots + \ldots \quad . \quad (2)$$

Then

$$\sum_{0}^{A} a\bar{P}_c = \left[\sum_{0}^{A_{z_1}} + \sum_{0}^{A_{z_2}} + \sum_{0}^{A_{a_1}} + \sum_{0}^{A_{a_2}}\right] a\bar{P}_c \quad . \quad . \quad (3)$$

For an aerial at ground level it is clear that $\sum_{0}^{A_{a_1}} a\bar{P}_c$ and $\sum_{0}^{A_{a_2}} a\bar{P}_c$ are zero, and in practice they would always be zero except for transmissions from high altitudes to low ground in the immediate vicinity, or from aeroplanes.

Evidently

$$\sum_{0}^{A} a\bar{P}_c \text{ approximates to } \int_0^A P_c da$$

4.3. Determination of the Value of K and hence of P_r the Power Density in watts per radian, per kilowatt Radiated.

W = total power radiated by antenna in watts.

If one kilowatt is radiated

$$W = 1{,}000$$

∴ from equation (1)

$$\int_0^{4\pi} P_r d\omega = \frac{K}{n}\int_0^A P_c da = 1{,}000$$

$$\therefore K = \frac{1{,}000n}{\int_0^A P_c da} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The method of determining $\int_0^A P_c da$ has been given in the last section, while n is obtained by dividing by π the area of the power density diagram (see Figs. 2 to 10/XVI:7) measured in square inches.

To guard against unequal paper shrinkages it is desirable that n should be recalculated for each chart, but as the area of the chart is measured each time a graphical integration is made, this is a small matter.

Having determined K from equation (4)

$$P_r = KP_c \quad . \quad . \quad . \quad . \quad (5)$$

4.4. Determination of *ED* per root kilowatt Radiated.

E = field strength in millivolts per metre.

D = distance in kilometres measured from the transmitting aerial along the radius vector to any point.

W_{cm} = power traversing each square centimetre of space normal to direction of propagation.

The power radiated per radian is distributed at any distance D over a spherical area which is numerically equal to D^2.

W_{cm}, the power density in watts per square centimetre of space normal to the direction of transmission, is therefore equal to $\frac{P_r}{D^2}$, where D is distance in centimetres, and to $\frac{P_r 10^{-10}}{D^2}$, where D is distance in kilometres.

But, as is shown in the following section, equation (8)

$$W_{cm} = 2{\cdot}65 \times 10^{-13}E^2$$

where E is the field strength in millivolts per metre.

$$\therefore\ 2{\cdot}65 \times 10^{-13}E^2 = \frac{P_r}{D^2} \times 10^{-10}$$

$$\therefore\ ED = \sqrt{\frac{P_r}{0{\cdot}00265}} = 100\sqrt{\frac{P_r}{26{\cdot}5}} = 100\sqrt{\frac{KP_c}{26{\cdot}5}}$$

Millivolts per metre × kilometres per root kilowatt . (6)

$$\text{Evidently } ED_{100} = 100\sqrt{\frac{K100}{26{\cdot}5}} = 1{,}000\sqrt{\frac{K}{26{\cdot}5}} \quad . \quad . \quad (7)$$

4.5. Derivation of Power Density in watts per square centimetre, Normal to Direction of Propagation, in a Plane Electromagnetic Wave.

E_s and E_v = field strength respectively in c.g.s. electrostatic units per centimetre and in volts per centimetre.

H_m = electromagnetic component of field in c.g.s. electromagnetic units.

ε = specific inductive capacity of space = 1 in Gaussian units.

μ = permeability of space = 1 in Gaussian units.

From Pierce, *Electric Oscillations and Electric Waves*, equation 16, p. 375, and equation 33, p. 387, respectively,

$$W_{cm} = \frac{cE_s}{4\pi} \times H_m \text{ and } H_m = \sqrt{\frac{\varepsilon}{\mu}}E_s$$

Eliminating H_m between these equations

$$\therefore\ W_{cm} = \frac{cE_s^2}{4\pi}\sqrt{\frac{\varepsilon}{\mu}} \text{ ergs per second}$$

$$= \frac{3 \times 10^{10}E_v^2}{4\pi \times 9 \times 10^4} \text{ ergs per second}$$

where E_v is in volts per centimetre (300 volts = 1 c.g.s. electrostatic unit).

$$\therefore\ W_{cm} = \frac{E_v^2}{4\pi \times 30} = 2{\cdot}65 \times 10^{-3}E_v^2 \text{ watts per square centimetre}$$

$$= 2{\cdot}65 \times 10^{-13}E^2 \text{ watts per square centimetre} \quad . \quad (8)$$

where E is field strength in millivolts per metre.

(1 millivolt per metre evidently $= 10^{-5}$ volts per centimetre.)

4.6. Determination of *K* and *ED* from Directivity. This does not involve any integration since the result of integration is implicit in the value of directivity. This method is evidently to be employed when the value of directivity is known.

The directivity of an aerial system is defined as the ratio of the power density radiated, in the direction of the maximum radiation, to the mean power density radiated over the surface of a sphere with the aerial at the centre. In other words, it is the maximum power gain compared with that of a source radiating equally in all directions. *It is therefore equal to the maximum gain of the array.*

δ = directivity.

$\hat{P}_c$ = maximum value of $P_c = 100$

Then the directivity $\delta = \dfrac{\hat{P}_c}{\dfrac{1}{A}\displaystyle\int_0^A P_c da}$

Also, for an antenna radiating 1 kilowatt, it is possible to write

$$\delta = \frac{\hat{P}_r}{\frac{1}{4\pi} \times 1{,}000} = \frac{K\hat{P}_c}{\frac{1}{4\pi} \times 1{,}000} = \frac{100K}{\frac{1}{4\pi} \times 1{,}000} = \frac{4\pi K}{10}$$

$$\therefore\ K = \frac{5\delta}{2\pi} \quad . \quad . \quad . \quad . \quad (9)$$

ED per root kilowatt then follows from formula (6)

$$ED = 100\sqrt{\frac{KP_c}{26{\cdot}5}}$$

Examples. On Fig. 4/XVI:7 the directivity $\delta = 6{\cdot}95$

$$\therefore\ K = \frac{5\delta}{2\pi} = 5{\cdot}54$$

$$\therefore\ ED = 100\sqrt{\frac{KP_c}{26{\cdot}5}} = 1{,}000\sqrt{\frac{5{\cdot}54}{26{\cdot}5}}\sqrt{\frac{P_c}{100}}$$

$$= 458\sqrt{\frac{P_c}{100}}\ \text{mV/metre} \times \text{kilometres}/\sqrt{\text{kW}}$$

On Fig. 9/XVI:7 attached the directivity $\delta = 90{\cdot}4$

$$K = \frac{500}{2\pi} = 72$$

$$ED = 1{,}000\sqrt{\frac{72}{26{\cdot}5}}\sqrt{\frac{P_c}{100}}$$

$$= 1{,}650\sqrt{\frac{P_c}{100}}\ \text{mV/metre} \times \text{kilometres}/\sqrt{\text{kW}}$$

CHAPTER CVII

FOURIER ANALYSIS

1. FOURIER ANALYSIS provides graphical and mathematical methods of analysing a periodic wave into its component frequencies. Its general scope is wider than this, but the discussion below is confined to a presentation of the relevant aids to harmonic analysis provided by this calculus.

The normal presentation of Fourier Analysis starts with a general proof that it is possible to represent any periodic wave by the sum of a series of sine waves, of appropriate amplitude and phase, plus a direct-current component of magnitude equal to the mean amplitude of the wave over one period. Such a proof, for instance, will be found in *The Theory of Alternating Currents*, by A. Russell.

In the whole of the following discussion it will be assumed that the periodic wave under consideration represents a quantity varying with time.

There are two methods of analytical representation of the series corresponding to a periodic wave.

The first is:

$$S = A_0 + A_1 \sin(\omega t + \phi_1) + A_2 \sin(2\omega t + \phi_2) + A_3 \sin(3\omega t + \phi_3) + \dots \quad (1)$$

where $\omega = 2\pi f$

$$f = \frac{1}{T}$$

T = the period of the wave measured from any point of amplitude A_0 and positive slope to the next similar point.

The second is:

$$S = A_0 + a_1 \cos \omega t + b_1 \sin \omega t + a_2 \sin 2\omega t + b_2 \cos 2\omega t + a_3 \sin 3\omega t + b_3 \sin 3\omega t + \dots \quad (2)$$

The coefficients and phase angles in (1) are evidently related to those in (2) by the equations:

$$A_1 = \sqrt{a_1^2 + b_1^2}, \quad A_2 = \sqrt{a_2^2 + b_2^2}, \quad A_3 = \sqrt{a_3^2 + b_3^2}, \text{ etc.} \quad (3)$$

$$\phi_1 = \tan^{-1}\frac{b_1}{a_1}, \quad \phi_2 = \tan^{-1}\frac{b_2}{a_2}, \quad \phi_3 = \tan^{-1}\frac{b_3}{a_3}, \text{ etc.} \quad (4)$$

A_0, the amplitude of the steady component (corresponding to the direct current or voltage component in a current or voltage wave), is the same in each series. The amplitude of each component frequency is given by A_1, A_2, A_3, etc.

Where the periodic wave is determined by a simple analytical expression it is usual to employ a mathematical method to determine the magnitudes of the coefficients A_1, A_2, A_3, etc., because the graphical methods are all very laborious. When this is not the case graphical methods have to be used.

2. Graphical Methods.

2.1. Simple Ordinate Method. This method involves finding the coefficients of all the frequencies of appreciable amplitude. Its accuracy depends on the amplitude of the remaining coefficients which are not found.

No rigid method of testing for the presence of higher harmonics exists which is simpler than the somewhat laborious process of applying the simple ordinate method at any frequency which is suspected to be present. Examination of the wave form may give some clues. If the wave form is periodic at a high harmonic frequency, that frequency is evidently present. If the wave form is not periodic at a high frequency, then the harmonic series can normally be assumed to be convergent (i.e. the amplitudes falling off with increase of harmonic frequency) at a rate at least as great as that of a square wave. The number of harmonics for which the amplitude is found should therefore be equal to the number of harmonics extending up to the highest harmonic of interest, or extending up to the highest harmonic which would have to be taken into account in a square wave to give the required accuracy, whichever is the greater. Any harmonic frequency which is obviously present is evidently a frequency of interest.

The value of A_0 in equations (1) and (2) is then given by the mean height of the curve above the axis of time, see Fig. 1, which shows the case of a square wave plus a D.C. component. The value of A_0 is equal to the D.C. component and may be found by dividing the total area between the curve and the axis of time by T. Areas below the axis of time are subtracted from areas above the axis of time. Evidently A_0 may be positive or negative.

A line parallel to the time axis should be drawn at a distance A_0 from the time axis. This represents the mean height of the curve above the axis of time and will be called the base line. It is shown as BB in Fig. 1.

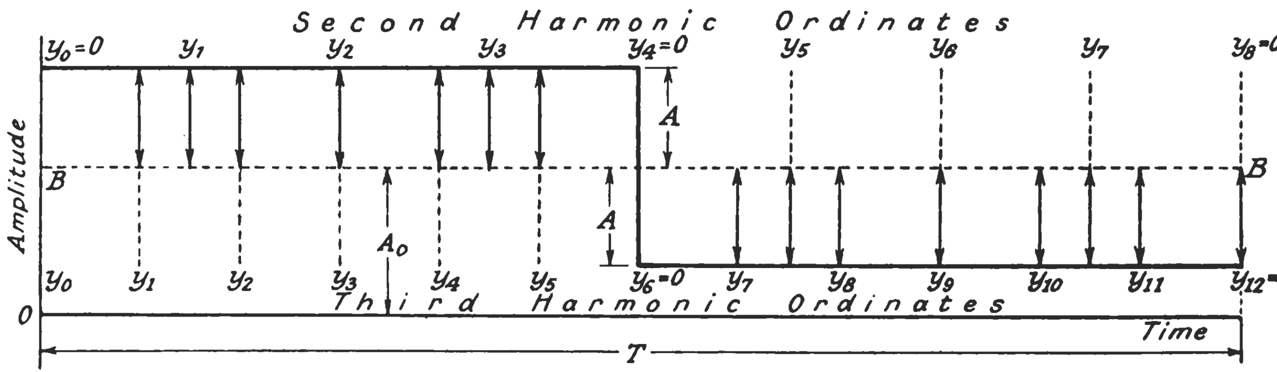


FIG. 1/CVII:2.—Graphical Fourier Analysis of a Square Wave.

The origin of time is located at any point, of zero amplitude with regard to the base line, and positive slope.

The highest harmonic required should be found first. For this purpose divide the base line into $4n$ equal parts, where n is the number of the harmonic sought : if the second harmonic is required $n = 2$, and so on. The beginning of the first part should be the point corresponding to the origin of time as defined above. The values of the ordinates measured from the base line to the curve at the beginning of each of these parts will be called y_0, y_1, y_2, etc.

Then evaluate

$$a'_n = \frac{1}{n}(y_0+y_4+y_8+\ \dots\ +y_{4n-4}) \qquad . \qquad . \quad (5a)$$

and

$$b'_n = \frac{1}{n}(y_1+y_5+y_9+\ \dots\ +y_{4n-3}) \qquad . \qquad . \quad (5b)$$

Then the corresponding coefficients in equation (2) are given by $a_n = a'_n$ and $b_n = b'_n$. And the corresponding coefficients in equation (1), if the series is required in this form, are given by

$$A_n = \sqrt{a_n^2+b_n^2} \text{ and } \phi_n = \tan^{-1}\frac{a_n}{b_n} \qquad . \qquad . \quad (5c)$$

The above procedure applies generally to the determination of the highest harmonic only. For a lower harmonic, of frequency m times the frequency of the function, i.e. of frequency $f_m = \frac{m}{T}$, *where T is the period of the function, the procedure is identical, except that the a and b coefficients of such higher harmonics must be added or subtracted from the values of* a'_m *and* b'_m *to obtain the values of* a_m *and* b_m, *in accordance with the following equations.*

$$a_m = a'_m - a_{3m} - a_{3m} - a_{4m} - a_{5m} \qquad . \qquad . \quad (6a)$$

$$b_m = b'_m + b_{3m} - b_{5m} + b_{7m} - b_{9m}\ . \qquad . \qquad . \quad (6b)$$

If required, A_m and ϕ_m are then given by

$$A_m = \sqrt{a_m^2+b_m^2} \quad \text{and} \quad \phi_m = \tan^{-1}\frac{a_m}{b_m} \qquad . \qquad . \quad (6c)$$

Example. As an example the function in Fig. 1 will be analysed graphically in the above way.

The series for this wave is known and is

$$S = \frac{4A}{\pi}\sin\omega t + \frac{4A}{3\pi}\sin 3\omega t + \frac{4A}{5\pi}\sin 5\omega t + \ \dots$$
$$+\frac{4A}{n\pi}\sin n\omega t + \ \dots \qquad . \qquad . \quad (7)$$

where $f = \frac{\omega}{2\pi} = \frac{1}{T}$.

The amplitude of the nth harmonic is evidently $1/n$ times the amplitude of the fundamental frequency component : the amplitude of the twenty-first harmonic is therefore 4·75 % of the fundamental, and the error involved by neglecting all frequencies above the twenty-first harmonic might be expected to be appreciable.

Since the curve is of a simple regular shape it is evidently not necessary to draw in the subdivisions and ordinates to obtain the values of the y's, although the subdivisions must be put in by eye. The ordinates for the second and third harmonics have, however, been drawn for purpose of illustration. It should be noticed that the amplitude of the ordinates at the amplitude transition points, where the amplitude is theoretically indeterminate, is made equal to the mean of the extreme amplitudes. With the particular curve shown the values of these ordinates are not used, but in a curve with several transition points of this type in its period it might not be possible to locate them all at points where the ordinates are neglected.

Since ordinates drawn below the base line BB are negative it is evident that the values of all even-order coefficients, as given by equations (5a), (5b) and (6a), (6b), are zero.

The values of the a coefficients are also zero.

The values of the b' components for the odd harmonics are equal to A/n (or A/m), where A is the amplitude of swing of the function about the base line BB. This follows because the number of positive ordinates required to be taken (as indicated by equation (5b)) is always greater by one than the number of negative ordinates.

If all the harmonics above the twenty-first are neglected, evidently the values of the coefficients are given as below. The correct value and the percentage error are also shown.

TABLE I

Coefficient	Value by graphical method	True Value	Error
b_{21}	$0{\cdot}0475A$	0·0605	− 21·5%
b_{19}	$0{\cdot}0526A$	0·0667	− 21·5%
b_{17}	$0{\cdot}0588A$	0·0746	− 21·5%
b_{15}	$0{\cdot}0667A$	0·0846	− 21·5%
b_{13}	$0{\cdot}0770A$	0·098	− 21·5%
b_{11}	$0{\cdot}091A$	0·116	− 21·5%
b_9	$0{\cdot}111A$	0·142	− 21·5%
b_7	$0{\cdot}143A + b_{21} = 0{\cdot}1905A$	0·182	+4·7%
b_5	$0{\cdot}2A + b_{15} = 0{\cdot}2667A$	0·255	+4·7%
b_3	$0{\cdot}333A + b_9 - b_{15} + b_{21} = 0{\cdot}4251A$	0·425	—
b_1	$A + b_3 - b_5 + b_7$, etc. $= 1{\cdot}2615A$	1·275	− 0·86%

For the particular curve chosen the accuracy of this method is therefore not very good at the higher harmonics, and in general for any curve the accuracy obtained in the evaluation of the ampli-

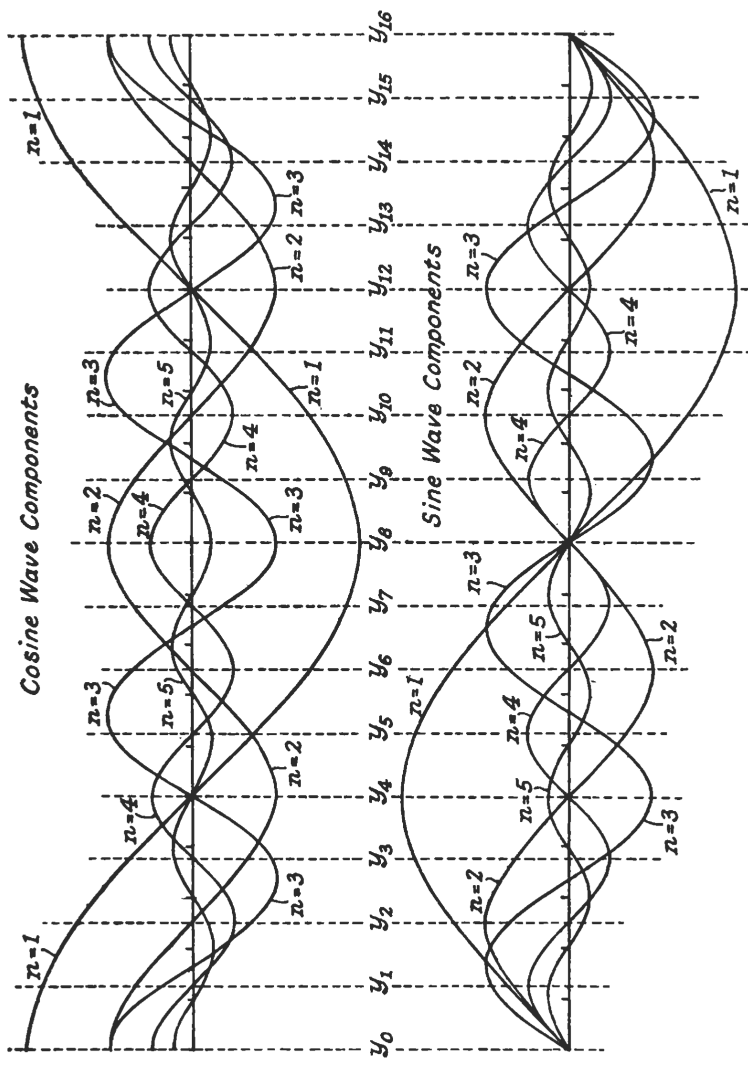


FIG. 2/CVII:2.—Cosine and Sine Components of a Hypothetical Periodic Wave. (y ordinates drawn in for fourth harmonic.)

tude of any harmonic depends on how many coefficients of higher harmonics are taken into account. It appears that if each component is evaluated as the sum of two components the error is about 5%. For instance, $b_{17} = 0{\cdot}0588A + b_{51} = 0{\cdot}0784$, which is within

5% of the true value. The error in the case of curves which are normally met in practice is hard to estimate, but will probably be smaller.

Validity of the Method. The validity of the method may be established by an examination of Fig. 2, which shows two sets of component frequencies : a set of cosine waves consisting of a fundamental frequency and all harmonics up to the fifth, and a set of sine waves consisting of a fundamental and all harmonics up to the fifth. The two sets of curves may be imagined to represent the component frequencies of a periodic wave which is not shown, but may be built up by adding the ordinates of the curves. If the procedure for obtaining the amplitudes of each component described above is applied to the two sets of curves, replacing the ordinates of the unknown periodic wave by the sum of the ordinates of the two sets of curves, the peak values of each component frequency will be obtained as the sum of one of the cosine waves in the figure plus one of the sine waves in the figure. This proof is inductive, but reasonably convincing. It should certainly be followed by anyone who is going to apply the method.

Rigorous Proof. To avoid confusion it should be made clear that m has been used to indicate a particular value of n. In the discussion below n is used in cases where m was used previously.

1. It will be noticed that the ordinates chosen by equations (5*a*) and (5*b*), *when determining the coefficients at any frequency* f_n, are located at all the points at which the cosine or sine component of the frequency in question is at its peak value, and the number of ordinates is always equal to n.

It follows that the sum of the ordinates in equation (5*a*) equals na_n, and in equation (5*b*) equals nb_n.

2. It will be shown that the contribution from all frequencies lower than f_n is zero and that the contributions of all frequencies higher than f_n is zero for all frequencies which are non-integral multiples of f_n.

3. In equation (5*a*) the sum of the ordinates contains the sum of the coefficients (i.e. peak amplitudes) of all component frequencies which are integral multiples of f_n.

4. In equation (5*b*) the sum of the coefficients of all component frequencies which are even integral multiples of f_n is zero, while the coefficients of odd integral multiples of f_n alternately subtract and add to provide their contribution.

It is evident that, if these four statements are true, equations (5*a*), (5*b*), (6*a*) and (6*b*) are true.

No. 1 can be seen to be true for any frequency by inspection of Fig. 2 with the y ordinates for that frequency drawn. The ordinates for the fourth harmonic of the fundamental frequency have been drawn. It will be seen that there are four ordinates called for in each equation and that y_0, y_4, y_8, and y_{12} all occur at the positive peak amplitude of the fourth harmonic in the cosine series, while y_1, y_5, y_9 and y_{13} all occur at the positive peak amplitude of the sine series. Similarly, if the ordinates for the nth harmonic were drawn, the equations would call for n ordinates each and the ordinates on each set of curves would all be located at the positive peak amplitudes of the nth harmonic in the respective set of waves. Hence, in each case, the sum of the y ordinates contains the sum of n wanted coefficients.

Proof of Statement No. 2. This follows from the corollary of a simple theorem, of which the enunciation is:

The sum of any system of vectors of equal magnitude and uniform angular distribution is zero.

Corollary. The sum of the components of the vectors in any direction is zero.

A system of vectors with uniform angular distribution is one in which, if the vectors are drawn so as to meet at a common point, the angles between neighbouring vectors are all equal.

The proof of the theorem is derived from the fact that in any regular polygon the sides are all equal and the external angles between any two adjacent sides are all equal.

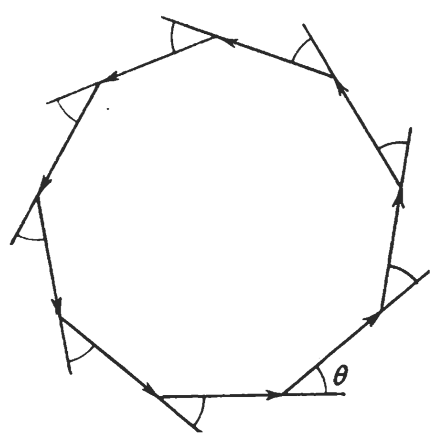


FIG. 3/CVII:2.—Vector Polygon Corresponding to n Vectors of Uniform Angular Distribution with $n = 9$.

Note that external angle $\theta = \frac{2\pi}{n}$ (= 40° in this case).

Any regular polygon of vectors with cyclic sense, as shown, for instance, in Fig. 3, therefore represents a system of equal magnitude vectors with uniform angular distribution, and evidently the resultant is zero. If the number of sides is n, the external angles θ are all equal to $2\pi/n$, and each external angle is the angle between adjacent vectors.

Evidently a regular polygon may be drawn with any number of sides, so that any system of equal magnitude vectors with uniform angular distribution may be represented by a closed regular polygon and has zero resultant.

Now designate the individual vectors of such a system of n vectors by $Ve^{j\phi}$, $Ve^{j(\phi+\theta)}$, $Ve^{j(\phi+2\theta)}$, etc.

Note that ϕ denotes any arbitrary angular position for the first vector, while θ is the angular difference between adjacent vectors. Note also that the last vector in the series is $Ve^{j(\phi+\overline{n-1}\theta)}$ and $n\theta = 2\pi$ or $m2\pi$, where n and m are integers. (If m is greater than unity, the order of the vectors necessary to form a non re-entrant polygon will have to be changed. The polygon is, however, always closed and the point is unimportant.)

Each of these vectors may be resolved into two vectors at right angles :

$$Ve^{j\phi} = V\cos\phi + jV\sin\phi \qquad (8)$$

$$Ve^{j(\phi+\theta)} = V\cos(\phi+\theta) + jV\sin(\phi+\theta) \qquad (9)$$

and so on.

The sum of the real components :

$$\sum_{\varepsilon=0}^{\varepsilon=\overline{n-1}\theta} V\cos(\phi+\varepsilon) = 0 \qquad (10)$$

And the sum of the imaginary components :

$$\sum_{\varepsilon=0}^{\varepsilon=\overline{n-1}\theta} V\sin(\phi+\varepsilon) = 0 \qquad (11)$$

where ε is successively put equal to 0, θ, 2θ, 3θ, $\overline{n-1}\theta$ in each term.

Hence, if the sum of any number of quantities can be represented by either of the expressions (10) or (11), that sum is equal to zero. The restriction on the value of θ $\left(\text{i.e. } \theta = \frac{2\pi}{n}\right)$ must be remembered.

Now consider the contribution to the sum of the y ordinates in equation (5a) provided by a frequency $f_m = \frac{m}{T} = mf$, where f is the frequency of the function ; in other words, by the mth harmonic when a_n, the coefficient of the nth harmonic, is being determined. Remembering that the ordinates are spaced $\frac{T}{4n}$ apart, the contributions of the mth harmonic are :

To y_0: a_m

To y_4: $a_m \cos m\frac{2\pi}{T}t \left(\text{where } t = \frac{4T}{4n}\right) = a_m \cos \frac{m}{n}2\pi$

To y_8: $a_m \cos m\frac{2\pi}{T}t \left(\text{where } t = \frac{8T}{4n}\right) = a_m \cos \frac{2m}{n}2\pi$

To y_{12}: $a_m \cos m\frac{2\pi}{T}t \left(\text{where } t = \frac{12T}{4\pi}\right) = a_m \cos \frac{3m}{n}2\pi$

and so on. (The series may, of course, stop before y_{12}, the last term being y_{4n-4}.)

When m is not an integral multiple of n the sum of these contributions is in the form of equation (10), where $\phi = 0$, $\theta = \frac{m}{n}2\pi$, and n is the order of the harmonic and also the number of ordinates, and therefore the number of vectors entering into the sum. This sum is therefore zero. This establishes statement No. 2 for equation (5*a*).

When m is an integral multiple of n the contributions to each ordinate are all the same and equal to a_m cos (an integral multiple of 2π), that is, they are equal to a_m. This establishes statement No. 3.

The contributions to the y ordinates in equation (5*b*) by the mth harmonic, when the nth harmonic is being determined are:

To y_1: $b_m \sin m\frac{2\pi}{T}t \left(\text{where } t = \frac{T}{4\pi}\right) = b_m \sin \frac{m}{n}\frac{\pi}{2}$

To y_5: $b_m \sin m\frac{2\pi}{T}t \left(\text{where } t = \frac{5T}{4\pi}\right) = b_m \sin \frac{5m}{n}\frac{\pi}{2}$

To y_9: $b_m \sin m\frac{2\pi}{T}t \left(\text{where } t = \frac{9T}{4n}\right) = b_m \sin \frac{9m}{n}\frac{\pi}{2}$

and so on. (The series may, of course, stop before y_9, the last term being y_{4n-3}.)

When m is not an integral multiple of n the sum of these contributions is in the form of equation (11), where $\phi = 0$, $\theta = \frac{m}{n}\frac{\pi}{2}$ and π is the number of the harmonic and also the number of ordinates, and therefore the number of vectors entering into the sum. The sum is therefore zero. This establishes statement No. 2 for equation (5*b*).

When m is an even integral multiple of n every contribution is zero and hence the sum is zero. When m is an odd integral multiple of n the contributions are all negative and equal to $-b_m$ for $\frac{m}{n} = 1$, 5, 9, 13 . . . etc., and are positive and equal to $+b_m$ for $\frac{m}{n} = 3$, 7, 11, 15 . . . etc. This establishes statement No. 4 in the specifically required form.

2.2. Basic Area Method. To describe this it is convenient to use a standard conventional method of representing the area bounded by a curve plotted on a field of co-ordinates, the axis of the independent variable, and any pair of chosen ordinates.

If $y = x^2$ or $mx+c$ or $\sin x$ or $\log x$ or any expression containing one variable x, then y is said to be a function of x. The general symbolic way of saying that y is a function of x is to write $y = f(x)$. If $y = f(x)$ it does not necessarily mean that a simple analytical

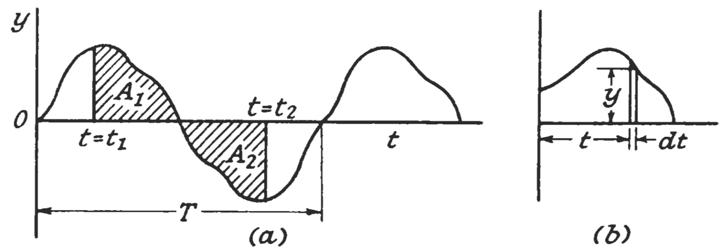


FIG. 4/CVII:2.—Curve $y = f(t)$ with area between $t = t_1$ and $t = t_2$ shaded.

expression exists for y in terms of x: the functional relationship may equally well be described by a curve as in Fig. 4, which shows y as a function of time t : $y = f(t)$. The area contained between the two ordinates $t = t_1$ and $t = t_2$, the curve and the axis of time t is shown shaded. On account of the form of the curve and the ordinates chosen, part of the area (A_1) lies above the curve and part (A_2) lies below the curve. A_1 is conveniently considered to be positive and A_2 is conveniently considered to be negative, so that the total area is equal to $A_1 - A_2$. The general way of describing this area is derived from assuming it to be made of a number of strips of height $y = f(t)$ and of width dt, where the d is put in front of the t to indicate that a vanishingly small width is to be taken. The area is the sum of all such strips between the chosen ordinates. See Fig. 4 (b).

The area between the ordinates $t = t_1$ and $t = t_2$ at (a) might be written $\sum_{t=t_1}^{t=t_2} y\, dt$ since $y\, dt$ is the area of each component strip. It is, however, usually written $\int_{t_1}^{t_2} y\,dt = \int_{t_1}^{t_2} f(t)\,dt$. The $\int$ sign may be taken as an elongated S, indicating that the sum of all the quantities $y\,dt$ is to be taken. This convention may appear cumbersome and useless, but it will appear later that in describing areas it saves a lot of words.

Now suppose that the curve $y = f(t)$ in Fig. 4 repeats itself every T seconds and that it is required to find the component frequencies contained in the wave. The zero frequency component, corresponding to the direct-current component in the case of a current wave, can evidently be found by finding the mean height of the curve during any period T. This is evidently equal to " the area under the curve " in any period T divided by T. In other words

$$A_0 = \frac{1}{T}\int_0^T f(t)\,dt \quad . \quad . \quad . \quad . \quad (12)$$

The amplitude of each component frequency may be found by a simple stratagem. Imagine that the wave is a current wave and that it is analysed by means of a dynamometer-type instrument as follows. The dynamometer consists of two air-core coils, one fixed and one arranged to rotate under the interaction of the coil fields when currents are passed simultaneously through both coils. The constant of the dynamometer is such that if a D.C. current I_1 is passed through coil 1 and a D.C. current I_2 through coil 2, the torque on the moving coil is $q = kI_1I_2$. Suppose further that the inertia of the instrument is such that it is incapable of responding to alternating forces.

If, now, the current $i = y = f(t)$ is passed through one coil and a unit amplitude current $i = \cos n\omega t$ $\left(\text{where } \omega = \frac{2\pi}{T}\right)$ is passed through the other coil, q_m, the instantaneous component of torque due to the harmonic of the mth order at any time t is equal to k times the product of the unit amplitude current and the sum of the sine and cosine components of the mth harmonic

$$\begin{aligned} q_m &= k \cos n\omega t(a_m \cos m\omega t + b_m \sin m\omega t) \\ &= k\frac{a_m}{2}(\cos \overline{m+n}\omega t + \cos \overline{m-n}\omega t) \\ &\quad + k\frac{b_m}{2}(\sin \overline{m+n}\omega t + \sin \overline{m-n}\omega t) \quad . \quad . \quad (13) \end{aligned}$$

When $m \neq n$, equation (13) represents sum and difference frequencies which will not register on the dynamometer owing to its inertia. In other words, any component frequency in the wave under analysis other than a frequency equal to the frequency in the second coil will cause no indication.

When $m = n$,

$$q_n = k\frac{a_n}{2}(\cos 2\omega t + \cos 0) + k\frac{b_n}{2}(\sin 2\omega t + \sin 0)$$

$$= k\frac{a_n}{2} + k\frac{a_n}{2}\cos 2\omega t + k\frac{b_n}{2}\sin 2\omega t \quad . \quad . \quad . \quad (14)$$

This corresponds to a steady torque of magnitude $k\frac{a_n}{2}$ plus an alternating torque of frequency $2\omega t$ to which the dynamometer does not respond. The indicated steady torque is therefore $k\frac{a_n}{2}$ = the mean value of the torque.

Since the function $i = f(t)$ repeated at intervals T can be represented by a series of sine and cosine waves, and it has been shown that only that component of frequency equal to the frequency of the current in the second coil gives rise to torque, it follows that the torque due to the whole current in the first coil is also $k\frac{a_n}{2}$.

The steady torque can be calculated in another way : consider the interaction between the whole current in the first coil and the current in the second coil. The instantaneous torque is

$$q = kf(t) \cos n\omega t$$

which may be plotted as a curve relating q and t : i.e. q as a function of t where the function of t is now $F(t) = kf(t) \cos n\omega t$. The mean value of this torque during any period T is

$$\bar{q} = \frac{1}{T}\int_0^T F(t)\,dt = \frac{k}{T}\int_0^T f(t)\cos n\omega t\,dt = k\frac{a_n}{2}$$

since the mean torque has already been shown to be equal to $k\frac{a_n}{2}$.

$$\therefore\ a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt \quad . \quad . \quad . \quad (15a)$$

Similarly,

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt \quad . \quad . \quad . \quad (15b)$$

where $\omega = \frac{2\pi}{T}$

Equation (15*a*) may appear formidable, but in fact what it says is : Divide the time base of the function under analysis (from 0 to T)

into any convenient number of parts and read off the value of t and $y = f(t)$ at the end of the first part and multiply the value of $f(t)$ by cos $n\omega t$. Repeat at the end of each of the other parts of the base line and plot the new set of ordinates as a function of t. Then find the area contained between the resultant curve, the axis of time and the ordinates $t = 0$ and $t = T$ and multiply it by $\frac{2}{T}$. The result is the value of a_n. The advantage in brevity of the method used for defining areas will now be apparent ; but unfortunately it does not provide help in avoiding the considerable labour of doing what it says.

This may be reduced to an amount commensurate with that required in the simple ordinate method by the application of one of the two approximations, for calculating areas which follow immediately.

(When using the approximations it is evidently not necessary to plot the resultant modulated curve, but only to find the amplitude of the relevant ordinates as described below.)

Rule for Calculating Areas of Complete Half-Cycles with Negligible Curvature in Original Wave. It will be appreciated that no positive area and no negative area under the cosine or sine modulated wave can have a time base greater than $T/2n$, although the time base may be less owing to the original function $f(t)$ crossing the time axis, i.e. changing from a positive to a negative value. In the case of areas on one side of the time base, of duration $T/2n$, *when the curvature of the function $f(t)$ in this interval is negligible*, the area is given by

$$\frac{2}{\pi}y_p \times \frac{T}{2n} = \frac{Ty_p}{n\pi}$$

where y_p is the value of $f(t)$ at the time the cosine or sine function in question is at its peak value, and therefore equal to unity, so that at this instant the value of the modulated wave is equal to plus or minus the original function.

The general problem cannot be further simplified, but particular problems can be very much simplified, particularly where the original function is represented by straight lines. Such cases are, however, best solved by analytical methods.

It is to be noted that the above method gives the values of the coefficients (a_n and b_n) of each frequency component quite independently of the other components. It is only necessary to perform the analysis for the particular frequencies which are required. The method is theoretically exact, but in practice is limited by the accuracy with which the relevant areas are determined.

Weddle's Rule for Determination of Areas.

In general, this rule may be applied to any positive area without discontinuities in it, or to any negative area with the same limitation. *In particular for the present purpose it is to be applied when the curvature of the function being analysed is appreciable* in any period of duration $T/2n$, and where positive or negative areas of time duration less than $T/2n$ are involved. Each positive area and each negative area must be determined by the application of this rule : the algebraic sum of the areas then gives the total area.

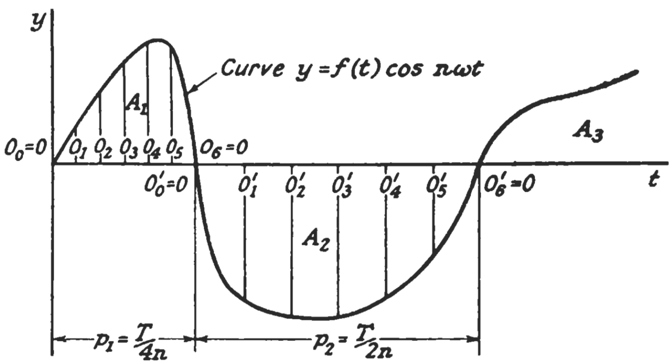


FIG. 5/CVII:2.—Application of Weddle's Rule.

The method is applicable (within the above limits) to the determination of an area bounded by a curve, the axis of the independent variable (time in the examples above) and two limiting ordinates.

Divide the base line of the area into six equal parts and measure the height of the seven ordinates o_0, o_1, o_2, o_3, o_4, o_5 and o_6. o_0 and o_6 are then the bounding ordinates.

The ordinates to be measured in the determination of a_n are illustrated in Fig. 5.

Then

$$\text{Area } A_1 = 0{\cdot}05 p_1(o_0+o_2+o_4+o_6+5(o_1+o_5)+6 \times o_3) \quad . \quad (16)$$
$$\text{Area } A_2 = -\, 0{\cdot}05 p_2(o_0'+o_2'+o_4'+o_6'+5(o_1'+o_5')+6 \times o_3') \quad . \quad (16a)$$

and so on.

The total area is evidently equal to the sum of all the positive areas minus the sum of all the negative waves.

The proof of equation (16) is given in *The Theory of Alternating Currents*, by A. Russell, Vol. II, p. 123.

3. Analytical Method.

This is only applicable where the wave to be analysed is capable of being expressed exactly or approximately by an analytical expression. If it can only be represented approximately by an analytical expression, the analysis is evidently approximate to the same degree.

The method consists in substituting for $f(t)$ in equations (15*a*) and (15*b*) the actual analytical expression defining the wave to be analysed and then performing the necessary gymnastics to obtain an explicit analytical expression. If the method fails it is always possible to plot the function and resort to graphical methods.

The above constitutes a complete description of the method. The application of the method involves integral calculus and is therefore only useful to those who are familiar with this.

Periodic Waves Defined by Different Functions of Time in Different Parts of the Period. If $y = f(t)$, the wave to be analysed is not continuous throughout its period T, but is represented by:

$$y_1 = f_1(t) \text{ in the range } 0 \text{ to } t_1$$
$$y_2 = f_2(t) \text{ in the range } t_1 \text{ to } t_2$$
$$y_n = f_n(t) \text{ in the range } t_n \text{ to } T$$

Then

$$A_0 = \frac{1}{T}\left[\int_0^{t_1} y_1\,dt + \int_{t_1}^{t_2} y_2\,dt + \ \ldots \ \int_{t_n}^{T} y_n\,dt\right]$$

$$a_n = \frac{2}{T}\left[\int_0^{t_1} y_1 \cos n\omega t\,dt + \int_{t_1}^{t_2} y_2 \cos n\omega t\,dt \ \ldots \ + \int_{t_n}^{T} y_n \cos n\omega t\,dt\right]$$

$$b_n = \frac{2}{T}\left[\int_0^{t_1} y_1 \sin n\omega t\,dt + \int_{t_1}^{t_2} y_2 \sin n\omega t\,dt \ \ldots \ + \int_{t_n}^{T} y_n \sin n\omega t\,dt\right]. \quad (1)$$

The above is self-evident since it amounts to analysing each part of the wave separately and adding up the result.

Example 1. Analyse the wave shown in Fig. 1 which consists

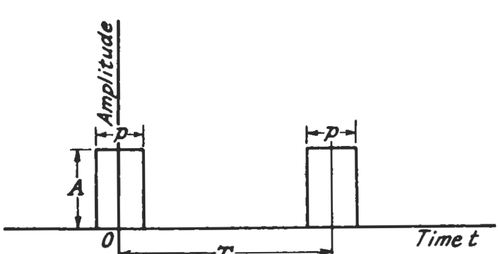


FIG. 1/CVII:3.—Periodic Wave Consisting of Square Pulses.

of a series of square pulses of amplitude A and duration p separated by intervals T.

Choose the origin of time in the centre of a pulse. Then

$$A_0 = \frac{1}{T}\int_0^{p/2} A\,dt + \frac{1}{T}\int_{p/2}^{T-\frac{p}{2}} 0 \times dt + \frac{1}{T}\int_{T-\frac{p}{2}}^{T} A\,dt$$

$$= \frac{pA}{2T} + 0 + \frac{pA}{2T} = \frac{pA}{T}$$

This result can, of course, be seen by inspection without resorting to calculation.

$$a_n = \frac{2}{T}\int_0^{p/2} A \cos n\frac{2\pi}{T}t\,dt + \frac{2}{T}\int_{p/2}^{T-\frac{p}{2}} 0 \times \cos n\frac{2\pi}{T}t\,dt + \frac{2}{T}\int_{T-\frac{p}{2}}^{T} A \cos n\frac{2\pi}{T}t\,dt$$

$$= \frac{2}{T} \times \frac{T}{2n\pi}A\left[\sin n\frac{2\pi}{T}t\right]_0^{p/2} + 0 + \frac{2}{T} \times \frac{T}{2\pi n}A\left[\sin n\frac{2\pi}{T}t\right]_{T-\frac{p}{2}}^{T} = \frac{2A}{n\pi}\sin\frac{n\pi p}{T} \quad . \quad (2)$$

This completes the analysis. If b_n is calculated in the same way it will be found to be zero.

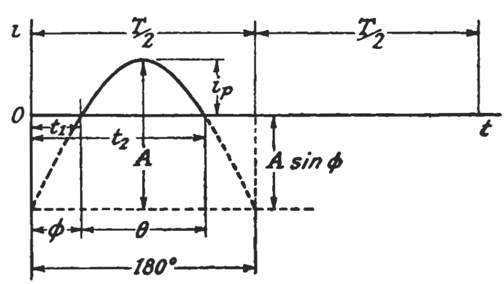


FIG. 2/CVII:3.—Current Wave in Class C Amplifier.

Example 2. Analyse the Current Wave in a Class C Amplifier with Sinusoidal Grid Drive. Fig. 2 shows the wave with the time origin arranged to make the wave constitute part of a sine wave during the period of current flow.

Conventions.

i = instantaneous current $= A \sin \omega t - A \sin \phi$.

i_p = peak current.

$\bar{i}$ = mean current = driven feed.

$\hat{i}_f$ = peak amplitude of fundamental frequency component of current.

A = equivalent value of peak current with 180° angle of current flow assuming linearity (e.g. of valve).

θ = angle of current flow.

ϕ = angle before current starts to flow.

t_1 = time at which current starts to flow.

t_2 = time at which current ceases to flow.

T = period of one cycle of the driving wave.

$\omega = 2\pi/T$.

Then

$$\omega t_1 = \phi \qquad \therefore t_1 = \frac{\phi}{\omega} \quad . \quad . \quad . \quad (3)$$

$$\omega t_2 = \pi - \phi \qquad \therefore t_2 = \frac{\pi}{\omega} - \frac{\phi}{\omega} \quad . \quad . \quad (4)$$

$$\hat{i}_p = A - A \sin \phi$$

$$\therefore A = \frac{\hat{i}_p}{1 - \sin \phi} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\phi = 90° - \frac{\theta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The wave to be analysed consists of three functions of time :

From 0 to t_1 : $i = 0$

„ t_1 to t_2 : $i = A \sin \omega t - A \sin \phi$

$$= \frac{\sin \omega t - \sin \phi}{1 - \sin \phi} \hat{i}_p$$

„ t_2 to T : $i = 0$

There is no point in writing down those integrals which are evidently zero because they correspond to the time during which no current flows.

The mean current or driven feed is then given by

$$\bar{i} = \frac{1}{T}\int_{t_1}^{t_2} \frac{\sin \omega t - \sin \phi}{1 - \sin \phi} \hat{i}_p \, dt$$

$$= \frac{\hat{i}_p}{T(1 - \sin \phi)} \int_{t_1}^{t_2} (\sin \omega t - \sin \phi) \, dt$$

$$= \frac{\hat{i}_p}{T(1 - \sin \phi)} \left[-\frac{1}{\omega} \cos \omega t - t \sin \phi \right]_{t_1}^{t_2}$$

Inserting the values of t_1 and t_2 from (3) and (4) and putting

$$T = \frac{2\pi}{\omega}$$

$$\bar{i} = \frac{i_p\omega}{2\pi(1 - \sin\phi)}\left[-\frac{1}{\omega}\cos(\pi - \phi) - \left(\frac{\pi}{\omega} - \frac{\phi}{\omega}\right)\sin\phi + \frac{1}{\omega}\cos\phi + \frac{\phi}{\omega}\sin\phi\right]$$

$$= \frac{2\cos\phi - (\pi - 2\phi)\sin\phi}{2\pi(1 - \sin\phi)}i_p \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$a_n = \frac{2i_p}{T(1 - \sin\phi)}\int_{t_1}^{t_2}(\sin\omega t - \sin\phi)\cos n\omega t\,dt \quad . \quad . \quad . \quad (8)$$

$$= \frac{2i_p}{T(1-\sin\phi)}\int_{t_1}^{t_2}\left(-\tfrac{1}{2}\sin\overline{n-1}\,\omega t + \tfrac{1}{2}\sin\overline{n+1}\,\omega t - \sin\phi\cos n\omega t\right)dt$$

$$= \frac{2i_p}{2\pi(1 - \sin\phi)}\left[\frac{\cos\overline{n-1}\,\omega t}{2n-2} - \frac{\cos\overline{n+1}\,\omega t}{2n+2} - \frac{\sin\phi\sin n\omega t}{n}\right]_{t_1}^{t_2}$$

Which, by analogy with the transformation used in the case of the b_n coefficient (see below), may be seen to be

$$a_n = \frac{i_p}{\pi(1 - \sin\phi)}\left[-\frac{\sin\frac{1}{2}(n-1)\pi\sin\frac{1}{2}(n-1)(\pi - 2\phi)}{n-1} + \frac{\sin\frac{1}{2}(n+1)\pi\sin\frac{1}{2}(n+1)(\pi-2\phi)}{n+1} - \frac{2}{n}\sin\phi\cos\frac{n\pi}{2}\sin\frac{n}{2}(\pi-2\phi)\right]. \quad (9)$$

Hence when n is odd $a_n = 0$.

When n is even, the trigonometrical functions of n and π not containing ϕ are equal to 1 or -1, so that substitution in (9) is quite simple in spite of its formidable appearance. For instance, when $n = 2$

$$a_n = a_2 = \frac{i_p}{\pi(1 - \sin\phi)}\left[-\sin\tfrac{1}{2}(\pi - 2\phi) - \tfrac{1}{3}\sin\tfrac{3}{2}(\pi - 2\phi) + \sin\phi\sin(\pi - 2\phi)\right]$$

The coefficients of the sine-wave components are given by

$$b_n = \frac{2i_p}{T(1 - \sin\phi)}\int_{t_1}^{t_2}(\sin\omega t - \sin\phi)\sin n\omega t\,dt \quad . \quad . \quad (10)$$

$$= \frac{2i_p}{T(1 - \sin\phi)}\int_{t_1}^{t_2}\left(\tfrac{1}{2}\cos\overline{n-1}\,\omega t - \tfrac{1}{2}\cos\overline{n+1}\,\omega t - \sin\phi\sin n\omega t\right)dt$$

$$= \frac{2i_p}{2\pi(1 - \sin\phi}\left[\frac{\sin\overline{n-1}\,\omega t}{2n-2} - \frac{\sin\overline{n+1}\,\omega t}{2n+2} + \frac{\sin\phi\cos n\omega t}{n}\right]_{t_1}^{t_2}$$

Inserting the values of t_1 and t_2

$$b_n = \frac{\hat{i}_p}{\pi(1 - \sin\phi)}\left[\frac{\sin(n-1)(\pi-\phi)}{2n-2} - \frac{\sin(n+1)(\pi-\phi)}{2n+2}\right.$$

$$\left. + \frac{\sin\phi\cos n(\pi-\phi)}{n} - \frac{\sin(n-1)\phi}{2n-2} + \frac{\sin(n+1)\phi}{2n+2} - \frac{\sin\phi\cos n\phi}{n}\right]$$

$$= \frac{\hat{i}_p}{\pi(1 - \sin\phi)}\left[\frac{\cos\frac{1}{2}(n-1)\pi\sin\frac{1}{2}(n-1)(\pi-2\phi)}{n-1}\right.$$

$$\left. - \frac{\cos\frac{1}{2}(n+1)\pi\sin\frac{1}{2}(n+1)(\pi-2\phi)}{n+1} - \frac{2}{n}\sin\phi\sin\frac{n\pi}{2}\sin\frac{n}{2}(\pi-2\phi)\right]. \quad (11)$$

When n is even $b_n = 0$. When n is odd (11) may be simplified for each value of n as in the case of (9).

When $n = 1$ the first term inside the brackets in (11) is equal to 0/0 and is therefore indeterminate.

For the case where $n = 1$ it is therefore necessary to put n equal to unity in equation (10) and start again. It will then be found that $a_n = 0$.

But

$$b_1 = \hat{i} = \frac{2\hat{i}_p}{T(1 - \sin\phi)}\int_{t_1}^{t_2}(\sin^2\omega t - \sin\phi\sin\omega t)\,dt$$

$$= \frac{2\hat{i}_p}{T(1 - \sin\phi)}\int_{t_1}^{t_2}(\tfrac{1}{2} - \tfrac{1}{2}\cos 2\omega t - \sin\phi\sin\omega t)\,dt$$

$$= \frac{2\hat{i}_p\omega}{2\pi(1 - \sin\phi)}\left[\tfrac{1}{2}t - \frac{\sin 2\omega t}{4\omega} + \frac{\sin\phi\cos\omega t}{\omega}\right]_{t_1}^{t_2}$$

$$= \frac{\hat{i}_p}{\pi(1 - \sin\phi)}\left[\frac{\pi}{2} - \phi - \frac{\sin 2(\pi-\phi)}{4} + \frac{\sin 2\phi}{4}\right.$$

$$\left. + \sin\phi\cos(\pi-\phi) - \sin\phi\cos\phi\right]$$

$$= \frac{\hat{i}_p}{\pi(1 - \sin\phi)}\left[\frac{\pi}{2} - \phi + \frac{\sin 2\phi}{2} - 2\sin\phi\cos\phi\right]$$

$$= \frac{\hat{i}_p}{\pi(1 - \sin\phi)}\left[\frac{\pi}{2} - \phi + \frac{\sin 2\phi}{2} - \sin 2\phi\right]$$

Hence

$$b_1 = \hat{i}_f = \frac{\pi - 2\phi - \sin 2\phi}{2\pi(1 - \sin\phi)}\hat{i}_p. \quad . \quad . \quad (11a)$$

From equations (7) and (11*a*) are derived the three important ratios for predicting the performance of Class C amplifiers.

From (7)

$$g = \frac{\bar{i}}{\hat{i}_p} = \frac{2\cos\phi - (\pi - 2\phi)\sin\phi}{2\pi(1 - \sin\phi)} \quad . \quad . \quad (12)$$

From (11*a*)

$$f = \frac{\hat{i}_f}{\hat{i}_p} = \frac{\pi - 2\phi - \sin 2\phi}{2\pi(1 - \sin\phi)} \quad . \quad . \quad . \quad (13)$$

From (12) and (13)

$$u = \frac{\hat{i}_f}{\bar{i}} = \frac{\pi - 2\phi - \sin 2\phi}{2\cos\phi - (\pi - 2\phi)\sin\phi} \quad . \quad . \quad (14)$$

In all the above $\phi = 90° - \frac{\theta}{2}$, where θ is the angle of current flow.

Relations (12), (13) and (14) with ϕ replaced by $90° - \frac{\theta}{2}$ are plotted in Fig. 1/X:22. Relations (9) and (11) are plotted in Figs. 1/X:22 and Figs. 6, 7 and 8/VIII:1.

CHAPTER CVIII

REPRESENTATION OF COMPLEX QUANTITIES AND RELATIONS BETWEEN THEM

1. Resistance-Reactance Representation of Impedances.

NORMALLY the magnitudes of the resistance and reactance components (whether series or parallel) of impedances vary with frequency. The most obvious, and probably the most common, method of representing such impedances is by two curves, one curve consisting of a plot of the resistance magnitude against frequency, and the other being a plot of the reactance magnitude against frequency. For many purposes such a form of representation is the most useful as it gives a continuous representation of the variation of each component with frequency, which is usually the relation of interest.

For some purposes, however, it is more useful to use another form of representation. For this purpose a plane of resistance and reactance (i.e. an axis of reactance crossing an axis of resistance at right angles) is used. At each of a number of chosen frequencies the value of the reactance is then plotted against the value of the resistance and the value of the frequency at which the corresponding impedance occurs is marked against each plotted point. Admittances can evidently be plotted in the same way.

When this is done it is found that the curve representing the impedance of the combination of a resistance in parallel with any combination of inductances and condensers is a circle of radius equal to half the resistance with centre on the real axis distant from the origin in a positive direction by an amount equal to half the resistance.

The admittance of such an arrangement is represented by a straight-line parallel to the imaginary axis drawn through a point on the real axis distant from the origin in a positive direction by an amount equal to the reciprocal of the resistance.

Similarly, the curve representing the conductance of a resistance in series with any combination of inductances and condensers is a circle of radius equal to half the reciprocal of the resistance with centre on the real axis distant from the origin in a positive direction by an amount equal to half the reciprocal of the resistance.

The impedance of such an arrangement is represented by a straight-line parallel to the imaginary axis drawn through a point on the real axis distant from the origin in a positive direction by an amount equal to the resistance.

If the combination of reactances in any of the above four cases is constituted by a single inductance or a single condenser, either the positive half or the negative half only of the circle is involved: the positive half for a positive reactance (e.g. inductance) and the negative half for a negative reactance (e.g. condenser).

If the combination of reactances contains two elements (which evidently must consist of one inductance and one capacity), the whole of the circle is involved.

If the combination of reactances contains more than two elements so that the total effective value of the reactance, as the frequency is varied, traverses the region between zero and minus infinity or zero and plus infinity more than once, the negative or the positive half of the curve is traversed once for each time the corresponding reactance region is traversed. Each point on the curve is then many valued as far as frequency is concerned: in other words, the same value of impedance occurs at several frequencies.

If the impedance of a network is represented by a straight line the inverse impedance is represented by a circle, and vice versa.

If the combination of reactances contains resistance in one more of its elements so arranged that the resistance component of the assembly of reactances cannot be represented at all frequencies by a constant resistance in series or parallel with the main resistance, then the resultant impedance is not represented by a straight line or a circle. For some purposes, however, it is still convenient to use this method of plotting.

When looking for an equivalent network to simulate an observed impedance, it is evident that, by plotting the impedance in the above way, it is possible to see at a glance whether it is capable of being simulated by a simple combination of one resistance and one or more reactances.

1.1. Proof. Before giving the extremely simple proof of the above statements it is instructive to derive the equation of a circle drawn in the resistance-reactance plane, which will be called the *ab* plane.

Fig. 1 shows a circle drawn in the *ab* plane. This is a plane in which resistances are plotted along the *a* axis and reactances are plotted along the *b* axis. Any plotted point of co-ordinates *a* and *b* then corresponds to an impedance $a+jb$.

It will be seen that the centre of the circle *c* is at the point: $a = a_c$, $b = b_c$ and its radius is *r*.

The point P on the circle (incidentally corresponding to a plot of the impedance $A+jB$) has co-ordinates $a = A$ and $b = B$.

The remainder of the construction and the values of the important lengths can be seen by inspection.

From the right-angled triangle CPQ it is immediately evident that

$$(A - a_c)^2+(B - b_c)^2 = r^2$$

For any other point on the circle of co-ordinates a and b,

$$(a - a_c)^2+(b - b_c)^2 = r^2 \quad . \quad . \quad . \quad (1)$$

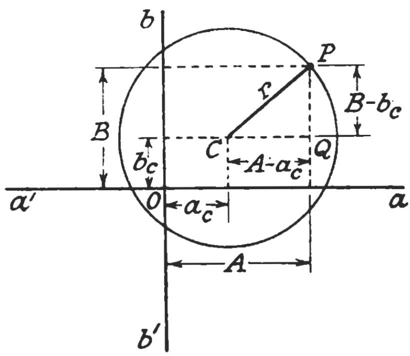


FIG. 1/CVIII:1.—Circle in *ab* Plane.

Since this condition must be satisfied for any point on the circle, and so for any values of a and b which specify a point on the circle, it follows that this equation gives sufficient information to enable the circle to be drawn, and is therefore said to be the equation to the circle in question.

Inspection of this equation shows that the co-ordinates of C, the centre of the circle, are respectively subtracted from a and b, inside the brackets defining the quantities to be squared ; while the square of the radius r appears on the right of the equation. Hence, if any equation involving a relation between a and b can be arranged in this form it defines a circle of prescribed centre and radius.

Reactance in Parallel with Resistance. If a resistance R is in parallel with a resistance X which may vary with frequency, whatever the value of X, the *impedance* is given by

$$Z = a+jb = \frac{1}{\frac{1}{R}+\frac{1}{jX}} = \frac{\frac{1}{R}}{\frac{1}{R^2}+\frac{1}{X^2}}+j\frac{\frac{1}{X}}{\frac{1}{R^2}+\frac{1}{X^2}}$$

Then

$$\frac{a}{b} = \frac{X}{R} \text{ (which incidentally is worth noting)}$$

$$\therefore X = \frac{aR}{b}$$

Hence

$$a = \frac{\frac{1}{R}}{\frac{1}{R^2}+\frac{1}{X^2}} = \frac{\frac{1}{R}}{\frac{1}{R^2}+\frac{b^2}{a^2R^2}} = \frac{R}{1+\frac{b^2}{a^2}} = \frac{a^2R}{a^2+b^2}$$

$$\therefore a^2+b^2 = aR$$

Adding $\frac{R^2}{4}$ to each side

$$\therefore a^2 - aR+\frac{R^2}{4}+b^2 = \frac{R^2}{4}$$

$$\therefore \left(a - \frac{R}{2}\right)^2+(b - 0)^2 = \frac{R^2}{4} \quad . \quad . \quad . \quad (2)$$

Equation (2) describes a circle with centre at $a = \frac{R}{2}$, $b = 0$, and radius $\frac{R}{2}$.

Fig. 2 shows such a circle with one value of $a+jb = A+jB$ plotted at the point P.

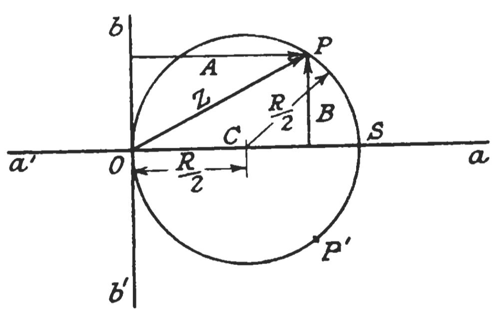


FIG. 2/CVIII:1.—Circle representing Impedance of Parallel Resistance and Reactance.

It is evident that if X is positive b can never be negative, so that no point such as P' on the negative half of the circle can exist. In this case, as the parallel reactance increases from zero to infinity, the point P travels round the upper half of the circle from the origin to the point S. Similarly, if X is negative, the point P travels round the lower half of the circle from O to S. The necessary discrimination (which selects one-half of the circle) is lost in the above analysis because it depends on the square of the reactance, which is positive, whether the reactance is negative or positive.

If, for instance, the parallel reactance is constituted by a condenser in parallel with an inductance, then, as the frequency increases from zero, the point P, representing the plot of the impedance, first travels round the upper half-circle from O to S and then round the lower half-circle from S to O. The behaviour of P for more complex arrangements of reactive elements in shunt with R can easily be determined.

The admittance of a reactance X varying with frequency in parallel with a resistance R is given by

$$Y = \frac{1}{R} - j\frac{1}{X} = a+jb$$

So that

$$a = \frac{1}{R} \text{ constant and } b = -\frac{1}{X} \text{ varying with frequency.}$$

Since a is constant the admittance is represented by a line at constant distance from the b axis equal to $\frac{1}{R}$.

This line is shown in Fig. 3. As before, the parts of this line which are traversed as the frequency varies, depends on the behaviour of the shunt reactance.

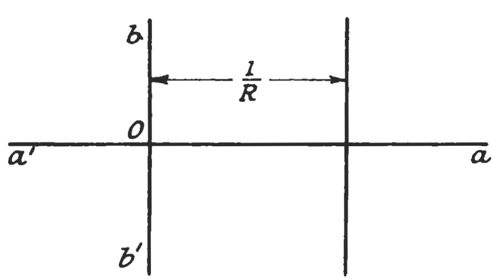


FIG. 3/CVIII:1.—Line representing Admittance of Parallel Resistance and Reactance.

Reactance in Series with a Resistance. The impedance of a resistance in series with a reactance X which varies with frequency is

$$Z = R+jX = a+jb$$

Since $a = R$, constant, this is represented by a line parallel to the axis of b at distance R. The diagram is identical with Fig. 3 except that $1/R$ is replaced by R.

The admittance of the same arrangement is given by

$$Y = \frac{1}{R+jX}$$

and comparison with the evolution of equation (2) shows that the

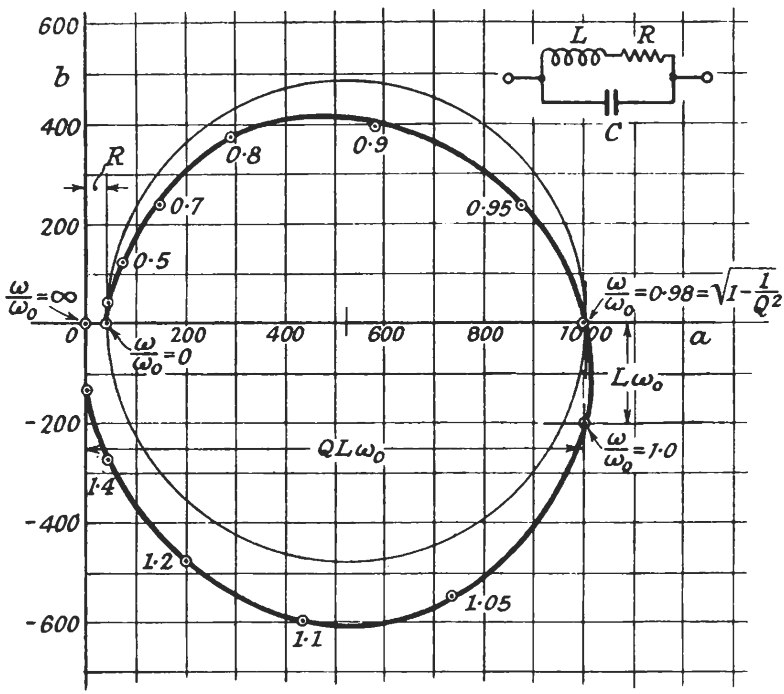


FIG. 4/CVIII:1.—Impedance of Parallel Resonant Circuit shown with

$$Q = \frac{L\omega_1}{R} = 5, \; L\omega_0 = \frac{1}{C\omega_0} = 200.$$

Values of a and b for given values of $\frac{\omega}{\omega_0}$.

$\frac{\omega}{\omega_0}$	a	b	$\frac{\omega}{\omega_0}$	a	b
0	40	0	0·98	1,000	− 0
0·1	40·8	19·4	1·0	1,000	−200
0·5	69·8	124	1·05	732	−550
0·6	93·7	169	1·1	433	−595
0·7	148	239	1·2	198	−475
0·8	297	377	1·4	41·7	−270
0·9	583	394	1·6	15·8	−192
0·95	877	237	2·0	4·3	−133

admittance is represented by a circle of radius $1/2R$ with centre at $a = 1/2R$, $b = 0$.

It can now be deduced that when the impedance is represented by a circle or a half-circle the admittance is represented respectively by a line or a " half "-line. The converse is evidently true.

Since the inverse of an impedance is the reciprocal of that impedance (i.e. the equivalent admittance), multiplied by the square of a resistance, it follows that the relation between an impedance and its inverse is also represented by a change from a circle to a line, and vice versa.

It will now be evident that the presence of a circle or a loop in an impedance plot of this kind indicates the presence of either a parallel resonance or of a series resonance seen through an impedance inverting network.

An impedance is only represented by a perfect circle when it is constituted by a resistance in parallel with a reactance, or is constituted by a resistance in series with a reactance and seen through an impedance inverting network.

In practice, a parallel resonant circuit normally consists of a condenser with a negligible resistance, and an inductance with a small but appreciable resistance : hence it is not represented by a perfect circle.

Fig. 4 shows the impedance curve of a practical parallel resonant circuit with a value of $Q = L\omega_0/R$ equal to 5. It will be seen that the impedance curve approximates to a circle of radius $(QL\omega_0 - R)/2$ with centre at $a = (QL\omega_0 + R)/2$, $b = 0$. As the value of Q increases the impedance curve approximates more and more closely to such a circle.

The curve in Fig. 4 is plotted from equation (18)/XXIV:2 and the values of ω/ω_0 are indicated for a number of points. These may be converted to frequency for any particular values of L and C by the obvious relation

$$f = \frac{f}{f_0} \times f_0 = \frac{\omega}{\omega_0} \times f_0 = \frac{\omega}{\omega_0} \times \frac{1}{2\pi\sqrt{LC}}$$

1.2. Generalized Impedance Diagram for Parallel Resonant Circuit. The development and the use of this diagram both depend on the value of Q being large. The error of the diagram is under 5% if Q is greater than 20 and under 1% if Q is greater than 100 ; errors for other values of Q are *pro rata*. For many purposes an error of 5% is quite tolerable.

Based on the concluding remarks of the last section, the assumption is made that the impedance curve of any parallel resonant circuit is represented by a circle in the ***ab*** **plane with** centre at $a = QL\omega_0/2$, $b = 0$, and of radius equal to $QL\omega_0/2$.

If, therefore, a circle is drawn with centre $a = \frac{1}{2}$, $b = 0$ and radius equal to $\frac{1}{2}$, this represents the impedance of all possible parallel resonant circuits of the form of the circuit in Fig. 4 to a scale of 1 to $QL\omega_0$, provided the value of Q is large enough to bring the error within tolerable limits.

Half of such a circle has been drawn in Fig. 5. If the value of

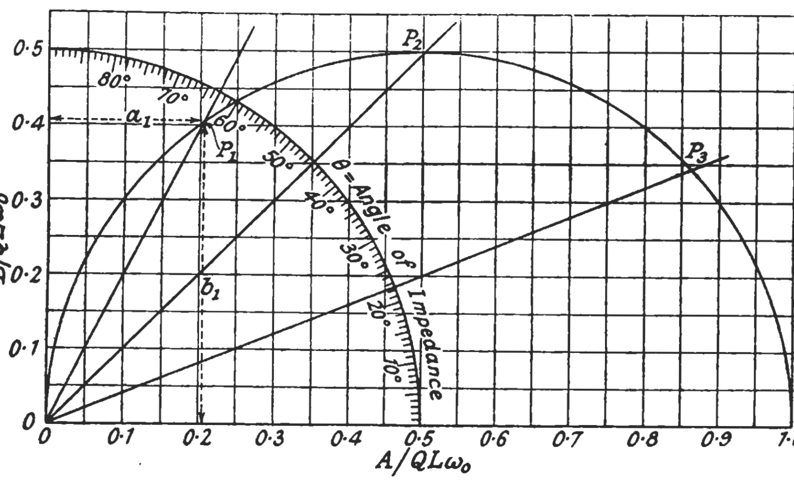


FIG. 5/CVIII:1.—Generalized Impedance of Parallel Resonant Circuit. When Q is greater than 20, error is less than 5% ; when Q is greater than 100, error is less than 1%.

the impedance at any frequency is $A + jB$, the scale markings of the axes of ordinates and abscissae evidently give the value of $A/QL\omega_0$ and $B/QL\omega_0$ respectively.

It is not necessary to draw the whole circle since the negative half of the diagram is the same as the positive half, and negative reactances can be read off the diagram as well as positive reactances.

The diagram is, however, of little use unless some means can be found for inserting a generalized scale of frequency so that the diagram can be used to give the impedance of any parallel circuit at any frequency.

This may be done as follows. Assuming that the impedance of the circuit is given by $Z = |Z|\underline{/\theta} = A + jB$, from equation (18)/XXIV:2

$$\tan\theta = \frac{B}{A} = Q\frac{f}{f_0}\left(1 - \frac{f^2}{f_0^2} - \frac{1}{Q^2}\right) \quad . \quad . \quad . \quad (3)$$

$$\therefore\ \theta = \tan^{-1}\left[Q\left(\frac{f}{f_0} - \frac{f_0^3}{f_0^3}\right) - \frac{1}{Q}\frac{f}{f_0}\right] \quad . \quad . \quad (4)$$

Use of Fig. 5 to give the Impedance of any Parallel Resonant Circuit at any Frequency.

1. Determine the value of Q, of f_0 the resonant frequency, and the value of $L\omega_0$ or $1/C\omega_0$.
2. Decide the frequency f at which the impedance is required, and calculate the value of f/f_0.
3. Enter the value of f/f_0 in equation (4) and so obtain the value of θ.
4. In Fig. 5 draw a line (or place a ruler) passing through the origin and making an angle θ with the horizontal axis.
5. At the point P, where the line or ruler cuts the semicircle, read off the values of $A/QL\omega_0$ and $B/QL\omega_0$. To obtain the values of A and B it is evidently necessary to multiply these values by $QL\omega_0$.
6. Alternatively, the magnitude of the impedance $|Z|$ may be obtained by multiplying the distance OP by $QL\omega_0$. (O here represents the origin.)

Example. A coil with a Q of 100 is tuned to resonance at 1 Mc/s by a capacity of 300 $\mu\mu$F. What is the impedance of the circuit at 990, 995 and 1,002 kc/s.

The corresponding values of f/f_0 are 0·990, 0·995 and 1·002.

Entering these in equation (4), the corresponding values of θ are 63°, 45° and − 22° 30′. Lines drawn through the origin making these angles with the horizontal axis cut the semicircle at P_1, P_2 and P_3. The scaled-down impedances (which must be multiplied by $QL\omega_0$ to give the actual impedance) are then read off as $a_1 + jb_1 = 0{\cdot}207 - j0{\cdot}406$, $a_2 + jb_2 = 0{\cdot}5 + j0{\cdot}5$, $a_3 + jb_3 = 0{\cdot}863 - j0{\cdot}344$. The negative sign is inserted in the last case because the value of θ is negative in this case.

The reactance of 300 $\mu\mu$F at 1 Mc/s is 530 ohms, which is substantially equal to the reactance of the inductance at 1 Mc/s and so to $L\omega_0$. The value of $QL\omega_0$ is therefore 53,000, and multiplying the values of a_1, b_1, etc., by 53,000, the values of impedance are:

At	990 kc/s	$10{,}970+j21{,}550$	$(10{,}650+j21{,}000)$
	995 kc/s	$26{,}500+j26{,}500$	$(26{,}600+j26{,}200)$
	1,002 kc/s	$45{,}700+j18{,}230$	$(45{,}700+j17{,}900)$

The figures in brackets give the values as calculated directly by slide rule.

2. Representation of Relations between Complex Quantities. Conformal Representation.

In the case of formulae involving complex quantities it may be necessary to show in graphical form the relation between two complex quantities.

Take the case of two inverse impedances $Z_1 = A_1+jB_1$ and $Z_2 = A_2+jB_2$, which are mutually inverted about a resistance R.

Then
$$Z_1 = \frac{R^2}{Z_2} \text{ or } A_1+jB_1 = \frac{R^2}{A_2+jB_2} \qquad . \quad . \quad (1)$$

If any value of A_2 and any value of B_2 is entered in equation (1) a particular value of A_1 and a particular value of B_1 results.

The graphical representation is required to show for a certain range of values of A_2 and B_2 the corresponding values of A_1 and B_1.

The form of representation used for this purpose is the most obvious, although the means used for drawing the curves may sometimes involve a certain amount of finesse. One of the variables such as B_2 is fixed at a useful integral value and the impedance Z_1 is then shown as a resistance-reactance representation on a plane of A_1 and B_1 (the Z_1 plane) by plotting A_1 against B_1 for continuous variation of A_2. This produces one curve on the Z_1 plane which corresponds to one value of B_2; the values of A_2, although they have been used in determining this curve are not marked, so that, as yet, the curve is of no value. This process is repeated for a series of integral values of B_2 and each curve is marked with the corresponding value of B_2.

In the same way, a series of impedance curves corresponding to constant integral values of A_2 is then drawn, by continuous variation of B_2. The second set of curves crosses the first set of curves.

The two families of curves then constitute a mesh defining the value of Z_2; this mesh is said to constitute the A_2B_2 or the Z_2 plane.

The frame of the Z_1 plane constituted by the mesh system, with regard to which co-ordinates on this plane are plotted, is a normal rectangular frame: the lines forming the mesh are constituted by two sets of parallel straight lines mutually at right angles. In general, the frame of the Z_2 plane is not necessarily constituted by straight lines, although it is generally constituted by two sets of lines mutually at right angles. Such sets of lines are said to constitute orthogonal trajectories.

The method of construction of the Z_2 plane has superimposed it on the Z_1 plane in such a way that at every point in the diagram the co-ordinates of a point, read on the Z_1 plane, and the co-ordinates of the same point read on the Z_2 plane, respectively define values of Z_1 and Z_2 which simultaneously satisfy equation (1). In other words, if either Z_1 or Z_2 is known, by plotting it on its appropriate plane, the corresponding value of Z_2 or Z_1 respectively may be read from the co-ordinates of the point on the other plane. The plane of Z_1 may be called the primitive plane, while the plane of Z_2 is conveniently called the derived plane.

This method of representing the relation between two complex quantities is called *Conformal Representation.*

In practice, when the mesh system of the derived plane is defined by lines which have some easily drawn geometrical configuration (e.g. circles), the process of drawing the mesh may be simplified by carrying the analysis to the point where the location on the Z_1 plane of the lines of the mesh of the Z_2 plane is defined by simple specific relations.

In the general case where, as above, the plane of Z_1 is the primitive plane and the plane of Z_2 is the derived plane, these specific relations may sometimes be obtained by eliminating A_2 and B_2 alternately. From the resulting relations A_1 is plotted against B_1 by varying continuously B_2 and A_2 respectively. This process is generally worth while, whether or not the resulting equations define mesh lines which may be drawn without plotting ; i.e. by geometrical construction.

The method will now be illustrated with regard to equation (1). From this equation, rationalizing the right-hand side and equating reals and imaginaries respectively :

$$A_1 = \frac{R^2 A_2}{A_2^2+B_2^2} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (2a)$$

$$B_1 = \frac{-R^2 B_2}{A_2^2+B_2^2} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (2b)$$

$$\therefore \frac{A_2}{A_1} = \frac{A_2^2+B_2^2}{R^2} = -\frac{B_2}{B_1}$$

$$\therefore A_2 = -\frac{A_1 B_2}{B_1} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (3a)$$

$$B_2 = -\frac{B_1 A_2}{A_1} \quad . \quad . \quad . \quad . \quad (3b)$$

Eliminating A_2 by substituting (3*a*) in (2*a*)

$$A_1 = \frac{-R^2 A_1 B_2}{B_1\left(\frac{A_1^2 B_2^2}{B_1^2}+B_2^2\right)} = \frac{-R^2 A_1 B_1}{A_1^2 B_2 + B_1^2 B_2}$$

$$\therefore \; B_2 B_1^2 + R^2 B_1 + B_2 A_1^2 = 0$$

$\therefore$ Dividing through by B_2 and adding $\frac{R^4}{4B_2^2}$ to each side

$$B_1^2 + \frac{R^2}{B_2} B_1 + \frac{R^4}{4B_2^2} + A_1^2 = \frac{R^4}{4B_2^2}$$

$$\therefore \; \left(B_1 + \frac{R^2}{2B_2}\right)^2 + A_1^2 = \left(\frac{R^2}{2B_2}\right)^2 \quad . \quad . \quad . \quad (4)$$

This is the equation of a circle of radius $\frac{R^2}{2B_2}$ and centre at a point on the Z_1 plane corresponding to the point plotted for an impedance with values $A_1 = 0$, $B_1 = -\frac{R^2}{2B_2^2}$. One such circle should be drawn for each chosen integral value of B_2: the set of circles then constitutes one set of lines defining the mesh of the Z_2 plane.

Eliminating B_2 by substituting (3*b*) in (2*a*)

$$A_1 = \frac{R^2 A_2}{A_2^2 + \frac{B_1^2 A_2^2}{A_1^2}} = \frac{R^2 A_1}{A_1 A_2 + \frac{B_1^2 A_2}{A_1}}$$

$$\therefore \; A_2 A_1^2 - R^2 A_1 + B_1^2 A_2 = 0$$

$$\therefore \; A_1^2 - \frac{R^2}{A_2} A_1 + \frac{R^4}{4A_2^2} + B_1^2 = \frac{R^4}{4A_2^2}$$

$$\therefore \; \left(A_1 - \frac{R^2}{2A_2}\right)^2 + B_1^2 = \left(\frac{R^2}{2A_2}\right)^2 \quad . \quad . \quad . \quad (5)$$

This is the equation of a circle of radius $\frac{R^2}{2A_2}$ and centre at a point on the Z_1 plane corresponding to the point plotted with values $A_1 = \frac{R^2}{2A_2}$, $B_1 = 0$. One such circle should be drawn for each chosen integral value of A_2: the set of circles so drawn constitutes the second set of lines defining the mesh of the Z_2 plane.

If the relation between Z_1 and Z_2 is other than that of equation (1) any resulting relations, such as (3) and (4), which are derived will not necessarily describe circles: they may describe ellipses, hyperbolas or other forms which are not easily drawn with the kind of instruments at present sold in boxes. In this case, each line of each family must be plotted on the Z_1 plane as a relationship between A_1 and B_1 defined by the equivalents of equations (3) and (4) respectively for chosen integral values of B_2 and A_2. This process is very laborious, so that the saving of time when circles are involved is evident.

Examples of conformal representation are constituted by Fig. 3/V:16 and Figs. 1 and 2/XVI:2.

The first does not relate two different impedances but relates the series and parallel representations of any one impedance. It therefore relates two complex quantities. The derivation of the necessary relations proceeds as follows:

From (left-hand) equation (2)/V:16, multiplying through by R_s

$$R_pR_s = R_s^2+X_s^2$$

$$\therefore\ R_s^2 - R_pR_s+\left(\frac{R_p}{2}\right)^2+X_s^2 = \left(\frac{R_p}{2}\right)^2$$

$$\therefore\ \left(R_s - \frac{R_p}{2}\right)^2+X_s^2 = \left(\frac{R_p}{2}\right)^2 \quad . \quad . \quad . \quad (6a)$$

Similarly from (right-hand) equation (2)/V:16

$$\left(X_s - \frac{X_p}{2}\right)^2+R_s^2 = \left(\frac{X_p}{2}\right)^2 \quad . \quad . \quad . \quad (6b)$$

Equations (6a) and (6b) describe circles of specified centres and radii so that they provide all the information necessary for constructing Fig. 3/V:16. This construction is due to Mr. R. Toombs.

Figs. 1 and 2/XVI:2 provide the same information as does the equation:

$$\frac{Z_s}{Z_0} = \frac{\dfrac{Z}{Z_0}+j\tan\dfrac{2\pi l}{\lambda}}{1+j\dfrac{Z}{Z_0}\tan\dfrac{2\pi l}{\lambda}} \quad . \quad . \quad . \quad . \quad (7)$$

where the meaning of the symbols is the same as is given at the beginning of XVI:1. They therefore give directly the sending-end impedance of a transmission line of length l terminated in an impedance Z, expressed as a fraction of the characteristic impedance Z_0 of the transmission line. They do not, however, constitute, as might be expected, the conformal relation between Z_s/Z_0 and Z/Z_0. If this were plotted, a separate diagram would be necessary for every value of l/λ. They represent a conformal relation between the value of Z_s/Z_0 and a complex quantity equal to $\dfrac{R}{Z_0}+j\tan\dfrac{2\pi l}{\lambda}$, where the relationship is defined by equation (7) with Z replaced by R, where R is a pure resistance. The justification for this is best explained in physical terms.

If a transmission line is terminated in a pure resistance R, the impedance at any point in the line looking towards the load (i.e. the terminating impedance R) is given by the value of Z_s in equation (7) with Z replaced by R. Z_s evidently depends on the two quantities $\frac{R}{Z_0}$ and $\tan\frac{2\pi l}{\lambda}$, and it is possible to produce various types of conformal representation relating the two complex quantities $\frac{Z_s}{Z_0}$ and $\frac{R}{Z_0}+j\tan\frac{2\pi l}{\lambda}$. In Fig. 2/XVI:2 the plane of Z_s/Z_0 is made the primitive plane and is plotted on normal rectangular (Cartesian) co-ordinates. The other quantity is then represented by the derived plane.

When this is done the diagram consists of a rectangular mesh defining the plane of Z_s/Z_0 on which are drawn a family of circles of constant distance (representing constant distance from the termination) expressed as l/λ, intersected by a family of circles of constant R/Z_0. The latter are represented by complete circles, and as any one of these is traversed completely the number of constant l/λ circles crossed is such that the corresponding distance along the transmission line is half a wavelength. This means that the impedance half a wavelength away from the load is the same as the load impedance, and that during the next half-wavelength along the line the same cycle of impedances will be observed as during the first half-wavelength. In other words, the diagram is cyclic or periodic, the period being half a wavelength.

Further, by going to the appropriate point on the diagram any sending-end impedance may be observed, and this may be treated as a terminating impedance for any length of the line farther away from the load. Hence, although the diagram is drawn by considering a transmission line to be terminated in a number of resistances, it can be used to find the sending-end impedance of a length of feeder terminated in any impedance whatever, so long as it lies within the range for which the diagram has been drawn. For use of this diagram reference should be made to XVI:2.

Derivation of Equations to the Circles in Fig. 2/XVI:2. Replace Z in equation (7) by R and make the substitutions

$$\frac{Z_s}{Z_0} = a+jb\,;\quad \frac{R}{Z_0} = A\,;\quad \tan\frac{2\pi l}{\lambda} = B.$$

Then equation (7) becomes

$$a+jb = \frac{A+jB}{1+jAB} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Hence

$$a = \frac{A+AB^2}{1+A^2B^2} \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

and

$$b = \frac{B - BA^2}{1+B^2A^2} \quad . \quad . \quad . \quad . \quad . \quad (8b)$$

$$\therefore\ B^2 = \frac{1 - \frac{a}{A}}{aA - 1} \quad . \quad . \quad . \quad . \quad . \quad (9a)$$

and

$$A^2 = \frac{1 - \frac{b}{B}}{bB+1} \quad . \quad . \quad . \quad . \quad . \quad (9b)$$

Substitute (9*b*) in (8*a*) and (9*a*) in (8*b*)

$$\therefore\ a = A(1+bB) \quad . \quad . \quad . \quad . \quad . \quad (10a)$$

and

$$b = B(1 - aA) \quad . \quad . \quad . \quad . \quad (10b)$$

$$\therefore\ A = \frac{a}{1+bB} \quad . \quad . \quad . \quad . \quad . \quad (11a)$$

and

$$B = \frac{b}{1 - aA} \quad . \quad . \quad . \quad . \quad . \quad (11b)$$

Substitute (11*b*) in (10*a*) and (11*a*) in (10*b*)

$$\therefore\ a = A\left(1+\frac{b^2}{1 - aA}\right) \quad . \quad . \quad . \quad (12a)$$

and

$$b = B\left(1 - \frac{a^2}{1+bB}\right) \quad . \quad . \quad . \quad (12b)$$

(12*a*) and (12*b*) are now multiplied out and the process of " completing the square " is performed on each to obtain the equations to circles in recognizable form.

(12*a*) then yields

$$\left[a - \frac{1}{2}\left(A+\frac{1}{A}\right)\right]^2+b^2 = \frac{1}{4}\left(A+\frac{1}{A}\right)^2 - 1 = \frac{1}{4}\left(A - \frac{1}{A}\right)^2 \quad . \quad (13a)$$

(12*b*) yields

$$a^2+\left[b - \frac{1}{2}\left(B - \frac{1}{B}\right)\right]^2 = 1+\frac{1}{4}\left(B - \frac{1}{B}\right)^2 = \frac{1}{4}\left(B+\frac{1}{B}\right)^2 \quad . \quad (13b)$$

Restoring the original values of A and B, equation (13a) describes a family of circles with centres at

$$a = \frac{1}{2}\left(\frac{R}{Z_0}+\frac{Z_0}{R}\right),\ b = 0 \text{ and radius } = \frac{1}{2}\left(\frac{R}{Z_0} - \frac{Z_0}{R}\right)$$

Equation (13b) describes a family of circles with centres at $a = 0$,

$$b = \tfrac{1}{2}\left(\tan\frac{2\pi l}{\lambda} - \cot\frac{2\pi l}{\lambda}\right) \text{ and radius } = \tfrac{1}{2}\left(\tan\frac{2\pi l}{\lambda}+\cot\frac{2\pi l}{\lambda}\right).$$

In both cases the radius should always be considered to be positive : the radius is derived as the square root of a quantity and strictly should have a $\pm$ sign in front of it which permits the positive sign to be chosen always.

It is useful to notice that the components of the complex quantity in the numerator of the right-hand side of equation (7) appear in the denominator multiplied together. In general, therefore, it may be possible to represent by a conformal relationship any two pairs of quantities even when these quantities are not always associated together in a consistent manner.

Since the above was first written Professor Willis Jackson and Dr. L. G. Huxley have published solutions of both Figs. 1 and 2/XVI:2, see Bibliography A4:10.

CHAPTER CIX

COUPLING CIRCUITS AS BAND-PASS FILTERS

1. IN Fig. 1 are shown a number of impedance matching or coupling circuits which are in general use. Various methods are employed to adjust the values of component circuit elements of these structures to give the performance required. It is often useful to design them as band-pass filters.

Any structure of loss-free reactances can, within limits, have such values assigned to its elements that, when terminated by

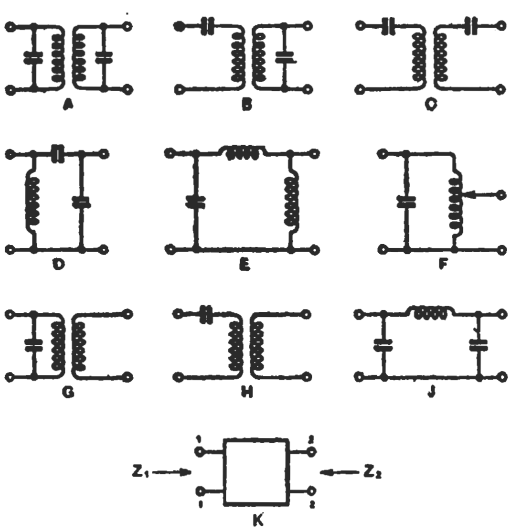


FIG. 1/CIX.—(*A*) Double parallel tuned mutual coupling; (*B*) series parallel tuned mutual coupling; (*C*) double series tuned mutual coupling; (*D*) capacitance tap coupling circuit; (*E*) inductance tap coupling circuit; (*F*) tapped inductance coupling circuit; (*G*) single parallel tuned circuit; (*H*) single series tuned circuit; (*J*) series inductance parallel capacitance π coupling circuit.

(By the courtesy of the Editor of *The Wireless Engineer*.)

appropriate impedances, it will pass certain bands of frequencies without attenuation, and will attenuate other bands. The frequencies at the ends of the pass bands are called cut-off frequencies. In one class of filter the conditions which limit the performance of the filter are largely those set by the size of the filter elements which is physically realizable. In another class of structure there is a form of limitation which appears as a functional relationship between the ratio of input and output impedances, and the ratio of the cut-off frequencies. Only structures designed to pass a single band of frequencies are here under discussion. The structures at *D*, *E*, *F* and *J* of Fig. 1 are subject to the last-named limitation : if the cut-off frequencies are determined the ratio of output to input impedances is determined, while if the ratio of output to input impedance is determined the ratio of the cut-off frequencies is determined. In the structures at *A*, *B*, *C*, *G* and *H*, this limitation is absent : the ratio of the cut-off frequencies is independent of the ratio of input and output impedances, and vice versa.

The design formulae for the elements of these structures operating as band-pass filters may be obtained by the general method described in XXV:11. The formulae for the first two may, however, be obtained more quickly by making use of existing design formulae for equivalent structures. As the process of deriving the required formulae by this method is instructive it is given immediately below.

2. Derivation of Formulae for the Double Parallel Tuned Mutual Coupling (Fig. 1*A*).

Equivalent π for Unity Ratio Mutual Coupling.

In the two circuits of Fig. 2, L_a and L_b are two equal inductances of magnitude L and coupling factor k : P and S are inductances of magnitude P and S respectively.

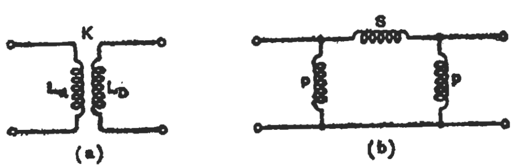


FIG. 2/CIX.

The two circuits are equivalent when the short circuit and open circuit impedances are respectively equal, i.e. when

$$(1 - k^2)L = \frac{PS}{P+S} \quad . \quad . \quad . \quad . \quad (1)$$

and

$$L = \frac{P(P+S)}{P+S+P} \quad . \quad . \quad . \quad (2)$$

$$\therefore \frac{1}{L} = \frac{1}{P} + \frac{1}{P+S} = \frac{1}{P} + \frac{(1 - k^2)L}{PS}$$

$$\therefore P = L + \frac{1}{S}(1 - k^2)L^2 \quad . \quad . \quad . \quad . \quad (3)$$

Substituting (3) in (1)

$$\therefore \ (1-k^2)LS+(1-k^2)[L+\frac{1}{S}(1-k^2)L^2]L = SL+(1-k^2)L^2$$

$$\therefore \ (1-k^2)S^2+(1-k^2)LS+(1-k^2)^2L^2 = S^2+(1-k^2)LS$$

$$\therefore \ S^2k^2 = (1-k^2)^2L^2$$

$$\therefore \ S = \pm\frac{(1-k^2)L}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$P = L\pm\frac{k}{(1-k^2)L}\cdot(1-k^2)L^2 = (1\pm k)L \quad . \quad . \quad (5)$$

Since S must be a real inductance the positive sign must be taken.

Solving for L and k in terms of P and S.

From (4) and (5)

$$\frac{P}{S} = \frac{k}{1-k} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\therefore \ k = \frac{P}{S} - k\frac{P}{S} = \frac{\frac{P}{S}}{1+\frac{P}{S}} = \frac{P}{S+P} \quad . \quad . \quad (7)$$

From (5)

$$L = \frac{P}{1+k} = \frac{P}{1+\frac{P}{S+P}} = \frac{P(S+P)}{S+2P} \quad . \quad . \quad (8)$$

Now consider the filter structure shown in Fig. 3 (*a*) and the double parallel tuned mutual inductance circuit in Fig. 3 (*b*). The former is a section of a type III_3, filter terminated at mid-shunt

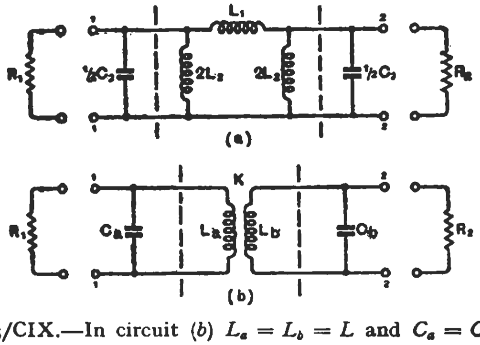


FIG. 3/CIX.—In circuit (*b*) $L_a = L_b = L$ and $C_a = C_b = C$.

when $$L_1 = \frac{R}{\pi(f_1+f_2)}, \quad L_2 = \frac{(f_2 - f_1)R}{4\pi f_1^2}$$

and $$C_2 = \frac{1}{\pi(f_2 - f_1)R}$$

and $$R = R_1 = R_2$$

See Fig. 2/XXV:5.

The structures between the chain dotted lines are equivalent when $P = 2L_2$ and $S = L_1$.

Hence the two structures in Fig. 3 are equivalent when the following relations hold:

$$\text{(i)}\ k = \frac{P}{S+P} = \frac{2L_2}{L_1+2L_2}$$

$$= \frac{\dfrac{2(f_2 - f_1)R}{4\pi f_1^2}}{\dfrac{R}{\pi(f_1+f_2)} + \dfrac{2(f_2 - f_1)R}{4\pi f_1^2}}$$

$$= \frac{f_2^2 - f_1^2}{2f_1^2+f_2^2 - f_1^2} = \frac{f_2^2 - f_1^2}{f_2^2+f_1^2} = \frac{\phi^2 - 1}{\phi^2+1} \qquad (9)$$

where $\phi = \dfrac{f_2}{f_1}$

$$\text{(ii)}\ L = \frac{P(S+P)}{S+2P} = \frac{2L_2(L_1+L_2)}{L_1+L_2}$$

$$= \frac{\dfrac{2(f_2 - f_1)R}{4\pi f_1^2}\left[\dfrac{R}{\pi(f_1+f_2)} + \dfrac{2(f_2 - f_1)R}{4\pi f_1^2}\right]}{\dfrac{R}{\pi(f_1+f_2)} + \dfrac{(f_2 - f_1)R}{\pi f_1^2}}$$

$$= \frac{R}{4\pi}\left[\frac{2(f_2 - f_1)\left(1 - \dfrac{f_2^2 - f_1^2}{2f_1^2}\right)}{f_1^2+f_1^2 - f_1^2}\right]$$

$$= \frac{R}{4\pi}\left[\frac{(f_2 - f_1)(2f_1^2+f_2^2 - f_1^2)}{f_1^2 f_2^2}\right]$$

$$= \frac{R}{4\pi}\,\frac{(f_2 - f_1)(f_1^2+f_2^2)}{f_1^2 f_2^2} \qquad (10)$$

$$= \frac{R(\phi - 1)(\phi^2+1)\phi}{4\pi f_1} \qquad (10a)$$

$$\text{(iii)}\ C = \tfrac{1}{2}C_2 = \frac{1}{2\pi(f_2 - f_1)R} \qquad (11)$$

The above equations define the performance for a unity ratio structure, i.e. when the inductances each side of the mutual are equal.

When the two inductances are unequal, let one be equal to $L_a = L$ and the other equal to $L_b = n^2L$, the coupling factor k being unchanged.

Then $$M = k\sqrt{L_aL_b} = k\sqrt{n^2L_a^2} = nkL.$$

The condition that the attenuation through the filter shall be unchanged, by change of L_b, from equality with L_a will be satisfied if the impedance presented at terminals 1,1 is unchanged.

Try putting $R_2 = n^2R_1$ and $C_b = \frac{1}{n^2}C_a$

When $L_b = L_a$, $R_2 = R_1$, *etc.*

Unit current through L_a generates an e.m.f. $Mj\omega$ in $L_b = kLj\omega$ and a secondary current flows of magnitude $j\frac{kL\omega}{Z}$ where Z is the total secondary series impedance.

This induces an e.m.f. into L_1 of magnitude and sense

$$= -\frac{k^2L^2\omega^2}{Z}$$

When $L_b = n^2L_a$, $R_2 = n^2R_1$, etc.

Unit current through L_a generates an e.m.f. in $L_b = nkLj\omega$ and a secondary current flows of magnitude

$$j\frac{nkL\omega}{n^2Z} = j\frac{kL\omega}{nZ}$$

This induces an e.m.f. into L_1 of magnitude and sense

$$= -nkL.\frac{kL\omega}{nZ} = -\frac{k^2L^2\omega^2}{Z}$$

Hence the impedance facing the input to the structure remains unchanged if the impedance of all elements on the secondary side is changed in the same ratio as L_b is changed.

It follows that to design an inequality ratio circuit, each side can be designed independently of the other : for the value of R in equations (9), (10) and (11) the value of R_a is substituted to determine the value of the primary impedance elements, and the value of R_b is substituted to determine the secondary impedance elements.

The relations of equations (9), (10), (10*a*) and (11) are presented graphically in Fig. 1/VII:14.

3. Derivation of Formulae for the Series-Parallel Tuned Mutual Coupling Circuit (Fig. 1 *B*).

For this purpose it is convenient to use another alternative circuit for the mutual coupling. This is shown in Fig. 4.

These two circuits are equivalent if the short circuit and open circuit impedances looking into both circuits at both ends are equal, i.e. if $Z_{11} = Z_{33}$ and $Z_{22} = Z_{44}$ for short circuit and open circuit conditions of the other terminals.

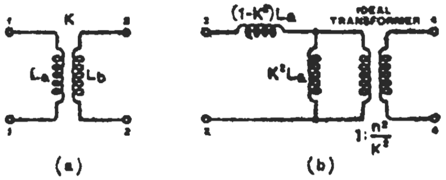


FIG. 4/CIX.

Check—

Open circuit condition

$$Z_{11} = L_a$$
$$Z_{22} = n^2 L_a$$
$$Z_{33} = (1 - k^2)L_a + k^2 L_a = L_a$$
$$Z_{44} = \frac{n^2}{k^2}.k^2 L_a = n^2 L_a$$

Short circuit condition

$$Z_{11} = (1 - k^2)L_a$$
$$Z_{22} = n^2(1 - k^2)L_a$$
$$Z_{33} = (1 - k^2)L_a$$
$$Z_{44} = \frac{n^2}{k^2}.\frac{k^2 L_a(1 - k^2)L_a}{(1 - k^2)L_a - k^2 L_a} = n^2(1 - k^2)L_a$$

Now consider the filter structure shown in Fig. 5 (*a*), neglecting the ideal transformer associated with it. This is a half-section of a Type IV_k filter (see Fig. 1/XXV:5) when

$$L_1 = \frac{R}{\pi(f_2 - f_1)}, \qquad C_1 = \frac{f_2 - f_1}{4\pi f_1 f_2 R}$$
$$L_2 = \frac{(f_2 - f_1)R}{4\pi f_1 f_2}, \text{ and } C_2 = \frac{1}{\pi(f_2 - f_1)R}.$$

The structure is evidently identical with that shown in Fig. 5 (*b*) if corresponding elements are equal. While, from the principle of equivalence established immediately above, the structures in *b* and *c* are also equivalent. Evidently *b* can be derived from *c* by replacing the coupled circuit in *c* by the equivalent structure as shown in Fig. 4 (*b*) and transferring C_b from one side of the ideal transformer to the other, changing its magnitude in the ratio of the ideal transformer.

Fig. 5 (*c*) is evidently in the form of the series-parallel tuned mutual coupling. Suffixes $_a$ and $_b$ are used instead of 1 and 2 to distinguish the elements from those of the half-section of Type IV_k filter shown at 5 (*c*).

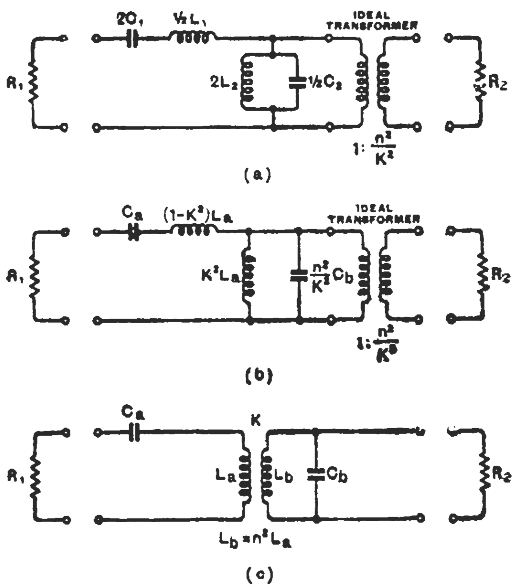


FIG. 5/CIX.

It is immediately possible to determine the values of the circuit elements in Fig. 5 (*c*) as follows, taking advantage of the fact that in a Type IV_k filter, at the geometric mid-band frequency, the mid-shunt image impedance is equal to the mid-series image impedance. These are both represented by the symbol R in the equations immediately above.

By inspection of Fig. 5 (*a*), (*b*) and (*c*), it can be seen that

$$C_a = 2C_1 = \frac{f_2 - f_1}{2\pi f_1 f_2 R_1} \quad . \quad . \quad . \quad (12)$$

Also

$$(1 - k^2)L_a = \tfrac{1}{2}L_1 = \frac{R_1}{2\pi(f_2 - f_1)}$$

$$\therefore\ L_a = \frac{R_1}{2\pi(f_2 - f_1)(1 - k^2)} \quad . \quad . \quad . \quad (13)$$

Further, $$k^2L_a = \frac{k^2}{n^2}L_b = 2L_2 = \frac{(f_2 - f_1)\frac{k^2}{n^2}R_2}{2\pi f_1 f_2}$$

$$\therefore\ L_b = \frac{(f_2 - f_1)R_2}{2\pi f_1 f_2} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Finally, $$\frac{n^2}{k^2}C_b = \tfrac{1}{2}C_2 = \frac{1}{\pi(f_2 - f_1)\frac{k^2}{n^2}R_2}$$

$$\therefore\ C_b = \frac{1}{\pi(f_2 - f_1)R_2} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Determination of Relation between *k* and Cut-off Frequencies.

By inspection of Figs. 5 (*a*) and 5 (*b*) :

$$k^2L_a = 2L_2 = \frac{(f_2 - f_1)R_1}{2\pi f_1 f_2} \quad . \quad . \quad . \quad (16)$$

From (13) and (16)

$$\frac{k^2}{1 - k^2} = \frac{(f_2 - f_1)^2}{f_1 f_2} = \frac{(\phi - 1)^2}{\phi}$$

$$\therefore\ \phi k^2 = \phi^2 - 2\phi + 1 - \phi^2 k^2 + 2\phi k^2 - k^2$$

$$\therefore\ (\phi^2 - \phi + 1)k^2 = \phi^2 - 2\phi + 1 \quad . \quad . \quad . \quad (17)$$

$$\therefore\ k = \frac{\phi - 1}{\phi^2 - \phi + 1} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and alternately from (17)

$$\phi = \frac{f_2}{f_1} = \frac{2 - k^2 + \sqrt{4k^2 - 3k^4}}{2(1 - k^2)} \quad . \quad . \quad (19)$$

The relations of equations (12), (13), (14), (15) and (18) are presented graphically in Fig. 2/VII:14.

(Chapter CIX is published by the courtesy of the Editor of *The Wireless Engineer*.)

BIBLIOGRAPHY

To make proper use of this bibliography the list of categories which follows immediately should be read, in order to find the headings under which information has been classified. It is quite impossible to produce a classification which will please everybody, since every one has different methods of pigeonholing information in his mind. The categories chosen are for the most part those in general current use, and by referring to the list below it is a simple matter to find the category under which any reference has been placed.

Although this list of references appears fairly extensive, it is very far from complete and makes no pretence to be systematic. With a view to obtaining a representative cross section of techniques developed in the past a very rapid scrutiny of a few periodicals was made and it is impossible to guarantee that important references have not been omitted. On the other hand, the inclusion of a reference does not guarantee that it is useful even in its own particular sphere. Most of the references have been chosen because they are useful, but in certain cases, particularly when they are highly mathematical, it is often hard to assess the value of references.

The periodicals which have been scrutinized are: *I.E.E. Journal*, *I.R.E. Proceedings*, *Bell System Technical Journal*, *E.W. and Wireless Engineer*, *Marconi Review*, and *Electrical Communication*. These periodicals have been scrutinized from their inception up to the middle of 1942.

The references to other periodicals have been inserted on a purely opportunistic basis and represent a chosen few—about one per cent.—of the entries in a bibliography started by the author about 1925 and only kept up to date to the year 1935. No attempt can be made to define the range of periodicals covered since the coverage on any one periodical was not continuous, and the great mass of literature examined in those years precluded any except a very small fraction of the references examined being entered.

Finally, it is almost impossible to define the scope of the bibliography, which is more extensive than that of this book, except to say that it covers the theory and practice of every branch of Radio Technique except Radar, Power Engineering, Television, Studio Technique, Valve Design, and Recording.

The bibliography does not constitute a documentation of this book—it was prepared after the book was more than three-quarters written. It does, however, constitute a valuable extension of the information provided in the book, with a considerable amount of overlap: to the extent that overlap occurs documentation is provided, but little attempt has been made to relate the bibliography to the book. An exception to this is constituted by the fact that some relevant references are cited in certain places in the text.

CATEGORIES

13. **Response Correction**
14. **R.M.A. Specification**
15. **Rocky Point Effect**

S

1. **Schotky Noise, Schot or Shott Effect :** *see* **Fluctuation Noise**
2. **Screening**
3. **Selenium Rectifiers**
4. **Series Modulation**
5. **Shielding**
6. **Silicones**
7. **Single Sideband :** *see* **Asymmetric Sideband Systems**
8. **Skip Distance**
9. **Sparking Distance :** *see* **Voltage Breakdown**
10. **Speech and Music**
11. **Stabilovolt :** *see* **Gas Discharge Potentiometers**
12. **Sub-Harmonics**
13. **Synchronized Operation of Two Transmitters on the same Wavelength**

T

1. **Tapered Transmission Lines**
2. **Telegon Effect :** *see* **Luxembourg Effect**
3. **Temperature Control**
4. **Thermal Agitation**
5. **Thyratrons**
6. **Thyrite**
7. **Tracking :** *see* **Ganging and Tracking**
8. **Transformers A.F., I.F., R.F.**
9. **Transformer Oil**
10. **Transients**
11. **Transmitters**
12. **Turn-over**

U

1. **U.H.F. Aerials**
2. **U.H.F. Amplifiers**
3. **U.H.F. Cavity Resonators**
4. **U.H.F. Feeders**
5. **U.H.F. General**
6. **U.H.F. Measurements**
7. **U.H.F. Oscillators**
8. **U.H.F. Propagation**
9. **U.H.F. Receivers**
10. **U.H.F. Transmitters**
11. **U.H.F. Valves**

V

1. **Valves**
2. **Valve Input Admittance**
3. **Voltage Breakdown**
4. **Volume Compression and Expansion**

W

1. **Wave Guides**
2. **Wave Resonance Tuning**
3. **Wide Band Amplifiers**

REFERENCES

A1—Acoustics

1. Velocity of Ultra-sonic Frequencies in Air. By C. D. Reid. *Physical Review*, April 1930, p. 814.
2. Velocity of Longitudinal Waves in Liquids at Audio Frequencies. By L. G. Pouler. *Physical Review*, April 1930, p. 832.
3. Definition of Acoustic Quantities. By L. Bouthillon. *Review D'Acoustique*, May 1935, p. 57.
4. Acoustic Delay Circuit. By W. P. Mason. *Bell Lab. Record*, May 1931, p. 430.

A2—Acoustic Filters

1. Variable Single Band Filter. By G. W. Stewart. *Physical Review*, Nov. 1923.
2. Acoustic Wave Filters. By G. W. Stewart. *Physical Review*, Dec. 1922, p. 528.
3. Acoustic Filters. By G. W. Stewart. *Physical Review*, Jan. 1925, p. 90.
4. Regular Combination of Acoustic Elements. By W. P. Mason. *Bell System Tech. Jl.*, April 1927, p. 258.

A3—Aerials

I.E.E. JOURNAL.

1. A Theoretical Investigation of the Phase Relations in Beam Systems. By R. M. Wilmotte and J. S. McPetrie. Vol. 3, 1928, p. 182.
2. The Distribution of Current in a Transmitting Antenna. By R. M. Wilmotte. Vol. 3, 1928, p. 136.
3. General Considerations of the Directivity of Beam Systems. By R. M. Wilmotte. Vol. 3, 1928, p. 188.
4. The Nature of the Field in the Neighbourhood of an Antenna. By R. M. Wilmotte. Vol. 3, 1928, p. 194.
5. The Design of Transmitting Aerials for Broadcasting Stations. By P. P. Eckersley. Vol. 4, 1929, p. 51.
6. The Action of a Reflecting Antenna. By L. S. Palmer and L. L. K. Honeyball. Vol. 4, 1929, p. 152.
7. General Formulae for the Radiation Distribution of Antenna Systems. By R. M. Wilmotte. Vol. 5, 1930, p. 218.
8. The Radiation Distribution of Antennae in Vertical Planes. By R. M. Wilmotte. Vol. 5, 1930, p. 235.
9. Beam Arrays and Transmission Lines. By T. Walmsley. Vol. 6, 1931, p. 30.

10. The Graphical Method for Determining the Magnitude and Phase of the Electric Field in the Neighbourhood of an Antenna carrying a known Distribution of Current. By J. S. McPetrie. Vol. 6, 1931, p. 40.
11. A Method for Determining the Effect of the Earth on the Radiation from Aerial Systems. By J. S. McPetrie. Vol. 7, 1932, p. 54.
12. An Experimental and Analytical Investigation of Earthed Receiving Aerials. By F. M. Colebrook. Vol. 7, 1932, p. 266.
13. An Investigation into the Factors Controlling the Economic Design of Beam Arrays. By T. Walmsley. Vol. 9, 1934, p. 115.
14. The Radiation Resistance of Aerials whose Length is Comparable with the Wavelength. By E. B. Moullin. Vol. 11, 1936, p. 93.
15. The Application of Transmission Line Theory to Closed Aerials. By F. M. Colebrook. Vol. 13, 1938, p. 273.
16. The Electric and Magnetic Fields of a Linear Radiator carrying a Progressive Wave. By F. M. Colebrook. Vol. 15, 1940, p. 17.
17. Aerial Characteristics. By N. Wells. Vol. 89, 1942, p. 76.

I.R.E. PROCEEDINGS.

18. On the Radiation Resistance of a Simple Vertical Antenna at Wavelengths below the Fundamental. 1924, p. 823.
19. On the Optimum Transmitting Wavelength for a Vertical Antenna over Perfect Earth. By S. Valentine. 1924, p. 833.
20. The Receiving System for Long-wave Transatlantic Radio Telephony (Beverage Wave Antenna). By Bailey, Dean and Wintringham. Dec. 1928.
21. Resistance of Beam Antennas. By Pistolkors. March, 1929.
22. Certain Factors affecting the Gain of Directive Antennas. By G. C. Southworth. Sept. 1930, p. 1502.
23. Calculation of Electric and Magnetic Field Intensities of Oscillating Straight Conductors. By Bechmann. March 1931.
24. Theoretical and Practical Aspects of Directional Transmitting Systems. By Sterba. July 1931, p. 1184.
25. On the Calculation of the Radiation Resistance of Antenna and Antenna Combinations. By Bechmann. Aug. 1931, p. 1471.
26. Inverted-V Aerials. By E. Bruce. Aug. 1931, p. 1406.
27. Development of Directive Transmitting Antennas by R.C.A. Communications Inc. By Carter, Hansell and Lindenblad. Oct. 1931, p. 1773.
28. Circuit Relations in Radiating Systems and Application to Antenna Problems. By Carter. June 1932, p. 1004.
29. The Action of Short-wave Frame Aerials. By L. S. Palmer and L. L. K. Honeyball. Aug. 1932, p. 1345.
30. Rectangular Short-wave Frame Aerials for Reception and Transmission. By L. S. Palmer and D. Taylor. Jan. 1934, p. 93.
31. Maintaining the Directivity of Arrays. By F. G. Kear. July 1934, p. 847.
32. Horizontal Rhombic Antennas. By E. Bruce, A. C. Beck and L. R. Lowry. Jan. 1935, p. 24.

33. SOME COMMENTS ON BROADCAST ANTENNAS. By Harmon. Jan. 1936, p. 36.
34. THE BROADCAST ANTENNA. By Chamberlain and Lodge. Jan. 1936, p. 11.
35. A CRITICAL STUDY OF THE CHARACTERISTICS OF BROADCAST ANTENNAS AS AFFECTED BY ANTENNA CURRENT DISTRIBUTION. By G. H. Brown. Jan. 1936, p. 48.
36. GROUND SYSTEMS AS A FACTOR IN ANTENNA EFFICIENCY. By G. H. Brown, R. F. Lewis and J. Epstein. June 1937, p. 753.
37. DIRECTIONAL ANTENNAS. By G. H. Brown. Jan. 1937, p. 78.
38. SIMPLE METHOD OF OBSERVING CURRENT AMPLITUDE AND PHASE RELATIONS IN ANTENNA ARRAYS. By J. F. Morrison. Oct. 1937, p. 1310.
39. RADIATION FROM RHOMBIC ANTENNAS. By Foster. Oct. 1937, p. 1327.
40. STANDARDS ON TRANSMITTERS AND ANTENNAS. Issued by I.R.E. 1938.
41. RADIO FREQUENCY VOLTAGES ACROSS INSULATORS IN TOWER ANTENNAS. By G. H. Brown. Sept. 1939, p. 566.
42. THEORY OF ANTENNAS OF ARBITRARY SIZE AND SHAPE. By S. A. Schelkunoff. Sept. 1941, p. 493.
43. HORIZONTAL-POLAR-PATTERN TRACER FOR DIRECTIONAL BROADCAST ANTENNAS. By F. A. Everest and W. S. Pritchett. May 1942, p. 227.
44. A MECHANICAL CALCULATOR FOR DIRECTIONAL ANTENNA. Patterns by W. G. Hutton and R. M. Pierce. May 1942, p. 233.
45. . . . RADIATION PATTERN OF TWO-ELEMENT BROADCAST ANTENNA ARRAYS. By K. Spangenberg. May 1942, p. 237.
46. THE INCLINED RHOMBIC ANTENNA. By C. W. Harrison. May 1942, p. 241.
47. THE SELF-IMPEDANCE OF A SYMMETRICAL ANTENNA. By R. King. July 1942, p. 335.

BELL SYSTEM TECHNICAL JOURNAL.

48. DIRECTIVE DIAGRAMS OF ANTENNA ARRAYS. By R. M. Foster. Vol. 5, 1926, p. 292.
49. ASYMPTOTIC DIPOLE RADIATION FORMULAS. By W. Howard Wise. Vol. 8, 1929, p. 662.
50. CERTAIN FACTORS AFFECTING THE GAIN OF DIRECTIVE ANTENNAS. By G. C. Southworth. Vol. 10, 1931, p. 63.

E.W. AND WIRELESS ENGINEER.

51. ANTENNA CONSTANTS: CAPACITY. By G. W. Ingram. May 1924, p. 439.
52. THE EFFECT OF THE SERIES CONDENSER IN A TRANSMITTING AERIAL. By E. H. Robinson. June 1924, p. 532.
53. NODAL POINTS OF AERIAL TUNING. By P. K. Turner. Sept. 1924, p. 723.
54. THE DIRECTIONAL PROPERTIES OF (INVERTED L) AERIALS. By R. L. Smith-Rose and R. H. Barfield. June 1925, p. 575.
55. THE POLAR CURVES OF RECEPTION FOR SPACED AERIAL SYSTEMS. By E. Green. Oct. 1925, p. 828.
56. THEORY OF THE FRANKLIN AERIAL. By J. A. Fleming. July 1927, p. 387.
57. CALCULATION OF THE POLAR CURVES OF EXTENDED AERIAL SYSTEMS. By E. Green. Oct. 1927, p. 587.

58. Potential Difference and Capacity in A.C. Problems. Editorial by G. W. O. Howe. March 1928, p. 113.
59. The Equivalent Inductance or Capacity of an Aerial. Editorial by G. W. O. Howe. June 1928, p. 297.
60. Polar Diagrams due to Plane Aerial Reflector Systems. By T. Walmsley. Oct. 1928, p. 575.
61. Theory of Receiving Aerials. By F. M. Colebrook. Nov. 1927, p. 657.
62. French System of Directional Aerials for Transmission on Short Waves. By H. Chireix. May 1929, p. 235.
63. Rhombic Transmitting Aerial—Increasing the Power Efficiency. By L. Lewin. May 1941, p. 180.
64. Extended Aerial Systems (Polar Diagrams of). By E. Green. May 1942, p. 195.
65. Radiation Energy and Earth Absorption for Dipoles and Antennae. By A. Sommerfeld and F. Renner. Sept. 1942, p. 414, and Oct. 1942, p. 457.
66. Current Distribution along a Straight Wire in a Uniform Field. By G. W. O. Howe. Nov. 1942, p. 495.

ELECTRONICS.

67. Tower Antennas. By E. A. Laport. Aug. 1934, p. 239.
68. Calculation of Vertical Antenna Radiation Pattern. By E. A. Laport. Feb. 1935, p. 47.
69. The Turnstile Antenna. By G. H. Brown. April 1936, p. 14.
70. Directional Antenna Design. By E. A. Laport. April 1936, p. 23.
71. Directional Array Field Strengths. By A. R. Rumble. Aug. 1937, p. 16.
72. Radiating Systems and Wave Propagation. By A. G. Kandoian. April 1942, p. 39.

MARCONI REVIEW.

73. Aerial Resistance and Aerial Termination. By N. Welles. No. 47, 1934, p. 13.
74. Elementary Principles of Aerial Design. By N. Welles. No. 55, 1935, p. 6.
75. The Marconi Folded Top Aerial for Medium Waves. By N. Welles. No. 63, 1936, p. 19.
76. The Series Phase Array. By A. W. Ladner. No. 67, 1937, p. 1.
77. Radiation Resistance of a Horizontal Dipole above Earth. By L. Lewin. No. 73, 1939, p. 13.

ELECTRICAL COMMUNICATION.

78. Short Wave Directive Transmitting and Receiving Antennas. By E. H. Ullrich and N. K. Fairbank. Vol. 8, 1929, p. 235.
79. The Budapest Anti-fading Antenna. By F. Holland. Vol. 12, 1933, p. 289.
80. Electromagnetic Fields of Radiating Conductors. By A. Alford. Vol. 15, 1936, p. 70.
81. Electrical Properties of Aerials for Medium and Long Wave Broadcasting. By W. L. McPherson. Vol. 16, 1937, p. 306.

82. ELECTRICAL PROPERTIES OF AERIALS FOR MEDIUM AND LONG WAVE BROADCASTING. By W. L. McPherson. Vol. 17, 1938, p. 44.

Other References.

83. SIMPLE TELEVISION ANTENNAS. By P. S. Carter. *R.C.A. Review*, Vol. 4, No. 2, 1939, p. 168.
84. SOME FACTORS IN THE DESIGN OF DIRECTIVE BROADCAST. ANTENNA SYSTEMS. By W. S. Duttera. *R.C.A. Review*, Vol. 2, No. 1, 1937, p. 81.
85. THE FLAT TOP BEAM ANTENNA. By J. D. Kraus. *Radio*, June 1938, p. 15.
86. THE BARRAGE ANTENNA. By R. L. Dawley. *Radio*, July 1938, p. 12.
87. THE THREE ELEMENT ROTARY. By P. Gioga and R. L. Dawley. *Radio*, Nov. 1938, p. 13.
88. PROTECTIVE CIRCUITS FOR ANTENNA COUPLING NETWORKS. By F. C. Ong. *Bell Lab. Record*, March 1938, p. 254.
89. DIRECTIONAL ANTENNAS WITH CLOSELY SPACED ELEMENTS. By J. D. Kraus. *QST*, Jan. 1938, p. 21.
90. ELECTRICAL OSCILLATIONS AND ELECTRIC WAVES. (RADIATION RESISTANCE OF SIMPLE VERTICAL ANTENNA ABOVE FUNDAMENTAL.) By G. W. Pierce. *Proc. Amer. Acad.*, Vol. 52, p. 192 (1916).
91. DIRECTIONAL AERIALS FOR TRANSMISSION. By L. Wildman. *Television*, July 1935, p. 389.

A4—Aerial Feeders

I.E.E. JOURNAL.

1. LONG FEEDERS FOR TRANSMITTING WIDE SIDEBANDS, WITH REFERENCE TO THE ALEXANDRA PALACE AERIAL FEEDER SYSTEM. By E. C. Cork and J. L. Pawsey. Vol. 14, 1939, p. 111.

I.R.E. PROCEEDINGS.

2. SINGLE WIRE TRANSMISSION LINES FOR SHORT WAVE ANTENNAS. By W. L. Everitt and J. F. Byrne. Oct. 1929, p. 1840.
3. TRANSMISSION LINES FOR SHORT WAVE RADIO SYSTEMS. By E. J. Sterba and C. B. Feldman. July 1932, p. 1163.
4. LOSSES IN TWISTED PAIR TRANSMISSION LINES AT RADIO FREQUENCIES. By C. C. Harris. March 1936, p. 425.
5. ON THE OPTIMUM LENGTH FOR TRANSMISSION LINES USED AS CIRCUIT ELEMENTS. By B. Salzburg. Dec. 1937, p. 1561.
6. RESONANT IMPEDANCE OF TRANSMISSION LINES. By L. S. Nergaard and B. Salzburg. Sept. 1939, p. 579.
7. TRANSMISSION LINES WITH EXPONENTIAL TAPER. By H. A. Wheeler. Vol. 27, Jan. 1939, p. 65.
8. COUPLED NETWORKS IN RADIO FREQUENCY CIRCUITS. By A. Alford. Feb. 1941, p. 55.
9. CHARACTERISTIC IMPEDANCE OF PARALLEL WIRES IN RECTANGULAR TROUGHS. By S. Frankel. April 1942, p. 182.
10. THE SOLUTION OF TRANSMISSION LINE PROBLEMS BY USE OF THE CIRCLE DIAGRAM OF IMPEDANCES. By Willis Jackson and L. G. H. Huxley.
11. IMPEDANCE MATCHING OF HALF WAVE DIPOLE. June 1945, p. 408.

BELL SYSTEM TECHNICAL JOURNAL.

12. The Natural Period of Linear Conductors (i.e. As Dependent on Phase Velocity). By C. R. Englund. Vol. 7, 1928, p. 404.
13. Transmission Lines for Short Wave Radio Systems. By E. J. Sterba and C. B. Feldman. Vol. 11, 1932, p. 411.
14. The Exponential Transmission Line. By C. R. Burrows. Vol. 17, 1938, p. 555.

ELECTRONICS.

15. U.H.F. Antenna Terminations. By W. C. Tinus. Aug. 1935, p. 239.
16. Antenna Terminations. By C. G. Dietsch. Sept. 1935, p. 15.
17. Terminating Concentric Lines. By C. G. Dietsch. Dec. 1936, p. 16.
18. Measurement of H.F. Lines. By Ballantines Laboratories. April 1938, p. 27.
19. Transmission Line Calculator (Circle Diagram). By P. H. Smith. Jan. 1939, p. 29.
20. New Coaxial (High Pressure Air Small Diameter) Transmission Line. By W. S. Duttera. March 1939, p. 30.

MARCONI REVIEW.

21. Marconi T.C.M. High Frequency Cables. By E. W. Smith and R. F. O'Neill. No. 70, 1938, p. 32.
22. Sending End Impedance of Feeders. By H. Cafferata. No. 64, 1937, p. 12, and No. 67, 1937, p. 21.

ELECTRICAL COMMUNICATION.

23. High Frequency Transmission Line Networks. By A. Alford. Vol. 17, 1938, p. 301.
24. R.F. High Voltage Phenomena. By A. Alford and S. Pickles. Oct. 1939, p. 135.
25. Radio Frequency High Voltage Phenomena. By A. Alford and S. Pickles. Vol. 18, 1939, p. 135.

Other References.

26. Graphical Solutions for a Transmission Line Terminated by an Arbitrary Complex Load Impedance. By W. van B. Roberts. *R.C.A. Review*, Vol. 3, No. 1, 1938, p. 107.
27. Charts for Transmission Line Measurements and Computations. By P. S. Carter. *R.C.A. Review*, Vol. 3, No. 3, 1939, p. 355.
28. The Behaviour of a Transmission Line at Radio Frequencies. By F. E. Nancarrow. *P.O.E.E. Journal*, July 1928, p. 165.

A5—Alignment: *see* Ganging and Tracking

A6—Ammeters

I.E.E. JOURNAL.

1. Ammeter for Very High Frequencies. By C. L. Fortescue and L. A. Moxon. Vol. 5, 1930, p. 102.
2. The Development of a Precision Ammeter for Very High Frequencies. By E. B. Moullin. Vol. 5, 1930, p. 133.

3. The Theory and Design of Hot-wire Ammeters for Frequencies of 25 to 100 Megacycles. By C. L. Fortescue. Vol. 11, 1936, p. 175.

I.R.E. PROCEEDINGS.

4. Thermocouple Ammeters for Ultra-high Frequencies. By J. M. Miller. Dec. 1936, p. 1567.
5. Frequency Errors in Radio Frequency Ammeters. By J. D. Wallace and A. H. Moore. March 1937, p. 327.
6. An Electrodynamic Ammeter for Use from 1 to 100 Megacycles. By H. M. Turner and P. Mitchel. Nov. 1937, p. 1367.
7. The Shielding of Radio Frequency Ammeters. By J. D. Wallace. Jan. 1941, p. 1.

A7—Amplifiers

I.R.E. PROCEEDINGS.

1. The Inverted Vacuum Tube, a Voltage Reducing Power Amplifier. By F. E. Terman. 1928, p. 447.
2. Circuit Analysis Applied to the Screen Grid Tube. By J. R. Nelson. Feb. 1929, p. 320.
3. Cascaded Direct Coupled Tube Systems Operated from A.C. By E. H. Loftin and S. Y. White. April 1930, p. 669.
4. Analysis and Reduction of Output Disturbances Resulting from the A.C. Operation of the Heaters of Indirectly Heated Cathode Triodes. By J. O. McNally. Aug. 1932, p. 1263.
5. Graphical Determination of Push-Pull Audio Amplifiers. By B. J. Thompson. April 1933, p. 591.
6. Determination of the Grid Driving Power in Radio Frequency Power Amplifiers. By H. P. Thomas. Aug. 1933, p. 1134.
7. Multiple Amplifier (Using Secondary Emission). By L. A. Kubetsky. April 1937, p. 421.

E.W. AND WIRELESS ENGINEER.

8. Reactive Anode Loads in Amplifiers. By E. Green and Others. Aug. 1927, p. 467.
9. Grid Current Compensation in Power Amplifiers. By W. Bagally. Feb. 1933.
10. Gas Discharge Tube as Intervalve Coupling. By H. Smith and E. G. Hill. July 1934, p. 359.

BELL SYSTEM TECHNICAL JOURNAL.

11. High Frequency Amplifiers. By A. T. Friis and Jensen. Vol. 3, 1924, p. 181.
12. The Operation of Vacuum Tubes as Class B and Class C Amplifiers. By C. E. Fay. Vol. 11, 1932, p. 28.

ELECTRONICS.

13. Bridge Type Push-Pull Amplifiers. By L. Tulauskus. May 1933, p. 134.

14. Practical Feedback Amplifiers. By J. R. Day and J. B. Russell. April 1937, p. 16.
15. An Amplifier without Phase Distortion. June 1937, p. 27.
16. Parasitic Circuits. By P. A. Ekstrand. Oct. 1938, p. 27.
17. The High-Gain Wide-Band High Frequency Amplifier. By F. A. Everest. Feb. 1939, p. 17.
18. The Inverted Amplifier. By C. E. Strong. July 1940, p. 14.
19. Wide-Band Amplifiers and Frequency Multiplication. By D. L. Jaffe. April 1942, p. 56.

Other Reference.

20. A Method for Realizing the Full Amplification Factor of High Mu Tubes. By O. H. A. Schmidt. *Rev. Sci. Inst.*, Dec. 1933, p. 661.

A8—Amplifiers, Class B

I.R.E. PROCEEDINGS.

1. Class B Amplifiers. By J. R. Nelson. June 1933, p. 858.
2. Optimum Operating Conditions for Class B Radio Frequency Amplifiers. By W. L. Everitt. Feb. 1936, p. 305.
3. Recent Developments of the Class B Audio and Radio F Amplifiers. By L. E. Barton. July 1936, p. 985.

Other Reference.

4. Necessity for Very Low Leakage of Output Transformer in Class B Amplifiers. By H. L. Short. *Radio Eng.*, May 1935, p. 12.

A9—Amplifiers, Class C

I.R.E. PROCEEDINGS.

1. Optimum Operating Conditions for Class C Amplifiers. By W. L. Everitt. Feb. 1934, p. 152.
2. The Calculation of Class C Amplifier and Harmonic Generator Performance of Screen-Grid and Similar Tubes. By F. E. Terman and J. H. Ferns. March 1934, p. 359.
3. Calculation and Design of Class C Amplifiers. By F. E. Terman and W. C. Rooke. April 1936, p. 620.

A10—Amplifiers—Wide Range

I.R.E. PROCEEDINGS.

1. Wide-Band Amplifiers. By H. A. Wheeler. July 1939, p. 429.

E.W. AND WIRELESS ENGINEER.

2. Oscillograph Amplifiers for Very Wide Frequency Ranges. By M. von Ardenne. Feb. 1936, p. 59.

Other Reference.

3. Analysis and Design of Video Amplifiers. By S. W. Seeley and C. N. Kimball. *R.C.A. Review*, Vol. 2, No. 2, 1937, p. 171, and Vol. 3, No. 3, 1939, p. 290.

A11—Amplitude Modulation

I.R.E. PROCEEDINGS.

1. An Experimental Study of the Tetrode as a Modulated Radio Frequency Amplifier. By H. A. Robinson. Jan. 1932, p. 131.
2. Analysis of Load-Impedance Modulation. By Hans Roder. June 1939, p. 386.

BELL SYSTEM TECHNICAL JOURNAL.

3. Grid Current Modulation. By E. Peterson and C. R. Keith. Vol. 7, 1928, p. 106.
4. Copper Oxide Modulation in Carrier Telephone Systems. By R. S. Carruthers. Vol. 18, 1939, p. 315.

ELECTRONICS.

5. The Bridge Modulator. By R. K. Hellman. March 1938, p. 29.

ELECTRICAL COMMUNICATION.

6. The Spread Sideband System on Short Wave Telephone Links. By L. T. Hinton. Vol. 10, p. 60, 1931.
7. The Single Sideband System Applied to Short Wave Telephone Links. By A. H. Reeves. Vol. 12, 1933, p. 90.

Other Reference.

8. Characteristics of Amplitude Modulated Waves. By E. A. Laport. *R.C.A. Review*, Vol. 1, No. 4, 1937, p. 26.

A12—Asymmetric Sideband Systems

I.E.E. JOURNAL.

1. The Single Sideband System Applied to Short Wave Telephone Links. By A. H. Reeves. Vol. 8, 1933, p. 142.
2. Asymmetric Sideband Broadcast Transmission. By P. P. Eckersley. Vol. 10, 1935, p. 177.
3. A Quantitive Study of Asymmetric Sideband Broadcasting. By P. P. Eckersley. Vol. 13, 1938, p. 137.
4. The Transmission Characteristics of Asymmetric Sideband Communication Networks. By E. Colin Cherry. Vol. 89, 1942, p. 19.

I.R.E. PROCEEDINGS.

5. Single Sideband Telephony Applied . . . By N. Koomans. Feb. 1938, p. 182.
6. Asymmetric Sideband Broadcasting. By P. P. Eckersley. Sept. 1938, p. 1041.
7. A Short Wave Single Sideband Radio Telephone System. By A. A. Oswald. Dec. 1938, p. 1431.
8. Asymmetric Sideband Broadcasting. By N. Koomans. Nov. 1939, p. 687.
9. The Solution of Unsymmetrical Problems with the Aid of the Zero-Frequency Carrier. By H. A. Wheeler. Aug. 1941, p. 446.

MARCONI REVIEW.

10. A System for Single Sideband Transmission. By F. M. G. Murphy. Sept.–Oct. 1934, p. 8.

ELECTRICAL COMMUNICATION.

11. The Single Sideband System Applied to Short Wavelengths. By A. H. Reeves. Vol. 10, 1931, p. 3.

A13—Attenuators

I.R.E. PROCEEDINGS.

1. Designing Resistive Attenuating Networks. By P. K. Elray. March and June 1935, pp. 213, 682.
2. Errors in the Calibrated Losses of Symmetrical Resistance Networks. By A. W. Melloh. July 1941, p. 387.

E.W. AND WIRELESS ENGINEER.

3. Resistance Networks—Complete Design Tables. By C. D. Colchester and M. W. Gough. May 1940, p. 206.

ELECTRONICS.

4. The Design of Attenuation Networks. By W. F. Lanterman. Feb. 1931, p. 509.
5. Impedance Matching Networks. By A. E. Thiessen. March 1931, p. 553.
6. A Universal Network Chart (for Design of Resistance L Networks). March 1935, p. 85.
7. Unsymmetrical Attenuators. By P. M. Honnell. Aug. 1942, p. 41.

A14—Automatic Frequency Control

I.R.E. PROCEEDINGS.

1. Automatic Frequency Control. By C. Travis. Oct. 1935, p. 1125.
2. Theory of the Discriminator Circuit for Automatic Frequency Control. By Hans Roder. May 1938, p. 590.

MARCONI REVIEW.

3. Automatic Frequency Correction. By O. E. Keall and G. Millington. No. 69, 1938, p. 1.

A15—Automatic Tuning

I.R.E. PROCEEDINGS.

1. Tuning, Simplified Circuits and Design Practice. By D. E. Foster and S. W. Seeley. March 1937, p. 289.

A16—Automatic Volume Control

E.W. AND WIRELESS ENGINEER.

1. The Design of A.V.C. Systems. By W. T. Cocking. Aug. 1934, p. 414; Sept. 1934, p. 476, and Oct. 1934, p. 542.
2. D.C. Amplified A.V.C. Circuit Time Constants. By K. R. Sturley. Sept. 1941, p. 353.

B1—Balanced and Unbalanced Circuits

I.R.E. PROCEEDINGS.

1. Reactance Networks for Coupling between Unbalanced and Balanced Circuits. By S. Frankel. Sept. 1941, p. 486.

Other Reference.

2. Unbalance in Circuits. By M. Reed. *Phil. Mag.*, Sept. 1929, p. 341.

B2—Band Spreading

E.W. AND WIRELESS ENGINEER.

1. Band Spreading. By F. H. Woodbridge. Sept. 1940, p. 394.

B3—Broadcast Coverage

1. Measurement of Broadcast Coverage and Antenna Performance. By W. A. Fitch and W. S. Duttera. *R.C.A. Review*, Vol. 2, No. 4, 1938, p. 396 ; Vol. 3, No. 3, 1939, p. 340, and Vol. 3, No. 4, 1939, p. 453.
2. The Service Range of Frequency Modulation. By M. G. Crosby. *R.C.A. Review*, Vol. 4, No. 3, 1940, p. 349.

C1—Capacity Neutralization : *see* Valve Input Admittance

C2—Cathode Ray Oscillographs

I.R.E. PROCEEDINGS.

1. U.H.F. Oscillography. By H. E. Hollman. May 1940, p. 213.

BELL SYSTEM TECHNICAL JOURNAL.

The Cathode Ray Oscillograph. By J. B. Johnson. Vol. 11, 1932, p. 1.

E.W. AND WIRELESS ENGINEER.

3. Inductance Linearized Time Base. By F. C. Williams and A. Fairweather. June 1941, p. 224.

ELECTRICAL COMMUNICATION.

4. Wave Form Examination with a Cathode Ray Oscillograph. By N. V. Kipping. Vol. 3, 1924, p. 69.

ELECTRONICS.

5. Time Axis for C.R.O. By W. L. Meier and P. A. Richards. April 1934, p. 111.
6. A Signal Synchronized Sweep Circuit for C.R.O. May 1935, p. 159.
7. Distortion of Saw-Tooth Wave Forms. By M. von Ardenne. Nov. 1937, p. 37.
8. Cathode Ray Wave Form Distortion at U.H.F. R. M. Bowie. Feb. 1938, p. 19.

9. The Stabilized Sweep Circuit Oscillator. By W. E. Kock. April 1939, p. 21.
10. Wave Form Circuits for Cathode Ray Tubes. By H. M. Lewis. July 1942, p. 44, and Aug. 1942, p. 48.

C3—Charts

ELECTRONICS.

1. R.F. Transition Loss Charts. By H. A. Wheeler. Jan. 1936, p. 26.
2. R.F. Resistance of Copper Wire. Feb. 1936, p. 38.
3. Directional Radiation Patterns. By A. J. Ebel. April 1936, p. 30.
4. Band-pass Characteristics. July 1936, p. 34.
5. Resistance-Coupling Design Charts. By G. Koehler. Aug. 1936, p. 25.
6. Universal Performance Curves for Tuned Transformers. By J. E. Maynard. Feb. 1937, p. 15.
7. Multiple-Circuit Pads. By F. Wheeler. March 1937, p. 33.
8. Universal Amplification Charts. By F. E. Terman. June 1937, p. 35.
9. Mutual Inductance Calculations. By D. Pollack. July 1937, p. 31.
10. Condenser Discharge Chart. By J. B. Hoag. Sept. 1937, p. 20.
11. Distributed Capacitance Chart. By P. H. Massaut. March 1938, p. 31.
12. Multilayer Coil Conductance Chart. By J. E. Maynard. Jan. 1939, p. 33.
13. Determination of Amplification Factor of Valve from its Geometrical Construction. By E. R. Jervis. June 1939, p. 45.
14. Value of Condenser Resonating with Straight Conductor. By R. L. Haskins. Aug. 1939, p. 35.
15. A (Two-Terminal) Network-selecting Chart. By P. J. Selgin. Oct. 1939, p. 30.
16. Short Wave Inductance Chart. March 1940, p. 33.
17. Resistance and Characteristic Impedance of Transmission Lines. By Salzburg. Jan. 1942, p. 47.
18. Impedance Determination of Eccentric Lines. By G. H. Brown. Feb. 1942, p. 49.
19. Simplified Inductance Chart. By E. S. Povington. Aug. 1942, p. 60.

C4—Chireix Modulation System

I.R.E. PROCEEDINGS.

1. High Power Outphasing Modulation. By H. Chireix. Nov. 1935, p. 1370.

C5—Coaxial Lines : *see* Concentric Feeders

C6—Coils, Chokes and Inductances

I.E.E. JOURNAL.

1. The Stability of Inductance Coils for Radio Frequencies. By H. A. Thomas. Vol. 11, 1936, p. 44.
2. The Dependence on Frequency of the Temperature Coefficient of Inductance of Coils. By H. A. Thomas. Vol. 14, 1939, p. 19.

3. The Electrical Stability of Tubular Inductance Coils with Deposited Conductors. By H. A. Thomas. Vol. 15, 1940, p. 106.

I.R.E. PROCEEDINGS.

4. A Study of the High Frequency Resistance of Single Layer Coils. By A. J. Palermo and F. W. Groves. Dec. 1930, p. 2041.
5. Distributed Capacity of Single Layer Coils (gives Results of Measurements on Typical Coils). By A. J. Palermo. July 1934, p. 897.
6. The Temperature Coefficient of Inductance, with Special Reference to the Valve Generator. By E. B. Moullin. Jan. 1935, p. 65.
7. Optimum Design of Toroidal Inductances. By G. Reber. Sept. 1935, p. 1056.
8. Some Possibilities of Low Loss Coils. By F. E. Terman. Sept. 1935, p. 1069.
9. The Design of the Universal Winding. By L. M. Hershey. Aug. 1941, p. 442.

E.W. AND WIRELESS ENGINEER.

10. The High Frequency Copper Losses in Inductance Coils. By S. Butterworth. July 1925, p. 613.
11. The Self Capacity of Inductance Coils. By H. J. Barton Chapple. Aug. 1925, p. 716.
12. The Design of Inductance Coils having Rectangular Winding Section. By S. Butterworth. Sept. 1925, p. 750.
13. (Inductance of Single Turn of Various Shapes, and Inductance of Straight Wire.) By A. P. Castellain. Jan. 1926, p. 46.
14. Effective Resistance of Inductance Coils at Radio Frequency. By S. Butterworth. April 1926, p. 203; May 1926, p. 309; July 1926, p. 417, and Aug. 1926, p. 483.
15. Inductance Coils Quantitatively Compared. By A. L. M. Sowerby. June 1926, p. 363, and April 1926, p. 220.
16. Losses in Inductance Coils. By S. Butterworth. May 1926, p. 267.
17. The Establishment of Formulae for the Self-Inductance of Single-Turn Circuits of Various Shapes. By R. G. Allen. May 1928, p. 259.
18. The Establishment of a General Formula for the Inductance of Single-Turn Circuits of any Shape. By V. I. Bashenoff. May 1929, p. 245.
19. The Effect of Screening Cans on the Effective Inductance and Resistance of Coils. By T. W. O. Howe. March 1934, p. 115.
20. The Inductance of Solenoids in Cylindrical Screen Boxes. By W. G. Hayman. April 1934, p. 189.
21. Recent Improvements in Air-Cored Inductances. By W. H. Griffiths. Jan. 1942, p. 8, and Feb. 1942, p. 56.

ELECTRONICS.

22. A Study of R.F. Choke Coils. By R. Lee. April 1934, p. 121.
23. Coil Design for Short Wave Receivers. Anonymous. June 1934, p. 175.

24. MULTI-BAND R.F. CHOKE COIL DESIGN. By H. P. Miller. Aug. 1935, p. 255.
25. WINDING THE UNIVERSAL COIL. Oct. 1936, p. 23.

MARCONI REVIEW.

26. IRON-DUST CORED COILS AT RADIO FREQUENCIES. By C. Austin and A. L. Oliver. No. 70, 1938, p. 17.

Other References.

27. THE DESIGN OF INDUCTANCES FOR FREQUENCIES BETWEEN 4 AND 25 MEGACYCLES. By D. Pollack. *R.C.A. Review*, Vol. 2, No. 2, 1937, p. 184.
28. MEASURING THE INDUCTANCE OF COILS WITH SUPERIMPOSED D.C. By H. T. Wilhelm. *Bell Lab. Record*, Dec. 1935, p. 131.

C7—Cold Cathode Valves

ELECTRONICS.

1. STATUS OF COLD CATHODE TUBES ABROAD. By I. J. Sarl. Jan. 1932, p. 17.
2. TUBES WITH COLD CATHODES. By A. Hund. Jan. 1933, p. 6.
3. A NEW COLD CATHODE GAS-TRIODE. By W. E. Bahls and C. H. Thomas. May 1938, p. 15.

C8—Communication on Infra-Red Wavelengths

1. INFRA-RED RADIATION TELEPHONY. By L. Rolla and L. Mazza. *Accad. Lincei*, Jan. 5, 1930, p. 19.

C9—Concentric Feeders

BELL SYSTEM TECHNICAL JOURNAL.

1. THE PROPORTIONING OF SHIELDED CIRCUITS FOR MINIMUM HIGH FREQUENCY ATTENUATION. By E. I. Green, F. A. Leibe and H. E. Curtis. Vol. 15, 1936, p. 248.

ELECTRICAL COMMUNICATION.

2. IRREGULARITIES IN TELEPHONE AND TELEVISION COAXIAL CABLES. By L. Brillouin. Vol. 17, 1938, p. 164.

C10—Condensers

I.E.E. JOURNAL.

1. THE ELECTRICAL STABILITY OF CONDENSERS. By H. A. Thomas. Vol. 11, 1936, p. 202.

I.R.E. PROCEEDINGS.

2. THE VARIATION OF THE RESISTANCE OF A RADIO CONDENSER WITH CAPACITY AND FREQUENCY. By R. R. Ramsay. July 1930, p. 1226.

E.W. AND WIRELESS ENGINEER.

3. COMMON ERRORS IN CONDENSER CALCULATIONS. By E. H. W. Banner. Sept. 1926, p. 556.
4. THE TEMPERATURE COMPENSATION OF CONDENSERS. By W. H. F. Griffiths. March 1942, p. 101, and April 1942, p. 148.

ELECTRONICS.

5. NITROGEN-FILLED CONDENSERS. April 1939, p. 17.
6. HIGH PRESSURE NITROGEN CONDENSERS. Aug. 1939, p. 60.

C11—Condensers—Electrolytic

I.E.E. JOURNAL.

1. ELECTROLYTIC CONDENSERS. By P. R. Coursey and S. N. Ray. Vol. 14, 1939, p. 203.

BELL SYSTEM TECHNICAL JOURNAL.

2. THE ALUMINIUM ELECTROLYTIC CONDENSER. By H. O. Siegmund. Vol. 8, 1929, p. 41.

C12—Condensers—Variable

I.E.E. JOURNAL.

1. A VARIABLE AIR CONDENSER WITH ADJUSTABLE COMPENSATION FOR TEMPERATURE. By H. A. Thomas. Vol. 12, 1937, p. 286.
2. THE DEVELOPMENT OF A SMALL VARIABLE AIR CONDENSER COMPENSATED FOR RAPID CHANGES OF TEMPERATURE. By H. A. Thomas. Vol. 14, 1939, p. 157.

E.W. AND WIRELESS ENGINEER.

3. LAWS OF VARIABLE AIR CONDENSERS. By W. H. F. Griffiths. Jan. 1926, p. 3.
4. COMPARISON BETWEEN SEMI-CIRCULAR AND SQUARE LAW VANES. By O. Hall. May 1926, p. 302.
5. . . . LAWS OF VARIABLE CONDENSERS. By W. H. F. Griffiths. Dec. 1926, p. 743.
6. THE "LAW CORRECTION" OF VARIABLE AIR CONDENSERS. By W. H. F. Griffiths. Aug. 1927, p. 479.
7. THE LOSSES IN VARIABLE AIR CONDENSERS. By W. H. F. Griffiths. March 1931, p. 124.

C13—Coupling Circuits: *see* Transformers; *also* Filters and Selective Circuits

MARCONI REVIEW.

1. COMPARISON OF PARALLEL AND SERIES COUPLING CIRCUITS FOR TRANSMITTERS. By E. Green. No. 68, 1938, p. 31, and No. 69, 1938, p. 22.

C14—Crystal Oscillators

I.E.E. JOURNAL.

1. The Application and Use of Quartz Crystals in Telecommunications. By C. F. Booth. Vol. 88, 1941, p. 97.

I.R.E. PROCEEDINGS.

2. Quartz Crystal Frequency Substandards. CT and DT cut Crystals. By S. C. Hight and G. W. Willard. May 1937, p. 549.
3. Observations on Modes of Vibration and Temperature Coefficients of Quartz Crystal Plates. By F. R. Lack. July 1929, p. 1123.
4. A New Quartz Crystal Plate, Designated the G.T., which Produces a very Constant Frequency over a Wide Temperature Range. By W. P. Mason. May 1940. Vol. 28, p. 220.

BELL SYSTEM TECHNICAL JOURNAL.

5. Observations on Modes of Vibration and Temperature Coefficients or Quartz Crystal Plates. By F. R. Lack. Vol. 8, 1929, p. 515.
6. The Bridge Stabilized Oscillator. By L. A. Meacham. Vol. 17, 1938, p. 574.

MARCONI REVIEW.

7. Analysis of Bridge Circuit for Quartz Resonators. By O. E. Keall. No. 59, 1936, p. 19.
8. Grid Bias for Crystal Oscillators. By D. A. Bell. No. 71, 1938, p. 20.

D1—Detection

I.E.E. JOURNAL.

1. Straight Line Detection with Diodes. Conditions for Distortionless Detection. By F. Roberts and F. C. Williams. Vol. 9, 1934, p. 325.

I.R.E. PROCEEDINGS.

2. Detection by Grid Rectification. By S. Ballantine. 1928, p. 593.
3. Some Principles of Grid-Leak Grid-Condenser Detection. By F. E. Terman. Oct. 1928, p. 1384.
4. Detection at High Signal Voltages. By S. Ballantine. July 1929, p. 1153.
5. Some Properties of Grid-Leak Power Detection. By F. E. Terman and N. R. Morgan. Dec. 1930, p. 2160.
6. Grid Circuit Power Rectification. By J. R. Nelson. March 1931, p. 489.
7. Some Notes on Demodulation. Hans Roder. Dec. 1932, p. 1946.
8. Diode Detection Analysis. By C. E. Kilgour and J. M. Glessner. July 1933, p. 931.
9. On Conversion Detectors (Mixers), with Bibliography. By M. J. O. Strutt. Aug. 1934, p. 981.

E.W. AND WIRELESS ENGINEER.

10. THE RECTIFYING DETECTOR. By F. M. Colebrook. May 1925, p. 459; March 1925, p. 330, and April 1925, p. 394.
11. THE APPARENT DE-MODULATION OF A WEAK STATION BY A STRONGER ONE. By R. T. Beatty. June 1928, p. 300.
12. THE RECTIFICATION OF SMALL RADIO FREQUENCY POTENTIAL DIFFERENCES BY MEANS OF TRIODE VALVES. By F. M. Colebrook. July 1925, p. 628; Nov. 1925, p. 865; Dec. 1925, p. 946; Jan. 1926, p. 34, and Feb. 1926, p. 90.
13. THE THEORY OF THE STRAIGHT LINE RECTIFIER. By F. M. Colebrook. Nov. 1930, p. 595.
14. THE APPARENT DE-MODULATION OF A WEAK STATION BY A STRONGER ONE. By F. M. Colebrook. Aug. 1931, p. 409.
15. DETECTION BY A STRAIGHT LINE RECTIFIER. By E. B. Moullin. July 1932, p. 378.

ELECTRONICS.

16. WEAK SIGNAL DEMODULATION. By E. V. Appleton. May 1932, p. 171.

MARCONI REVIEW.

17. THE REDUCTION OF DIODE DETECTOR DISTORTION BY POSITIVE BIAS. Anonymous. No. 73, 1939, p. 10.

D2—Dewey Classification

I.R.E. PROCEEDINGS.

1. CLASSIFICATION OF RADIO SUBJECTS: AN EXTENSION OF THE DEWEY DECIMAL SYSTEM. Aug. 1930, p. 1433.

D3—Dielectrics : *see* Insulators

D4—Diffraction

I.R.E. PROCEEDINGS.

1. RADIO PROPAGATION OVER SPHERICAL EARTH. By C. R. Burrows. May 1935, p. 470.
2. DIFFRACTION MEASUREMENTS AT ULTRA-HIGH FREQUENCIES. By H. Salvidge. Jan. 1941, p. 10.
3. THE EFFECT OF THE EARTH'S CURVATURE ON GROUND-WAVE PROPAGATION. By C. R. Burrows and M. C. Gray. Jan. 1941, p. 16.
4. MEASUREMENT (CALIBRATION) OF LOOP-ANTENNA RECEIVERS. By W. O. Swinyard. July 1941, p. 382.

BELL SYSTEM TECHNICAL JOURNAL.

5. RADIO PROPAGATION OVER SPHERICAL EARTH. By C. R. Burrows Vol. 14, 1935, p. 477.

D5—Direction Finding

I.E.E. JOURNAL.

1. The Cause and Elimination of Night Errors in Radio Direction Finding. By R. L. Smith-Rose. Vol. 1, 1926, p. 145.
2. Recent Developments in Direction Finding Apparatus. By R. H. Barfield. Vol. 5, 1930, p. 169.
3. A Note on the Theory of Night Errors in Adcock Direction Finding Systems. By J. F. Coales. Vol. 7, 1932, p. 292.
4. Some Principles Underlying the Design of Spaced Aerial Direction Finders. By R. H. Barfield. Vol. 10, 1935, p. 55.
5. An Improved Medium Wave Adcock Direction Finder. By R. H. Barfield and R. A. Fereday. Vol. 13, 1938, p. 33.
6. A Short Wave Adcock Direction Finder. By R. H. Barfield and W. Ross. Vol. 13, 1938, p. 39.
7. The Measurement of the Lateral Deviation of Radio Waves by Means of a Spaced-Loop Direction Finder. By R. H. Barfield. Vol. 13, 1938, p. 314.
8. Direction Finding on Wavelengths between 6–10 Metres (Frequencies 50 to 30 Mc/sec.). By R. L. Smith-Rose. Vol. 13, 1938, p. 303.
9. A Sense-Finding Device for Use with Spaced-Aerial Direction Finders. By R. A. Fereday. Vol. 14, 1939, p. 14.
10. The Calibration of Four-Aerial Adcock Direction Finders. By W. Ross. Vol. 14, 1939, p. 229.
11. Symposium on Radio Direction Finding. Vol. 14, 1939, p. 240.
12. The Performance and Limitation of the Compensated Loop Direction Finder. By R. H. Barfield. Vol. 15, 1940, p. 108.
13. Radio Direction Finding on Wavelengths between 2 and 3 Metres. By H. G. Hopkins and R. L. Smith-Rose. Vol. 15, 1940, p. 161.

I.R.E. PROCEEDINGS.

14. Radio Direction Finding. By R. L. Smith-Rose. March 1929, p. 425.

E.W. AND WIRELESS ENGINEER.

15. Directive Radio Telegraphy and Telephony. By R. L. Smith-Rose. Dec. 1923, p. 119; Jan., Feb., March 1924, pp. 192, 251 and 307.
16. Direction Finders (Abstract of Wireless Section of I.E.E. Meeting). May 1924, p. 487.
17. A Short Wave Cathode Ray Direction-Finding Receiver. By N. P. L. Staff. Aug. 1938, p. 432.
18. (Direction Finder for 6–11 Metres.) By G. H. Hopkins. Dec. 1938, p. 651.
19. Automatic Radiogoniometers. By J. Marique. March 1939, p. 121.
20. The Screened Loop-Aerial. By E. R. Burgiss. Oct. 1939, p. 492.

ELECTRICAL COMMUNICATION.

21. The Automatic Radio Compass. By H. Busignies. Vol. 15, p. 157, 1936.
22. Reduction of Night Error. By H. Busignies. Vol. 16, 1937, p. 213.

MARCONI REVIEW.

23. Elimination of Night Effect with a Pulse Transmitter. By T. L. Eckersley. No. 46, 1934, p. 12.
24. Marconi Direction Finder. By H. A. Ewen and F. Woods. No. 48, 1934, p. 1. (Type D.F.G. 9.C.)
25. The Night Performance of the Marconi-Adcock Direction Finder. (Type D.F.G.8.) By S. P. Smith. No. 50, 1934, p. 1.
26. Scattering, Polarization Errors and the Accuracy of Short Wave Direction Finding. By T. L. Eckersley. No. 53, 1935, p. 1, and No. 55, 1935, p. 20.
27. Recent Developments in Marconi-Adcock Direction Finding. By S. B. Smith and G. F. Hatch. No. 58, 1936, p. 1.
28. Directional Errors from Local Transmitters. By T. L. Eckersley. No. 60, 1936, p. 21.
29. The Extended Feeder Marconi-Adcock Direction Finder. By S. B. Smith. No. 66, 1937, p. 23.
30. Errors in Radiogoniometers. By S. B. Smith and G. F. Hatch. No. 71, 1938, p. 9.

Other Reference.

31. Small Vessel Direction Finders. By H. B. Martin. Vol. 2, No. 1, 1937, p. 69.

D6—Distortion

I.R.E. PROCEEDINGS.

1. Distortion Tests by the Intermodulation Method. By J. K. Hilliard. Dec. 1941, p. 614.

E.W. AND WIRELESS ENGINEER.

2. (The Buzz Effect in Pentodes.) By K. A. Macfadyen. June 1938, p. 310.

D7—Diversity Reception

I.R.E. PROCEEDINGS.

1. Diversity Receiving System, etc. By H. H. Beverage and Others. April 1931, pp. 531, 562.

Other Reference.

2. Recent Developments in Diversity Receiving Equipment. By J. B. Moore. *R.C.A. Review*, Vol. 2, No. 1, 1937, p. 94.

D8—Doherty Type Transmitters

I.R.E. PROCEEDINGS.

1. A New High Efficiency Power Amplifier for Modulated Waves. By W. H. Doherty. Sept. 1936, p. 1163.
2. A Phase Opposition System of Amplitude Modulation. By L. F. Gaudernack. Aug. 1938, p. 983.

3. A HIGH EFFICIENCY GRID MODULATED AMPLIFIER. By F. E. Terman and J. R. Woodyard. Aug. 1938, p. 929.
4. A HIGH EFFICIENCY MODULATING SYSTEM. By A. W. Vance. Aug. 1939, p. 506.
5. A FIFTY KILOWATT . . . DOHERTY AMPLIFIER. By W. H. Doherty and G. W. Turner. Sept. 1939, p. 531.

BELL SYSTEM TECHNICAL JOURNAL.

6. A NEW HIGH EFFICIENCY POWER AMPLIFIER FOR MODULATED WAVES. By W. H. Doherty. Vol. 15, 1936, p. 469.

ELECTRICAL COMMUNICATION.

7. AN EXPERIMENTAL 5-KILOWATT DOHERTY AMPLIFIER. By C. E. Strong and G. C. Sampson. Vol. 16, 1937, p. 233.

E1—Earth Conductivity

ELECTRICAL COMMUNICATION.

1. A SEARCH-COIL METHOD OF MEASURING THE A.C. RESISTIVITY OF THE EARTH. By J. Collard. Vol. 14, 1935, p. 270.

E2—Earth Connections

I.R.E. PROCEEDINGS.

1. THE PHASE AND MAGNITUDE OF EARTH CURRENTS NEAR RADIO TRANSMITTING ANTENNAS. By G. N. Brown. Feb. 1935, p. 168.

E.W. AND WIRELESS ENGINEER.

2. . . . AERIAL AND EARTH SYSTEMS FOR RECEPTION. By R. L. Smith-Rose and F. M. Colebrook. Jan. 1925, p. 207.

E3—Eddy Current Tuning

E.W. AND WIRELESS ENGINEER.

1. EDDY CURRENT TUNING. By C. C. Eaglesfield. May 1942, p. 202.

E4—Electrical Effects

I.R.E. PROCEEDINGS.

1. ELECTRO-ELASTIC AND PYRO-ELECTRIC PHENOMENA. By W. G. Cady. July 1930, p. 1247.

Other References.

2. RECENT INVESTIGATIONS ON ELECTRETS. By A. Gemant. *Phil. Mag.*, Nov. 1935, p. 929.
3. THE HALL EFFECT. By E. Stevens. *Phil. Mag.*, April 1930, p. 547.
4. THE PERMANENT ELECTRET. By M. Eguchi. *Phil. Mag.*, Jan. 1925, p. 178.

5. A Compilation of Named Effects and Laws. By L. C. Krueger and H. R. Hosmer. *Gen. El. Review*, March 1921, p. 258. (See also *Television*, Dec. 1925, p. 712.)
6. Johnson-Rahbek Effect. By T. Hans. *Ann. der Physik.*, June 30, 1928, p. 353.
7. Johnson-Rahbek Effect. By P. Boning. *Zeit fur Fernmeldtchnik.*, April 29, 1929.

E5—Electron Multipliers

I.R.E. PROCEEDINGS.

1. The Secondary Emission Multiplier. By V. K. Zworykin and Others. March 1936, p. 351.
2. The Voltage-Controlled Electron Multiplier. By B. J. Thompson. Nov. 1941, p. 583.
3. The Behaviour of Electrostatic Electron Multipliers as a Function of Frequency. By L. Malter. Nov. 1941, p. 587.

ELECTRONICS.

4. An Electron Multiplier. Anonymous. Aug. 1934, p. 242.
5. Secondary Emission Electron Multipliers. Anonymous report on Zworykin's work. Nov. 1935, p. 10.

E6—Equalization : *see* Response Correction

F1—Fading

I.R.E. PROCEEDINGS.

1. Notes on the Random Fading of 50 Megacycles over Non-Optical Paths. By K. G. Maclean and G. S. Wickizer. Aug. 1939, p. 501.
2. Fading Curves and Weather Conditions. By R. C. Colwell. Jan. 1929, p. 143.
3. Observations of Short Period Fading. By T. Parkinson. June 1929, p. 1042.

F2—Feedback

I.R.E. PROCEEDINGS.

1. Regeneration Theory and Experiment. By E. Petersen, J. G. Kreer and L. A. Ware. Oct. 1934, p. 1191.
2. Balanced Feedback Amplifiers. By E. L. Ginztun. Nov. 1938, p. 1367.
3. Control of the Effective Internal Impedance of an Amplifier by Feedback. By H. E. Mayer. Feb. 1939, p. 213.
4. A Distortion-free Amplifier. By P. O. Pedersen. Vol. 27, Feb. 1939, p. 59.

BELL SYSTEM TECHNICAL JOURNAL.

5. Regeneration Theory. By H. Nyquist. Vol. 11, 1932, p. 126.
6. Stabilized Feedback Amplifiers. By H. S. Black. Vol. 13, 1934, p. 1.
7. Regeneration Theory and Experiment. By E. Peterson and Others. Vol. 13, 1934, p. 680.

E.W. AND WIRELESS ENGINEER.

8. SOME PROPERTIES OF NEGATIVE FEEDBACK AMPLIFIERS. By L. I. Farren. Jan. 1928, p. 23.
9. FEEDBACK. By E. K. Sandeman. Aug. 1940, p. 342.

ELECTRONICS.

10. FEEDBACK AMPLIFIERS. July 1936, p. 31.
11. A CONTROLLED FEEDBACK COMPRESSION AMPLIFIER. By H. H. Stewart and H. S. Pollock. Feb. 1940, p. 19.

MARCONI REVIEW.

12. GRAPHICAL TREATMENT OF BACK COUPLED CIRCUITS. No. 74, 1939, p. 35.

F3—Feeders : *see* Aerial Feeders

F4—Field Distribution

I.R.E. PROCEEDINGS.

1. A STUDY OF THE ELECTROMAGNETIC FIELD IN THE VICINITY OF A RADIATOR. By F. R. Stansel. May 1936, p. 802.

ELECTRONICS.

2. DETERMINING FIELD DISTRIBUTION. By E. D. McArthur. June 1932, p. 193.

F5—Field Strength Determination

I.E.E. JOURNAL.

1. A METHOD OF USING HORIZONTALLY POLARIZED WAVES FOR THE CALIBRATION OF SHORT WAVE FIELD-STRENGTH MEASURING SETS BY RADIATION. By J. S. McPetrie and B. G. Pressey. Vol. 13, 1938, p. 267.
2. THE DESIGN AND CONSTRUCTION OF A SHORT WAVE FIELD-STRENGTH MEASURING SET. By F. M. Colebrook and A. C. Gordon-Smith. Vol. 14, 1939, p. 146.
3. THEORY AND EXPERIMENTAL CONFIRMATION OF CALIBRATION OF FIELD-STRENGTH MEASURING SETS BY RADIATION. By J. S. McPetrie and J. A. Saxton. Vol. 88, 1941, p. 11.
4. A METHOD OF CALIBRATING A FIELD-STRENGTH MEASURING SET. By F. M. Colebrook. Vol. 88, 1941, p. 15.

I.R.E. PROCEEDINGS.

5. EXPERIMENTS IN RECORDING RADIO SIGNAL INTENSITY. By L. W. Austin. July 1929, p. 1192.
6. THE PROBLEMS CENTERING ABOUT THE MEASUREMENTS OF FIELD INTENSITY. By S. W. Edwards and J. E. Brown. Aug. 1929, p. 1377.
7. USE OF AUTOMATIC RECORDING EQUIPMENT IN RADIO TRANSMISSION RESEARCH. By P. A. de Mars and Others. Sept. 1931, p. 1618.

8. On the Use of Field Intensity Measurements for the Determination of Broadcast Station Coverage. By C. M. Jansky and S. L. Bailey. Jan. 1932, p. 40.
9. Field Intensity Measurements at Frequencies from 285 to 5400 Kc/s. By S. S. Kirby and K. A. Norton. May 1932, p. 841.
10. A Method of Calculation of Field-Strengths in High-Frequency Radio Transmission. By S. Namba and T. Tsukada. July 1933, p. 1003.
11. A Compact Radio Field-Strength Meter. By P. B. Taylor. Feb. 1934, p. 191.
12. A Continuous Recorder of Field-Strength Intensities. By K. A. Norton and S. E. Reymer. Feb. 1934, p. 266.
13. Precise Measurements of Electromagnetic Fields. By H. G. Smith. Jan. 1938, p. 45.
14. The Calculation of Ground Wave Field Intensity over a Finitely Conducting Spherical Earth. By K. A. Norton. Dec. 1941, p. 623.

ELECTRICAL COMMUNICATION.

15. Signal Strength Measurements. By E. H. Ullrich. Vol. 5, 1926, p. 22.

MARCONI REVIEW.

16. Marconi U.S.W. Field-Strength Measuring Equipment. By F. M. Wright. No. 67, 1937, p. 14.

F6—Field Strength Measurement : *see* Field Strength Determination

F7—Filters and Selective Circuits

I.E.E. JOURNAL.

1. A Nomogram for the Band Pass Filter (Zobel Type). By A. T. Starr and H. C. Hall. Vol. 10, 1935, p. 41.

I.R.E. PROCEEDINGS.

2. The Band Pass—Low Pass Analogy. By V. D. Landon. Dec. 1936, p. 1582.
3. Two-Mesh Tuned Coupled Circuit Filters. By C. B. Aiken. Feb. 1937, p. 230.
4. A New Type of Selective Circuit (Uses no Inductance). By H. N. Scott. Feb. 1938, p. 226.
5. A Receiver with Automatic Selectivity Control Responsive to Interference. By J. F. Farrington. April 1939, p. 239.

BELL SYSTEM TECHNICAL JOURNAL.

6. Physical Theory of the Electric Wave-Filter. By G. A. Campbell. Vol. 1, 1922, p. 1.
7. Theory and Design of Uniform and Composite Electric Wave Filters. By O. J. Zobel. Vol. 2, 1923, p. 1.
8. Transmission Characteristics of Wave Filters. By O. J. Zobel. Vol. 3, 1924, p. 567.

9. MUTUAL INDUCTANCE IN WAVE FILTERS WITH AN INTRODUCTION ON FILTER DESIGN. By K. S. Johnson and T. E. Shea. Vol. 4, 1925, p. 52.
10. ACOUSTIC FILTERS. By W. P. Mason. Vol. 6, 1927, p. 258.
11. IMPEDANCE CORRECTION ON WAVE FILTERS. By E. B. Payne. Vol. 9, 1930, p. 770.
12. EXTENSIONS TO THE THEORY AND DESIGN OF ELECTRIC WAVE FILTERS. By O. J. Zobel. Vol. 10, 1931, p. 284.
13. PRECISION METHODS IN CONSTRUCTING ELECTRIC WAVE FILTERS FOR CARRIER SYSTEMS. By G. R. Harris. Vol. 11, 1932, p. 264.
14. ELECTRICAL WAVE FILTERS EMPLOYING QUARTZ CRYSTALS AS ELEMENTS. By W. P. Mason. Vol. 13, 1934, p. 405.
15. IDEAL WAVE FILTERS. By H. W. Bode and R. L. Dietzold. Vol. 14, 1935, p. 215.
16. THE USE OF COAXIAL AND BALANCED TRANSMISSION LINES IN FILTERS AND WIDE BAND TRANSFORMERS FOR HIGH RADIO FREQUENCIES. By W. P. Mason and R. A. Sykes. Vol. 16, 1937, p. 275.
17. RESISTANCE COMPENSATED BAND PASS CRYSTAL FILTERS FOR USE IN UNBALANCED CIRCUITS. By W. P. Mason. Vol. 16, 1937, p. 423.

E.W. AND WIRELESS ENGINEER.

18. ON THE THEORY OF FILTER AMPLIFIERS. By S. Butterworth. Oct. 1930, p. 536.
19. A BAND PASS FILTER OF VARIABLE WIDTH WITH CONSTANT FORM FACTOR AND MIDDLE FREQUENCY. Editorial by G. W. O. Howe. Aug. 1939, p. 381.
20. COUPLING CIRCUITS AS BAND PASS FILTERS. By E. K. Sandeman. 1941, Sept., p. 361; Oct., p. 406; Nov., p. 450; Dec., p. 492.
21. TRANSFORMED NETWORKS. By H. J. Griese. Oct. 1942, p. 463.

ELECTRONICS.

22. ELECTRIC WAVE FILTERS FOR HIGH FREQUENCIES. By C. L. Frederick. March 1934, p. 85.
23. POWER SUPPLY FILTER (ATTENUATION CURVES). By W. W. Waltz. Dec. 1935, p. 31.
24. BAND PASS AND BAND REJECTION FILTERS (INCLUDING DISSIPATIVE CASE). By H. Holubow. Aug. 1942, p. 54.

ELECTRICAL COMMUNICATION.

25. THE DESIGN OF FILTERS FOR CARRIER PROGRAMME CIRCUITS. By F. Ralph. Vol. 11, 1932, p. 204.

Other References.

26. (INSERTION LOSS FORMULAE FOR PROTOTYPE L.P. and H.P. FILTERS IN TERMS OF f/f_c.) By O. S. Meixell. *R.C.A. Review*, Vol. 5, No. 3, 1941, p. 337.
27. M-DERIVED BAND PASS FILTERS WITH RESISTANCE CANCELLATION. By V. D. Landon. *R.C.A. Review*, Vol. 1, No. 2, 1936, p. 93.
28. GENERAL THEORY OF ELECTRIC WAVE FILTERS. By H. W. Bode. *Jl. Math. Phys.*, Vol. 13, p. 275.

F8—Fluctuation Noise

1. FLUCTUATIONS IN SPACE-CHARGE LIMITED CURRENTS AT MODERATELY HIGH FREQUENCIES. By B. J. Thompson, D. O. North and W. A. Harris. *R.C.A. Review*, Vol. 4, No. 3, 1940, p. 269; Vol. 4, No. 4, 1940, p. 441; Vol. 5, No. 1, 1940, p. 106; Vol. 5, No. 2, 1940, p. 244; Vol. 5, No. 3, 1941, p. 371; Vol. 5, No. 4, 1941, p. 505.

F9—Frequency Measurement

I.E.E. JOURNAL.

1. THE TECHNIQUE OF FREQUENCY MEASUREMENT AND ITS APPLICATION TO TELECOMMUNICATION. By E. Thwaites and F. J. M. Laver. Vol. 89, 1942, p. 139.

I.R.E. PROCEEDINGS.

2. A DIRECT READING RADIO FREQUENCY METER. By R. C. Hitchcock. Jan. 1929, p. 24.
3. A CONVENIENT METHOD FOR REFERRING SECONDARY FREQUENCY STANDARDS TO A STANDARD TIME INTERVAL. By L. M. Hull and J. K. Clapp. Feb. 1929, p. 252.
4. A SYSTEM OF FREQUENCY MEASUREMENTS BASED ON A SINGLE FREQUENCY. By E. L. Hall. Feb. 1929, p. 272.
5. A DIRECT READING FREQUENCY BRIDGE FOR THE AUDIO-FREQUENCY RANGE. By C. I. Soucey and B. de F. Bayly. May 1929, p. 834.
6. A HIGH PRECISION STANDARD OF FREQUENCY. By W. A. Marrison. July 1929, p. 1103.
7. INTERPOLATION METHODS FOR USE WITH HARMONIC STANDARDS. By J. K. Clapp. Sept. 1930, p. 1575.
8. A PRECISE AND RAPID METHOD OF MEASURING FREQUENCIES FROM 5 TO ZERO CYCLES PER SECOND. By N. P. Case. Sept. 1930, p. 1586.
9. ACCURATE METHOD OF MEASURING TRANSMITTED WAVE FREQUENCIES AT 5,000 AND 20,000 Kc/s. By E. L. Hall. Jan. 1931, p. 35.
10. SOME METHODS OF MEASURING THE FREQUENCY OF SHORT WAVES (USE OF LUMINOUS QUARTZ RESONATORS). By Hans Mogel. Feb. 1931, p. 195.
11. A DEVICE FOR THE PRECISE MEASUREMENT OF HIGH FREQUENCIES (5–30 Mc/s). By F. A. Polkinghorn and A. A. Roetken. June 1931, p. 937.
12. MEASUREMENT OF THE FREQUENCY OF ULTRA RADIO WAVES. By J. B. Hoag. Jan. 1933, p. 29.

BELL SYSTEM TECHNICAL JOURNAL.

13. A HIGH PRECISION STANDARD OF FREQUENCY. By W. A. Marrison. Vol. 8, 1929, p. 493.

E.W. AND WIRELESS ENGINEER.

14. MEASUREMENT OF WAVELENGTHS OF BROADCASTING STATIONS (AN ACCOUNT OF THE WORK OF THE BRUSSELS CHECKING STATION OF THE U.I.R.). By R. Braillard and E. Divoire. Aug. 1929, p. 412.

MARCONI REVIEW.

15. Frequency Measuring Equipment. By N. Lea and K. K. Sturley. No. 70, p. 1, 1938.

F10—Frequency Modulation

I.R.E. PROCEEDINGS.

1. Frequency Modulation. By B. Van der Pol. July 1930, p. 1194.
2. Amplitude, Phase and Frequency Modulation. Hans Roder. Dec. 1931, p. 2145.
3. A Method of Reducing Disturbances in Radio Signalling by a System of Frequency Modulation. By E. N. Armstrong. May 1936, p. 689.
4. Frequency Modulation Propagation Characteristics. By M. G. Crosby. June 1936, p. 898.
5. Frequency Modulation Noise Characteristics. By M. G. Crosby. April 1937, p. 472.
6. Application of the Autosynchronized Oscillator to Frequency Demodulation. (Enables a Carrier to be Amplitude and Frequency Modulated Respectively with Two Different Programmes.) By J. R. Woodyard. May 1937, p. 612.
7. Armstrong's Frequency Modulator. By D. L. Jaffe. April 1938, p. 475.
8. Effects of Tuned Circuits upon a Frequency Modulated Signal. By Hans Roder. Dec. 1937, p. 1617.
9. The Application of Negative Feedback to Frequency Modulation Systems. By J. G. Chaffee. May 1939, p. 317.
10. A Frequency Modulation Monitoring System. By R. J. Pieracci. Vol. 28, April 1940, p. 374.
11. A New Broadcast Transmitter Circuit for Frequency Modulation. By J. F. Morrison. Vol. 28, Oct. 1940, p. 444.
12. Two Signal Cross Modulation in a Frequency Modulation Receiver. By H. A. Wheeler. Vol. 28, Dec. 1940, p. 537.
13. Short Bibliography. March 1941, p. 97.
14. Drift Analysis of the Crosby Frequency Modulated Transmitter Circuit. By E. S. Winlund. July 1941, p. 390.
15. Observations of Frequency Modulation Propagation on 26 Megacycles. By M. G. Crosby. July 1941, p. 398.
16. A Commercial 50 kW Frequency Modulation Broadcast Transmitting Station. By H. P. Thomas and R. H. Williamson. Oct. 1941, p. 537.
17. Intermediate Frequency Valves for Frequency Modulated Wave Receivers. By D. E. Foster. Oct. 1941, p. 546.
18. Factory Alignment Equipment for Frequency Modulation Receivers. By H. E. Rice. Oct. 1941, p. 551.
19. Inductively Coupled Frequency Modulator. By B. E. Montgomery. Oct. 1941, p. 559.
20. Common-Channel Interference between Two Frequency Modulated Signals. By H. A. Wheeler. Jan. 1942, p. 34.

21. A Stabilized Frequency Modulation System. By R. J. Pieracci. Feb. 1942, p. 76.
22. The Use of Vacuum Tubes as Variable Impedance Elements. By H. J. Reich. June 1942, p. 288.
23. A New Frequency Modulator Broadcasting Transmitter. By A. A. Skene and N. C. Olmstead. July 1942, p. 330.
24. The Zero Beat Method of Frequency Discrimination. By C. F. Sheaffer. Aug. 1942, p. 365.
25. Transients in Frequency Modulation. By H. Salinger. Aug. 1942, p. 378.
26. F.M. Distortion due to Multipath Transmission. By M. S. Carrington. Dec. 1945, p. 878.

BELL SYSTEM TECHNICAL JOURNAL.

27. Theory of Frequency Modulation. By J. R. Carson and T. C. Fry. Vol. 16, 1937, p. 513.
28. Frequency Modulation: Theory of the Feedback Receiving Circuit. By J. R. Carson. Vol. 18, 1939, p. 395.
29. The Application of Negative Feedback to Frequency Modulation Systems. Vol. 18, 1939, p. 404. By J. G. Chaffee.

E.W. AND WIRELESS ENGINEER.

30. Interference in Relation to Amplitude, Phase and Frequency Modulated Systems. By O. E. Keall. Jan. 1941, p. 6.
31. Reduction of Band Width in F.-M. Receivers. By D. A. Bell. Nov. 1942, p. 497.
32. F.-M. Communication Systems. By D. A. Bell. May 1943.

ELECTRONICS.

33. Phase-Frequency Modulation. Anonymous Report on Armstrong's Work. Nov. 1935, p. 17.
34. Noise in Frequency Modulation. By Hans Roder. May 1937, p. 22.
35. A Cathode Ray Frequency Modulation Generator. By R. E. Shelley. Feb. 1940, p. 14.
36. Frequency Modulation in Television. A Symposium. Feb. 1940, p. 26.
37. Wide Band *v.* Narrow Band in F.-M. Reception. By M. L. Levy. June 1940, p. 26.
38. F.-M. Receivers—Design and Performance. By M. Hobbs. Aug. 1940, p. 22.
39. Modulation Limits in F.-M. By L. J. Black and H. J. Scott. Sept. 1940, p. 30.
40. A Modern 10 kW F.-M. Transmitter. By E. S. Winlund and C. S. Perry. March 1942, p. 40.
41. Amplitude, Frequency and Phase Modulation Relations. By A. Hund. Aug. 1942, p. 48.

ELECTRICAL COMMUNICATION.

42. "What is F.M.?" (Contains Bibliography.) By W. H. Capen. Vol. 19, 1940, p. 99.

Other References.

43. A METHOD OF MEASURING FREQUENCY DEVIATION. By M. G. Crosby. *R.C.A. Review*, Vol. 4, No. 4, 1940, p. 473.
44. (N.B.C.) F.-M. FIELD TESTS. By R. F. Guy. *R.C.A. Review*, Vol. 5, No. 2, 1940, p. 190.
45. FREQUENCY MODULATION. By S. W. Seeley. (Contains Bibliography.) *R.C.A. Review*, Vol. 5, No. 4, 1941, p. 468.

F11—Frequency Multiplication and Division

I.E.E. JOURNAL.

1. QUASI-STABLE FREQUENCY DIVIDING CIRCUITS. By R. L. Fortescue. Vol. 14, 1939, p. 171.

I.R.E. PROCEEDINGS.

2. FREQUENCY MULTIPLICATION BY SHOCK EXCITATION (USE OF SATURATED IRON CORES). By E. A. Guillemin and P. T. Rumsey. April 1929, p. 629.
3. FREQUENCY DIVISION. By J. Groskowski. Nov. 1930, p. 1960.
4. THE ADJUSTMENT OF THE MULTIVIBRATOR FOR FREQUENCY DIVISION. By V. J. Andrews. Nov. 1931, p. 1911.
5. THE REQUIRED MINIMUM FREQUENCY SEPARATION BETWEEN CARRIER WAVES OF BROADCAST STATIONS. By P. P. Eckersley. Feb. 1933, p. 193.
6. FREQUENCY MULTIPLICATION AND DIVISION. By H. Sterk. Sept. 1937, p. 1153.
7. FRACTIONAL FREQUENCY GENERATION USING REGENERATIVE MODULATION. By R. L. Miller. July 1939, p. 446.
8. A STABILIZED FREQUENCY DIVIDER. By W. Builder. April 1941, p. 177.

BELL SYSTEM TECHNICAL JOURNAL.

9. MAGNETIC GENERATION OF A GROUP OF HARMONICS. By E. Petersen and Others. Vol. 16, 1937, p. 437.

ELECTRONICS.

10. WIDEBAND AMPLIFIERS AND FREQUENCY MULTIPLICATION. By D. L. Jaffe. April 1942, p. 56.

F12—Frequency Multipliers

ELECTRONICS.

1. IMPROVEMENTS IN FREQUENCY MULTIPLIERS. By E. G. Watts. April 1932, p. 125.

F13—Frequency Range Necessary

BELL SYSTEM TECHNICAL JOURNAL.

1. TRANSMITTED FREQUENCY RANGE FOR CIRCUITS IN BROAD BAND SYSTEMS. By H. A. Affel. Vol. 16, 1937, p. 487.

F14—Fundamentals

I.R.E. PROCEEDINGS.

1. The Equivalent Generator Theorem. By V. D. Landon. Feb. 1930, p. 294.

BELL SYSTEM TECHNICAL JOURNAL.

2. Impedance Properties of Electron Streams. By L. C. Peterson. Vol. 18, 1939, p. 465.

ELECTRONICS.

3. The Plate Circuit Theorem. By W. Richter. March 1936, p. 19.
4. Time Delay in Resistance-Capacity Circuits. By E. W. Kellogg and W. D. Phelps. Feb. 1937, p. 23.
5. Time Delay in Resistance-Capacity Circuits. By E. W. Kellogg and W. D. Phelps. Feb. 1938, p. 27.
6. Electrical Concepts at Extremely High Frequencies. By S. Ramo. Sept. 1942, p. 34.

Other Reference.

7. Thevenin-Pleijel Theorem and its Application. By F. Herlitz. *A.S.E.A. Jl.*, April 1927, p. 53.

G1—Gamma

1. Gamma and Range in Television. By I. G. Maloff. *R.C.A. Review*, Vol. 3, No. 4, 1939, p. 409.

G2—Ganging and Tracking in Radio Receivers

E.W. AND WIRELESS ENGINEER.

1. Ganging the Tuning Controls of a Superheterodyne Receiver. By A. L. M. Sowerby. Feb. 1932, p. 70.
2. Ganging Superhet. By C. P. Singer. June 1936, p. 307.
3. Superheterodyne Padding Capacities. By W. T. Cocking. May 1937. p 246.
4. Ganging Superheterodyne Receivers. By M. Wald. March 1940, p. 105.
5. Ganging Superheterodyne Receivers. By M. Wald. April 1941, p. 146.
6. Superheterodyne Checking Charts. By A. L. Green and R. Payne-Scott. June 1942, p. 243; July 1942, p. 290.
7. Notes on Tracking Circuits. By A. Bloch. Nov. 1942, p. 508.

ELECTRONICS.

8. The Padding Condenser. By B. E. McNamee. May 1932, p. 161.
9. Superheterodyne Tracking. By V. D. Landon and E. A. Sveen. Aug. 1932, p. 251.
10. Beat Frequency Oscillator Control. (Shape of Condenser Vanes.) By G. F. Lampkin. Dec. 1932, p. 369.

11. A SUPERHETERODYNE TRACKING SOLUTION. By R. de Cola. Feb. 1942, p. 28.

Other Reference.

12. GANGING AND TRACKING. By R. Payne-Scott and A. L. Green. *A.W.A. Tech. Review*, Vol. 5, p. 77.

G3—Gas Discharge Potentiometers

MARCONI REVIEW.

1. THE MARCONI STABILOVOLT SYSTEM. No. 66, 1937, p. 31. (Reprint from a paper, by L. Koros and R. Seidelbach in *Helios*, Vol. 42, No. 34, Aug. 1936.)

Other Reference.

2. POTENTIAL DIVIDER OF GLOW TYPE. By L. Koros and R. Seidelbach. *Arch. Fur. Elektrot.*, Aug. 3, 1932, p. 539.

G4—Grid Controlled Rectifiers

ELECTRONICS.

1. GRID CONTROLLED GLOW AND ARC DISCHARGE TUBES. By D. B. Knowles and J. P. Sashoff. July 1930, p. 183.

H1—Harmonic Analysis

I.R.E. PROCEEDINGS.

1. A THERMIONIC VOLTMETER METHOD FOR THE HARMONIC ANALYSIS OF ELECTRICAL WAVES. By C. G. Suits. Jan. 1930, p. 178.
2. A SIMPLE METHOD OF HARMONIC ANALYSIS FOR USE IN ENGINEERING PRACTICE. By Hans Roder. Aug. 1931, p. 1481.
3. A NEW ELECTRICAL METHOD OF FREQUENCY ANALYSIS AND ITS APPLICATION TO FREQUENCY MODULATION. By W. L. Barrow. Oct. 1932, p. 1626.
4. A FOURIER ANALYSIS OF RADIO FREQUENCY POWER AMPLIFIER WAVE FORMS. By L. B. Hallman. Oct. 1932, p. 1640.

BELL SYSTEM TECHNICAL JOURNAL.

5. AN OPTICAL HARMONIC ANALYSER. By H. C. Montgomery. Vol. 17, 1938, p. 406.

E.W. AND WIRELESS ENGINEER.

6. HARMONICS. THEIR EFFECT ON WAVE FORM. By J. F. Herd. July 1926, p. 407.

ELECTRONICS.

7. GRAPHICAL HARMONIC ANALYSIS. By J. A. Hutcheson. Jan. 1936, p. 17.
8. COMBINATION TONES IN NON-LINEAR SYSTEMS. By F. Massa. Sept. 1938, p. 21.
9. ORDINATE SCHEDULES FOR HARMONIC ANALYSIS. By R. P. G. Denman. Sept. 1942, p. 44.

H2—Hearing

I.R.E. PROCEEDINGS.

1. Hearing the Determining Factor in High-Fidelity Transmission. By H. Fletcher. June 1942, p. 266.

BELL SYSTEM TECHNICAL JOURNAL.

2. Physical Measurements of Audition. By H. Fletcher. Vol. 2, 1923, p. 145.
3. Useful Numerical Constants. By H. Fletcher. July 1925.

ELECTRICAL COMMUNICATION.

4. The Physical Characteristics of Audition and Dynamical Analysis of the External Ear. By R. L. Wegel. Nov. 1922, Vol. 1, p. 43.

Other References.

5. The Relative Sensitivity of the Ear at Different Levels of Loudness. By D. Mackenzie. *Phys. Review*, Oct. 1922, p. 331.
6. Auditary Masking of One Pure Tone by Another. By R. L. Wegel and W. E. Lane. *Phys. Review*, Feb. 1924.

I1—Impedance Measurement

I.R.E. PROCEEDINGS.

1. Measurement of Radio Frequency Impedance with Networks Simulating Lines. By W. L. Barrow. July 1935, p. 807.
2. Methods of Measuring High Impedances. By D. B. Sinclair. Dec. 1938, p. 1466.

BELL SYSTEM TECHNICAL JOURNAL.

3. Classification of Bridge Methods of Measuring Impedances. By J. G. Ferguson. Vol. 12, 1933, p. 452.
4. Theorems Regarding the Driving Point Impedance of Circuits. By R. M. Foster. Vol. 3, 1924, p. 651.

I2—Insulators

I.E.E. JOURNAL.

1. The Measurement of the Permittivity and Power Factor of Dielectrics at Frequencies from 10^4 to 10^8 Cycles per Second. By L. Harshorn and W. H. Ward. Vol. 12, 1937, p. 6.

I.R.E. PROCEEDINGS.

2. Notes on the Design of Radio Insulators. By T. Walmsley. 1928, p. 361.
3. Properties and Application of Mycalex to Radio Apparatus. By W. W. Brown. Aug. 1930, p. 1307.
4. Properties of Mycalex. By W. W. Brown. Sept. 1933, p. 1339.

ELECTRONICS.

5. THE PROPERTIES OF CERAMIC MATERIALS. By H. Thurnauer. April 1939, p. 33.

Other References.

6. DIELECTRIC PROPERTIES OF MICA. *Phys. Soc. Proc.*, Dec. 1934, p. 42.
7. DIELECTRIC PROPERTIES OF WATER FOR CONTINUOUS WAVES. By G. C. Southworth. *Phys. Review*, Vol. 23, No. 5, p. 631.

I3—Interference and Noise

I.E.E. JOURNAL.

1. THE INTERFERENCE OF ELECTRICAL PLANT WITH THE RECEPTION OF RADIO BROADCASTING. By A. Morris. Vol. 9, 1934, p. 39.
2. THE SPONTANEOUS BACKGROUND NOISE IN AMPLIFIERS DUE TO THERMAL-AGITATION AND SHOT EFFECTS. By E. B. Moullin and H. D. N. Ellis. Vol. 9, 1934, p. 81.
3. INTERIM REPORT OF I.E.E. COMMITTEE ON ELECTRICAL INTERFERENCE WITH BROADCASTING. Vol. 10, 1935, p. 49.
4. FLUCTUATION NOISE IN VACUUM TUBES WHICH ARE NOT TEMPERATURE-LIMITED. By F. C. Williams. Vol. 11, 1936, p. 120.
5. ELECTRICAL INTERFERENCE WITH RADIO RECEPTION. By A. J. Gill and S. Whitehead. Vol. 13, 1938, p. 209.
6. THE MEASUREMENT OF INTERFERENCE AT ULTRA-HIGH FREQUENCIES. By L. H. Daniel and G. Noble. Vol. 38, 1941, p. 41.

I.R.E. PROCEEDINGS.

7. RADIO FREQUENCY INTERFERENCE. By J. G. Allen. May 1929, p. 882.
8. HUM IN ALL-ELECTRIC RADIO RECEIVERS. By B. F. Miessner. Jan. 1930, p. 137.
9. A STUDY OF NOISES IN VACUUM TUBES AND ATTACHED CIRCUITS. By F. B. Llewellyn. Feb. 1930, p. 243.
10. LOCATING RADIO INTERFERENCE WITH THE OSCILLOGRAPH. By J. K. McNeely and P. J. Kowke. July 1930, p. 1216.
11. A RAPID METHOD OF ESTIMATING THE SIGNAL-TO-NOISE RATIO OF A HIGH GAIN RECEIVER. By F. B. Llewellyn. March 1931, p. 416.
12. A SYSTEM FOR SUPPRESSING HUM BY A NEW FILTER ARRANGEMENT. By P. H. Craig. April 1931, pp. 864 and 908.
13. THEORY AND OPERATION OF TUNED RADIO FREQUENCY COUPLING SYSTEMS. By H. A. Wheeler and W. A. MacDonald. May 1931, p. 738.
14. HIGH FREQUENCY ATMOSPHERIC NOISE. By R. K. Potter. Oct. 1931, p. 1731.
15. METHODS OF MEASURING INTERFERING NOISE. By L. Espenschied. No. 1931, p. 1951.
16. AN ESTIMATE OF THE FREQUENCY DISTRIBUTION OF ATMOSPHERIC NOISE. By R. K. Potter. Sept. 1932, p. 1512.
17. SOME NOTES ON ADJACENT CHANNEL INTERFERENCE. By I. J. Kaar. March 1934, p. 295.

18. A Study of the Characteristics of Noise. By V. D. Landon. Nov. 1935, p. 1514.
19. Rain Static. By H. K. Morgan. July 1936, p. 959.
20. Minimum Noise Levels obtained on Short Wave Radio Receiving Systems. By Karl G. Jansky. Dec. 1937, p. 1517.
21. A New Antenna System for Noise Reduction. By V. D. Landon and J. D. Reid. Vol. 27, March 1939, p. 189.
22. Field-Strength of Motor Car Ignition between 40 and 540 Megacycles. By R. W. George. Vol. 28, Sept. 1940, p. 409.
23. Fluctuations Induced in Vacuum-Tube Grids at High Frequency. By D. O. North and W. R. Ferris. Feb. 1941, p. 49.
24. The Distribution of Amplitude with Time in Fluctuation Noise. By V. D. Landon. Feb. 1941, p. 50.
25. The Effect of Fluctuation Voltages on the Linear Detector. By J. R. Ragazzini. June 1942, p. 277.

BELL SYSTEM TECHNICAL JOURNAL.

26. Status of Joint Development and Research on Noise Frequency Induction. By H. L. Wills & O. B. Blackwell. Vol. 10, 1931, p. 184.
27. Some Theoretical and Practical Aspects of Noise Induction. By R. F. Davis and H. R. Huntley. Vol. 12, 1933, p. 469.
28. Fluctuation Noise in Vacuum Tubes. By G. L. Pearson. Vol. 13, 1934, p. 634.
29. Limits to Amplification (due to Circuit Noise). By J. B. Johnson and F. B. Llewellyn. Vol. 14, 1935, p. 85.
30. Some Aspects of Low Frequency Induction between Power and Telephone Circuits. By H. R. Huntley and E. J. O'Connell. Vol. 14, 1935, p. 573.
31. Magnetic Shielding of Transformers at Audio Frequencies. By W. G. Gustafson. Vol. 17, 1938, p. 416.
32. Devices for Controlling Amplitude Characteristics of Telephone Signals. By A. C. Norwine. Vol. 17, 1938, p. 539.

E.W. AND WIRELESS ENGINEER.

33. Whistling Notes in Superheterodyne Receivers. By M. J. O. Strutt. April 1935, p. 194.
34. Noise Suppression in the Receiver. By W. N. Weeden. July 1936, p. 365.
35. Noise Reduction by Means of Photo-Electric Multipliers. By F. Preisach. April 1939, p. 169.
36. Noise Suppression by Means of Amplitude Limiters. By M. Wald. Oct. 1940, p. 432.

ELECTRONICS.

37. Suppressor Resistors and Their Effects on Automobile Operation. Anonymous. March 1934, p. 71.
38. A Shielded Loop for Noise Reduction. By S. Goldman. Oct. 1938, p. 21.
39. High Frequency Pre-Emphasis. By J. L. Hathaway. Nov. 1939, p. 29.

MARCONI REVIEW.

40. The Microvolt and Receiver Noise Level. By S. B. Smith. No. 60, 1936, p. 26.

ELECTRICAL COMMUNICATION.

41. Capacity Unbalance in Telephone Cables and its Effect on Noise Due to External Induction. By J. Collard. Vol. 11, 1932, p. 59.
42. Noise Limits in Long International Circuits. By J. Collard. Vol. 13, 1934, p. 125.
43. Multiple Earthing and its Effect on Induced Noise. By J. Collard. Vol. 13, 1934, p. 130.
44. Cross Talk Between Coaxial Transmission Lines. By S. A. Schelkunoff and T. M. Odarenko. Vol. 16, 1937, p. 39.

Other Reference.

45. Noise Measurements (Levels of Typical Sounds). By J. C. Steinberg. *Elec. Eng.*, Jan. 31, 1935, p. 43.

I4—Ionosphere

I.E.E. JOURNAL.

1. Wireless Studies of the Ionosphere. By E. V. Appleton. Vol. 7, 1932, p. 257.
2. Measurement of the Angle of Incidence at the Ground of Downcoming Short Waves from the Ionosphere. Vol. 9, 1934, p. 154.
3. The Emission of Special Radio Signals for the Study of the Ionosphere. By H. A. Thomas. Vol. 9, 1934, p. 310.

I.R.E. PROCEEDINGS.

4. A New Method of Determining the Height of the Kennelly-Heaviside Layer. By C. B. Mirick and E. R. Hentschel. June 1923, p. 1034.
5. Experiments and Observations Concerning the Ionized Regions of the Atmosphere. By R. A. Heising. Jan. 1928, p. 75.
6. Measurements of the Effective Heights of the Conducting Layer and the Disturbances of. By O. Dahl and L. A. Gebhardt. Aug. 1927, p. 290.
7. Determination of the Ionization in the Upper Atmosphere. By J. G. Schelling. Nov. 1928, p. 1471.
8. Measurements of the Height of the Kennelly-Heaviside Layer. By G. W. Kenrick and C. K. Jen. April 1929, p. 711.
9. Further Studies of the Kennelly-Heaviside Layer by the Echo Method. By I. R. Hafstad and M. A. Tuve. Sept. 1929, p. 1519.
10. Further Observations of Radio Transmission and the Heights of the Kennelly-Heaviside Layer. By G. W. Kenrich and C. K. Jen. Nov. 1929, p. 2034.
11. Wireless Telegraphy and the Ionization in the Upper Atmosphere. By E. O. Hulbert. July 1930, p. 1231.
12. Kennelly-Heaviside Layer Studies. By P. A. Mars and Others. Jan. 1931, p. 106.

13. Bibliography on Propagation up to 1930 including Ionosphere is given in *I.R.E.*, June 1931, p. 1034.
14. Kennelly-Heaviside Layer Studies Employing a Rapid Method of Virtual Height Determination. By J. P. Schafer and W. M. Goodall. July 1932, p. 1131.
15. General Theory on the Propagation of Radio Waves in the Ionosphere. By S. Namba. Feb. 1933, p. 238.
16. Continuous Measurements of the Virtual Heights of the Ionosphere. By T. R. Gilliland. Oct. 1933, p. 1463.
17. Studies of the Ionosphere and Their Application to Radio Transmission. By S. S. Kirby and Others. April 1934, p. 481.
18. Continuous Recording of Retardation and Intensity of Echoes from the Ionosphere. By L. C. Verman and Others. July 1934, p. 906.
19. Report of Ionosphere Investigations at the Huancayo Magnetic Observatory (Peru) during 1933. Sept. 1934, p. 1102.
20. Ionosphere Measurements in Polar Regions. By M. A. B. Bruewitch. Sept. 1934, p. 1124.
21. Diurnal and Seasonal Variations in the Ionosphere during the Years 1933 and 1934. By J. P. Schafer and W. M. Goodall. June 1935, p. 670.
22. Recent Studies of the Ionosphere. By S. S. Kirby and E. B. Judson. July 1935, p. 733.
23. Multifrequency Ionosphere Recording and its Significance. By T. R. Gilliland. Sept. 1935, p. 1076.
24. Characteristics of the Ionosphere and their Application to Radio Transmission. By T. R. Gilliland, S. S. Kirby and S. E. Reymer. July 1937, p. 823.
25. Sudden Disturbances of the Ionosphere. By J. H. Dellinger. Oct. 1937, p. 1253.
26. The Ionosphere, Skip Distances of Radio Waves, and the Propagation of Micro-Waves. By E. O. Hulbert. Dec. 1935, p. 1492.
27. Characteristics of the Ionosphere at Washington. By T. R. Gilliland and Others. Sept. 1937, pp. 1174, 1185 and 1188.
28. Measuring the Reflecting Regions in the Troposphere. By A. W. Friend and R. C. Colwell. Dec. 1937, p. 1531.
29. Standards on Electronics, 1938. Issued by *I.R.E.*
30. Abnormal Ionization in the E Region of the Ionosphere. By J. A. Pierce. July 1938, p. 892.
31. Maximum Useable Frequencies for Radio Sky-Wave Transmission, 1933–1937 (Gives Application to Simple and Complex Transmission Paths). By T. R. Gilliland and Others. Nov. 1938, p. 1347.
32. Non-Existence of Continuous Intense Ionization in the Troposphere and Lower Stratosphere. By O. H. Gish and H. G. Booker. Feb. 1939, p. 117.
33. The Relation of Radio Sky-Wave Transmission to Ionosphere Measurements. By Newburn Smith. May 1929, p. 332.
34. The Heights of the Reflecting Regions in the Troposphere. By A. W. Friend and R. C. Colwell. Oct. 1939, p. 626.
35. The Troposphere and Radio Waves. By R. C. Colwell. July 1940, p. 299.

BELL SYSTEM TECHNICAL JOURNAL.

36. Analysis of the Ionosphere. By K. K. Darrow. Vol. 19, July 1940, pp. 455–488.

MARCONI REVIEW.

37. The Ionosphere. By K. W. Tremellen. No. 72, 1939, p. 1.

Other References.

38. Some Notes on Wireless Methods of Investigating the Electrical Structure of the Upper Atmosphere. By E. V. Appleton. *Phys. Soc. Proc.*, June 1930, p. 321.
39. The Relation between Ionospheric Transmission Phenomena at Oblique Incidence and those at Vertical Incidence. By G. Millington. *Phys. Soc. Proc.*, Sept. 1938, p. 801.
40. Regularities and Irregularities in the Ionosphere. By E. V. Appleton. *Roy. Soc. Proc.*, Oct. 1937, p. 451.

J1—Johnson Noise : *see* Thermal Agitation

K1—Klystron

1. Klystron Oscillations. By D. L. Webster. *Jl. of Applied Physics*, Dec. 1939, p. 864.

L1—Level Measurement

ELECTRONICS.

1. " A New Volume Indicator and Reference Level." (Describes American Volume Unit, which is Directly Equivalent to B.B.C. Level in db. above One Milliwatt.) By H. A. Affel, H. A. Chinn and R. M. Morris. Feb. 1939, p. 28.

L2—Lines

BELL SYSTEM TECHNICAL JOURNAL.

1. Wide Band Open Wire Programme System. By H. S. Hamilton. Vol. 13, 1934, p. 351.

ELECTRICAL COMMUNICATION.

2. Irregularities in Loaded Telephone Circuits. By G. Grisson. Vol. 4, 1925, p. 98.
3. Methods of Locating Cross-Talk Faults on Loaded Cables. By K. E. Lattimer. Vol. 8, 1929, p. 45.
4. The Standard Carrier Broadcast System. By K. G. Hodgson, F. Ralph and B. B. Jacobsen. Vol. 13, 1934, p. 197.

Other Reference.

5. Transmission Characteristics of Open Wire Lines. By I. Green. *Amer. I.E.E. Jl.*, July 1930, p. 566.

L3—Loudness

BELL SYSTEM TECHNICAL JOURNAL.

1. Loudness: Its Definition, Measurement and Calculation. By H. Fletcher and W. A. Munson. Vol. 12, 1933, p. 377.

Other References.

2. The Direct Comparison of the Loudness of Pure Tones. By B. A. Kingsbury. *Phys. Review*, Feb. 1927, p. 373, and April 1927, p. 588.
3. The Dependence of the Loudness of a Complex Sound upon the Energy in the Various Frequency Regions of the Sound. By H. Fletcher and J. C. Steinberg. *Phys. Review*, Sept. 1929.

L4—Luxembourg Effect

I.R.E. PROCEEDINGS.

1. The Generation of Spurious Signals by Non-Linearity of the Transmission Path. By A. V. Eastman and L. C. Horle. Oct. 1940, p. 439.
2. . . . Spurious Radiation in the Field of Two Strong Signals. By A. J. Ebel. Feb. 1942, p. 81.

E.W. AND WIRELESS ENGINEER.

3. Measurements of the Luxembourg Effect. (Editorial.) April 1938, p. 187.

M1—Magnetic Materials

BELL SYSTEM TECHNICAL JOURNAL.

1. Permalloy, a New Magnetic Material of Very High Permeability. By H. D. Arnold and G. W. Elmen. Vol. 2, 1923, p. 101.
2. Harmonic Production in Ferromagnetic Materials at Low Frequencies and Low Flux Densities. By E. Peterson. Vol. 7, 1928, p. 762.
3. Magnetic Alloys of Iron, Nickel and Cobalt. By G. W. Elmen. Vol. 8, 1929, p. 435.
4. Magnetic Alloys of Iron, Nickel and Cobalt. By G. W. Elmen. Vol. 15, 1936, p. 113.
5. Survey of Magnetic Materials in the Telephone System. By V. E. Legg. Vol. 18, 1939, p. 438.

E.W. AND WIRELESS ENGINEER.

6. Permeability at Very High Frequencies. Editorial by G. W. O. Howe. Nov. 1939, p. 541.
7. Permanent Magnets. Editorial by G. W. O. Howe. July 1942, p. 287.

ELECTRONICS.

8. The New Permanent Magnet Alloys. May 1936, p. 30.

ELECTRICAL COMMUNICATION.

9. PERMALLOY. By G. W. Elmen and H. D. Arnold. Vol. 11, 1923, p. 226.
10. PERMALLOYS AND RELATED FERROMAGNETIC ALLOYS—A REVIEW. By J. C. Chaston. Vol. 15, 1936, p. 38.

Other References.

11. HYPERNIK. By D. Jensen. *Electrical Jl.*, June 1931, p. 386.
12. MATERIALS USED IN RADIO MANUFACTURE. By V. O. Stokes. *Marconi Review*, No. 59, 1936, p. 7.

M2—Magnetism

I.E.E. JOURNAL.

1. DESIGN OF PERMANENT MAGNETS. By S. Evershed. Sept. 1920, p. 780, and Aug. 1925, p. 725.

Other References.

2. MAGNETIC MEASUREMENTS. By Spooner. *Amer. I.E.E.J.*, Jan. 1923, also *Jap. I.E.E.J.*, Oct. 1924, and *Electrical Jl.*, Feb. and March 1925.
3. A series of fundamental and extremely valuable articles by T. Spooner, covering the whole range of Magnet Design appeared in the issues of *The Electrical Jl.* from Feb. 1925 to March 1926 inclusive.
4. METHOD OF STUDY, LAWS OF AGEING AND MAGNETIC REJUVENATION OF IRONS EMPLOYED IN TELEPHONY. *Tech. Mitt.*, Dec. 1, 1925.
5. LAWS OF MAGNETIZATION. By Gokhale. *Amer. I.E.E. Jl.*, Sept. 1926.
6. THE MAGNETIC STABILITY OF PERMANENT MAGNETS. By R. C. Gray. *Phyl. Mag.*, Sept. 1926, p. 521.
7. MAGNETIC DEFINITIONS AND UNITS. V.D.E. Committee. *Electrician*, July 11, 1930, p. 57.

M3—Magnetrons

I.E.E. JOURNAL.

1. AN INVESTIGATION OF THE MAGNETRON SHORT WAVE OSCILLATOR. By E. C. S. Megaw. Vol. 8, 1933, p. 72.
2. AN EXPERIMENTAL INVESTIGATION OF RESONANCE AND ELECTRONIC OSCILLATIONS IN MAGNETRONS. By J. S. McPetrie. Vol. 15, 1940, p. 27.
3. A MAGNETRON OSCILLATOR WITH A COMPOUND FIELD WINDING. By L. H. Ford. Vol. 15, 1940, p. 37.
4. THE IMPEDANCE OF THE MAGNETRON IN DIFFERENT REGIONS OF THE FREQUENCY SPECTRUM. By A. F. Harvey. Vol. 15, 1940, p. 41.

I.R.E. PROCEEDINGS.

5. ON THE SHORT WAVE LIMIT OF MAGNETRON OSCILLATIONS. By K. Okabe. April 1929, p. 652.

E.W. AND WIRELESS ENGINEER.

6. MAGNETRONS. (Editorial.) July 1936, p. 347.
7. MAGNETRON OSCILLATORS. By G. W. O. Howe. Nov. 1936, p. 573.
8. THE MAGNETRON AND THE GENERATION OF ULTRA SHORT WAVES. Editorial. Jan. 1938, p. 1.

ELECTRONICS.

9. Producing Very High Frequencies by Means of Magnetrons. By W. C. White. April 1930, p. 34.

M4—Magnets : *see* Magnetism

M5—Mathematics

I.R.E. PROCEEDINGS.

1. The Response of Electrical Networks to Non-Sinusoidal Periodic Waves. By N. Marchand. June 1941, p. 330.
2. (New Type Fourier Analysis.) By R. V. L. Hartley. March 1942, p. 144.

E.W. AND WIRELESS ENGINEER.

3. Mathematics for Wireless Amateurs. By F. M. Colebrook.

May 1926, p. 283	Dec. 1926, p. 756	June 1927, p. 349
June ,, p. 373	Jan. 1927, p. 24	July ,, p. 412
July ,, p. 446	Feb. ,, p. 81	Sept. ,, p. 559
Aug. ,, p. 493	March ,, p. 169	Oct. ,, p. 605
Sept. ,, p. 559	April ,, p. 223	Nov. ,, p. 684
Oct. ,, p. 613	May ,, p. 279	Dec. ,, p. 746
Nov. ,, p. 674		

M6—Measurements

I.R.E. PROCEEDINGS.

1. A Condenser Bridge for Factory Inspection of Variable Condensers. By R. A. Braden and H. C. Forbes. Jan. 1930, p. 123.
2. The Campbell-Shackelton Shielded Ratio Box. By L. Behr and A. J. Williams. June 1932, p. 969.
3. Design of Resistors for Precise High Frequency Measurements. By L. Behr and R. E. Tarpley. July 1932, p. 1101.
4. The Determination of Dielectric Properties at Very High Frequencies. By J. G. Chaffee. Aug. 1934, p. 1009.
5. A 60-Cycle Bridge for the Study of Radio Frequency Power Amplifiers (Simulating Operating Conditions on Power). By A. Noyes. July 1935, p. 785.
6. Radio Frequency Measurements with the Quadrant Electrometer. By C. I. Bradford. Aug. 1935, p. 958.
7. Electrical Measurements at Wavelengths less than Two Metres. By L. S. Nergaard. Sept. 1935, p. 1207.
8. Direct Measurement of the Loss and Conductance of Condensers at High Frequencies. By M. Buella. April 1938, p. 421.
9. The Twin T. (R.F. Bridge up to 30 Mc/s.) By D. B. Sinclair. Vol. 28, July 1940, p. 310.
10. Bridged T and Parallel T Null Circuits for Measurement at Radio Frequencies. By W. N. Tuttle. Vol. 28, Jan. 1940, p. 23.
11. Bridged T Measurement of High Resistances at Radio Frequencies. By P. M. Hunnell. Vol. 28, Feb. 1940, p. 88.

12. A Direct Current and Audio Frequency Amplifier. By L. J. Black and H. J. Scott. Vol. 28, June 1940, p. 269.
13. A Radio Frequency Bridge for Impedance Measurements from 400 Kc/s to 60 Mc/s. By D. B. Sinclair. Vol. 28, Nov. 1940, p. 497.
14. Voltage Measurements at Very High Frequencies. By E. C. S. Feb., p. 65, and March, p. 135, 1936.

BELL SYSTEM TECHNICAL JOURNAL.

15. A Shielded Bridge for Inductive Impedance Measurements at Speech and Carrier Frequencies. By W. J. Shackleton. Vol. 6, 1927, p. 142.
16. An Analyser for the Voice Frequency Range. By C. R. Moore and A. S. Curtis. Vol. 6, 1927, p. 217.
17. Analyser for Complex Electric Waves. By A. G. Landken. Vol. 6, 1927, p. 230.
18. Measurement of Inductance by the Shielded Owen Bridge. By J. G. Ferguson. Vol. 6, 1927, p. 375.
19. Electrical Measurement of Communication Apparatus. By W. J. Shackleton and J. G. Ferguson. Vol. 7, 1928, p. 70.
20. Bridge for Measuring Small Time Intervals. By J. Herman. Vol. 7, 1928, p. 343.
21. The Measurement of Capacitance in Terms of Resistance and Frequency. By J. G. Ferguson. Vol. 7, 1928, p. 420.
22. Bridge Methods for Locating Resistance Faults on Cable Wires. By T. C. Henneberger and G. Edwards. Vol. 10, 1931, p. 382.
23. New Transmission Measuring Systems. By F. H. Best. Vol. 17, 1938, p. 1.

E.W. AND WIRELESS ENGINEER.

24. Errors in Measurements. By P. K. Turner. March 1925, p. 360; May 1925, p. 512, and June 1925, p. 547.
25. Thermo-Junctions at High Radio Frequencies. By F. M. Colebrook. July 1931, p. 356.

ELECTRONICS.

26. Precision Condenser Calibration at Radio Frequencies. By E. L. Hall and W. D. George. Oct. 1934, p. 319.
27. Automatic Line Measuring Equipment of the German Broadcast System. By R. W. P. Leonhardt. Nov. 1934, p. 347.
28. Measurement of Minute Changes of Capacitance and Inductance. By S. C. Leonard. March 1938, p. 19.

MARCONI REVIEW.

29. Cathode Ray Oscillograph Ellipses. By G. Millington. No. 62, 1936, p. 8.

ELECTRICAL COMMUNICATION.

30. The Measurement of Small Values of Inductance and Effective Resistance. By J. K. Webb and C. Brookes-Smith. Vol. 13, 1934, p. 9.

Other References.

31. (Lamps as High Power Loads.) By E. G. Linder. *R.C.A. Review*, Vol. 4, No. 1, 1939, p. 83.
32. On the Avoidance of Errors due to the " Leads " in Radio Frequency Measurements. By R. M. Davies. *Phil. Mag.*, July 1935, p. 75.
33. Radio Instruments and Measurements. Circular No. 74 of American Bur. of Standards.

M7—Measurements of Ground Constants

I.E.E. JOURNAL.

1. Some Measurements of the Electrical Constants of the Ground at Short Wavelengths by the Wave-Tilt Method. By R. H. Barfield. Vol. 9, 1934, p. 286.
2. Electrical Measurements on Soil with Alternating Currents. By R. S. Smith-Rose. Vol. 9, 1934, p. 293.

M8—Measuring Apparatus

I.R.E. PROCEEDINGS.

1. The Design of Radio Frequency Signal Generators. By J. R. Bird. March 1931, p. 438.
2. Measurement of Resistance and Impedance at High Frequencies. By J. W. Labus. March 1931, p. 452.

MARCONI REVIEW.

3. Marconi Ultra Short Wave Standard Signal Generator. By F. M. Wright. No. 61, 1936, p. 30.
4. A High Frequency Resistance Bridge. By C. Austin and A. L. Oliver. No. 63, 1936, p. 22.

M9—Mercury Arc Rectifiers

I.R.E. PROCEEDINGS.

1. Hot Cathode Mercury-Vapor Rectifier Tubes. By H. C. Steiner and H. T. Maser. Jan. 1930, p. 67.

M10—Miscellaneous

I.E.E. JOURNAL.

1. The Design and Distribution of Wireless Broadcasting Stations for a National Service. By P. P. Eckersley. Vol. 3, 1928, p. 108.

I.R.E. PROCEEDINGS.

2. Sinusoidal Variation of a Parameter in a Simple Series Circuit. By F. J. Maginnis. Jan. 1941, p. 25.

ELECTRICAL COMMUNICATION.

3. Production and Utilization of Micro-Rays. By A. G. Clavier. Vol. 12, 1933, p. 3.

M11—Mixers

I.R.E. PROCEEDINGS.

1. On the Theory of Tubes with Two Control Grids. By A. H. Wing, March 1941, p. 121.
2. . . . Mixers for Superhet Reception. By E. W. Herold. Feb. 1942, p. 84.

E.W. AND WIRELESS ENGINEER.

3. Heptode Frequency Changers. By R. J. Wey. Dec. 1934, p. 642.

M12—M.K.S. Units

E.W. AND WIRELESS ENGINEER.

1. Nov. 1935, p. 600.

Other References.

2. *Elect. Eng.*, Dec. 1935, p. 1373 (contains Bibliography).
   Proc. Nat. Acad Sci., p. 579, October 1935.

M13—Modulated Amplifiers

I.R.E. PROCEEDINGS.

1. Linear Plate Modulation of Triode Radio Frequency Amplifiers. By Chao-Ying Meng. Vol. 28, Dec. 1940, p. 563.

M14—Modulation

MARCONI REVIEW.

1. Errors in Heising Method of Measuring Depth of Modulation. By E. Green. No. 58, 1936, p. 15.

M15—Multivibrators

ELECTRICAL COMMUNICATION.

1. A New Hard Valve Relaxation Oscillator. By D. H. Black. Vol. 18, 1939, p. 50.

M16—Musa

I.R.E. PROCEEDINGS.

1. A Multiple Unit Steerable Antenna for Short Wave Reception. By H. T. Friis and C. B. Feldman. July 1937, p. 841.
2. A Single Sideband Musa Receiving System for Commercial Operation on Transatlantic Radio Telephone Circuits. By F. A. Polkinghorn. Vol. 28, April 1940, p. 157.

BELL SYSTEM TECHNICAL JOURNAL.

3. Experiments with Directivity Steering for Fading Reduction. By E. Bruce and A. C. Beck. Vol. 14, 1935, p. 195.

4. A Multiple Unit Steerable Antenna for Short Wave Reception. By H. T. Friis and C. B. Feldman. Vol. 16, 1937, p. 337.

Other References.

5. Principles of the Musa. By C. B. Feldman. *Bell Lab. Record,* Jan. 1938, p. 148.
6. Musa Apparatus. By W. M. Sharpless. *Bell Lab. Record,* Jan. 1938, p. 153.
7. The Musa from Outside. By L. R. Lowry. *Bell Lab. Record.* Feb. 1938, p. 203.

M17—Mutual Impedance

BELL SYSTEM TECHNICAL JOURNAL.

1. Mutual Impedance of Parallel Wires. By R. S. Hoyt, S. P. Hoyt and S. P. Mead. Vol. 14, 1935, p. 509.

M18—Mutual Inductance

I.R.E. PROCEEDINGS.

1. Tables for the Calculation of the Mutual Inductance of any Two Coaxial Single Layer Coils. By F. W. Grower. July 1933, p. 1039.

N1—Negative Impedance

1. Negative Lengths of Telephone Line. By A. C. Bartlett. *Phil. Mag.,* Feb. 1934, p. 230.
2. U.S. Patents Nos. 1,779,382; 1,779,126; 1,779,380; 1,836,809; 1,836,844; 1,776,310.

N2—Negative Resistance

I.R.E. PROCEEDINGS.

1. Negative Resistance and Devices for Obtaining it, with Bibliography. By E. W. Herold. Oct. 1935, p. 1201.

BELL SYSTEM TECHNICAL JOURNAL.

2. Negative Impedances and the Twin 21-Type Repeater. By G. Grisson. Vol. 10, 1931, p. 485.

N3—Networks

I.R.E. PROCEEDINGS.

1. Analysis of A.C. Networks (Circle Diagrams). By K. Spangenberg. April 1936, p. 657.
2. Lattice Attenuating Networks. By G. C. Omer. May 1937, p. 620.

BELL SYSTEM TECHNICAL JOURNAL.

3. Constant Resistance Networks with Application to Filter Groups. By E. L. Norton. Vol. 16, 1937, p. 178.

E.W. AND WIRELESS ENGINEER.

4. GENERALIZED CHARACTERISTICS OF LINEAR NETWORKS. By E. K. Sandeman. Dec. 1936, p. 637.

ELECTRONICS.

5. L TYPE IMPEDANCE TRANSFORMING CIRCUITS. By P. H. Smith. March 1942, p. 48.

MARCONI REVIEW.

6. DISSYMMETRICAL T PURE REACTANCE NETWORKS. By H. Cafferata. No. 73, 1939, p. 27; No. 74, 1939, p. 24.

Other References.

7. GENERAL THEORY OF FOURPOLES. By A. Feige. *T.F.T.*, 1927, Sept., p. 247, and Dec., p. 253.
8. CONSTANTS OF PASSIVE FOURPOLES. *Rev. Gen. D'Electrique*, Oct. 1927, p. 493.

N4—Neutrodyning: *see* Valve Input Admittance

N5—Noise and Interference: *see* Interference and Noise

N6—Non-Linear Distortion

I.R.E. PROCEEDINGS.

1. THE CALCULATION OF HARMONIC PRODUCTION IN THERMIONIC VALVES WITH RESISTIVE LOADS. By D. C. Epsley. Oct. 1933, p. 1439.
2. GRAPHICAL ANALYSIS FOR DETERMINING MODULATION DISTORTION IN AMPLIFIER TUBES. By W. R. Ferris. May 1935, p. 150.

BELL SYSTEM TECHNICAL JOURNAL.

3. MODULATION IN VACUUM TUBES USED AS AMPLIFIERS. By E. Peterson and H. P. Evans. Vol. 6, 1927, p. 442.

O1—Oscillators

I.R.E. PROCEEDINGS.

1. MAGNETOSTRICTION OSCILLATORS. By G. W. Pierce. Jan. 1929, p. 42.
2. THE PIEZO-ELECTRIC OSCILLATOR. By J. W. Wright. Jan. 1929, p. 127.
3. ON THE MECHANISMS OF ELECTRON OSCILLATIONS IN A TRIODE. (DESCRIBES DIFFERENT TYPES OF OSCILLATION.) By H. E. Hollman. Feb. 1929, p. 229.
4. THE VAN DER POL FORM OF ELECTRODE TUBE RELAXATION OSCILLATION CIRCUIT. By R. M. Page and W. F. Curtis. Nov. 1930, p. 1921.
5. THE NON-LINEAR THEORY OF ELECTRIC OSCILLATIONS (WITH BIBLIOGRAPHY). By B. Van der Pol. Sept. 1934, p. 1051.
6. THE BRIDGE-STABILIZED OSCILLATOR. By L. A. Meacham. Oct. 1938, p. 1278.
7. PHASE SHIFT OSCILLATORS. By E. L. Ginzton and L. M. Hollingsworth. Feb. 1941, p. 43.
8. REFLEX OSCILLATORS. By J. R. Price. Feb. 1945, p. 112.

BELL SYSTEM TECHNICAL JOURNAL.

9. CONSTANT FREQUENCY OSCILLATORS. By F. B. Llewellyn. Vol. 11, 1932, p. 67.
10. SOME IMPROVEMENTS IN QUARTZ CRYSTAL CIRCUIT ELEMENTS. By F. R. Lack, G. W. Willard and I. E. Fain. *E.W. and Wireless Engineer*, Vol. 13, 1934, p. 453.
11. THE NEON LAMP AS AN OSCILLATION GENERATOR. By H. St. G. Anson. May 1924, p. 467.

ELECTRONICS.

12. RESISTANCE STABILIZED OSCILLATORS. By F. E. Terman. July 1933, p. 191.
13. A SINGLE TUBE BEAT FREQUENCY OSCILLATOR. By L. Podolsky and E. M. Bride. Nov. 1934, p. 357.
14. THE CONTROLLED TRANSITRON OSCILLATOR. By S. R. Jordan. July 1942, p. 42.

P1—Parasitic Oscillations

I.E.E. JOURNAL.

1. THE BALANCING AND STABILIZING OF HIGH FREQUENCY AMPLIFIERS WITH SPECIAL REFERENCE TO POWER AMPLIFIERS FOR RADIO TRANSMITTERS. By W. Ure, E. J. Grainger and H. R. Cantelo. Vol. 8, 1933, p. 108.
2. SOME NOTES ON THE STABILIZING OF HIGH FREQUENCY POWER AMPLIFIERS. By J. Greig. Vol. 10, 1935, p. 103.

I.R.E. PROCEEDINGS.

3. PARASITES AND INSTABILITY IN RADIO TRANSMITTERS. (*See* Discussion in February Issue, 1936, p. 328.) By G. W. Fyler. Sept. 1935, p. 985.

E.W. AND WIRELESS ENGINEER.

4. . . . STABILITY . . . OF AMPLIFIERS. By O. F. Brown. April 1925, p. 434.
5. THE SUPPRESSION OF PARASITIC OSCILLATIONS. By M. Reed. Dec. 1927, p. 725.
6. INTERACTION IN AMPLIFIERS. By L. Bainbridge-Bell. Jan. 1931, p. 18.

Other Reference.

7. PARASITIC OSCILLATIONS IN BROADCAST TRANSMITTERS. By A. D. Ring. *Radio Eng:*, June 1932, p. 17.

P2—Peak Voltmeters

I.E.E. JOURNAL.

1. THERMIONIC PEAK VOLTMETERS FOR USE AT VERY HIGH FREQUENCIES. By C. L. Fortescue. Vol. 10, 1935, p. 262.

P3—Permeability: *see* Magnetic Materials

P4—Phase Distortion

I.R.E. PROCEEDINGS.

1. PHASE SHIFT IN RADIO TRANSMITTERS. By W. A. Fitch. May 1932, p. 863.
2. DISSIPATION IN PHASE-COMPENSATING NETWORKS. By A. T. Starr. Sept. 1935, p. 1103.

BELL SYSTEM TECHNICAL JOURNAL.

3. PHASE DISTORTION AND PHASE DISTORTION CORRECTION. By S. P. Mead. Vol. 7, 1928, p. 195.
4. PHASE DISTORTION IN TELEPHONE APPARATUS. By C. E. Lane. Vol. 9, 1930, p. 493.
5. MEASUREMENT OF PHASE DISTORTION. By H. Nyquist and S. Brand. Vol. 9, 1930, p. 522.
6. EFFECTS OF PHASE DISTORTION ON TELEPHONE QUALITY. By J. C. Steinberg. Vol. 9, 1930, p. 550.

E.W. AND WIRELESS ENGINEER.

7. SIDEBAND PHASE DISTORTION. By D. M. Johnstone and E. E. Wright. (*See also Editorial*, G. W. O. H., for Oct.) Oct. 1936, p. 534.
8. PHASE ADJUSTER. By K. Kreielsheimer. Oct. 1940, p. 439.

ELECTRICAL COMMUNICATION.

9. PHASE COMPENSATION (EXPLANATION AND DESIGN OF NETWORKS AND MEASURING EQUIPMENT). By I. L. Turnbull, A. R. A. Rendall and E. K. Sandeman. Vol. 7, 1928, p. 309.
10. THE DETERMINATION OF THE DESIRABLE ATTENUATION FREQUENCY CHARACTERISTIC OF A LONG TOLL CABLE CIRCUIT. By A. R. A. Rendall. Vol. 8, 1929, p. 320.
11. METHODS AND APPARATUS FOR MEASURING PHASE DISTORTION. By M. Levy. Vol. 18, 1939, p. 206.

Other References.

12. THE PERCEPTIBILITY OF BUILDING-UP TIMES. By F. Strecker. *T.F.T.*, Jan. 1925, p. 1.
13. THE INCREASE OF THE RANGE OF LOADED CIRCUITS BY ECHO SUPPRESSION AND PHASE BALANCING. By K. Küpfmüller. *E.N.T.*, Vol. 3, No. 3, 1926.
14. EQUALIZATION AND PHASE COMPENSATION. By H. Decker. *E.N.T.*, April 1928, p. 163.
15. SUR LES PHÉNOMÉNES TRANSITOIRES DANS LES LIGNES PUPINISÉES ET LE MOYEN D'Y REMEDIER. By K. Küpmüller and H. F. Mayer. *Jl. Télégraphique*, June 1928, pp. 93 and 113.
16. RELATION BETWEEN FREQUENCY CHARACTERISTICS AND TRANSIENT PHENOMENA IN LINEAR SYSTEMS. By K. Küpfmüller. *Annales de Poste*, Sept. 1929, p. 836.

P5—Phase Modulation

I.R.E. PROCEEDINGS.

1. COMMUNICATION BY PHASE MODULATION. By M. G. Crosby. Vol. 27, Feb. 1939, p. 126.
2. AMPLITUDE, PHASE AND FREQUENCY MODULATION. By Hans Roder. Dec. 1931, p. 2145.

P6—Piezo Electricity

I.R.E. PROCEEDINGS.

1. BIBLIOGRAPHY ON PIEZO-ELECTRICITY. By W. G. Cady. 1928, p. 521.
2. PIEZO-ELECTRIC CRYSTAL CONFERENCE. By W. G. Cady. Dec. 1930, p. 2128.
3. PIEZO-ELECTRIC TERMINOLOGY. By W. G. Cady. Dec. 1930, p. 2136.

P7—Plastics

I.R.E. PROCEEDINGS.

1. A REVIEW OF PLASTICS. By H. L. Brouse. Dec. 1945, p. 825.

P8—Polarization

I.R.E. PROCEEDINGS.

1. THE LIMITING POLARIZATION OF DOWNCOMING RADIO WAVES TRAVELLING OBLIQUELY TO THE EARTH'S MAGNETIC FIELD. By W. A. Baker and A. L. Green. Aug. 1933, p. 1003.
2. THE POLARIZATION OF SKY WAVES IN THE SOUTHERN HEMISPHERE. By A. L. Green. March 1934, p. 324.

ELECTRONICS.

3. VERTICAL VERSUS HORIZONTAL POLARIZATION. By G. H. Brown. Oct. 1940, p. 20.

P9—Polyphase Broadcasting

I.R.E. PROCEEDINGS.

1. EXPERIMENTAL POLYPHASE BROADCASTING. By P. Loyet. May 1942, p. 213.

P10—Preferred Numbers

I.R.E. PROCEEDINGS.

1. PREFERRED NUMBERS. By A. Van Dyck. Feb. 1936, p. 159.

P11—Propagation

I.R.E. PROCEEDINGS.

1. SUMMARY OF PROGRESS IN THE STUDY OF RADIO WAVES PROPAGATION PHENOMENA. By G. W. Kenrick and G. W. Pickard. March 1930, p. 649.

2. Report of Commission II Radio Wave Propagation. By J. H. Dellinger. Oct. 1939, p. 645.

3. A Report of Committee on Radio Propagation Data. Oct. 1933, p. 1419.

4. Report of Committee on Radio Wave Propagation. Oct. 1938, p. 1193.

5. Bibliography on Propagation up to 1930 is given in *I.R.E.* June 1931, p. 34.

6. Graphs to Prof. Sommerveld's Attenuation Formula for Radio Waves. (*See* A. Sommerveld, "Ueber die Ausbreitung der Wellen in der Drahtlosen Telegraphie", *Annalen der Physik*, 4, 28, 665; 1909, *and* A. Sommerveld, "Ausbreitung der Wellen in der Drahtlosen Telegraphie", *Jahrbuch der Drahtlosen Telegraphie*, 4, 157; 1911.) By B. Rolf. March 1930, p. 391.

7. Studies of High Frequency Radio Wave Propagation. By A. H. Taylor and L. C. Young. 1928, p. 561.

8. Reception Experiments in Mount Royal Tunnel. By A. S. Eve and Others. Feb. 1929, p. 347.

9. Some Experiments in Short-Distance Short-Wave Radio Transmission. By J. K. Clapp. March 1929, p. 479.

10. Double and Multiple Signals with Short Waves. By E. Quäck and H. Mögel. May 1929, p. 791.

11. East–West and North–South Attenuations of Long Radio Waves on the Pacific. By E. Yokayama and T. Nakai. July 1929, p. 1240.

12. The Propagation of Low Power Short Waves in the 1,000 Kilometre Range. By K. Krüger and H. Plenal. Aug. 1929, p. 1296.

13. Long-Wave Radio Receiving Measurements at the Bureau of Standards in 1929. By L. W. Austin. Sept. 1929, p. 1481.

14. Studies of Echo Signals. By A. H. Taylor and L. C. Young. Sept. 1929, p. 1491.

15. On the Daylight Transmission Characteristics of Horizontally and Vertically Polarized Waves from Airplanes. By F. H. Drake and R. M. Wilmotte. Dec. 1929, p. 2242.

16. Multiple Signals in Short Wave Transmission. By T. A. Eckersley. Jan. 1930, p. 106.

17. Propagation of 3 Metre Waves. By A. Esau and W. N. Hahnemann. March 1930, p. 471.

18. Transmission Characteristics of a Short Wave Telephone Circuit. By R. K. Potter. April 1930, p. 581.

19. Some Problems in Short Wave Telephone Transmission. By J. C. Schelling. June 1930, p. 913.

20. The Effect of Rain and Fog on the Propagation of Very Short Radio Waves. By J. A. Stratton. June 1930, p. 1064.

21. The Relation of Radio Propagation to Disturbances in Terrestrial Magnetism. By I. J. Wymore. July 1930, p. 1206.

22. Low Frequency Radio Transmission. By A. de Mari and Others. Sept. 1930, p. 1488.

23. Note on the Accuracy of Rolf's Graphs. By W. H. Wise. Nov. 1930, p. 1971.

24. The Diurnal and Seasonal Performance of High Frequency Radio Transmission over Various Long Distance Circuits. By M. L. Prescott. Nov. 1930, p. 1797.
25. Some Experiences with Short Wave Wireless Telegraphy. By N. H. Edes. Dec. 1930, p. 2011.
26. Note on High Frequency Transmission during the Summer of 1930. By G. W. Kendrick and Others. Feb. 1931, p. 252.
27. Propagation of Very Short Waves. By R. Jouaust. March 1931, p. 479.
28. Notes on Radio Transmission. By C. N. Anderson. July 1931, p. 1050.
29. Attenuation of Electromagnetic Waves from 200 to 2,000 Metres. By H. Fassfender and Others. Aug. 1931, p. 1446.
30. The Propagation of Short Radio Waves over the North Atlantic. By C. R. Burrows. Sept. 1931, p. 1634.
31. Direct Ray Broadcast Transmission. By T. L. Eckersley. Oct. 1932, p. 1555.
32. Transoceanic Reception of High Frequency Telephone Signals. By R. M. Morris and W. A. R. Brown. Jan. 1933, p. 63.
33. Short Wave Transmission to South America. By C. R. Burrows and E. J. Howard. Jan. 1933, p. 102.
34. A Study of the Propagation of Wavelengths between 3 and 8 Metres. By L. F. Jones. March 1933, p. 349.
35. Propagation of Waves Below 10 Metres in Length. By B. Trevor and P. S. Carter. March 1933, p. 387.
36. Propagation of Waves of 150 to 2,000 Kc/s at Distances between 50 and 2,000 Kilometres. By B. Van der Pol, T. L. Eckersley, J. H. Dellinger and P. C. Corbeiller. July 1933, p. 996.
37. Attenuation of Overland Radio Transmission in the Frequency Range 1·5 to 3·5 Mc/s. By C. N. Anderson. Oct. 1933, p. 1447.
38. The Determination of the Direction of Arrival of Short Radio Waves. By H. T. Friis and Others. Jan. 1934, p. 47.
39. Notes on Propagation at a Wavelength of 73 Centimetres. By B. Trevor and R. W. George. May 1935, p. 461.
40. Ultra Short Wave Propagation Over Land. By C. R. Burrows and Others. Dec. 1935, p. 1507.
41. Radio Broadcast Coverage. By G. D. Gillett and M. Eager. Feb. 1936, p. 190.
42. A Study of Ground Wave Radio Transmission. By R. C. Higgy and E. D. Shipley. March 1936, p. 483.
43. U.S.W. Propagation along the Curved Earth's Surface. By P. von Handel and W. Pfister. March 1937, p. 345.
44. Characteristics of the Ionosphere and Their Application to Radio Transmission. By T. R. Gilliland and Others. July 1937, p. 823.
45. The Propagation of Radio Waves Over the Surface of the Earth and in the Upper Atmosphere. By K. A. Norton. Sept. 1937, p. 1203.
46. Field-Strength Observations of Transatlantic Signals 40 to 50 Mc/s. By H. O. Peterson and D. R. Goddard. Oct. 1937, p. 1291.

47. The Propagation of Radio Waves Over the Surface of the Earth and in the Upper Atmosphere. By K. A. Norton. Oct. 1938, p. 1367.
48. Meteorological Influences on Long-Distance Long-Wave Reception. By E. Yokayama and T. Nakai. June 1930, p. 1075.
49. The Calculation of the Service Area of Broadcast Stations. By P. P. Eckersley. July 1930, p. 1160.
50. Observations on Sky Wave Transmission on Frequencies Above 40 Megacycles. By D. R. Goddard. Jan. 1939, p. 12.
51. A Study of U.H.F. Wide Band Propagation Characteristics. By A. W. George. Jan. 1939, p. 28.
52. High Frequency Propagation Characteristics. By F. Hamburger and Others. April 1940, p. 175.
53. U.S.W. Transmission Over a 39-Mile Optical Path. By C. R. Englund and Others. Aug. 1940, p. 360.
54. Experiments on the Propagation of Ultra Short Radio Waves. By A. H. Weynick. Oct. 1940, p. 468.
55. Standards on Radio Wave Propagation: Definitions of Terms, 1942; Standards on Radio Wave Propagation; Measuring Methods, 1942. Both issued by I.R.E.

I.E.E. JOURNAL.

56. Short Wave Wireless Telegraphy. By T. L. Eckersley. Vol. 2, 1927, p. 85.
57. Short Distance Observations on Long Wave Phenomena. By R. Naismith. Vol. 6, 1931, p. 121.
58. Field-Strength Measurements on Daventry 5XX. By R. Naismith. Vol. 6, 1931, p. 127.
59. Studies in Radio Transmission. By T. L. Eckersley. Vol. 7, 1932, p. 137.
60. Some Characteristics of Short Wave Propagation. By J. Hollingworth. Vol. 8, 1933, p. 35.

BELL SYSTEM TECHNICAL JOURNAL.

61. Radio Propagation Over Plane Earth. Field Strength Curves. By C. R. Burrows. Vol. 16, 1937, p. 45.

E.W. AND WIRELESS ENGINEER.

62. Some Measurements on Wireless Wave-Fronts. By R. L. Smith-Rose and R. H. Barfield. Sept. 1925, p. 737.

ELECTRONICS.

63. Sommerveld's Formula. Sept. 1936, p. 23.

Other References.

64. Some Notes on Ultra High Frequency Propagation. By H. H. Beverage. *R.C.A.*, Vol. 1, No. 3, 1937, p. 76.
65. Short Wave Radio Transmission and Geomagnetism. By H. E. Halborg. *R.C.A. Review*, Vol. 5, No. 4, 1941, p. 395.

P12—Pulse Generation

ELECTRONICS.

1. SIMPLE PULSE GENERATING CIRCUITS. By S. P. Sashoff and W. K. Roberts. Sept. 1940, p. 40.

MARCONI REVIEW.

2. THE ONGAR PULSE CIRCUIT. By G. Millington. No. 63, 1936, p. 1.

R1—Radiation

E.W. AND WIRELESS ENGINEER.

1. THE FIELD IN THE IMMEDIATE NEIGHBOURHOOD OF A TRANSMITTING AERIAL. By G. W. O. Howe. March 1932, p. 119.

R2—Receivers

I.E.E. JOURNAL.

1. MICROPHONIC FEED-BACK PHENOMENA IN RADIO RECEIVERS. By H. A. Brooke. Vol. 7, 1932, p. 53.
2. DEVELOPMENTS IN THE TESTING OF RADIO RECEIVERS. By H. A. Thomas. Vol. 7, 1932, p. 106.
3. BROADCAST RECEIVERS: A REVIEW. By N. M. Rust, O. E. Keall, J. F. Ramsey and K. R. Sturley. Vol. 88, 1941, p. 59.

I.R.E. PROCEEDINGS.

4. THE RECEIVING SYSTEM FOR LONG WAVE TRANSATLANTIC RADIO TELEPHONY. By A. Bailey and Others. 1928, p. 1645.
5. THE EFFECT OF REGENERATION ON THE RECEIVED SIGNAL STRENGTH. By B. Van der Pol. Feb. 1929, p. 339.
6. REDUCTION OF DISTORTION AND CROSS-TALK IN RADIO RECEIVERS BY MEANS OF VARIABLE MU TETRODES. By S. Ballantine and H. A. Snow. Dec. 1930, p. 2102.
7. PROBLEMS IN SELECTIVE RECEPTION. By M. V. Callender. Sept. 1932, p. 1427.
8. FERRO INDUCTORS AND PERMEABILITY TUNING. By W. J. Polydonoff. May 1933, p. 690.
9. THE DESIGN AND TESTING OF MULTIRANGE RECEIVERS. By D. E. Harnett and N. P. Case. June 1935, p. 578.
10. RADIO FREQUENCY DISTRIBUTING SYSTEMS. By F. X. Rettenmeyer. Nov. 1935, p. 1286.
11. AUTOMATIC SELECTIVITY CONTROL. By G. L. Beers. Dec. 1935, p. 1425.
12. CHARACTERISTICS OF AMERICAN BROADCAST RECEIVERS . . . By A. Van Dyck and D. E. Foster. April 1937, p. 387.
13. STANDARDS ON RADIO RECEIVERS, 1938. Issued by I.R.E.
14. HIGH FIDELITY RECEIVERS WITH EXPANDING SELECTORS (i.e. VARIABLE I.F. FILTER SELECTIVITY). By H. A. Wheeler and J. K. Johnson. June 1935, p. 594.

E.W. AND WIRELESS ENGINEER.

15. The Performance of Reflexed Valves. By D. Kingsbury. Sept. 1927, p. 547.
16. Band Pass Filters in Radio Receivers. By G. W. O. Howe. May 1931, p. 233.
17. The Design of the Band Pass Filter. By N. R. Bligh. Feb. 1932, p. 61.
18. Band Pass Filters for Radio Receivers. By C. W. Oatley. Nov. 1932, p. 608.
19. Connecting Several Receivers to One Aerial. By M. Reed. Aug. 1934, p. 428.
20. Variable Selectivity and the I.F. Amplifier. By W. T. Cocking. March 1936, p. 119.
21. Receiver Aerial Coupling Circuits. By K. R. Sturley. April 1941, p. 137, and May 1941, p. 190.

ELECTRONICS.

22. An Improved Pre-Selector Circuit for Radio Receivers. By E. A. Uehling. Sept. 1930, p. 279.
23. Inter Carrier Noise Suppression. By N. E. Wunderlich. Jan. 1933, p. 13.
24. Circuits for Amplified A.V.C. Anonymous. Jan. 1933, p. 16.
25. Notes on Dual Band Receiver Design. By E. Messing. Nov. 1933, p. 300.
26. A.V.C. Applied to A.F. Amplifier Tubes. By J. R. Nelson. Feb. 1934, p. 51.
27. Superheterodyne Tuning Condenser Design. By H. Schwartzmann and L. G. Burnell. June 1934, p. 181.
28. Vibrator Power Supply from Dry Cells. By W. B. Roberts. July 1934, p. 215.
29. A.F.C. Design. By S. V. White. Sept. 1936, p. 29.
30. Improvements in Automatic Frequency Control Circuits. Nov. 1936, p. 21.
31. Automatic Selectivity Control. By H. F. Mayer. Dec. 1936, p. 33.
32. The Band-Spread Problem. By R. C. Woodhead. Jan. 1937, p. 29.
33. A Condenser Tracking Test Set. By J. L. Roemisch. March 1937, p. 12.
34. The Padding Condenser. By L. B. Sklar. May 1937, p. 41.
35. Thermal Drift in Superheterodyne Receivers. By J. M. Miller. Nov. 1937, p. 25.
36. Frequency Drift Compensation. By M. L. Levy. May 1939, p. 15.
37. A Receiver for Frequency Modulation. By J. R. Day. June 1939, p. 33.

ELECTRICAL COMMUNICATION.

38. The Community-Aerial System of Broadcast Distribution. By C. W. Earp and S. Hill. Vol. 15, p. 129, 1936.

Other References.

39. New Features in Broadcast Receiver Design. By B. R. Carson and Others. *R.C.A.*, Vol. 2, No. 1, 1937, p. 45.

40. DESIGN OF SUPERHETERODYNE I.F. CIRCUITS. By F. E. Spaulding. *R.C.A.*, Vol. 4, No. 4, 1940, p. 485.
41. AMERICAN STANDARD: STANDARD VACUUM-TUBE BASE AND SOCKET DIMENSIONS. "A.S.A. Standards (A.S.A. C16.3–1939)."
42. AMERICAN STANDARD: MANUFACTURING STANDARDS APPLYING TO BROADCAST RECEIVERS. "A.S.A. Standards (A.S.A. C16.3–1939)."

R3—Receivers—Reflex

E.W. AND WIRELESS ENGINEER.

1. THE REFLEX. Anonymous. March 1924, p. 331.

ELECTRONICS.

2. REFLEX CIRCUIT CONSIDERATIONS. By J. M. Stinchfield. June 1933, p. 152.

R4—Receivers—Superheterodyne

I.R.E. PROCEEDINGS.

1. RECENT DEVELOPMENTS IN SUPERHETERODYNE RECEIVERS. By G. L. Beers and W. L. Carlson. March 1929, p. 501.
2. CONSIDERATIONS IN SUPERHETERODYNE DESIGN. By E. G. Watts. April 1930, p. 690.
3. IMAGE SUPPRESSION IN SUPERHET RECEIVERS. By H. A. Wheeler. June 1935, p. 569.
4. INTERFERING RESPONSES IN SUPERHETERODYNE. By H. K. Morgan. Oct. 1935, p. 1164.

R5—Receivers—Super-Regenerative

I.R.E. PROCEEDINGS.

1. THE BASIC PRINCIPLES OF SUPER-REGENERATIVE RECEPTION. By F. W. Frink. Jan. 1938, p. 76.

E.W. AND WIRELESS ENGINEER.

2. THE SUPER-REGENERATIVE RECEIVER. By M. G. Scroggie. Nov. 1936, p. 581.
3. SUPER-REGENERATION WITH REFERENCE TO BROADCAST RECEIVERS. By D. Maurice. Jan. 1938, p. 4.

ELECTRONICS.

4. A STUDY OF SUPER-REGENERATION. Anonymous. Feb. 1934, p. 42.

R6—Receivers, Testing of

I.E.E. JOURNAL.

1. A METHOD OF MEASURING THE OVERALL PERFORMANCE OF RADIO RECEIVERS. By H. A. Thomas. Vol. 5, 1930, p. 81.

I.R.E. PROCEEDINGS.

2. RADIO RECEIVER TESTING EQUIPMENT. By K. W. Jarvis. April 1929, p. 664.

3. Proposed Standard Tests of Broadcast Radio Receivers. (Committee.) Aug. 1930, p. 1282.
4. Problems Involved in the Design and Use of Apparatus for Testing Radio Receivers. By P. O. Farnham and A. W. Barber. Aug. 1930, p. 1338.

R7—Reception

1. Baldock Receiving Station. By G. T. Evans and J. E. Lockhurst. *P.O.E.E. Jl.*, Jan. 1940, p. 256.

R8—Rectifiers

I.E.E. JOURNAL.

1. Hot Cathode Mercury-Vapor Rectifier Circuits. By C. R. Dunham. Vol. 9, 1934, p. 275.

I.R.E. PROCEEDINGS.

2. Polyphase Rectification Special Connections. By R. W. Armstrong. Jan. 1931, p. 78.
3. The Use of the Copper-Oxide Rectifier for Instrument Purposes. By J. Sahagen. Feb. 1931, p. 233.
4. Copper-Oxide Rectifier Used for Radio Detection and Automatic Volume Control. By L. O. Grondahl and W. P. Place. Oct. 1932, p. 1599.
5. Full Wave Voltage-Doubling Rectifier Circuit. By D. L. Wardelich. Oct. 1941, p. 554.

E.W. AND WIRELESS ENGINEER.

6. Mercury Arc-Rectifiers for Wireless Stations. By K. Kotschubey. March 1934, p. 130.

ELECTRONICS.

7. Mercury Rectifiers for Radio Stations. By S. R. Durand. Jan. 1934, p. 4.
8. Half-Wave Gas Rectifier Circuits. By C. M. Wallis. Oct. 1938, p. 12.
9. Behaviour of Half-Wave Rectifiers. Sept. 1939, p. 33.
10. Full Wave Rectifier Analysis. By C. M. Wallis. March 1940, p. 19.

ELECTRICAL COMMUNICATION.

11. The Selenium Rectifier. By Erich Kipphan. Vol. 16, 1937, p. 21.
12. Harmonic Voltage Generation in Polyphase Rectifier Circuits. By H. Rissik. Vol. 18, 1939, p. 271.
13. Harmonic Current Generation in Polyphase Rectifier Circuits. By H. Rissik. Vol. 19, 1940, p. 94.
14. Harmonic Power Consumption of Polyphase Rectifier Systems. By H. Rissik. Vol. 19, 1940, p. 118.

R9—Reflection and Refraction of Radio Waves

I.E.E. JOURNAL.

1. ULTRA-SHORT-WAVE REFRACTION AND DIFFRACTION. By T. L. Eckersley. Vol. 12, 1937, p. 42.
2. THE REFLECTION COEFFICIENT OF THE EARTH'S SURFACE FOR RADIO WAVES. By J. S. McPetrie. Vol. 13, 1938, p. 48.
3. REFLECTION CURVES AND PROPAGATION CHARACTERISTICS OF RADIO WAVES ALONG THE EARTH'S SURFACE. By J. S. McPetrie and (Miss) A. C. Stickland. Vol. 15, 1940, p. 142.

I.R.E. PROCEEDINGS.

4. EARTH REFLECTION OF ULTRA SHORT RADIO WAVES. By E. H. Lange. April 1929, p. 745.
5. REFLECTION OF RADIO WAVES FROM THE SURFACE OF THE EARTH. By L. C. Verman. Aug. 1930, p. 1396.
6. THE MULTIPLE REFRACTION AND REFLECTION OF SHORT WAVES. By N. Edes. June 1931, p. 1024.
7. THE OPTICS OF RADIO TRANSMISSION. By E. Merritt. Jan. 1932, p. 29.
8. THE OPTICAL BEHAVIOUR OF THE GROUND FOR SHORT RADIO WAVES. By C. B. Feldman. June 1933, p. 764.

E.W. AND WIRELESS ENGINEER.

9. EFFECT OF THE GROUND ON DOWNCOMING PLANE SPACE WAVES. By E. T. Glas. Dec. 1929, p. 663.
10. THE REFRACTIVE INDEX OF SPACES WITH FREE ELECTRONS. By P. O. Pedersen. Jan. 1930, p. 16.

Other Reference.

11. APPLICATION OF ABELIAN FINITE GROUP THEORY TO ELECTROMAGNETIC REFRACTION. By R. A. Whiteman. *R.C.A. Review*, Vol. 4, No. 3, 1940, p. 372.

R10—Resistance

I.E.E. JOURNAL.

1. THE A.C. RESISTANCE OF PARALLEL CONDUCTORS OF CIRCULAR CROSS SECTION. By A. H. M. Arnold. July 1935, p. 49.
2. THE A.C. RESISTANCE OF TUBULAR CONDUCTORS. By A. H. M. Arnold. May 1936, p. 580.

E.W. AND WIRELESS ENGINEER.

3. A VARIABLE RESISTANCE FOR RADIO FREQUENCIES. By R. M. Wilmotte. Aug. 1925, p. 684.
4. THE HIGH FREQUENCY RESISTANCE OF TOROIDAL COILS. By S. Butterworth. Jan. 1929, p. 13.
5. THE BEHAVIOUR OF HIGH RESISTANCE AT HIGH FREQUENCIES: BOELLA EFFECT. By O. S. Puckle. June 1935, p. 303.
6. RESISTANCE AT HIGH RADIO FREQUENCIES. By G. W. O. Howe. June 1935, p. 295, and August 1935, p. 413.

7. The Behaviour of Resistors at High Frequencies. Editorial by G. W. O. Howe. Nov. 1940, p. 471.

ELECTRONICS.

8. Skin Effect Formulas. By J. R. Whinnery. Feb. 1942, p. 44.

R11—Resistance Tuning

I.R.E. PROCEEDINGS.

1. Resistance Tuning. By C. Brunette and P. Weiss. June 1941, p. 333.

R12—Resonant Line Oscillators

MARCONI REVIEW.

1. Concentric Tube Lines. By B. J. Witt. No. 58, 1936, p. 20.

R13—Response Correction

I.E.E. JOURNAL.

1. . . . New Method of Frequency Compensation of Single Stages. By G. W. Edwards and E. C. Cherry. Vol. 15, 1940, p. 204.

I.R.E. PROCEEDINGS.

2. Linear Distortions in Broadcast Receivers and their Compensation by Low Frequency Equalization Devices. By A. Clausing and W. Kautter. Sept. 1932, p. 1456.

BELL SYSTEM TECHNICAL JOURNAL.

3. Distortion Correction in Electrical Circuits with Constant Resistance Recurrent Networks. By O. J. Zobel. Vol. 7, 1928, p. 438.
4. Variable Equalizers. By H. W. Bode. Vol. 17, 1938, p. 229.

ELECTRONICS.

5. Audio Frequency Compensating Circuits. By S. Cutler. Sept. 1942, p. 63.

MARCONI REVIEW.

6. Correction Circuits for Amplifiers. By O. A. Keall. No. 54, p. 1, and No. 56, p. 9, 1935.
7. Design of Series and Parallel Resonant Circuits. By L. E. Q. Walker. No. 63, 1936, p. 7.
8. General Equalizer Theory, Two Terminal and Constant Resistance Structures. By F. M. G. Murphy. No. 64, 1937, pp. 20 and 65.

ELECTRICAL COMMUNICATION.

9. Controlling Quality in a Broadcasting System. By E. K. Sandeman. Vol. 7, p. 33.
10. Generalized Characteristics of Linear Networks. By E. K. Sandeman. Vol. 15, 1936, p. 115.

R14—R.M.A. Specification

I.E.E. JOURNAL.

1. R.M.A. Specification for Testing and Expressing the Overall Performance of Radio Receivers. Vol. 12, 1937, p. 179.

R15—Rocky Point Effect

I.E.E. JOURNAL.

1. The Flash-Arc in High-Power Valves. By B. S. Gossling. Vol. 7, 1932, p. 192.

S1—Schotky Noise, Schot or Shott Effect : *see* Fluctuation Noise

S2—Screening

E.W. AND WIRELESS ENGINEER.

1. The Electromagnetic Screening of Radio Apparatus. By R. L. Smith-Rose. April 1924, p. 375.
2. The Screening of Radio Receiving Apparatus. By R. H. Barfield. July 1924, p. 570.
3. The Screening of Small Variable Condensers. By D. A. Oliver. Dec. 1925, p. 970.

S3—Selenium Rectifiers

I.E.E. JOURNAL.

1. The Characteristics and Application of the Selenium Rectifier. By E. A. Richards. Vol. 88, 1941, p. 238.

S4—Series Modulation

I.R.E. PROCEEDINGS.

1. Series Modulation. By C. A. Culver. May 1935, p. 481.

S5—Shielding

I.R.E. PROCEEDINGS.

1. The Shielding of Electric and Magnetic Fields. By J. H. Morecraft and Alva Turner. Aug. 1925, p. 477.
2. Experiments on Electromagnetic Shielding at Frequencies between One and Thirty Kilocycles. By W. Lyons. April 1933, p. 574.
3. Electromagnetic Shielding Effect of an Infinite Plane Conducting Sheet Placed between Circular Coaxial Coils. By S. Levy. June 1936, p. 923.

BELL SYSTEM TECHNICAL JOURNAL.

4. Shielding in High Frequency Measurements. By J. G. Ferguson. Vol. 8, 1929, p. 560.

Other References.

5. MAGNETIC SHIELDS. By W. B. Eltwood. *Bell Lab. Record,* Nov. 1938, p. 93.
6. MAGNETIC SHIELDS FOR TRANSFORMERS. By W. G. Gustafson. *Bell Lab. Record,* Nov. 1938, p. 97.

S6—Silicones

I.R.E. PROCEEDINGS.

1. SILICONES. By S. L. Bass and T. A. Kauppi. July 1945, p. 441.

S7—Single Sideband: *see* Asymmetric Sideband Systems

S8—Skip Distance

I.R.E. PROCEEDINGS.

1. SKIP DISTANCE EFFECTS ON SUPER FREQUENCIES. By A. H. Taylor. Jan. 1931, p. 103.
2. THE RELATION CONNECTING SKIP DISTANCE WAVELENGTH, AND THE CONSTANTS OF THE IONIZED LAYERS. By N. H. Edes. Sept. 1931, p. 1663.

S9—Sparking Distance: *see* Voltage Breakdown

S10—Speech and Music

BELL SYSTEM TECHNICAL JOURNAL.

1. USEFUL NUMERICAL CONSTANTS OF SPEECH AND HEARING. By H. Fletcher. Vol. 4, 1925, p. 375.
2. THE SOUNDS OF SPEECH. By I. B. Crandall. Vol. 4, 1925, p. 586.
3. SPEECH POWER AND ITS MEASUREMENT (GIVES SPECTRUM OF SPEECH). By L. J. Syvian. Vol. 8, 1929, p. 646.
4. SOME PHYSICAL CHARACTERISTICS OF SPEECH AND MUSIC. By H. Fletcher. Vol. 10, 1931, p. 349.
5. AUDIBLE FREQUENCY RANGES OF MUSIC, SPEECH AND NOISE. By W. B. Snow. Vol. 10, 1931, p. 616.

ELECTRICAL COMMUNICATION.

6. THE NATURE OF SPEECH AND ITS INTERPRETATION. By Harvey Fletcher. Vol. 1, 1922, p. 41.

Other References.

7. ABSOLUTE AMPLITUDES AND SPECTRA OF CERTAIN MUSICAL INSTRUMENTS AND ORCHESTRAS. By L. J. Syvian, H. K. Dunn and S. D. White. *Jl. Acous. Soc. America,* 2, 330, Jan. 1931, p. 330.
8. SOME PHYSICAL CHARACTERISTICS OF SPEECH AND MUSIC. By H. Fletcher. *Jl. Acous. Soc. America,* Supplement to 3, 1931, p. 23.
9. STATISTICAL MEASUREMENTS ON CONVERSATIONAL SPEECH. By H. K. Dunn and S. D. White. *Jl. Acous. Soc. America,* 11, 278 (1940).
10. SPECTRA OF SPEECH AND MUSIC. *R.C.A. Review,* Vol. 3, No. 3, 1939, pp. 324 and 325.

S11—Stabilovolt : *see* Gas Discharge Potentiometers

S12—Sub-Harmonics

1. NON-LINEAR IMPEDANCE GENERATES SUB-FREQUENCIES. By E. Petersen. *Bell Lab. Record,* Feb. 1929, p. 231.
2. SUB-HARMONICS IN FORCED OSCILLATIONS IN DISSIPATIVE SYSTEMS. By P. O. Pedersen. *Jl. Acous. Soc. America,* Vol. 6, p. 227, 1935, and Vol. 7 (July), 1935, p. 64.

S13—Synchronized Operation of Two Transmitters on the same Wavelength

I.E.E. JOURNAL.

1. THE OPERATION OF SEVERAL BROADCASTING STATIONS ON THE SAME WAVELENGTH. By P. P. Eckersley. Vol. 4, 1929, p. 81.

I.R.E. PROCEEDINGS.

2. ON THE SIMULTANEOUS OPERATION OF DIFFERENT BROADCAST STATIONS ON THE SAME CHANNEL. By P. P. Eckersley. Feb. 1931, p. 175.
3. SOME DEVELOPMENTS IN COMMON FREQUENCY BROADCASTING. By G. D. Gillett. Aug. 1931, p. 1347.
4. SUPERPOSITION OF TWO MODULATED RADIO FREQUENCIES. By Hans Roder. Dec. 1932, p. 1962.
5. THEORY OF THE DETECTION OF TWO MODULATED WAVES BY A LINEAR RECTIFIER. By C. B. Aiken. April 1933, p. 601.
6. A STUDY OF RECEPTION FROM SYNCHRONIZED BROADCAST STATIONS. By C. B. Aiken. Sept. 1933, p. 1265.
7. AUTOMATIC SYNTRACTION OF TWO BROADCAST CARRIERS. By V. V. Gunsolley. March 1935, p. 244.

BELL SYSTEM TECHNICAL JOURNAL.

8. SOME DEVELOPMENTS IN COMMON FREQUENCY BROADCASTING. By G. D. Gillett. Vol. 10, 1931, p. 577.
9. THE EFFECT OF BACKGROUND NOISE IN SHARED CHANNEL BROADCASTING. By C. B. Aiken. Vol. 13, 1934, p. 333.

T1—Tapered Transmission Lines

BELL SYSTEM TECHNICAL JOURNAL.

1. THE EXPONENTIAL TRANSMISSION LINE. By C. R. Burrows. Vol. 7, 1938, p. 555.

Other Reference.

2. THE EXPONENTIAL TRANSMISSION LINE. By C. R. Burrows. *Bell Lab. Record,* Feb. 1940, p. 174.

T2—Telegon Effect: *see* Luxembourg Effect

T3—Temperature Control

I.E.E. JOURNAL.

1. . . . DESIGN (OF) . . . ISOTHERMAL CHAMBER CONSTANT TO ONE-THOUSANDTH OF A DEGREE CENTIGRADE. By L. B. Turner. Vol. 12, 1937, p. 281.

I.R.E. PROCEEDINGS.

2. THERMOSTAT DESIGN FOR FREQUENCY STANDARDS. By W. A. Marrison. 1928, p. 976.

T4—Thermal Agitation

1. THERMAL AGITATION IN CONDUCTORS. By J. B. Johnson. *Phys. Review*, April 1927, p. 614, and July 1928, p. 97.
2. THERMAL AGITATION OF ELECTRIC CHARGE IN CONDUCTORS. By H. Nyquist. *Phys. Review*, July 1928, p. 110.
3. THE E.M.F. OF THERMAL AGITATION. By E. K. Sandeman and L. H. Bedford. *Phil. Mag.*, May 1929, p. 774.

T5—Thyratrons

I.R.E. PROCEEDINGS.

1. SOME CHARACTERISTICS OF THYRATRONS. By J. C. Warner. Sept. 1931, p. 1561.

Other Reference.

2. GAS-FILLED RELAYS. By C. R. Dunham. *Television*, Feb. 1934, p. 71.

T6—Thyrite

1. THYRITE. By K. B. McEachron. *G.E.C. Review*, Feb. 1930, p. 92.
2. THE CALCULATION OF CIRCUITS CONTAINING THYRITE. *G.E.C.*, May 1934, p. 218.

T7—Tracking: *see* Ganging and Tracking

T8—Transformers A.F., I.F., R.F.

BELL SYSTEM TECHNICAL JOURNAL.

1. TRANSFORMER COUPLING CIRCUITS FOR HIGH FREQUENCY AMPLIFIERS. By A. J. Christofer. Vol. 11, 1932, p. 608.
2. IMPROVEMENTS IN COMMUNICATION TRANSFORMERS. By A. G. Ganz and A. G. Laird. Vol. 15, 1936, p. 136.

E.W. AND WIRELESS ENGINEER.

3. DESIGN OF AUDIO FREQUENCY INPUT AND INTERVALVE TRANSFORMERS. By J. G. Story. Feb. 1938, p. 69.

ELECTRONICS.

4. Litz Wire Coils for I.F. and A.F. Transformers. Anonymous. Nov. and Dec. 1933, pp. 303 and 342.
5. Economic Design for Radio Set Transformers. By C. A. Hultberg. Sept. and Oct. 1934, pp. 287 and 311.

MARCONI REVIEW.

6. New Design of (A.F.) Transformer. By F. M. G. Murphy and C. D. Colchester. No. 58, 1936, p. 26.

ELECTRICAL COMMUNICATION.

7. Telephone Transformers. By W. L. Casper. Vol. 2, p. 262, 1923.

T9—Transformer Oil

I.E.E. JOURNAL.

1. Conduction through Transformer Oil at High Field Strengths. By J. F. Gillies. June 1935, p. 647.

T10—Transients : *see also* Phase Distortion

I.R.E. PROCEEDINGS.

1. Transient Response. By H. E. Kallinais and R. E. Spencer. March 1945, p. 169.

BELL SYSTEM TECHNICAL JOURNAL.

2. Transient Oscillations in Electric Wave Filters. By J. R. Carson and O. Zobel. July 1923, Vol. 2, No. 3, p. 1.
3. The Building up of Sinusoidal Currents in Long Periodically Loaded Lines. By J. R. Carson. Vol. 3, 1924, p. 558.
4. The Practical Application of the Fourier Integral. By G. A. Campbell. Vol. 7, 1928, p. 639.
5. A New Method for Obtaining Transient Solutions of Electrical Networks. By W. P. Mason. Vol. 8, 1929, p. 109.

E.W. AND WIRELESS ENGINEER.

6. The Distortionless Amplification of Electrical Transients. By C. W. Oatley. May 1931, p. 244, and June 1931, p. 307.

Other References.

7. Theory of Transient Oscillations of Electrical Networks and Transmission Systems. By J. R. Carson. *Amer. I.E.E. Proc.*, Vol. 38, 1919, p. 345.
8. The Building up of Voltage and Current in a Long Circuit. By A. E. Kennelly. *Proc. Acad. Arts. Sciences*, Vol. 42, p. 70.

T11—Transmitters: *see also* Chireix and Doherty

I.E.E. JOURNAL.

1. A Wireless Broadcasting Transmitting Station for Dual Programme Service. By Sir Noel Ashbridge and P. P. Eckersley. Vol. 5, 1930, p. 193.
2. The Droitwich Broadcasting Station. By Sir Noel Ashbridge, H. Bishop and B. N. MacLarty. Vol. 10, 1935, p. 202.
3. The Empire Service Broadcasting Station at Daventry. By L. W. Hayes and B. N. MacLarty. Vol. 14, 1939, p. 332.

I.R.E. PROCEEDINGS.

4. The Suppression of Radio Frequency Harmonics in Transmitters. By J. W. Labus and Hans Roder. June 1931, p. 949.
5. (Symposium.) The WLW 500 Kilowatt Broadcast Transmitter. Oct. 1934, p. 1151.
6. Analysis of the Operation of Vacuum Tubes as Class C Amplifiers—Describes the Use of Constant Current Valve Characteristics. By I. E. Mouromtseff and H. N. Kozanowaski. July 1935, p. 752.
7. Simplified Methods for Computing Performance of Transmitter Tubes. By W. G. Wagener. Jan. 1937, p. 47.
8. Standards of Transmitters and Antennas, 1938. Issued by I.R.E.
9. Grid-Current Flow as a Factor in the Design of Vacuum Tube Power Amplifiers. By W. L. Everitt and K. Spangenberg. May 1938, p. 612.
10. High Efficiency Modulation Systems. By R. B. Dome. Aug. 1938, p. 963.

ELECTRONICS.

11. Suppression of Transmitter Harmonics. By C. O. Dutsch. June 1933, p. 167.

MARCONI REVIEW.

12. 100 kW Short Wave Broadcasting Transmitter. Type S.W.B. 14 and 18. By E. Green and L. T. Moody. No. 74, 1939, p. 1.

ELECTRICAL COMMUNICATION.

13. Transatlantic Radio Telephony (Description of Transatlantic Transmitter at Rugby). By E. M. Deloraine. Vol. 5, 1926, p. 3.

T12—Turn-over

E.W. AND WIRELESS ENGINEER.

1. The Problem of Turn-over. By M. Reed. June 1929, p. 310.

U1—U.H.F. Aerials

I.R.E. PROCEEDINGS.

1. Beam Transmission of U.S.W. By H. Yagi. 1928, p. 715.
2. An Experimental Study of Parasitic Wire Reflectors on 2·5 Metres. By A. W. Nagy. Feb. 1936, p. 233.

3. THE SECTORAL ELECTROMAGNETIC HORN. By W. L. Barrow and F. Lewis. Jan. 1939, p. 41.
4. THEORY OF THE ELECTROMAGNETIC HORN. By W. L. Barrow and L. J. Chu. Jan. 1939, p. 51.
5. METAL HORNS AS DIRECTIVE RECEIVERS OF ULTRA SHORT WAVES. By G. O. Southworth and A. P. King. Feb. 1939, p. 95.
6. BICONICAL ELECTROMAGNETIC HORNS. By W. L. Barrow and Others. Dec. 1939, p. 769.
7. MULTIUNIT ELECTROMAGNETIC HORNS. By W. L. Barrow and C. Shulman. March 1940, p. 130.

I.E.E. JOURNAL.

8. THE RADIATION RESISTANCE OF SURFACES OF REVOLUTION SUCH AS CYLINDERS, SPHERES AND CONES. By E. B. Moullin. Vol. 88, 1941, p. 5.

ELECTRICAL COMMUNICATION.

9. U.H.F. LOOP ANTENNA. By A. Alford and A. G. Kandoian. Vol. 18, 1939, p. 255.

U2—U.H.F. Amplifiers

I.R.E. PROCEEDINGS.

1. A POWER AMPLIFIER FOR ULTRA HIGH FREQUENCIES. By A. L. Samuel and N. E. Sowers. Nov. 1936, p. 1464.

BELL SYSTEM TECHNICAL JOURNAL.

2. A POWER AMPLIFIER FOR ULTRA HIGH FREQUENCIES. By A. L. Samuel and A. V. Sowers. Vol. 16, 1937, p. 10.

Other Reference.

3. EFFECT OF ELECTRON TRANSIT TIME ON EFFICIENCY OF A POWER AMPLIFIER. By A. V. Haeff. *R.C.A. Review,* Vol. 4, No. 1, 1939, p. 114.

U3—U.H.F. Cavity Resonators

I.R.E. PROCEEDINGS.

1. NATURAL OSCILLATIONS OF ELECTRICAL CAVITY RESONATORS. By W. L. Barrow and W. W. Miehen. April 1940, p. 184.

U4—U.H.F. Feeders

I.R.E. PROCEEDINGS.

1. RADIATION RESISTANCE OF CONCENTRIC CONDUCTOR TRANSMISSION LINES. By R. Whitmer. Sept. 1933, p. 1342.

U5—U.H.F. General

I.R.E. PROCEEDINGS.

1. RADIO TELEGRAPHY AND RADIOTELEPHONY ON HALF METRE WAVES. By S. Uda. June 1930, p. 1047.

2. The Effect of Rain and Fog on the Propagation of Very Short Radio Waves. By J. A. Stratton. June 1930, p. 1064.
3. U.S.W. for Limited Range Communication. By W. J. Brown. July 1930, p. 1129.
4. Application of Frequencies above 30,000 Kc/s to Communication Problems. By H. H. Beverage and Others. Aug. 1931, p. 1313.
5. Communication with Quasi Optical Waves (in Range 0·001 mm. to 10 Metres Scattering and Absorption Permit only Use of 5 cm. to 10 m. and 0·0008 to 0·002 mm.). By E. Karplus. Oct. 1931, p. 1715.
6. Historical Review of U.S.W. Progress. By R. A. Heising. Jan. 1932, p. 77.
7. A New Field of Application for U.S.W. By E. Kramer. Nov. 1933, p. 1519.
8. Seventy-five Centimetre Radio Communication Tests. By W. D. Hershberger. July 1934, p. 870.
9. Transmission and Reception of Centimetre Waves. By I. Wolf and Others. Jan. 1935, p. 11.
10. Electrical Measurements at Ultra High Frequencies. By R. King. Aug. 1935, p. 885.
11. Thermocouple Ammeters for U.H.F. By J. H. Miller. Dec. 1936, p. 1567.
12. Concentric Narrow Band Elimination Filter. By L. M. Leeds. May 1938, p. 576.
13. A General Reciprocity Theorem for Transmission Lines at U.H.F. By R. King. May 1940, p. 223.
14. High Gain Amplifier for 150 Megacycles. By G. Rodwin. Vol. 28, June 1940, p. 257.
15. Transversal Filters. By H. E. Kallmann. July 1940, p. 302.
16. The Measurement of Coil Reactance in the 100 Megacycle Region. By F. Hamburger and C. E. Miller. Oct. 1940, p. 475.

ELECTRONICS.

17. Cathode Rays for U.H.F. By L. L. Libby. Sept. 1936, p. 15.

ELECTRICAL COMMUNICATION.

18. The Anglo-French Micro-Ray Link between Lympne and St. Inglevert. By A. G. Clavier and L. C. Gallant. Vol. 12, 1933, p. 222.
19. Micro-Ray Communication. By W. L. McPherson and E. H. Ullrich. Vol. 14, 1935, p. 1340.
20. Ultra Short Wave Communication. By E. H. Ullrich. Vol. 16, 1937, p. 64.
21. Some Problems of Hyperfrequency Technique. By A. G. Clavier and E. Rostas. Vol. 16, 1937, p. 254.

U6—U.H.F. Measurements

I.R.E. PROCEEDINGS.

1. Measurement of the Frequency of Ultra Radio Waves. By J. B. Hoag. Jan. 1933, p. 29.

2. MEASUREMENTS OF CURRENTS AND VOLTAGES DOWN TO A WAVELENGTH OF 20 CENTIMETRES. By M. J. O. Strutt. Dec. 1939, p. 783.
3. A U.H.F. MEASURING ASSEMBLY (FOR FREQUENCY MEASUREMENT). By S. Sabaroff. Feb. 1939, p. 208.

E.W. AND WIRELESS ENGINEER.

4. MEASUREMENT OF CABLE ATTENUATION AT 300 MEGACYCLES PER SECOND. Editorial by G. W. O. Howe. Feb. 1942, p. 47.

Other References.

5. A SURVEY OF U.H.F. MEASUREMENTS. By L. S. Nergaard. *R.C.A. Review*, Vol. 3, No. 2, 1938, p. 156.
6. MEASUREMENTS OF ADMITTANCES AT U.H.F. By J. M. Miller and B. Salzburg. *R.C.A. Review*, Vol. 3, No. 4, 1939, p. 486.
7. FIELD-STRENGTH MEASURING EQUIPMENT FOR WIDE BAND U.H.F. TRANSMISSION. By R. W. George. *R.C.A. Review*, Vol. 3, No. 4, 1939, p. 431.
8. FIELD STRENGTH MEASURING EQUIPMENT AT 500 MEGACYCLES. By R. W. George. *R.C.A. Review*, Vol. 5, No. 1, 1940, p. 69.

U7—U.H.F. Oscillators

I.E.E. JOURNAL.

1. ELECTRONIC OSCILLATIONS. By E. C. S. Megaw. Vol. 8, 1933, p. 59.

I.R.E. PROCEEDINGS.

2. AN EXPERIMENTAL STUDY OF REGENERATIVE U.S.W. OSCILLATORS. By W. H. Wenstrom. Jan. 1932, p. 113.
3. A NEW CIRCUIT FOR THE PRODUCTION OF U.S.W. OSCILLATIONS. By H. N. Kozanowski. June 1932, p. 957.
4. A NEW TYPE OF U.S.W. OSCILLATOR. By I. E. Mouromtseff and H. V. Noble. Aug. 1932, p. 1328.
5. MAGNETO-STATIC OSCILLATORS FOR GENERATION OF U.S.W. By G. R. Kilgore. Nov. 1932, p. 1741.

BELL SYSTEM TECHNICAL JOURNAL.

6. VACUUM TUBES AS HIGH FREQUENCY OSCILLATORS (e.g. FOR 100 Mc/s). By M. J. Kelly and A. L. Samuel. Vol. 14, 1935, p. 97.

E.W. AND WIRELESS ENGINEER.

7. THE RETARDING FIELD OSCILLATOR. By W. Alexander. April 1942, p. 143.

MARCONI REVIEW.

8. CONCENTRIC TUBE LINES (AS OSCILLATORY CONTROL CIRCUITS). By B. J. Witt. Jan.–Feb. 1936, p. 20.

Other Reference.

9. U.H.F. OSCILLATOR FREQUENCY STABILITY CONSIDERATIONS. By S. W. Seeley and E. I. Anderson. *R.C.A. Review*, Vol. 5, No. 1, 1940, p. 77.

U8—U.H.F. Propagation

I.E.E. JOURNAL.

1. The Attenuation of Short Wireless Waves at the Surface of the Earth. By G. H. Munro. Vol. 7, 1932, p. 126.
2. An Experimental Investigation of the Propagation of Radiation having Wavelengths of 2 or 3 Metres. By J. S. McPetrie and J. A. Saxton. Vol. 15, 1940, p. 153.

I.R.E. PROCEEDINGS.

3. U.S.W. Propagation. By J. C. Schelleng, C. R. Englund and Others. March 1933, pp. 427 and 464.
4. Some Characteristics of U.H.F. Transmission. By H. Muyskens and J. D. Kraus. Sept. 1933, p. 1302.
5. U.H.F. Wave Propagation over Plane Earth and Fresh Water. By R. C. Colwell and A. W. Friend. Jan. 1937, p. 32.

E.W. AND WIRELESS ENGINEER.

6. U.S.W. Propagation. By R. L. Smith-Rose and (Miss) A. C. Stickland. March 1939, p. 111.

Other References.

7. U.H.F. Propagation Formulas. By H. O. Peterson. *R.C.A. Review,* Vol. 4, No. 2, 1939, p. 162.
8. U.H.F. Propagation through Woods and Underbrush. By B. Trevor. *R.C.A. Review,* Vol. 5, No. 1, 1940, p. 97.

U9—U.H.F. Receivers

I.R.E. PROCEEDINGS.

1. On Super-Regeneration of an Ultra-Short Wave Receiver. By H. Ataka. Aug. 1935, p. 841.
2. A Uni-Control Receiver for U.H.F. using Concentric Lines as Interstage Couplers. By F. W. Dunmore. June 1936, p. 837.

ELECTRONICS.

3. U.H.F. Reception and Receivers. By B. Dudley. April 1942, p. 51.

ELECTRICAL COMMUNICATION.

4. Superheterodyne Reception of Micro-Rays. By A. H. Reeves and E. H. Ullrich. Vol. 16, 1937, p. 153.

U10—U.H.F. Transmitters

I.R.E. PROCEEDINGS.

1. Electron Oscillations without Tuned Circuits. By W. H. Moore. Aug. 1934, p. 1021.
2. Barkhausen-Kurz Oscillator Operation with Positive Plate Potentials. By L. F. Dytrt. March 1935, p. 241.
3. Development of Transmitters for Frequencies above 300 Mc/s. By N. E. Lindenblad. Sept. 1935, p. 1013.

4. A Negative Grid Triode Oscillator and Amplifier for U.H.F. (e.g. 100 to 1,000 Mc/s). By A. L. Samuel. Oct. 1937, p. 1243.
5. Stability of Two-Metre Waves. By C. R. Burrows and Others. May 1938, p. 516.
6. Generation and Utilization of U.S.W. in Radio Communication. By F. A. Kolster. Dec. 1934, p. 1335.

ELECTRONICS.

7. (Transmitters Using Resonant Line Tank Circuits.) By C. J. Franks. Oct. 1935, p. 33.
8. Spherical Tank U.H.F. Oscillator. By H. E. Hollman. Dec. 1938, p. 27.
9. A U.H.F. Power Amplifier of Novel Design. By A. V. Haeff. Feb. 1939, p. 30.

ELECTRICAL COMMUNICATION.

10. Ultra Short Wave Oscillators. By D. H. Black. Vol. 17, 1938, p. 325.

Other References.

11. U.H.F. Equipment for Relay Broadcasting. By W. A. R. Brown. *R.C.A. Review*, Vol. 3, No. 2, 1938, p. 133.
12. A 500 Megacycles Radio Relay Distribution System for Television. By F. H. Kroger, B. Trevor and J. E. Smith. *R.C.A. Review*, Vol. 5, No. 1, 1940, p. 31.

U11—U.H.F. Valves

I.R.E. PROCEEDINGS.

1. Vacuum Tube Electronics at U.H.F. By F. B. Llewellyn. Nov. 1933, p. 1532.
2. Vacuum Tubes of Small Dimensions for Use at Extremely High Frequencies. By B. J. Thompson and G. M. Rose. Dec. 1933, p. 1707.
3. Phase Angle of Vacuum Tube Transconductances at Very High Frequencies. By F. B. Llewellyn. Aug. 1934, p. 947.
4. Vacuum Tubes for Generating Frequencies above 100 Mc/s. By C. E. Fay and L. M. Samuel. March 1935, p. 199.
5. The Operation of Electron Tubes at High Frequencies. By H. Rothe. July 1940, p. 325.

V1—Valves

I.E.E. JOURNAL.

1. The Holweck Demountable Type Valve. By C. F. Elwell. Vol. 2, 1927, p. 155.
2. Continuously Evacuated Valves and Their Associated Equipment. By C. R. Burch and C. Sykes. Vol. 10, 1935, p. 111.

I.R.E. PROCEEDINGS.

3\. . . . Constant Current Valve Characteristics. July 1935, p. 752.

4. A Direct-Capacity Bridge for Vacuum Tube Measurements. By L. Walsh, 1928, p. 482.
5. A Bridge Method for the Measurement of Inter-Electrode Admittance in Vacuum Tubes. By E. T. Hoch. 1928, p. 487.
6. Use of an Oscillograph for Recording Valve Characteristics. By W. A. Schneider. 1928, p. 674.
7. The Measurement of Direct Interelectrode Capacitance of Vacuum Tubes. By A. V. Loughren and H. W. Parker. June 1929, p. 957.
8. Equivalent Circuits of a Triode. By E. L. Chaffee. Sept. 1929, p. 1633.
9. Power Output Characteristics of the Pentode. By S. Ballantine and H. L. Cobb. March 1930, p. 450.
10. Graphical Analysis of Output Tube Performance. By C. E. Kilgour. Jan. 1931, p. 42.
11. Vacuum Tube Characteristics in the Positive Grid Region by an Oscillograph Method. By H. N. Kozanowski and I. E. Mouromtseff. Aug. 1933, p. 1082.
12. Harmonic Production and Cross Modulation in Thermionic Valves with Resistive Loads. By D. C. Espley. June 1934, p. 781.
13. Recent Developments in Miniature Tubes (Acorn Type). By B. Salzberg and D. G. Burnside. Oct. 1935, p. 1142.
14. This Matter of Contact Potential! By R. M. Bowie. Nov. 1935, p. 1501.
15. Electron Beams and Their Application in Low Voltage Devices. By H. C. Thompson. Oct. 1936, p. 1276.
16. Oscillograph Method for Measuring Positive Grid Characteristics. By O. W. Livingstone. Vol. 28, 1940, p. 267.
17. Water and Forced Air Cooling of Vacuum Tubes. By I. E. Mouromtseff. April 1942, p. 190.

BELL SYSTEM TECHNICAL JOURNAL.

18. Thermionic Vacuum Tubes and Their Application. By R. W. King. Vol. 2, 1923, p. 31.
19. Theory of Multi-Electrode Vacuum Tubes. By H. A. Pidgeon. Vol. 14, 1935, p. 44.
20. Equivalent Networks of Negative Grid Vacuum Tubes at Ultra High Frequencies. By F. B. Llewellyn. Vol. 15, 1936, p. 575.

E.W. AND WIRELESS ENGINEER.

21. The Algebraic Representation of Triode Valve Characteristics. By W. A. Barclay. April 1929, p. 178.
22. The Triode Equivalent Network. By F. M. Colebrook. Sept. 1929, p. 486.
23. The Variation of the Resistance and Interelectrode Capacities of Thermionic Valves with Frequency. By L. Hartshorne. Aug. 1931, p. 413.
24. Electron Transit Time. By B. E. Benham. Dec. 1939, p. 598.

ELECTRONICS.

25. MEASUREMENT OF VACUUM IN RADIO TUBES. By M. D. Sarbey. April 1921, p. 594.
26. RADIO TUBES AS PHOTOCELLS. By W. P. Koechal. Dec. 1932, p. 373.
27. MEASURED INPUT LOSSES OF VACUUM TUBES. By C. J. Franks. July 1935, p. 223.
28. COOLING TUBES WITH FORCED AIR DRAFT. By E. M. Osteund. June 1940, p. 36. (*See also* T11/6 above.)

V2—Valve Input Admittance

I.R.E. PROCEEDINGS.

1. AN ANALYSIS OF ADMITTANCE NEUTRALIZATION BY MEANS OF NEGATIVE TRANSCONDUCTANCE VALVES. By E. W. Herald. Nov. 1927, p. 1399.
2. INPUT RESISTANCE OF VACUUM TUBES AS ULTRA HIGH FREQUENCY AMPLIFIERS. By W. R. Ferris. Jan. 1936, p. 82.
3. THE CAUSES FOR THE INCREASE OF THE ADMITTANCE OF MODERN HIGH FREQUENCY AMPLIFIER TUBES ON SHORT WAVES. By M. J. C. Strutt and A. Van der Ziel. Aug. 1938, p. 1011.

ELECTRONICS.

4. A SOURCE OF LOSS IN HIGH FREQUENCY VALVE CIRCUITS. By St. Clair Finlay. May 1924, p. 469.
5. CIRCLE DIAGRAMS OF VALVE INPUT ADMITTANCE. By F. M. Colebrook. Dec. 1933, p. 657.
6. INPUT CONDUCTANCE. By F. Preisach and I. Zakarias. April 1940, p. 147.
7. INPUT CONDUCTANCE NEUTRALIZATION. By R. L. Freeman. Oct. 1939, p. 23.

V3—Voltage Breakdown

I.E.E. JOURNAL.

1. THE ELECTRIC STRENGTH OF AIR AT HIGH FREQUENCIES. By E. W. Seward. Vol. 14, 1939, p. 31.

I.R.E. PROCEEDINGS.

2. RADIO FREQUENCY SPARKOVER IN AIR. By P. A. Ekstrand. Vol. 28, 1940, p. 262.

ELECTRICAL COMMUNICATION.

3. R.F. HIGH VOLTAGE PHENOMENA. By S. Pickles. Oct. 1939, p. 135.

Other Reference.

4. SPARKING POTENTIAL (ITS DEPENDENCE ON TEMPERATURE). By E. F. Burton. *Phil. Mag.*, Jan. 1926, p. 219. (*See also* VI/3 above.)

V4—Volume Compression and Expansion

I.R.E. PROCEEDINGS.

1. PROGRAMME-OPERATED LEVEL-GOVERNING AMPLIFIER. By W. L. Black. Nov. 1941, p. 573.

BELL SYSTEM TECHNICAL JOURNAL.

2. The Compandor. By A. C. Mathers and S. B. Wright. Vol. 13, 1934, p. 315.
3. Amplitude Range Control. By S. B. Wright. Vol. 17, 1938, p. 520.

E.W. AND WIRELESS ENGINEER.

4. The Effect of Volume Compression on the Tolerable Noise Level. By E. L. E. Pawley. Jan. 1937, p. 12.
5. Low Distortion Volume Expansion. By B. J. Stevens. March 1938, p. 143.

ELECTRONICS.

6. Practical Volume Expansion. By C. M. Sinnett. Nov. 1935, p. 15.
7. An Automatic Volume Expander. By W. N. Weeden. June 1935, p. 185.
8. Practical Volume Compression. By L. B. Hallman. June 1936, p. 15.

MARCONI REVIEW.

9. Voice Frequency Volume Control. By H. Jefferson and C. D. Colchester. No. 72, 1939, p. 15.

W1—Wave Guides

I.E.E. JOURNAL.

1. Electromagnetic Waves in Metal Tubes of Rectangular Cross Section. By J. Kemp. Vol. 88, 1941, p. 213.

I.R.E. PROCEEDINGS.

2. Transmission of Electromagnetic Waves in Hollow Tubes of Metal. By W. L. Barrow. Oct. 1936, p. 1298.
3. Some Fundamental Experiments with Wave Guides. By G. C. Southworth. July 1937, p. 807.
4. Rectangular Hollow Pipe Radiators. By W. L. Barrow and F. M. Greene. Dec. 1938, p. 1498.
5. Electromagnetic Waves in Hollow Metal Tubes of Rectangular Cross Section. By L. J. Chu and W. L. Barrow. Dec. 1938, p. 1520.
6. Damped Electromagnetic Waves in Hollow Metal Pipes. By A. W. Melloh. April 1940, p. 179.

BELL SYSTEM TECHNICAL JOURNAL.

7. Hyper-Frequency Wave Guides—General Considerations and Experimental Results. By G. C. Southworth. Vol. 15, 1936, p. 284.
8. Hyper-Frequency Wave Guides—Mathematical Theory. By J. A. Carson, S. P. Meade and S. A. Schelkunoff. Vol. 15, 1936, p. 310.

E.W. AND WIRELESS ENGINEER.

9. Electromagnetic Waves in Rectangular Metal Tubes. Editorial by G. W. O. Howe. March 1942, p. 93.

ELECTRICAL COMMUNICATION.

10. General Properties of Dielectric Guides. By J. Saphores. Vol. 16, 1937, p. 346.
11. Theoretical Study of Dielectric Cables. By L. Brillouin. Vol. 16, 1937, p. 350.
12. Theoretical Relationships of Dielectric Guides (Cylindrical) and Coaxial Cables. By A. G. Clavier. Vol. 17, 1938, p. 276.
13. Experimental Researches on the Propagation of Electromagnetic Waves in Dielectric (Cylindrical) Guides. By A. G. Clavier and V. Altovsky. Vol. 18, 1939, p. 81.
14. Hyper-Frequency Waves and Their Practical Use. By G. L. Brillouin. Vol. 19, 1940, p. 118.

W2—Wave Resonance Tuning

I.R.E. PROCEEDINGS.

1. Circuit Tuning by Wave Resonance. By L. Cohen. Oct. 1929, p. 1868.
2. Wave Resonance Tuning. By W. R. Blair and L. Cohen. Oct. 1929, p. 1893.

W3—Wide Band Amplifiers

1. A Wide Band Video Amplifier for a C.R.O. By A. Preisman. *R.C.A. Review*, Vol. 3, No. 4, 1939, p. 473.

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The numbering of sections and subsections of chapters is normally on the decimal principle, e.g. 5.1 is a subsection of section 5; subsubsection 5.14 is the fourth subdivision of subsection 5.1, etc.

When, however, it is required to divide section 5 into, say, 14 subsections, the subsection numbering after subsection 5.9 is: 5.10, 5.11, 5.12, 5.13, 5.14. Such numbering is rare, but it introduces an element of ambiguity. All ambiguity can however be quickly cleared by reference to the contents list for the chapter in question.

Example: Suppose the reference is to XVIII:5.22.11. Reference to the contents list for chapter XVIII shows that 5.22.11 occurs immediately after 5.22.10 which follows 5.229, the last being a normal decimal reference.

Any reference to a chapter evidently includes all sections; similarly, any reference to a section includes all subsections of that section.

Reference to the introductory matter at the beginning of a chapter, which sometimes is introduced before section 1, is effected by means of the symbol '0', e.g. VI:0 refers to the beginning of chapter VI.

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