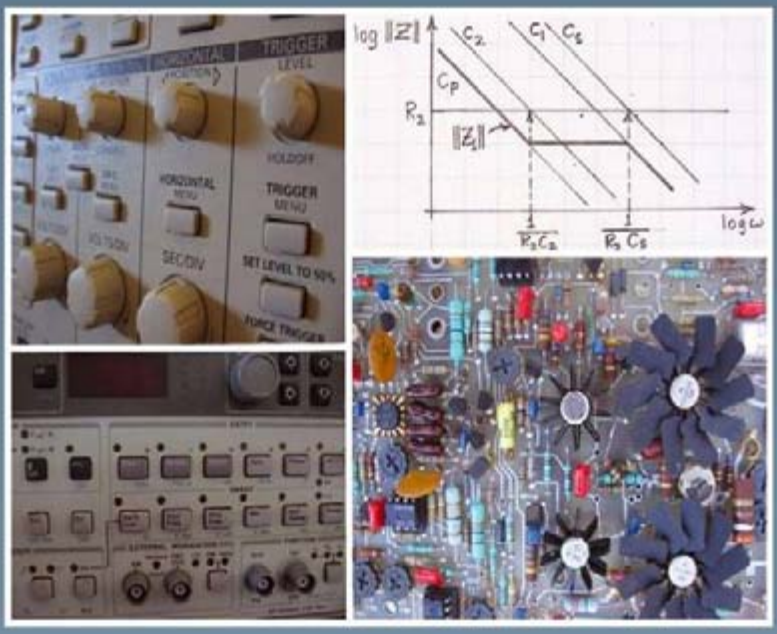
Volume 1

Analog Circuit Design Series

DESIGNING AMPLIFIER CIRCUITS



D. Feucht

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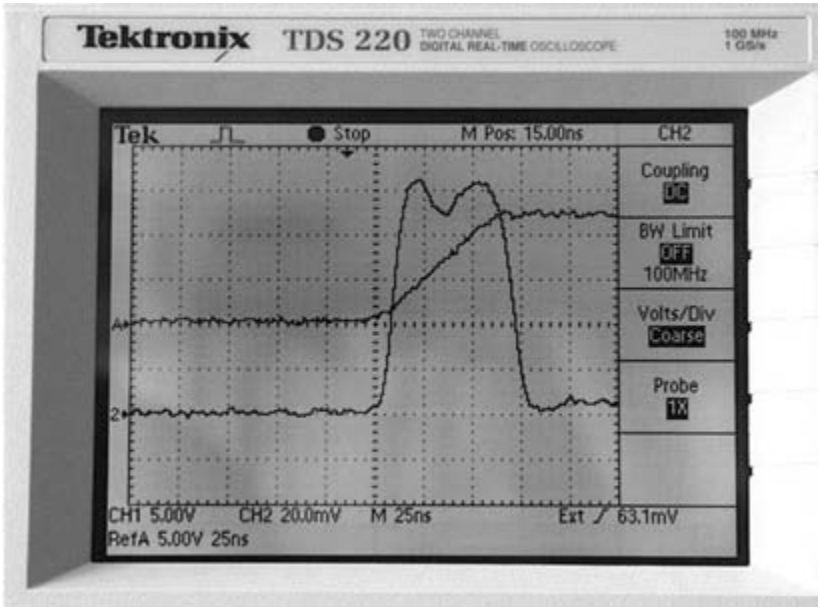

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Contents

Preface

Solid-state electronics has been a familiar technology for almost a half century, yet some circuit ideas, like the transresistance method of finding amplifier gain or identifying resonances above an amplifier's bandwidth that cause spurious oscillations, are so simple and intuitively appealing that it is a wonder they are not better understood in the industry. I was blessed to have encountered them in my earlier days at Tektronix but have not found them in engineering textbooks. My motivation in writing this book, which began in the late 1980s and saw its first publication in the form of a single volume published by Academic Press in 1990, has been to reduce the concepts of analog electronics as I know them to their simplest, most obvious form, which can be easily remembered and applied, even quantitatively, with minimal effort.

The behavior of most circuits is determined most easily by computer simulation. What circuit simulators do not provide is knowledge of what to compute. The creative aspect of circuit design and analysis must be performed by the circuit designer, and this aspect of design is emphasized here. Two kinds of reasoning seem to be most closely related to creative circuit intuition:

1. Geometric reasoning: A kind of visual or graphic reasoning that applies to the topology (component interconnection) of circuit diagrams and to graphs such as reactance plots.
2. Causal reasoning: The kind of reasoning that most appeals to our sense of understanding of mechanisms and sequences of events. When we can trace a chain of causes for circuit behavior, we feel we understand how the circuit works.

These two kinds of reasoning combine when we try to understand a circuit by causally thinking our way through the circuit diagram. These insights, obtained

by *inspection*, lie at the root of the quest. The sought result is the ability to write down accurate circuit equations by inspection. Circuits can often be analyzed multiple ways. The emphasis of this book is on development of an intuition into how circuits work with a perspective that can be applied more generally to circuits of the same class.

In this first volume of the Analog Circuit Design series, basic transistor amplifier circuits are given a design-oriented analysis, using the simple but effective T model of the bipolar junction transistor (BJT) and field-effect transistor (FET). It is delightful to be able to write down from inspection rather involved gain and port impedances that, when evaluated, give accuracies comparable to SPICE simulations. *Designing Amplifier Circuits* remains focused on quasistatic (low-frequency ac) analysis and leaves the additional complication of reactance and dynamic analysis for succeeding volumes.

Consequently, feedback analysis – a topic that I never found satisfactory treatment of in textbooks – is presented with insights and from angles that hopefully will reduce it to analysis by inspection for readers. Some circuit transformations that I call the β transform and the μ transform, its dual, are especially helpful in reducing circuits to simpler forms for analysis. They are usefully applied in considering transistor circuits for which collector-emitter (or drain-source) resistance is not negligible, a topic often omitted in the coverage of amplifier circuits.

Coverage of the list of basic amplifier stages, including two-transistor combinations and their interactions when connected, results in enough material for a book – this book.

Much of what is in this book must be credited in part to others from whom I picked up essential ideas about circuits at Tektronix, mainly in the 1970s. I am particularly indebted to Bruce Hofer, a founder of Audio Precision Inc.; Carl Battjes, who founded and taught the Tek Amplifier Frequency and Transient Response (AFTR) course; Laudie Doubrava, who investigated power supply topics; and Art Metz, for his clever contributions to a number of designs, some extending from the seminal work on translinear circuits by Barrie Gilbert, also at Tek at the same time. Then there is Jim Woo, who, like Battjes, is another oscilloscope vertical amplifier designer; Ian Getreu and Bob Nordstrom, from whom I learned transistors; and Mike Freiling, an artificial intelligence researcher in Tektronix Laboratories whose work in knowledge

representation of physical systems influenced my broader understanding of electronics.

In addition, in no particular order, are Fred Beckett, Lee Jalovec, Wayne Kelsoe, Cal Diller, Marv LaVoie, Keith Lofstrom, Peter Staric, Erik Margan, Tim Sauerwein, George Ermini, Jim Geddes, Carl Hollingsworth, Chuck Barrows, Dick Hung, Carl Matson, Don Hall, Phil Crosby, Keith Ericson, John Taggart, John Zeigler, Mike Cranford, Allan Plunkett, Neldon Wagner, and Paul Magerl. These and others I have failed to name have contributed personally to my knowledge as an engineer and indirectly to this book. Most of all, I am indebted to the creator of our universe, who made electronics possible. Any errors or weaknesses in this book, however, are my own.

1

Electronic Design

Electronic Design

Design is a creative activity that begins with a definition of the problem to be solved or specification of the device to be built. Solving the problem or specifying the device in enough detail to build it is the goal of the designer. Usually more than one alternative solution or design is possible. Sometimes they are already known, and the problem consists mainly in adapting a known general solution to a particular application for it. This is "standard engineering" practice.

Other problems have no known solution and require a novel search, which can include novel adaptation of existing solutions to similar problems. This is "state-of-the-art" engineering, sometimes called engineering research and development (R&D). When a solution is found, it is then refined and specified for use.

Design is largely a matter of achieving a desired function within given behavioral constraints. Therefore, a significant aspect of electronics circuit design skill is the ability to understand how circuit constraints affect desired function. Some analytic techniques, especially those best executed by computer, give the designer little insight into the relationship between circuit structure and function. The techniques developed here are intended to provide maximum insight into circuit function and how it relates to circuit behavior. Unlike the specific analysis that computer simulators perform on a specific circuit, design is the search among many circuits for the optimal one. Besides methods of analysis, a designer must be familiar with many existing circuits that can be used in a design.

In R&D projects, not enough is known about the detailed hierarchical levels of complexity of a design to proceed purely topdown from the system specifica-

tion. Experimentation with circuits and components is often necessary. When the details are adequately understood, the system-level design can then be clarified. Complexity is handled in electronics (as in software design) through *modularity*. A *module* is a subsystem that can be defined purely in terms of its interactions with other subsystems. Instead of passing objects, parameters, or pointers to data structures as in software, electrical connections are made between input and output ports of modular subsystems. Just as software parameter passing must be done according to a protocol, electrical connections between modules must take into account module interactions such as impedance matching, dynamic range, and loading effects.

Two kinds of reasoning in electronics seem to be most closely related to creativity in electronic design:

1. *Geometric reasoning*: This is a kind of visual or graphical reasoning that applies to circuit diagrams and to graphs.
2. *Causal reasoning*: This kind of reasoning most appeals to our sense of understanding of mechanisms. When we can trace a sequence of causes for circuit behavior, we intuit an understanding of how the circuit works.

These two kinds of reasoning combine when we try to understand a circuit by causally thinking our way through the circuit diagram. Because they involve seeing the whole in the parts, they rely extensively on intuitive insight.

Product Development

Electronic design is often executed according to an overall plan. If the problem must be solved many times, the device that solves the problem is manufactured. The process of creating devices and specifying them for manufacture is often called *product development*. The major steps in development are given next.

Product Development Process

1. *Concept phase:* Clarify the idea for a new product with a one-page description of it and a quickly built functional prototype device that demonstrates the product idea. This phase is completed upon product or *project approval*.
2. *Design phase:* Specify the performance parameters of the new product and design the product more carefully to meet the specifications. Build a few

models of this design, characterize their performance by testing, and refine the design to meet specifications where performance is inadequate. This phase ends upon *design completion.*

3. *Evaluation phase:* Evaluate the design more extensively by building several units of the product using the materials and processes that will be used in manufacturing, and testing them rigorously in the laboratory for performance under all anticipated operating conditions. Design refinement proceeds until the design meets the specification or the models cannot be modified further yet continue to embody the design. This phase is completed at *prototype release.*
4. *Verification phase:* Verify that the design meets performance and reliability specifications under the conditions of its intended use. (This is also called *field testing.*) Build a statistically significant number of units and subject them to environmental testing. Refinements to the design in this phase should be minimal and testing maximal. This phase is completed when all documentation that specifies the design for manufacture is acceptable at *engineering release* (or, to manufacturing personnel, "manufacturing acceptance").

After this, a pilot or *pre-production run* – the manufacture of a batch of product units using the design documentation – is carried out by manufacturing personnel to test the documented design for production flaws. Engineers may be required to correct these flaws.

Design-Driven Analysis

The behavior of circuits is determined most easily by computer simulation. Simulators *analyze* circuits but do not determine *what* to compute. They do not *design.* Simulator results apply to particular circuits. The kind of analysis that is useful for design is more general. Parameters in equations can take on a range of possible design values. We need methods for easily writing down such equations from schematic diagrams.

Managing Complexity

Electronic *systems* are often so complex that we cannot think about all the details at once. Systems are often organized into hierarchies consisting of levels of manageable complexity. Electronic systems can be described by a multilevel

hierarchy of concepts. At the most concrete level are the physical circuits themselves, represented commonly by a schematic (wiring) diagram or netlist. These are *structural* descriptions of the circuit. From these, various electrical (and thermal or mechanical) behaviors are deduced through a causal theory of circuits, by analysis. When analyzed, a *behavioral* description of the circuit results. At the next more abstract level of description, these behaviors are explained in terms of a *functional* theory that leads to a functional description.

Three Levels of Description in Electronics

- A *structural* description (schematic diagram) of a circuit describes what it is.
- A causal or *behavioral* description (waveforms) of a circuit describes what it does.
- A *functional* description (specification) of a circuit describes what it is for.

(For a software-oriented description of electronics, see de Kleer, 1985, pp. 205–280.) Each of these descriptions may be complex enough to require a hierarchical organization. For example, a structural description of a system consisting of hundreds of parts is too unwieldy to handle directly. Systems are consequently organized into *subsystems*, graphically described by a *block diagram*. These subsystems consist of *circuits* that, in turn, are composed of circuit *elements*, which are *components* in actual circuits. It is common for electronic systems to be structurally described by this kind of three-level hierarchy.

Electronics System Hierarchy

- *Subsystems* consist of circuits.
- *Circuits* consist of elements.
- *Elements* are idealized components.

Structural descriptions are often presented in a way that makes the causal and functional descriptions explicit. Block diagrams not only show which parts are grouped together but also represent various subsystem functions that help to show the overall function of the system.

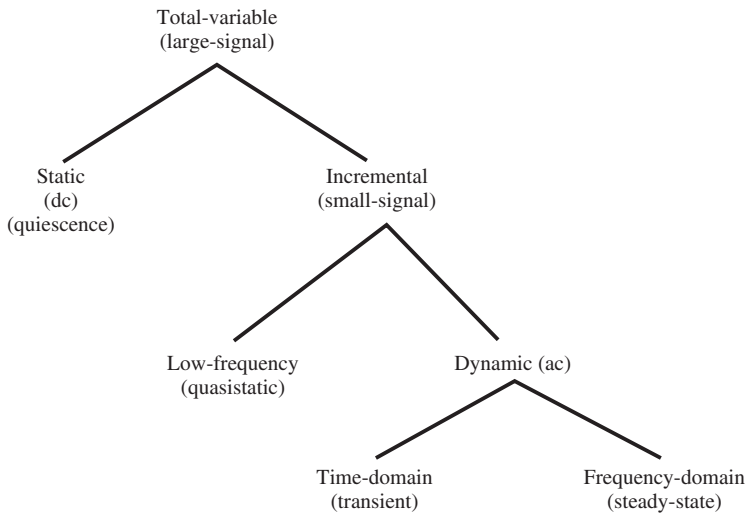
Design begins with a functional description or *specification* of the goals that the designed system is intended to accomplish. It describes function in terms of how the device or system should behave. The designer converts the specifica-

tion into a structural specification of the system that achieves the goal. Analysis is about how to go from a given structure to its behavior. Design goes from function to structure. Because function is described in the language of behavior, the relationship between behavior and structure is also a major aspect of design.

Nonlinear Circuit Analysis

Nonlinear systems can be analyzed by either solving for the desired result from the nonlinear (total-variable) model and linearizing the result or by first linearizing the nonlinear elements of the system and then solving it as a linear system. The second method is often easier.

Nonlinear circuits result from nonlinear devices, and solid-state devices are nonlinear. To analyze them, we use linear approximations to their behavioral models. With a linear model, the well-established techniques of linear circuit analysis can be applied. Nonlinear device models are often linearized by selecting an *operating point* for the device. Constant values for model variables are chosen, and small variations around those *quiescent* values are analyzed. The *total-variable* device model includes both the fixed operating point and the behavior due to small changes around it. The structure of this kind of analysis is shown below.



Linear approximation is valid as long as the excursions from the operating point are small or *incremental*. Incremental models are also called *small-signal* models. In contrast the total-variable (large-signal) model is the exact (nonlinear) model and does not depend on an operating point to be valid.

Static (dc) and quasistatic (low-frequency) quantities and behavior are different for nonlinear devices. To illustrate the difference between static and quasistatic device behavior, consider the voltage-current (v–i) relationship for a diode:

$$i = I_S\left(e^{v/V_T} - 1\right)$$

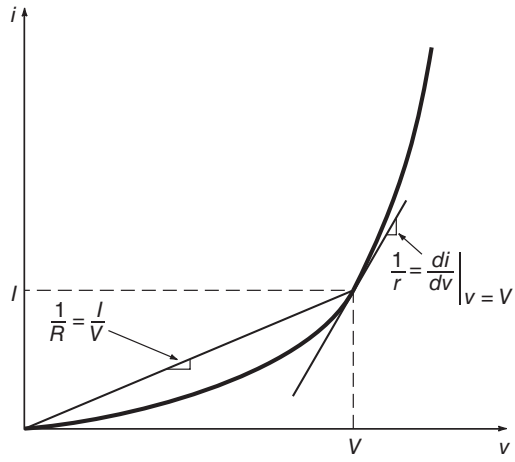
V_T is the *thermal voltage*, defined as

$$V_T = \frac{kT}{q_e}$$

where k = Boltzmann's constant, q_e = electron charge, and T = absolute temperature.

$$V_T \cong 26\,\text{mV at } 298\,\text{K } (25°\text{C})$$

The diode current is plotted below as a function of its voltage.



For a fixed operating point, $Q = (V, I)$, and the *static* (or dc) resistance of the diode is

$$R = \frac{V}{I} = \frac{V_T \ln(i/I_S)}{i}, \quad i >> I_S$$

This is shown as 1/slope of the line from the origin to Q. For a typical silicon diode operating at 1 mA, $Q = (0.6\,\text{V}, 1\,\text{mA})$ and $R = 0.6\,\text{V}/1\,\text{mA} = 600\,\Omega$.

For small excursions around Q, R varies slightly. A linear approximation to $i(v)$ at Q is a line tangent to the curve. Its 1/slope is

$$r = \frac{dv}{di} \cong \frac{\Delta V}{\Delta I} \cong \frac{V_T}{i}, \quad i >> I_S$$

This is the *quasistatic* (or low-frequency) resistance of the diode at Q. A small change in v will result in a small change in i of about $\Delta V/r$. For 1 mA operation,

$$r = 26\,\text{mV}/1\,\text{mA} = 26\,\Omega$$

and is considerably less than the static resistance. For a linear device such as a resistor, small- and large-signal behavior is identical for both small and large variations in variables.

The two circuit analyses just shown are static, or *dc*, and small-signal, low-frequency, or *quasistatic*. They correspond to two major aspects of circuit design.

Circuit Design Procedure

1. *Static design*: Set the nonlinear devices to operate at the desired operating point, or *bias*.
2. *Quasistatic design*: Determine circuit parameters to achieve low-frequency performance, such as amplification, and input and output resistances.
3. *Dynamic design*: Determine circuit parameters for desired dynamic response, such as risetime and bandwidth.

Quasistatic analysis, which involves only real-number mathematics, is sometimes confusingly referred to as dynamic analysis in electronics literature. More

correctly, it is usually called *low-frequency* analysis. Quasistatic (low-frequency) and dynamic analyses are *not* the same, and they are distinguished here.

Dynamic analysis applies to circuits with reactive elements (inductance and capacitance). Analyses involving reactive elements use imaginary numbers. Circuits with a combination of reactive and resistive (dissipative) elements are described with complex numbers. The unifying quantity is the variable *s*, the *complex frequency*. The real part of *s* in the analysis involves response in time (the *transient response*), which decays to zero at infinite time. The imaginary part is the frequency response. Together, they result in the total dynamic response. These analyses are done on linearized functions at an operating point.

The nomenclature of electronics literature uses upper case characters to stand for mathematical constants, which are static quantities (as in the diode example above), lower-case characters for quasistatic variables, and upper case characters with lower case subscripts for dynamic quantities in the *s*-domain. Total-variable quantities are represented by lower case characters with upper case subscripts. In this volume the nomenclature is simplified somewhat. When a distinction is useful between static and quasistaic variables, the upper case and lower case distinction will be retained. For dynamic quantities, however, either may be used and the domain specified (as *s* or *t*) in functional notation. Standard device modeling terminology is retained.

2

Amplifier Circuits

Bipolar Junction Transistor T Model

For a bipolar junction transistor (BJT), the quasistatic resistance of the base-emitter junction under forward bias is

$$r_e = \frac{dv_{BE}}{di_E} = \frac{v_{be}}{i_e}$$

BJT transconductance, g_m, can be inverted as a *transresistance*,

$$r_m = \frac{1}{g_m} = \frac{dv_{BE}}{di_C} = \frac{v_{be}}{i_c}$$

For the BJT, static current gain is defined as

$$\beta_0 = \frac{I_C}{I_B}$$

and quasistatic current gain as

$$\beta = i_c / i_b$$

By Kirchhoff's current law (KCL),

$$i_e = i_c + i_b$$

Then, combining r_e and β,

$$r_e = \frac{v_{be}}{i_e} = \frac{v_{be}}{(\beta+1)\cdot i_b}$$

and

$$r_m = \frac{v_{be}}{i_c} = \frac{v_{be}}{\beta \cdot i_b}$$

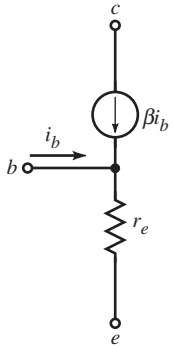
Substituting the previous equation into r_e results in

$$r_e = \left(\frac{\beta}{\beta+1}\right)\cdot r_m = \alpha \cdot r_m$$

where

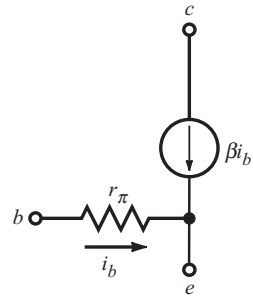
$$\alpha = \frac{i_c}{i_e} = \frac{\beta}{\beta+1}$$

β and r_e are quasistatic BJT parameters that are used in a simple quasistatic BJT model, the *T model*, shown in the following figure.



THE β TRANSFORM

Another commonly used BJT model is the hybrid-π model. It differs from the T model in that it has a base resistance, r_π, instead of the T-model emitter resistance, r_e.



The two models are equivalent. The T-model emitter resistance r_e is related to the hybrid-π base resistance r_π. It is one of the more interesting transistor relationships. Both r_e and r_π are across the same nodes, base and emitter. It might seem at first that they must be the same resistance. They differ, however, in the connection of the collector current source. In the hybrid-π model, it is connected to the emitter, whereas in the T model, it is connected to the base. Consequently, both base and collector current flow through r_e, but only base current flows through r_π. By definition,

$$r_\pi = \frac{v_{be}}{i_b}$$

From the definition of β,

$$i_e = (\beta + 1) \cdot i_b$$

v_{be} causes $(\beta + 1)$ times as much current to flow through r_e as r_π. With $(\beta + 1)$ times as much current flowing in the emitter as in the base for the same applied voltage, the resistance on the base side of the base-emitter loop can be transformed into an equivalent emitter resistance by the β transform.

β Transform

BJT base resistance, R_B, can be referred to the emitter as r_E:

$$r_E = \frac{R_B}{(\beta + 1)}$$

Similarly, emitter resistance, R_E, can be transformed to an equivalent base resistance:

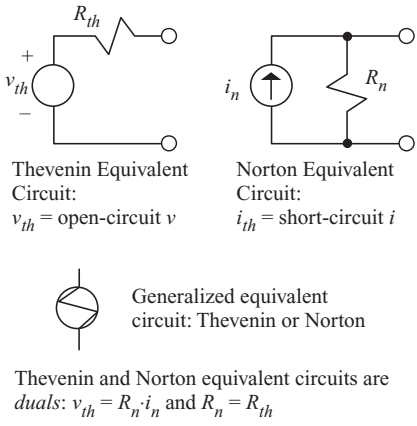
$$r_B = (\beta + 1) \cdot R_E$$

This transform is extremely useful in transistor circuit analysis. It lets us place all resistances on either the base or emitter side of a circuit loop containing the base-emitter junction. This results in elimination of one of the variables i_b or i_e from the analysis.

TWO-PORT NETWORKS

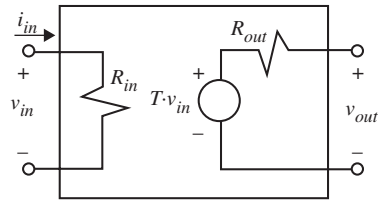
An electrical *port* is a pair of terminals of a network. The terminals are marked for polarity of both port voltage and current. Current into the positive terminal is defined as positive.

A *two-port network* has two ports. The circuitry at each port can be represented by either a Thevenin or Norton equivalent circuit, as shown below. All networks can be reduced to one or the other of these equivalent circuits, which themselves are duals.



To represent amplifiers as two-port networks, one port is designated as the input and the other as the output. The output-port source has a value $T \cdot x_{in}$ dependent on (or *controlled* by) an input port quantity x_{in} (where x is either voltage or current). T is the *transmittance* or *amplification* or the *gain*. The two-port network shown below depicts voltage amplification.

Amplifier represented as a two-port network



The controlling variables of port output sources are the voltages or currents of the input port. The behavior of a two-port network is fully determined by its port quantities and network parameters.

Transmittance, T, can be one of four kinds, based on the current and voltage combinations of the two ports:

$$\text{Voltage gain} = A_v = v_{out}/v_{in}$$

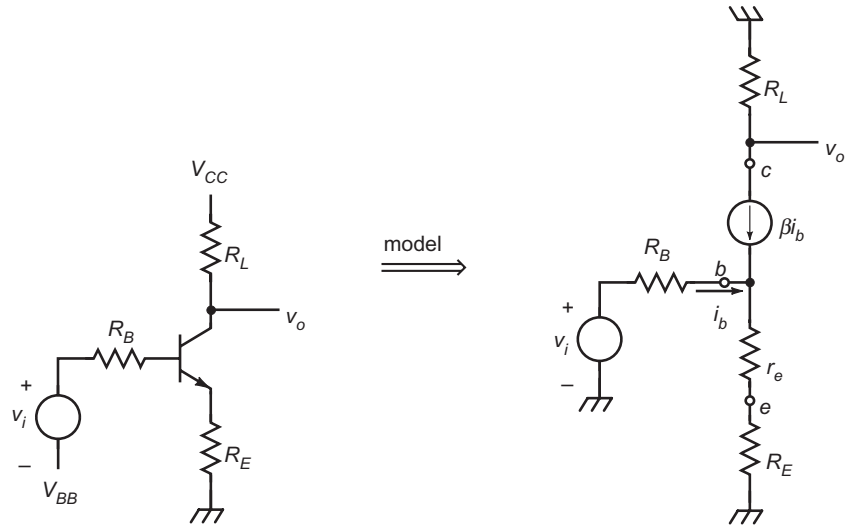
$$\text{Current gain} = A_\text{i} = i_{out}/i_{in}$$

$$\text{Transresistance} = R_\text{m} = v_{out}/i_{in}$$

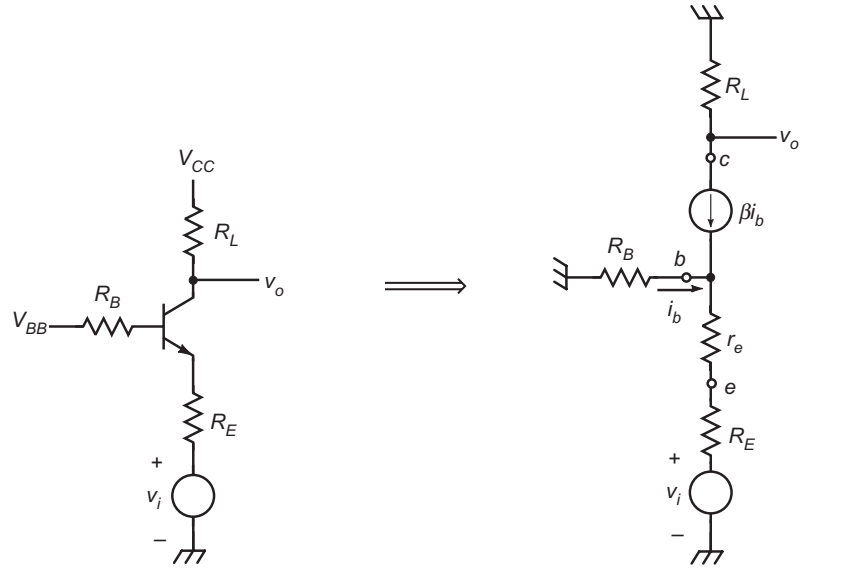
$$\text{Transconductance} = G_\text{m} = i_{out}/v_{in}$$

Amplifier Configurations

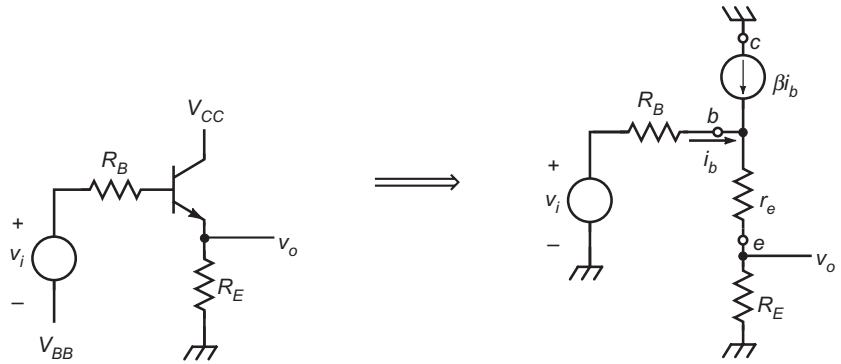
A single transistor can be configured as an amplifier in three ways when viewed as a two-port network with a common terminal. As a three-terminal device, one of the transistor terminals must be common to both input and output circuits, resulting in three basic transistor amplifier configurations. For BJTs, they are as shown below.



Common-emitter (CE) configuration



Common-base (CB) configuration



Common-collector (emitter-follower) configuration

Equivalently, for field-effect transistors (FETs), the three configurations are

Common source (CS)

Common gate (CG)

Common drain (or source-follower) (CD)

The circuit models shown above involve only quasistatic changes.

The total variables for input and output voltages,

$$v_I = V_{BB} + v_i; \quad v_O = V_{CC} + v_o$$

are replaced by incremental variables v_i and v_o, respectively. The bias supplies, V_{BB} and V_{CC}, set the operating point, Q. Static analysis yields emitter current, from which r_e can be calculated. Also, β varies somewhat with I_E in actual transistors. For now, we will use the simple T model, for it can produce accurate quasistatic gain and resistances if its two parameters are accurate. r_e is approximately constant when $i_e \ll I_E$, for

$$r_e \cong \frac{V_T}{|I_E|}$$

Of major interest in the static analysis, which is based on static circuit quantities, is not only the operating point but also its stability. With significant operating-point change, incremental parameters can vary too widely, resulting

in unacceptable performance. This is caused by *temperature drift* and changing values of *aging* components. The goal is to minimize sensitivity of the operating point to component-value variations.

THE TRANSRESISTANCE METHOD

The quantities of usual interest about an amplifier are its amplification (or gain) and its input and output resistances. The three transistor configurations can be analyzed using a common procedure. More complex circuits can also be analyzed by the same procedure by decomposing them into the three basic configurations.

The procedure is based on identification of two circuit loops or nodes, one relating to the input and the other to the output. In the common-emitter (CE) configuration, the input loop consists of v_i, R_B, r_e, and R_E. The currents that flow in this loop, i_b and i_e, are caused by the input voltage source v_i. Similarly, the output loop consists of R_L, the $\beta \cdot i_b$ current source, r_e, and R_E. The associated currents are i_c and i_e. As a CE circuit, i_e is common to both input and output loops and is the key to relating input to output. The procedure – the *transresistance method* – is as follows.

Transresistance Method

1. Refer all input circuit quantities to a common terminal by use of the β transform. Calculate a variable common to both input and output circuits.
2. Calculate the output from the common variable and output circuit quantities.

The effect of this procedure is to calculate forward from the input source to the output. Consider again the CE amplifier.

Step 1: By referring R_B to the emitter side of the circuit using the β transform, it becomes an emitter resistance of value $R_B/(\beta+1)$. Then calculate the common variable i_e as

$$i_e = \frac{v_i}{R_B/(\beta+1) + r_e + R_E}$$

Step 2: The output quantity v_o is

$$v_o = -R_L \cdot i_c = -R_L \cdot \alpha \cdot i_e$$

Substituting the variable common to both input and output, i_e, from the previous equation,

$$A_v(\text{CE}) = \frac{v_o}{v_i} = -\alpha \frac{R_L}{R_B/(\beta+1) + r_e + R_E}$$

The voltage-gain expression can be interpreted as a ratio of two resistances through which the common current i_e (adjusted by α) flows. The numerator is the resistance across which the common current develops the output voltage. The denominator is the *transresistance*, the resistance across which the input source voltage develops the common current. The i_e-to-i_c current factor α must be included and the sign of the gain deduced from the circuit topology. The essence of the method is to develop the following relationships in the order

$$x_i \Rightarrow x_{common} \Rightarrow x_o$$

For the CE, this amounts to $v_i \Rightarrow i_e \Rightarrow i_c \Rightarrow v_o$. The additional middle ($\Rightarrow i_c$) step accounts for α.

An alternative derivation based on the same approach is to refer the resistances r_e and R_E in the emitter circuit to the base and to calculate i_b as the common variable. Then the form of A_v is

$$A_v = -\frac{\beta \cdot R_L}{R_B + (\beta+1) \cdot (r_e + R_E)}$$

If $(\beta + 1)$ is factored from the denominator (thus transforming this resistance to an emitter-referred transresistance), the gain expression is the same as before.

The common-base (CB) amplifier can be analyzed by first using the β transform to refer R_B to the emitter circuit. Then the emitter current generated by v_i is

$$i_e = -\frac{v_i}{R_B/(\beta+1) + r_e + R_E}$$

The denominator is the transresistance. The collector current is $i_c = \alpha \cdot i_e$, and output voltage is

$$v_o = -i_c \cdot R_L$$

Combining these equations gives the CB voltage gain.

$$A_v(\text{CB}) = \alpha \cdot \frac{R_L}{R_B/(\beta+1) + r_e + R_E}$$

For the common-collector (CC) amplifier, the order of variables is

$$v_i \Rightarrow i_e \Rightarrow v_o$$

and the transresistance is

$$r_M = \frac{R_B}{\beta+1} + r_e + R_E$$

From $i_e = v_i/r_M$ and $v_o = i_e \cdot R_E$, the CC voltage gain is

$$A_v(\text{CC}) = \frac{R_E}{R_B/(\beta+1) + r_e + R_E}$$

For the CB and CC, A_v is also a ratio of resistances, adjusted by α and polarity. For the CC, the previous equation can be interpreted as a voltage divider with input v_i and output v_o. The top resistance of the divider is $R_B/(\beta + 1) + r_e$, and the bottom resistor is R_E.

Input and Output Resistances

Besides gain, the quasistatic input and output resistances, r_{in} and r_{out}, can be found using the β transform. For the CE, r_{in} is a resistance referred to the base side of the input loop and is

$$r_{in}(\text{CE}) = \frac{v_i}{i_i} = \frac{v_i}{i_b} = R_B + (\beta + 1)\cdot(r_e + R_E)$$

The base-side resistances are equivalently emitter-side resistances $(\beta + 1)$ times larger. This results in a relatively high input resistance when R_E is large.

Using similar analysis for the CB and CC configurations,

$$r_{in}(\text{CB}) = \frac{v_i}{i_i} = \frac{v_i}{i_e} = R_E + r_e + \frac{R_B}{(\beta + 1)}$$

$$r_{in}(\text{CC}) = \frac{v_i}{i_i} = \frac{v_i}{i_b} = R_B + (\beta + 1)\cdot(r_e + R_E)$$

Both CE and CC circuits have the same r_{in}, whereas $r_{in}(\text{CB})$ is smaller by a factor of $(\beta + 1)$.

The output resistance of the CE and CB configurations is

$$r_{out}(\text{CE}) = r_{out}(\text{CB}) = R_L$$

For the CC,

$$r_{out}(\text{CC}) = R_E \left\| \left(r_e + \frac{R_B}{\beta + 1} \right) \right.$$

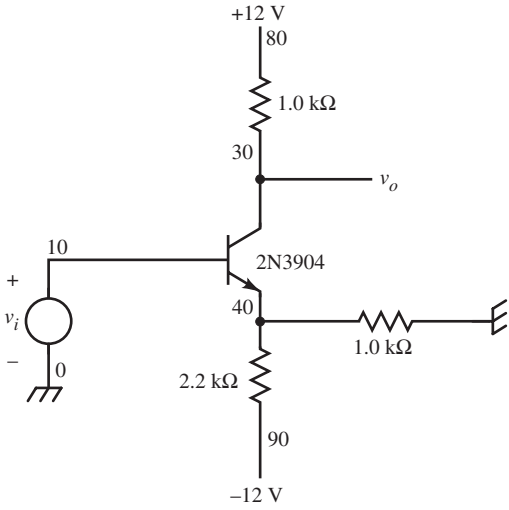
where ‖ designates a mathematical operator that, for resistances, means "in parallel with." Because the right-side resistance is relatively small, $r_{out}(\text{CC})$ is small.

Example: CE Amplifier

Assume the BJT CE amplifier (shown below) has the following parameters:

$$\beta + 1 = 100$$

$$I_S = 10^{-16}\ \text{A}$$



The saturation current I_S determines the v-i relationship of the base-emitter junction. For a first guess for $I_C = 12\text{ V}/2.2\text{ k}\Omega = 5.5\text{ mA}$,

$$V_{BE} \cong V_T \ln\left(\frac{I_E}{I_S}\right), \quad I_C >> I_S$$

and $V_{BE} = 0.82$ V. This gives a place to start for the static analysis. First, the emitter circuit can be simplified by Thevenin's theorem. The result is a –3.75 V source and 688 Ω resistance. Next, find I_E to determine r_e. Estimate I_E by assuming $V_{BE} = 0.82$ V. Then,

$$I_E \cong \frac{3.75\,\text{V} - 0.8\,\text{V}}{688\,\Omega} = 4.3\,\text{mA}$$

Then recalculating, $V_{BE} = 0.81$ V. I_E can be recalculated using this more refined value for V_{BE}. After only two iterations, the numbers converge to

$$V_{BE} = 0.81\,\text{V}, \quad I_E = 4.27\,\text{mA}$$

Because V_{BE} is logarithmically related to I_E, it is relatively insensitive to I_E variation. (This is why convergence was rapid.) Now solve for r_e:

$$r_e = \frac{26\,\text{mV}}{4.3\,\text{mA}} = 6.1\,\Omega$$

The transresistance method can now be applied to determine voltage gain:

$$\frac{v_o}{v_i} = -(0.99)\frac{1.0\,\text{k}\Omega}{6\,\Omega + 688\,\Omega} = -1.43$$

Input resistance is

$$r_{in} = (\beta + 1)\cdot(r_e + R_E) = (100\cdot[6\,\Omega + 688\,\Omega]) = 69.4\,\text{k}\Omega$$

and output resistance is $r_{out} = 1.0\ \text{k}\Omega$.

These results agree with those of the SPICE circuit simulation to the two significant digits of the manual calculations.

```
CE Amplifier

.OPT NOMOD OPTS NOPAGE
.DC VI -0.25 0.25 0.05
.TF V(30) VI

VI 10 0 DC 0V
VCC 80 0 DC 12
VEE 90 0 DC -12
RE1 40 90 2.2K
RE2 40 0 1.0K
RL 80 30 1.0K
Q1 30 10 40 BJT1

.MODEL BJT1 NPN (BF=99)
.END

NODE VOLTAGE

(30) 7.7686 (40) -.8115

VOLTAGE SOURCE CURRENTS

NAME CURRENT
VI -4.274E-05
VCC -4.231E-03
VEE 5.086E-03

TOTAL POWER DISSIPATION 1.12E-01 WATTS
```

```
V(30)/VI = -1.427E+00
INPUT RESISTANCE AT VI = 6.936E+04
OUTPUT RESISTANCE AT V(30) = 1.000E+03
```

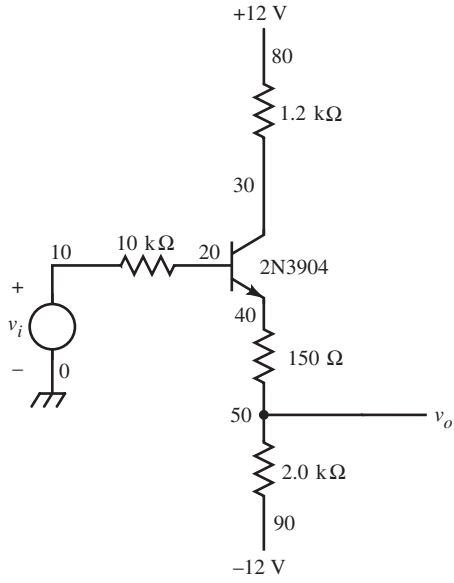
The simulation uses the same idealized T model of the analysis in this example. How do the results compare for a more realistic BJT model? The parameters of a 2N3904 were used, and the simulation was rerun. The results show good agreement except for r_{in}. A typical 2N3904 β of 150 is about 50% larger than the value of 99 used, and the discrepancy between the r_{in} values is also 50%. Therefore, the simple T model can produce accurate (typically <1% error) results.

```
CE Amplifier with 2N3904 model
.OPT NOMOD OPTS NOPAGE
.DC VI -0.25 0.25 0.05
.TF V(30) VI

VI 10 0 DC 0V
VCC 80 0 DC 12
VEE 90 0 DC -12
RE1 40 90 2.2K
RE2 40 0 1.0K
RL 80 30 1.0K
Q1 30 10 40 BJT1

.MODEL BJT1 NPN (BF=150 IS=1E-16 VA=110 RB=15 RE=2)
.END

SMALL SIGNAL BIAS SOLUTION

TEMPERATURE = 27.000 DEG C

NODE VOLTAGE

(30) 7.7627 (40) -.8187

VOLTAGE SOURCE CURRENTS

NAME CURRENT

VI -2.639E-05
VCC -4.237E-03
VEE 5.082E-03

TOTAL POWER DISSIPATION 1.12E-01 WATTS

V(30)/VI = -1.427E+00
INPUT RESISTANCE AT VI = 1.060E+05
OUTPUT RESISTANCE AT V(30) = 9.995E+02
```

The use of a second resistor in the emitter circuit allows a choice of Thevenin equivalent emitter supply voltages other than −12 V and any equivalent R_E. To maintain wide linear range for both input and output circuit (to avoid both saturation and cutoff) and to keep the operating point from shifting appreciably with operation, R_E must be kept much larger than r_e so that it dominates the transresistance in the gain expression. For gain magnitude much greater than one, the equivalent emitter supply must be less in voltage magnitude than the collector supply. But if made too small, then R_E no longer dominates as the input-circuit resistance (in r_M), and gain stability suffers.

Example: CC Amplifier

The emitter-follower has a voltage divider at its output. Assuming the same BJT parameters and using a static analysis as in the CE example, $V_{BE} = 0.82$ V and $I_E = 4.95$ mA. Consequently, $V_C = 6.1$ V, but this does not affect the small-signal amplifier parameters (using the T model).



Next, $r_e = 5.2\,\Omega$ and the quasistatic parameters of interest are

$$\frac{v_o}{v_i} = \frac{2.0\,\text{k}\Omega}{2.0\,\text{k}\Omega + 160\,\Omega + 5\,\Omega + 10\,\text{k}\Omega/100} = 0.883$$

$$r_{in} = 10\,\text{k}\Omega + (100)\cdot(5\,\Omega + 160\,\Omega + 2.0\,\text{k}\Omega) = 227\,\text{k}\Omega$$

These compare to the SPICE results to three digits. Except for arithmetic round-off, there is no difference between these results. A more accurate calculation of the operating point is necessary to produce a more accurate value of r_e, however.

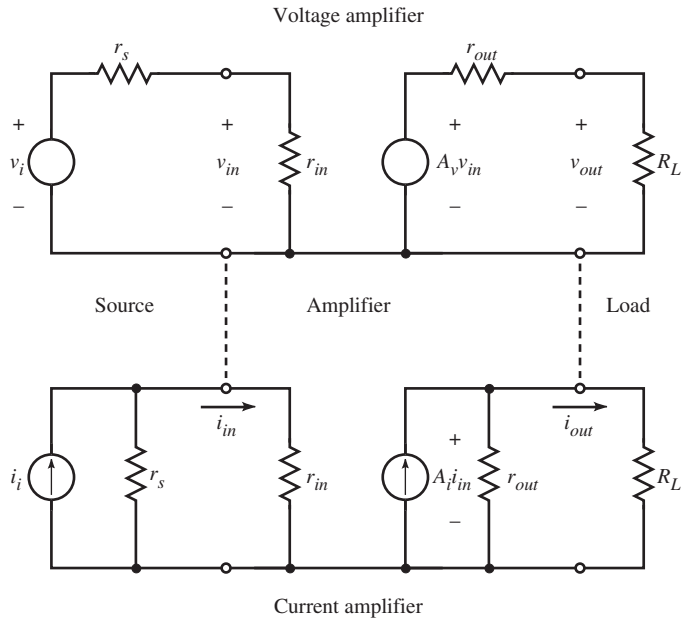
```
CC Amplifier

.OPT NOMOD OPTS NOPAGE
.DC VI -0.25 0.25 0.05
.TF V(50) VI

VI 10 0 DC 0V
VCC 80 0 DC 12
VEE 90 0 DC -12
RB 10 20 10K
RE1 40 50 160
RE2 50 90 2.0K
RC 80 30 1.2K
Q1 30 20 40 BJT1

.MODEL BJT1 NPN (BF=99)
.END

SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C

NODE VOLTAGE NODE VOLTAGE NODE VOLTAGE NODE VOLTAGE
( 10) 0.0000 ( 20) -.4949 ( 30) 6.1206 ( 40) -1.3102
( 50) -2.1020 ( 80) 12.0000 ( 90) -12.0000

VOLTAGE SOURCE CURRENTS

NAME CURRENT

VI -4.949E-05
VCC -4.899E-03
VEE 4.949E-03

TOTAL POWER DISSIPATION 1.18E-01 WATTS
```

```
SMALL-SIGNAL CHARACTERISTICS
V(50)/VI = 8.829E-01

INPUT RESISTANCE AT VI = 2.265E+05
OUTPUT RESISTANCE AT V(50) = 2.342E+02
```

The input and output resistances of the three configurations can be summarized in a table.

Configuration	Input Resistance	Output Resistance
CE	large	medium
CB	small	medium
CC	large	small

The large input resistances of the CE and CC cause them to appear like open circuits to the voltage sources driving them. In the three basic circuit configurations, the internal (Thevenin equivalent) resistances of the sources are omitted, but actual sources have a nonzero resistance. This source resistance forms a voltage divider with the input resistance of the amplifier circuit causing attenuation of v_i, as shown below.



If the voltage-source resistance, r_S, is variable or unknown, the attenuation of the divider and the overall voltage gain will be too. To avoid this, the ideal amplifier input resistance is infinite, so that $v_{in} = v_i$ independent of r_S.

Similarly, for the voltage divider at the output, formed by the nonzero amplifier output resistance and the load resistance, $A_v \cdot v_{in} = v_{out}$ and is independent of R_L when $r_{out} = 0$. The ideal voltage amplifier therefore has infinite r_{in} and zero r_{out}. In actual amplifier circuits, the input and output dividers must be taken into account when calculating the voltage gain.

For a current amplifier, current dividers at input and output similarly affect the current gain unless $r_{in} = 0$ and $r_{out} \to \infty$, the conditions for an ideal current amplifier. Considering the four basic amplifier types, the ideal terminal resistances are tabulated.

Ideal Amplifier Type	Input Resistance	Output Resistance
Voltage	∞	0
Transconductance	∞	∞
Current	0	∞
Transresistance	0	0

When these ideal properties are compared with the three configurations, the optimal matches can be made.

Ideal Amplifier Type	Optimal Configurations
Voltage	CC,CE
Transconductance	CE
Current	CB,CE
Transresistance	CB

In this table, none of the configurations is ideal. Although the CC resistances approach the ideal, the CC has a maximum voltage gain of only one. Similarly, the CB has a good resistance match but also has a maximum current gain of one. In these cases, the CE is the best choice because it provides useful voltage and current gain. It also is optimal for transconductance amplification because its resistances match best.

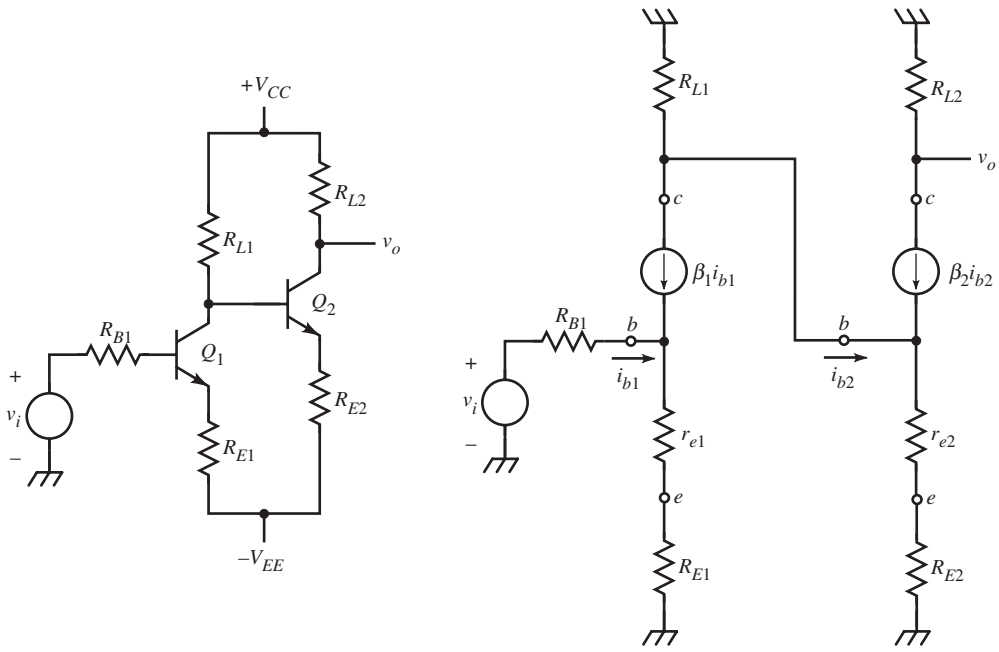
The CB is best for transresistance amplifiers for the same reason. Overall, the CE is the most versatile configuration and is used the most in practice. When

these basic configurations are combined in pairs, the resulting two-transistor configurations exceed the basic configurations in approaching the ideal.

THE CASCADE AMPLIFIER

The next step beyond single-transistor amplifier circuits is two- and three-transistor combinations. The three basic configurations can be combined in various ways to produce circuits with useful properties. These properties are not found in either of the configurations alone, just as molecules have properties different from their constituent atoms. Consequently, these circuits can be considered basic building blocks in themselves.

The most common combination of multiple-transistor amplifiers is the *cascade amplifier*, which consists of two CE amplifiers, the output of the first driving the input of the second. Each of these CE amplifiers is called a *stage* of amplification. Any unit of a sequence of consecutive amplifier circuits is a stage. This amplifier can be analyzed by the transresistance method. The only additional complication results from the interconnection of the two stages.



If the base of the second stage is disconnected from the collector of the first stage, the gain of the first stage is

$$A_{v1} = -\alpha_1 \cdot \frac{R_{L1}}{R_{B1}/(\beta_1 + 1) + r_{e1} + R_{E1}}$$

The output of the first stage can be represented as a Thevenin equivalent circuit and connected to the input of the second stage. The voltage source has a value of $A_{v1} \cdot v_i$ with a Thevenin resistance of R_{L1}. The calculation of the second stage gain is similar to that of the first stage, resulting in a total gain of

$$\begin{aligned} A_v &= A_{v1} \cdot A_{v2} \\ &= \left(-\alpha_1 \cdot \frac{R_{L1}}{R_{B1}/(\beta_1 + 1) + r_{e1} + R_{E1}} \right) \cdot \left(-\alpha_2 \cdot \frac{R_{L2}}{R_{L1}/(\beta_2 + 1) + r_{e2} + R_{E2}} \right) \end{aligned}$$

An alternative view of the interaction of the stages is to consider the collector of the first stage to be loaded by the input resistance of the second stage so that the collector load resistance is

$$R_{C1} = R_{L1} \| (\beta_2 + 1) \cdot (r_{e2} + R_{E2})$$

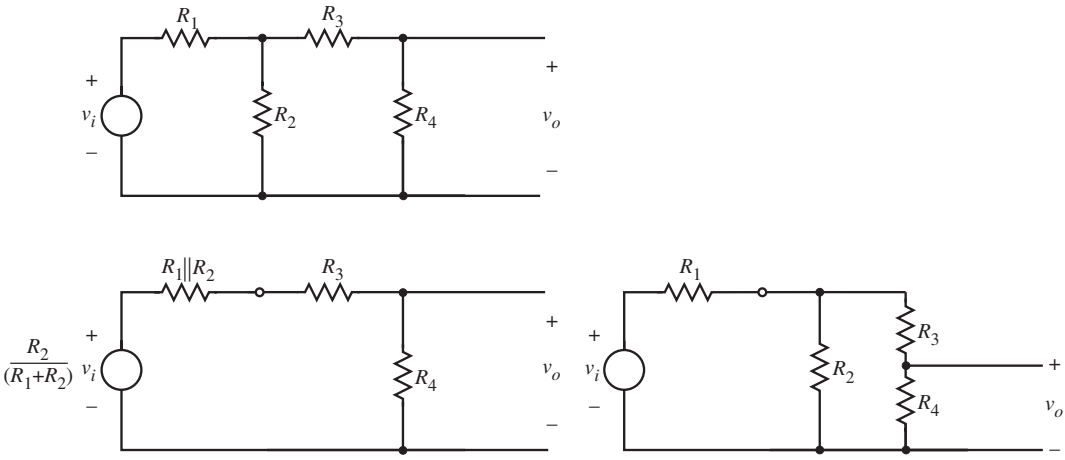
Then the gain formula for the second stage does not include base resistance since it is already taken into account in the first-stage collector resistance. With R_{C1} in the first-stage gain formula, the output is at the base terminal of the second stage, not at the Thevenin equivalent voltage source as in the first approach. The second approach is explicit in the gain formula when it is expressed as

$$A_v = \left(-\alpha_1 \cdot \frac{R_{L1} \| (\beta_2 + 1) \cdot (r_{e2} + R_{E2})}{R_{B1}/(\beta_1 + 1) + r_{e1} + R_{E2}} \right) \cdot \left(-\alpha_2 \cdot \frac{R_{L2}}{r_{e2} + R_{E2}} \right)$$

These two equations for A_v are equivalent but have different algebraic forms. Two alternative views of stage interaction follow from them. Often, the most difficult step in gaining insight into a new circuit is to express the equations

from circuit analysis in a meaningful form, one that reveals a simple equivalent circuit topology suggestive of higher circuit principles, such as the β transform.

The stage-interaction phenomenon of the *loaded divider* occurs often and can be generally demonstrated by analyzing the cascade attenuator shown below.



The Thevenin equivalent circuit (left) and loaded-divider (right) methods achieve the same result. For the Thevenized approach,

$$\frac{v_o}{v_i} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot \left(\frac{R_4}{R_1 \| R_2 + R_3 + R_4}\right)$$

and for the loaded-divider approach,

$$\frac{v_o}{v_i} = \frac{R_2 \| (R_3 + R_4)}{R_2 \| (R_3 + R_4) + R_1} \cdot \left(\frac{R_4}{R_3 + R_4}\right)$$

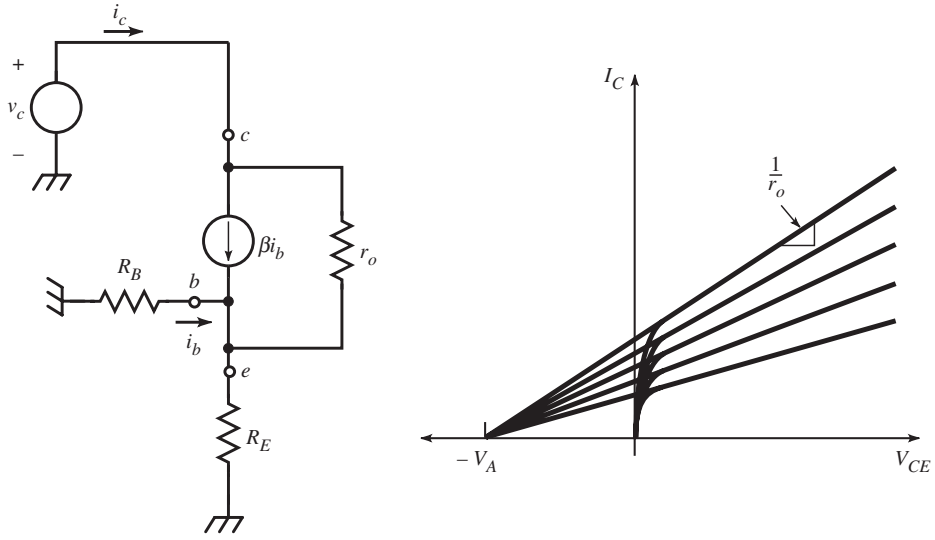
As in the case of the cascade amplifier voltage gain, the two equations are equivalent.

BJT Output Resistance

The simple BJT T model used in the preceding sections is now extended by considering the output resistance r_o, as shown in the following equation. In the BJT Ebers-Moll three (EM3) model (of which the T model is a simplification), r_o is defined as

$$r_o = \left.\frac{V_A + |V_{BC}|}{|I_C|}\right|_{v_{be}} = 0$$

where V_A is the *Early voltage*. This relationship is represented graphically by the collector family of curves as displayed by a curve tracer, as shown.



BJT T model with r_o

Whenever $v_{be} = 0$, r_e can be neglected. Then, from the extended T model,

$$v_c = R_E \cdot (i_c + i_b) + r_o \cdot (i_c - \beta \cdot i_b)$$

$$-i_b \cdot R_B = R_E \cdot (i_c + i_b)$$

Solving these equations for r_c results in

$$r_c = \frac{v_c}{i_c} = R_E \| R_B + r_o \cdot \left(1 + \beta \cdot \frac{R_E}{R_B + R_E}\right)$$

$$\Uparrow$$

fraction of i_c into R_B

This expression for r_c can be understood in terms of the T-model circuit. As indicated in the previous equation, the current divider formed by R_E and R_B determines the fraction of i_c that flows into R_B. When R_B is much larger than R_E (or the base terminal approaches an open circuit), then

$$r_c\big|_{R_B \to \infty} = R_E + r_o$$

and for a shorted base,

$$r_c\big|_{R_B = 0} = (\beta + 1) \cdot r_o$$

Thus, r_c increases as R_B decreases or R_E increases. Consider four limiting cases:

$$R_B \to \infty \Rightarrow r_c = r_o + R_E$$

$$R_B = 0 \Rightarrow r_c = (\beta + 1) \cdot r_o$$

$$R_E \to \infty \Rightarrow r_c = R_B + (\beta + 1) \cdot r_o$$

$$R_E = 0 \Rightarrow r_c = r_o$$

To envision r_c, begin at the *b-e* node where R_B and R_E are in parallel. This is accounted for by the first term in r_c. This parallel resistance is in series with r_o and the current source $\beta \cdot i_b$, accounted for by the second term of r_c. If $\beta \cdot i_b$ has no effect, then r_o is in series with $R_E \parallel R_B$, and the second term is only r_o. In this case, $\beta \cdot i_b = 0$ when $i_b = 0$. This occurs when R_B is infinite.

When all of the current through r_o flows in the base ($R_B = 0$), then both $\beta \cdot i_b$ and the current through r_o, i_o, flow in the collector, and

$$i_c = \beta \cdot i_b + i_o$$

With the base shorted to ground, no current flows in R_E and

$$i_c = -i_b$$

Current flowing out of the base is opposite in polarity to the indicated direction for the $\beta \cdot i_b$ current source and causes $\beta \cdot i_b$ to flow toward the collector terminal. Consequently, $\beta \cdot i_b$ contributes to i_o and adds to i_c, flowing down through r_o and out the base terminal. The effect is that most of i_o comes from $\beta \cdot i_b$ instead of being supplied as i_c. Because v_c causes a current of $v_c/r_o = i_o$, most of this current is supplied as $\beta \cdot i_b$. Substituting i_b and solving for i_c gives

$$i_c = \frac{i_o}{\beta + 1} = \frac{v_c / r_o}{\beta + 1} = \frac{v_c}{(\beta + 1) \cdot r_o}$$

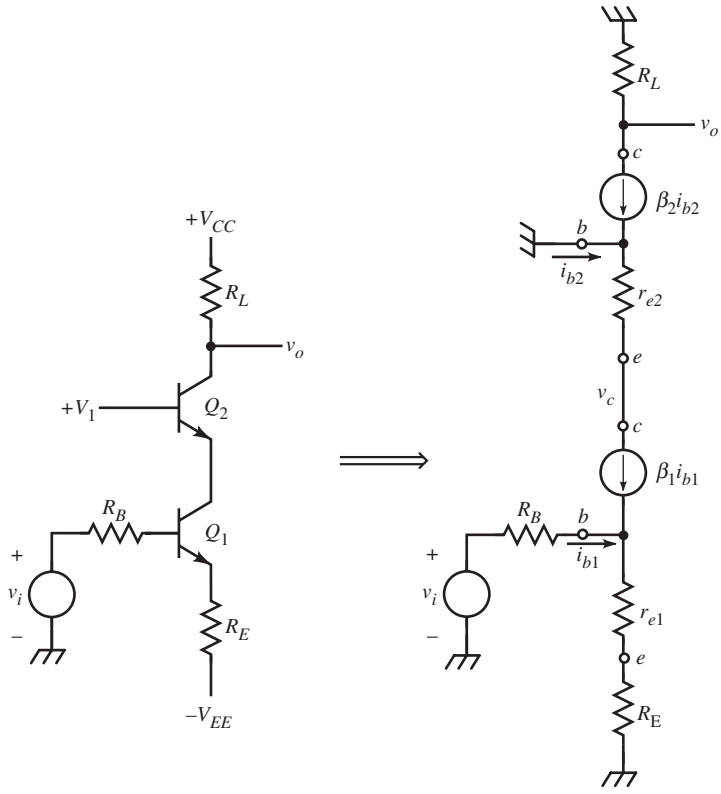
The collector resistance, v_c/i_c is $(\beta + 1) \cdot r_o$, as the equation for r_c with $R_B = 0$ indicates. The main insight here is the following:

> Current through r_o that flows in the base causes r_o to appear $(\beta + 1)$ times larger at the collector.

This results from the equation for r_c. The fraction of i_c that becomes base current causes r_o to be multiplied by β. The influence of r_o on amplifier performance can be significant because the collector node can affect collector current. The collector current is no longer isolated from the input circuit that causes it.

The Cascode Amplifier

A *cascode* amplifier is a CE stage followed by a CB stage, as shown.



Because the output is a CB stage with zero base resistance, r_{out} is high, yet r_{in} is also high due to a CE input stage. This results in a good transconductance amplifier with higher output resistance than a CE amplifier alone. When the cascode amplifier is analyzed by the transresistance method, the voltage gain is

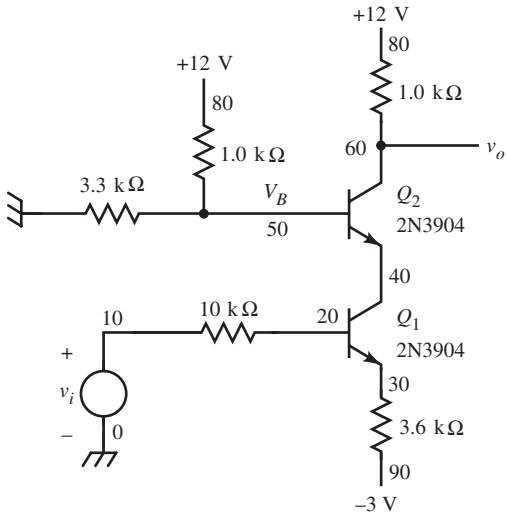
$$A_v = G_m \cdot R_L = -\alpha_1 \cdot \alpha_2 \cdot \frac{R_L}{R_B/(\beta+1) + r_{e1} + R_E}$$

The $\alpha_1 \cdot \alpha_2$ factor is due to loss of base current in both transistors; otherwise, the analysis holds no surprises. When the CB transistor r_o is taken into account, the numerator of the previous equation is modified so that

$$A_v = -\alpha_1 \cdot \alpha_2 \cdot \frac{R_L \| [(\beta_2+1) r_{o2}]}{R_B/(\beta_1+1) + r_{e1} + R_E}$$

Because of the CB output stage, not only is the output resistance higher but so is the voltage gain. The input resistance is that of a CE amplifier.

Example: Cascode Amplifier



A typical cascode amplifier is shown. Use a minimum-specified β of 99. The static calculation of Q_1 emitter current yields 600 μA. The emitter current of Q_2 is the collector current of Q_1, or 594 μA. Then the quasistatic emitter resistances are

$$r_{e1} = 43.3\ \Omega$$

$$r_{e2} = 43.3\ \Omega$$

Applying the cascode gain formula, the voltage gain is $A_v = -0.26$. This gain compares well to the SPICE results. The output resistance (assuming infinite r_{o2}) is $R_L = 1.0\ \text{k}\Omega$. The input resistance is

$$r_{in} = R_B + (\beta_1 + 1)\cdot(r_{e1} + R_E) = 374.3\ \text{k}\Omega$$

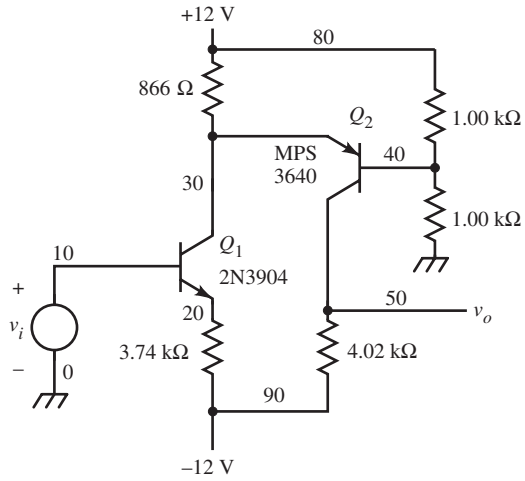
The SPICE output agrees.

```
Cascode Amplifier

.OPT NOMOD OPTS NOPAGE
.DC VI -0.25 0.25 0.05
.OP
.TF V(60) VI

VI 10 0 DC 0V

VCC 80 0 DC 12
VEE 90 0 DC -3
RI 10 20 10K
RE 30 90 3.6K
RB1 80 50 1.0K
RB2 50 0 3.3K
RL 80 60 1.0K
Q1 40 20 30 BJT1
Q2 60 50 40 BJT1

.MODEL BJT1 NPN (BF=99)
.END

SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C

NODE VOLTAGE

(20) -.0605 (30) -.8215 (40) 8.4440 (50) 9.2047
(60) 11.4070

BIPOLAR JUNCTION TRANSISTORS

NAME Q1 Q2
MODEL BJT1 BJT1

IB 6.05E-06 5.99E-06
IC 5.99E-04 5.93E-04
VBE 7.61E-01 7.61E-01
VBC -8.50E+00 -2.20E+00
VCE 9.27E+00 2.96E+00
BETADC 9.90E+01 9.90E+01
GM 2.32E-02 2.29E-02

V(60)/VI = -2.619E-01
INPUT RESISTANCE AT VI = 3.743E+05
OUTPUT RESISTANCE AT V(60) = 1.000E+03
```

This particular example of a cascode amplifier does not have a useful (>1) voltage gain, but it can function as a static voltage translator. If R_E is reduced, the gain (magnitude) increases, but the static emitter current also increases and input resistance decreases. The difficulty here is partly due to the values of the available power supplies. For a smaller R_E, V_{EE} must also be made smaller for the same bias current. But a decreasing V_{EE} makes the bias current more sensitive to V_{BE1}.

To achieve both a stable operating point and higher gain, use a large V_{EE} for stable bias current and construct a Thevenin equivalent source by placing another resistor from the emitter to ground. Two emitter resistors give the freedom needed to choose both a Thevenin equivalent supply voltage and an emitter resistance.

Example: Complementary Cascode Amplifier



A variation on the cascode amplifier is the *complementary cascode*, as shown. It is complementary because the two BJTs of the cascode are of different polarity (NPN and PNP).

The output transistor Q_2 is of opposite polarity to Q_1. The cascode gain formula does not exactly apply because some current is lost in the 866 Ω biasing resistor that shunts the emitter of Q_2. A static solution for I_{E1} is 3.0 mA. Then, $r_{e1} = 8.7\ \Omega$. For Q_2, node 40 is driven by a Thevenin equivalent circuit of 6 V and 500 Ω. Solving for the static solution of the base-emitter circuit of Q_2 is simplified by referring the 500 Ω equivalent base resistance to the emitter (as 5 Ω) and offsetting the 12 V supply by $(866\ \Omega)(I_{C1}) = 2.60$ V. Then the familiar diode-resistance circuit can be iteratively solved. This produces $I_{E2} = 3.05$ mA and $r_{e2} = 8.5\ \Omega$. Knowing the quasistatic emitter resistances, the voltage gain can be found. For Q_1, the transresistance is 3.75 kΩ.

While we are at it, input resistance is

$$r_{in} = (\beta_1 + 1)\cdot(r_{e1} + R_E) = 374.9\,\text{k}\Omega$$

The collector current of Q_1 is $\alpha_1 \cdot (v_i/3.75\ \text{k}\Omega)$. For $\beta = 99$, $\alpha = 0.99$. Next, the 866 Ω resistor forms a current divider with the emitter circuit of Q_2, and

$$i_{e2} = \left(\frac{866\,\Omega}{866\,\Omega + r_{e2} + 500\,\Omega/(\beta_2 + 1)}\right)\cdot\frac{0.99}{3.75\,\text{k}\Omega}\cdot v_i = \frac{0.975}{3.75\,\text{k}\Omega}\cdot v_i$$

Then $-i_{e2}\cdot R_L = v_o$. Combining this with $\alpha_2 = 0.99$ and i_{e2} gives the gain

$$A_v = -1.03$$

This agrees with the SPICE gain.

```
Complementary Cascode Amplifier

.OPT NOMOD OPTS NOPAGE
.DC VI -0.25 0.25 0.05
.OP
.TF V(50) VI

VI 10 0 DC 0V

VCC 80 0 DC 12
VEE 90 0 DC -12
RE1 20 90 3.74K
RB1 80 40 1.00K
```

```
RB2  40  0  1.00K
R0  30  80  866
RL  50  90  4.02K
Q1  30  10  20  BJT1
Q2  50  40  30  BJT2

.MODEL BJT1 NPN (BF=99)
.MODEL BJT2 PNP (BF=99)
.END

SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C

NODE VOLTAGE

(20) -.8023 (30) 6.8176 (40) 6.0151 (50) .0196

BIPOLAR JUNCTION TRANSISTORS

NAME Q1 Q2
MODEL BJT1 BJT2

IB 2.99E-05 -3.02E-05
IC 2.96E-03 -2.99E-03
VBE 8.02E-01 -8.03E-01
VBC -6.82E+00 6.00E+00
VCE 7.62E+00 -6.80E+00
BETADC 9.90E+01 9.90E+01
GM 1.15E-01 1.16E-01

V(50)/VI = -1.035E+00
INPUT RESISTANCE AT VI = 3.749E+05
OUTPUT RESISTANCE AT V(50) = 4.020E+03
```

The static output voltage is, according to SPICE, about 20 mV, or nearly zero volts – the same as the input. This amplifier is an inverting, (nearly) non offsetting $\times(-1)$ amplifier. Its output resistance of R_L is high for a voltage-output amplifier. The addition of a CC stage and static modification (to correct for V_{BE} of the CC) would result in a more acceptable inverting voltage amplifier.

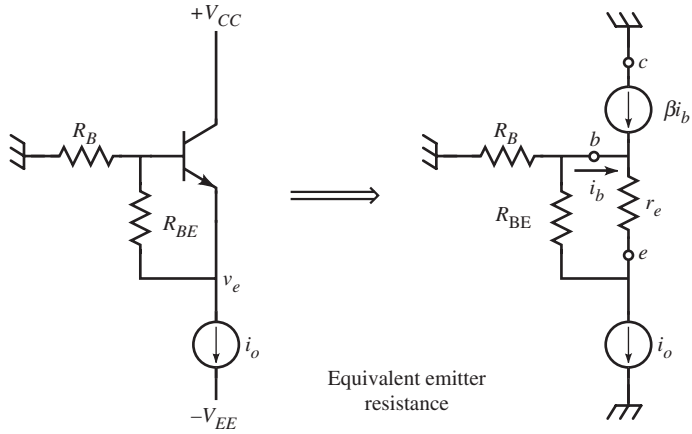
The Effect of Base-Emitter Shunt Resistance

An analysis similar to that for BJT output resistance can be applied to a base-emitter shunt resistance, as shown in the circuit diagram below.

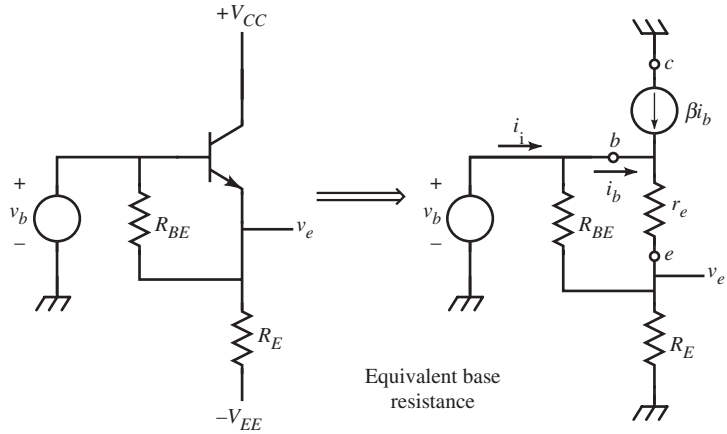
The equivalent resistance for both emitter and base can be derived from the equivalent small-signal circuits. Beginning with the emitter side first, we find the nodal equations for base and emitter to be

$$\frac{v_b}{R_B}+\frac{v_b-v_e}{R_{BE}}+\frac{v_b-v_e}{(\beta+1)\cdot r_e}=0, \quad \text{base node}$$

$$\frac{v_b-v_e}{R_{BE}}+\frac{v_b-v_e}{r_e}=i_o, \quad \text{emitter node}$$



(a)



(b)

Solving for emitter resistance from the previous equation results in

$$\frac{v_e}{i_o} = \frac{v_e}{\left(\dfrac{v_b - v_e}{R_{BE}} + \dfrac{v_b - v_e}{r_e}\right)} = \frac{v_e}{(v_b - v_e)\cdot\left[\dfrac{1}{R_{BE}} + \dfrac{1}{r_e}\right]}$$

Solving for v_b at the base node gives

$$v_b\cdot\left[\frac{1}{R_B} + \frac{1}{R_{BE}} + \frac{1}{(\beta+1)\cdot r_e}\right] = v_e\cdot\left[\frac{1}{R_{BE}} + \frac{1}{(\beta+1)\cdot r_e}\right]$$

$$v_b = v_e\cdot\frac{R_B}{R_B + R_{BE}\|(\beta+1)\cdot r_e}$$

Substituting for v_b in v_e/i_o and solving yields

$$\frac{v_e}{i_o} = R_{BE}\|r_e + \frac{R_B}{\beta+1}\cdot\left(\frac{R_{BE}}{r_e + R_{BE}}\right) + R_B\cdot\left(\frac{r_e}{r_e + R_{BE}}\right)$$

$\Uparrow$ fraction of i_o through r_e causing R_B to be $R_B/(\beta+1)$

$\Uparrow$ fraction of i_o through R_{BE} that also flows through R_B at e

Emitter resistance with a shunt R_{BE} can be understood in terms of the β transform. The first term is the parallel resistance of R_{BE} and r_e on the emitter side of the base-emitter loop. The drive current i_o divides between the emitter current and the shunt R_{BE}. The fraction of i_o that is emitter current results in base current that flows through R_B and causes R_B to appear $1/(\beta + 1)$ times smaller from the emitter. This fraction is determined by the current-divider factor in the second term of the previous equation. The third term accounts for the fraction of i_o that flows through R_{BE}. The β transform does not apply to it, and this current flows through R_B without being scaled down. This result is intuitively appealing since it can be constructed by use of the β transform and inspection of the circuit.

The current gain of this circuit, also derived from the basic circuit equations, is

$$A_i = \frac{i_c}{i_o} = \frac{\beta \cdot i_b}{i_o} = \alpha \cdot \left(\frac{R_{BE}}{R_{BE} + r_e} \right)$$

Without R_{BE}, the current gain would be α, but a fraction of i_o is lost to R_{BE}. The remaining fraction that is emitter current is expressed by the current divider factor in the previous equation.

The lower circuit shown above can also be analyzed from the base side. Writing the nodal equations at base and emitter and solving for v_e results in

$$v_e = v_b \cdot \frac{R_E}{R_E + R_{BE} \| r_e}$$

Substituting this into the emitter nodal equation yields

$$\frac{v_b}{i_i} = R_{BE} \| (\beta + 1) \cdot r_e + (\beta + 1) \cdot R_E \cdot \left(\frac{R_{BE}}{R_{BE} + (\beta + 1) \cdot r_e} \right) + R_E \cdot \left(\frac{(\beta + 1) \cdot r_e}{R_{BE} + (\beta + 1) \cdot r_e} \right)$$

$\Uparrow$ due to i_b — $\Uparrow$ due to i through R_{BE}

This result is similar to the emitter equation in form, as might be expected. The first term is the parallel combination of R_{BE} and r_e from the base side. The second term is due to the β transform effect of the base current according to the fraction of input current i_i that flows in the base. The shunt-current fraction of i_i that flows through R_{BE} contributes the third term, where R_E appears unscaled by $\beta + 1$.

The transconductance is

$$\frac{i_c}{v_b} = \frac{\beta \cdot i_b}{v_b} = \alpha \cdot \left(\frac{R_{BE}}{R_{BE} + r_e} \right) \cdot \left(\frac{1}{R_E + R_{BE} \| r_e} \right)$$

$\Uparrow$ fraction of current in R_E that is i_e — $\Uparrow$ v_b times this is the current in R_E

These equations can be better understood by considering the extremes of R_{BE}. For $R_{BE} = 0$, no current flows through r_e and no $\beta + 1$ scaling occurs. In this case, a passive resistive network results, and the second terms in the equation for v_e/i_o and the equation for v_b/i_i are zero. When $R_{BE} \to \infty$, the analysis using the β transform applies completely, and the third terms of these two equations are eliminated.

The shunt R_{BE} circuit has the useful property that the input resistance is higher than if R_{BE} were returned to ground instead of the emitter. R_{BE} is *bootstrapped* because its bottom terminal voltage follows that of the top terminal. This causes v_{be} across it to be less than the v_b that would be across a grounded R_{BE}. R_{BE} appears to be larger than its actual value because with less voltage across it, its current is reduced. To show this, consider first that $R_{BE} \| (\beta + 1) \cdot r_e$ (not R_{BE} alone) is the bootstrapped resistance. The equivalence factor for this resistance can be found by expressing the equation for v_b/i_i as

$$\text{BJT } r_{in} = \left[R_{BE} \| (\beta + 1) \cdot r_e\right] \cdot \left[1 + \alpha \cdot \frac{R_E}{r_e}\right] + R_E$$

The interpretation of this expression is that $[R_{BE} \| (\beta + 1) \cdot r_e]$ has an equivalent value that is larger by the bootstrap factor

$$\left(1 + \alpha \cdot R_E / r_e\right)$$

than its actual value.

This result, or that of the equation for v_b/i_i, can be modified for a FET from that of a BJT by allowing $\beta \to \infty$ (or $\alpha = 1$) and let R_{BE} become R_{GS}. Then FET $i_g = 0$ would be equivalent to setting i_b to zero. From the equation of v_b/i_i,

$$\frac{v_g}{i_i} = R_{GS} + R_S \cdot \frac{R_{GS}}{r_m} + R_S$$

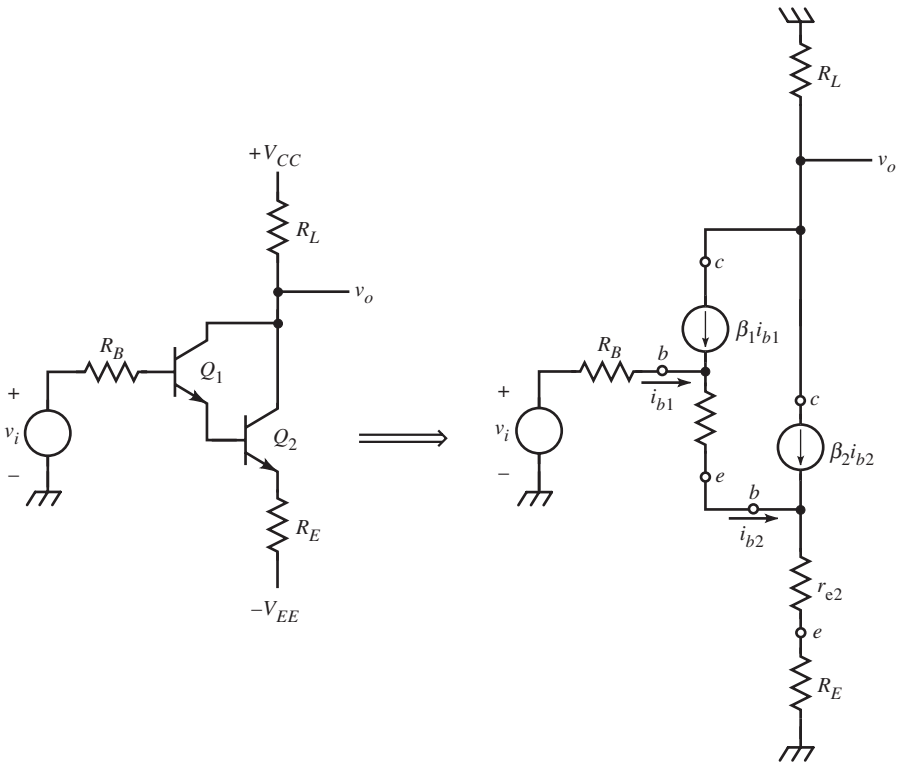
Here it is apparent that for infinite β, $r_e \to r_m$ and R_S replaces R_E. This result can be given physical meaning by factoring R_{GS} from the first two terms on the right. Then the equivalent R_{GS} due to bootstrapping is

$$R_{GS_{equiv}} = R_{GS} \cdot \left[1 + \frac{R_S}{r_m}\right]$$

The equivalence factor is the reciprocal of the divider formed by r_m and R_S and is identical in form to the second factor of the equation for BJT r_{in}. The difference is that for FETs, only R_{GS} is bootstrapped because $(\beta + 1) \cdot r_e \rightarrow \infty$. For a FET circuit with $R_{GS} = 100\,\text{k}\Omega$, $r_m = 100\,\Omega$, and $R_S = 1\,\text{k}\Omega$, then $R_{GSequiv} \cong 1\,\text{M}\Omega$.

THE DARLINGTON AMPLIFIER

Another two-transistor amplifier with useful properties is the *Darlington* or *compound amplifier* (sometimes called the "Darlington configuration"), as shown below. It consists of a CE stage emitter driving a second CE stage, except that the collectors are connected. This is a three-terminal amplifier that resembles a single transistor with some improved properties.



This circuit presents a challenge in using the transresistance method. Note that it has a single-input loop containing the transresistance but generates two output currents in parallel (from the two collectors). This results in a two-term expression for the voltage gain, one term per transistor. For the input transistor, the voltage gain is

$$A_{v1} = -\alpha_1 \cdot \frac{R_L}{(\beta_2 + 1)\cdot(r_{e2} + R_E) + r_{e1} + R_B/(\beta_1 + 1)}$$

The first term of the denominator is the external emitter resistance of the input transistor and is the input resistance of the driven BJT. The second term in the voltage gain is of the driven transistor:

$$A_{v2} = -\alpha_2 \cdot \frac{R_L}{R_E + r_{e2} + r_{e1}/(\beta_2 + 1) + R_B/(\beta_2 + 1)\cdot(\beta_1 + 1)}$$

For Q_2, the emitter resistance of Q_1 is its base resistance and is divided by $(\beta_2 + 1)$. The sum of A_{v1} and A_{v2} is the Darlington voltage gain:

$$A_v \cong -\frac{R_L}{R_E + r_{e2} + r_{e1}/(\beta_2 + 1) + R_B/(\beta_1 + 1)\cdot(\beta_2 + 1)}$$

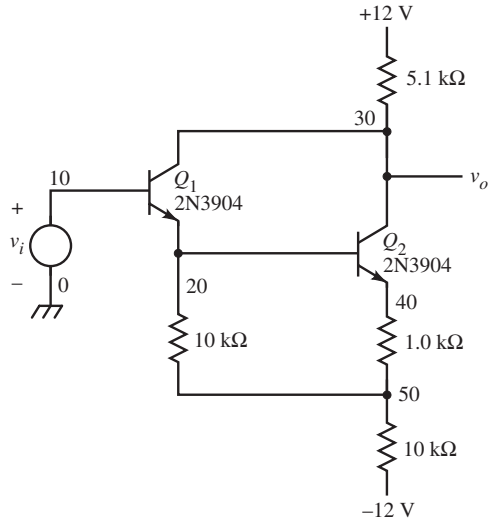
and $\beta_1, \beta_2 \gg 1$. This gain is slightly less than that of a comparable CE amplifier. Its advantage is its input resistance,

$$\begin{aligned} r_{in} &= R_B + (\beta_1 + 1)\cdot[r_{e1} + (\beta_2 + 1)\cdot(r_{e2} + R_E)] \\ &\cong (\beta + 1)^2 \cdot [r_{e2} + R_E],\ \beta_1 = \beta_2 = \beta \end{aligned}$$

which is larger than that of the CE by about a factor of $(\beta + 1)$. A Darlington amplifier makes a good input stage for voltage and transconductance amplifiers because of its high input resistance. When transistor r_o is taken into account, output resistance involves two parallel collector resistances with relatively low r_c, especially for the driven transistor.

Example: Darlington Amplifier

The figure shows a Darlington amplifier with a shunt emitter resistor on Q_2 terminating at a common bootstrapping resistor of 10 kΩ at node 50.



The static analysis follows a development similar to the quasistatic analysis. Therefore, the static solution will be taken from the SPICE model and only the quasistatic solution worked out. The emitter resistances of the BJTs are

$$r_{e1} = 156\,\Omega, \quad r_{e2} = 31.9\,\Omega$$

Because the input of Q_2 is bootstrapped, the input resistance to Q_2 (from the base of Q_2, including the 10 kΩ resistor between nodes 20 and 50) can be found by using the v_b/i_i equation. Substituting values,

$$\begin{aligned} r_{in2} &= 10\,\text{k}\Omega \| (100)\cdot(31.9\,\text{k}\Omega + 1.0\,\text{k}\Omega) \\ &\quad + (100)\cdot(10\,\text{k}\Omega)\cdot\left[\frac{10\,\text{k}\Omega}{10\,\text{k}\Omega + (100)\cdot(31.9\,\Omega + 1.0\,\text{k}\Omega)}\right] \\ &\quad + 10\,\text{k}\Omega\cdot\left[\frac{(100)\cdot(31.9\,\text{k}\Omega + 1.0\,\text{k}\Omega)}{10\,\text{k}\Omega + (100)\cdot(31.9\,\Omega + 1.0\,\text{k}\Omega)}\right] \\ r_{in2} &= 10\,\text{k}\Omega \| (100)\cdot(31.9\,\Omega + 1.0\,\text{k}\Omega) \\ &\quad + (100)\cdot(10\,\text{k}\Omega)\cdot\left[\frac{10\,\text{k}\Omega}{10\,\text{k}\Omega + (100)\cdot(31.9\,\Omega + 1.0\,\text{k}\Omega)}\right] \\ &\quad + 10\,\text{k}\Omega\cdot\left[\frac{(100)\cdot(31.9\,\Omega + 1.0\,\text{k}\Omega)}{10\,\text{k}\Omega + (100)\cdot(31.9\,\Omega + 1.0\,\text{k}\Omega)}\right] \\ &= 9116.53\,\Omega + 88347\,\Omega + 9116.43\,\Omega = 106580\,\Omega \end{aligned}$$

Now the first-stage gain formula is used to solve for the gain due to Q_1:

$$A_{v1} = -\alpha_1 \cdot \frac{R_L}{r_{e1} + r_{in2}} = -0.0473$$

The gain of Q_2 is found as follows:

$$A_{v2} = \frac{v_{b2}}{v_i} \cdot \frac{i_{c2}}{v_{b2}} \cdot (-R_L) = \left(\frac{r_{in2}}{r_{e1} + r_{in2}} \right) \cdot \frac{i_{c2}}{v_{b2}} \cdot (-R_L)$$

$$\Uparrow$$

$$i_c / v_b \text{ for shunt } b\text{-}e$$

The bootstrapping equation is used to find the transconductance of the Q_2 stage. Substituting values into this equation,

$$A_{v2} = -(0.09985) \cdot (0.4185) = -0.4179$$

Thus the total gain is

$$A_v = A_{v1} + A_{v2} = -0.4652$$

The input resistance is

$$r_{in} = (\beta_1 + 1) \cdot (r_{e1} + r_{in2}) = (100) \cdot (156\,\Omega + 106580\,\Omega) = 10.67\,\text{M}\Omega$$

The SPICE result agrees.

```
Darlington Amplifier

.OPT NOMOD OPTS NOPAGE
.DC VI -0.25 0.25 0.05
.OP
.TF V(30) VI

VI 10 0 DC 0V
VCC 80 0 DC 12
VEE 90 0 DC -12
```

```
RE1 20 50 10K
RE2 40 50 1.0K
RE3 50 90 10K
RL 80 30 5.1K
Q1 30 10 20 BJT1
Q2 30 20 40 BJT1

.MODEL BJT1 NPN (BF=99)
.END

SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C

NODE VOLTAGE

(20) -.7275 (30) 7.0652 (40) -1.4961 (50) -2.3073

BIPOLAR JUNCTION TRANSISTORS

NAME Q1 Q2
MODEL BJT1 BJT1

IB 1.66E-06 8.11E-06
IC 1.64E-04 8.03E-04
VBE 7.28E-01 7.69E-01
VBC -7.07E+00 -7.79E+00
VCE 7.79E+00 8.56E+00
BETADC 9.90E+01 9.90E+01
GM 6.36E-03 3.11E-02

V(30)/VI = -4.652E-01
INPUT RESISTANCE AT VI = 1.067E+07
OUTPUT RESISTANCE AT V(30) = 5.100E+03
```

With a gain of less than one, the purpose of this amplifier stage is to reduce input loading rather than to achieve voltage gain. The gain can be readily increased, however. (See the cascode example.) The high input resistance is desirable for a voltage-input amplifier.

THE DIFFERENTIAL (EMITTER-COUPLED) AMPLIFIER

For *single-ended* amplifiers, the input and output ports share a common (ground) node. Amplifiers with ports for which neither terminal is grounded are *differential amplifiers* (or *diff-amps* for short). Usually, an amplifier with differential input

(and differential or single-ended output) is called a differential amplifier and has an output of

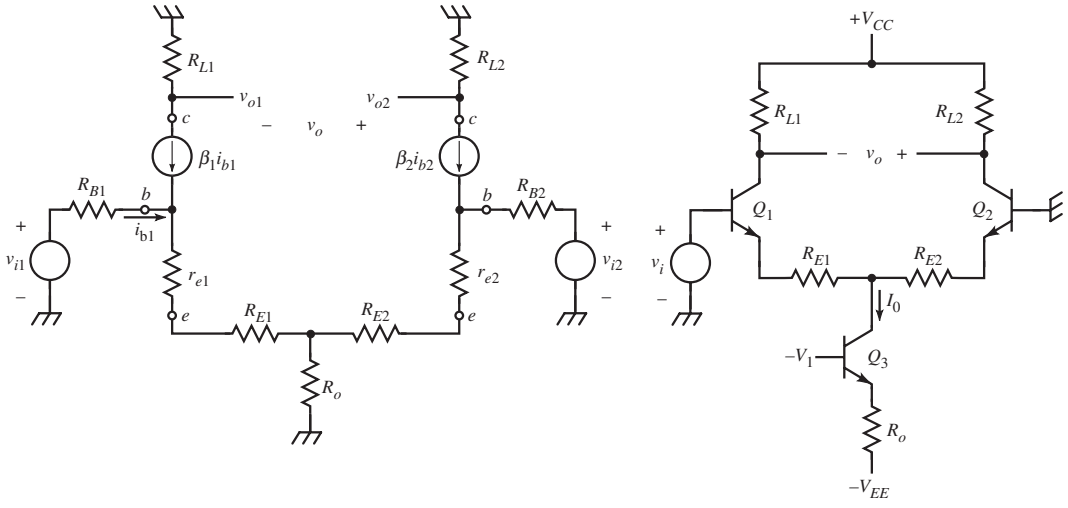
$$v_o = A_v \cdot (v_{i2} - v_{i1})$$

for a voltage amplifier. Some amplifiers have single-ended inputs and differential outputs. Differential output voltage is

$$\text{differential } v_o = v_{o2} - v_{o1}$$

A differential amplifier can be built from two CE amplifiers that share current-source resistor, R_o, as shown. Because the emitters are coupled, it is sometimes called an *emitter-coupled amplifier*. To achieve true differential amplification, the circuit must be symmetric so that the gains of each input to the output are the same in magnitude and opposite in sign. The output voltage for a general two-input voltage-difference amplifier is

$$v_o = A_{v2} \cdot v_{i2} - A_{v1} \cdot v_{i1}$$



The condition for differential amplification is that $A_{v2} = A_{v1}$. The voltage gain of the diff-amp (shown as a small-signal model) is found by the transresistance method and superposition. Because of its symmetric topology, we need only to find A_{v1} and to rewrite it for A_{v2} since it will have the same form:

$$A_{v1} = A_{v1+} - A_{v1-} = \frac{v_{o2}}{v_{i1}} - \frac{v_{o1}}{v_{i1}}$$

Beginning with A_{v1-},

$$A_{v1-} = -\alpha_1 \cdot \frac{R_{L1}}{R_{B1}/(\beta_1 + 1) + r_{e1} + R_{E1} + R_o \| [R_{E2} + r_{e2} + (R_{B2}/(\beta_2 + 1)]}$$

A_{v1+} is somewhat more complicated in that it involves the input transistor, operating as a CC, driving the output transistor as a CB with a cascaded attenuator in the emitter circuit. With the loaded-divider approach, the gain can be factored into two gains, using center node with voltage v_e as a splitting point:

$$A_{v1+} = \frac{v_{o2}}{v_{i1}} = \frac{v_e}{v_{i1}} \cdot \frac{v_{o2}}{v_e}$$

Defining

$$R_1 = \frac{R_{B1}}{\beta_1 + 1} + r_{e1} + R_{E1}$$

$$R_2 = \frac{R_{B2}}{\beta_2 + 1} + r_{e2} + R_{E2}$$

then,

$$A_{v1+} = \frac{R_o \| R_2}{R_o \| R_2 + R_1} \cdot \left(\alpha_2 \cdot \frac{R_{L2}}{R_2} \right)$$

The first factor of the previous equation is the loaded divider; multiplied by v_{i1}, it produces v_e. The remaining factor is the voltage gain of the output transistor. Because calculation of v_e took R_2 into account, it is the input voltage to R_2 when calculating gain.

The Thevenin circuit approach breaks R_{e2} at R_o and solves for the gain from v_{i1} to v_e. Because the loading of R_2 is neglected, a Thevenin equivalent circuit must then drive R_2. The alternative expression for A_{v1+} is

$$A_{v1+} = \left(\frac{R_o}{R_1 + R_o} \right) \cdot \left(\alpha_2 \frac{R_{L2}}{R_1 \| R_o + R_2} \right)$$

The Thevenin resistance appears in the transresistance in the second factor as $R_1 \| R_o$. This equation for A_{v1+} and the previous equation for A_{v1+} are equivalent. The total gain, A_{v1}, according to the equation of A_{v1}, is

$$\begin{aligned} A_{v1} &= \alpha_2 \cdot \frac{R_o \| R_2}{R_o \| R_2 + R_1} \cdot \left(\frac{R_{L2}}{R_2} \right) + \alpha_1 \cdot \frac{R_{L1}}{R_1 + R_o \| R_2} \\ &= \alpha_2 \cdot \frac{R_{L2}}{R_o \| R_2 + R_1} \cdot \left(\frac{R_o}{R_o + R_2} \right) + \alpha_1 \cdot \frac{R_{L1}}{R_o \| R_2 + R_1} \end{aligned}$$

Calculated thus far is the gain for a single-input diff-amp in that v_{i2} is shorted (by superposition). Before completing the derivation of the total differential gain, note that with $v_{i2} = 0$, this is a single-ended input, differential output amplifier. This circuit is common and useful; vertical input amplifiers of oscilloscopes use this (with FETs) as an input stage from the probe.

To produce a *balanced differential output* ($v_{o1} = -v_{o2}$), the magnitudes of the gains to both outputs must be equal. The required conditions (that the terms of the equation for A_{v1} be equal) are

$$\alpha_1 = \alpha_2$$

$$R_{L1} = R_{L2}$$

$$\frac{R_o}{R_o + R_2} = 1$$

The first condition requires matched transistors. For high-β transistors, this condition is not critical and is easily met. The third condition can be satisfied either by letting $R_2 = 0$ or $R_o \to \infty$. The first alternative is not physically realizable (because $r_{e2} > 0$); a finite R_o causes an imbalance. R_o is often replaced by a current source, thereby satisfying the condition. In practice, this can be the

collector of another transistor generating the current I_o, as shown in the above circuit on the right.

Returning to the full diff-amp gain derivation, by symmetry of the circuit topology, the gains to both outputs due to v_{i2} (with v_{i1} shorted) have the same form but with corresponding components from the other side of the circuit:

$$A_{v2} = \alpha_1 \cdot \frac{R_{L1}}{R_o \| R_1 + R_2} \cdot \left(\frac{R_o}{R_o + R_1} \right) + \alpha_2 \cdot \frac{R_{L2}}{R_o \| R_1 + R_2}$$

The total gain is

$$A_v = \frac{v_o}{v_i} = \frac{v_{o2} - v_{o1}}{v_{i2} - v_{i1}} = \frac{A_{v2} \cdot v_{i2} - A_{v1} \cdot v_{i1}}{v_{i2} - v_{i1}}$$

The condition for differential amplification is that

$$A_{v2} = A_{v1}$$

This condition can be met in two ways:

$$A_{v1+} = -A_{v2+},\ A_{v1-} = -A_{v2-} \text{ (antisymmetric)}$$

$$A_{v1+} = A_{v2-},\ A_{v1-} = A_{v2+} \text{ (symmetric)}$$

The first approach leads to the circuit component conditions

$$R_1 = R_2 = R$$

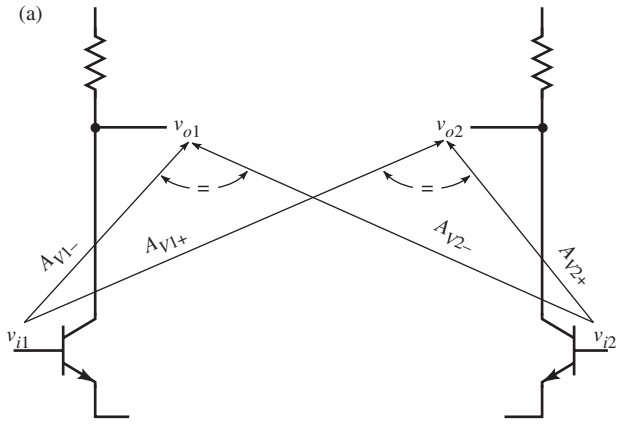
$$R_o \to \infty$$

and the second to the conditions

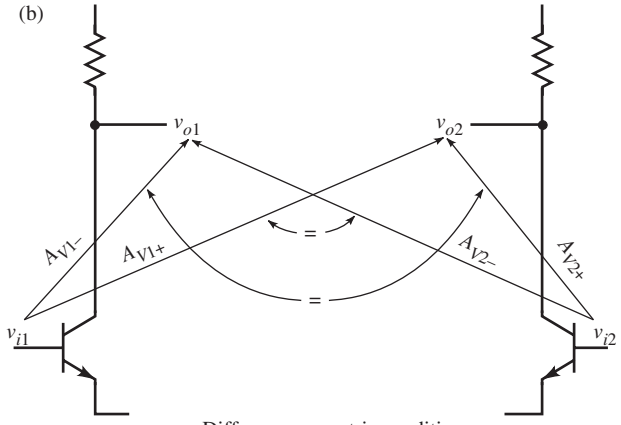
$$\alpha_1 = \alpha_2 = \alpha$$

$$R_{L1} = R_{L2} = R_L$$

$$R_1 = R_2 = R$$



Diff-amp antisymmetric conditions



Diff-amp symmetric conditions

The symmetric conditions require that the gains from the two inputs be the same to their corresponding inverting and non-inverting outputs, as shown graphically. The antisymmetric conditions are illustrated in the upper circuit drawing. Here, the gains from the two inputs to a given output must be equal. Neither approach to differential amplification necessarily satisfies the condition for balance. Either way, the differential gain is

$$A_v = 2 \cdot A_{v1} = 2 \cdot A_{v2}$$

When the circuit is differential and balanced, components on corresponding sides are equal, and the gain reduces to

$$A_v = 2 \cdot \alpha \cdot \frac{R_L}{R}$$

With symmetric circuit topology, $v_{i1} = -v_{i2}$ and $v_e = 0$ because the two superimposed inputs at v_e are equal and opposite. In this case, the v_e node is a *virtual ground*, and the gains of each side of the amplifier can be calculated under this assumption. Consequently, the transresistance is R on each side, and the gain for each side (v_{o1}/v_{i1} and v_{o2}/v_{i2}) is $\alpha \cdot R_L/R$ in magnitude. For differential outputs, the gain is twice that of a single side.

The *common-mode rejection ratio* (CMRR) is the measure of how differential an amplifier is. It is the change in output when both inputs are changed the same amount and is defined as

$$\text{CMRR} = \frac{v_o/(v_{i2} - v_{i1})}{v_o/(v_{i2} + v_{i1})}$$

The numerator is the *differential-mode gain* of the amplifier; the denominator is the *common-mode gain*. Two arbitrary inputs, v_{i1} and v_{i2}, can be combined into differential-mode v_d and common-mode v_c voltages:

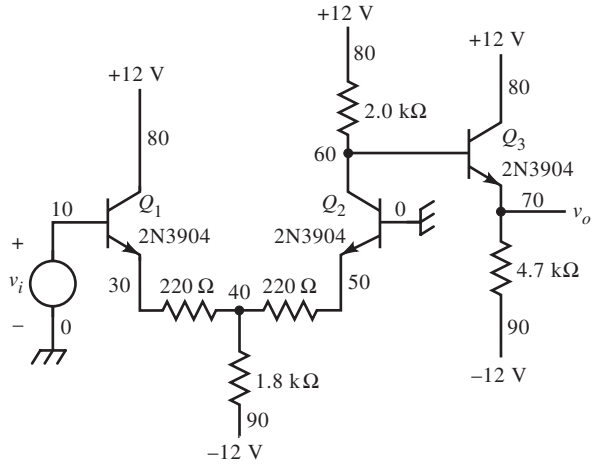
$$v_d = \frac{v_{i2} - v_{i1}}{2}$$

$$v_c = \frac{v_{i2} + v_{i1}}{2}$$

A purely differential input occurs when $v_{i2} = -v_{i1}$. Then $v_d = v_{i2}$ and $v_c = 0$. When $v_{i2} = v_{i1}$, the input is purely common mode and $v_c = v_{i2}$ whereas $v_d = 0$. An infinite CMRR is ideal because then the amplifier amplifies only the differential-mode voltage. CMRR is a measure of how well the conditions of antisymmetry or symmetry are achieved.

Example: Differential Amplifier with CC Output

The figure is that of a differential-amplifier stage buffered by an emitter-follower, Q_3. Both bases of the diff-amp are at the same static voltage (0 V), and



their emitter resistors have the same value. Then it is reasonable to assume that their currents are equal. Assume they both conduct

$$(12\,\text{V} - 0.8\,\text{V})/(220\,\Omega + 1.8\,\text{k}\Omega \times 2) \cong 3\,\text{mA}$$

Then

$$I_{E1} = I_{E2} = \frac{12\,\text{V} - (0.8\,\text{V} + (3\,\text{mA})(220\,\Omega))}{220\,\Omega + 2(1.8\,\text{k}\Omega)} = 2.76\,\text{mA}$$

Using this current, we find that V_{BE} agrees with the assumed value of 0.80 V and that the bias point has converged. Then $r_{e1} = r_{e2} = 9.6\,\Omega \cong 10\,\Omega$. Continuing the static analysis at the output collector,

$$V_{C2} = 12\,\text{V}\left(\frac{470\,\text{k}\Omega}{470\,\text{k}\Omega + 2.0\,\text{k}\Omega}\right) - (2.7\,\text{mA})(2.0\,\text{k}\Omega \| 470\,\text{k}\Omega) = 6.6\,\text{V}$$

and

$$I_{E3} \cong \frac{6.6\,\text{V} + 12\,\text{V} - 0.8\,\text{V}}{4.7\,\text{k}\Omega} = 3.8\,\text{mA}$$

Then $r_{e3} = 6.8\,\Omega$. Solving for the usual amplifier parameters gives:

$$\frac{v_o}{v_i} = \left(\frac{1.8\,\text{k}\Omega}{1.8\,\text{k}\Omega + 220\,\Omega + 10\,\Omega} \right) \times \left((0.99) \cdot \frac{2.0\,\text{k}\Omega}{10\,\Omega + 220\,\Omega + 1.8\,\text{k}\Omega \| (220\,\Omega + 10\,\Omega)} \right) \times \left(\frac{4.7\,\text{k}\Omega}{4.7\,\text{k}\Omega + 7\,\Omega + 2\,\text{k}\Omega/100} \right) = (0.887)(4.56)(0.994) = 4.02$$

$$r_{in} = (100) \cdot [10\,\Omega + 220\,\Omega + 1.8\,\text{k}\Omega \| (220\,\Omega + 10\,\Omega)] = 43\,\text{k}\Omega$$

$$r_{out} = 4.7\,\text{k}\Omega \| (7\,\Omega + 2\,\text{k}\Omega/100) = 27\,\Omega$$

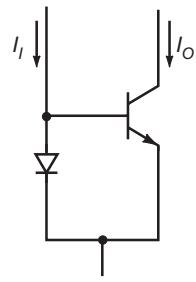
Again, compared to the simulation, these results are right on.

```
Differential Amplifier with CC Output

.OPT NOMOD OPTS NOPAGE
.DC VI -0.25 0.25 0.05
.OP
.TF V(70) VI

VI 10 0 DC 0V
VCC 80 0 DC 12
VEE 90 0 DC -12
RE1 30 40 220
RE2 50 40 220
R0 40 90 1.8K
RL 80 60 2.0K
RE3 70 90 4.7K
Q1 80 10 30 BJT1
Q2 60 0 50 BJT1
Q3 80 60 70 BJT1

.MODEL BJT1 NPN (BF=99)
.END

SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C
NODE VOLTAGE

(30) -.8018 (40) -1.4467 (50) -.8018
(60) 6.1220 (70) 5.3143

BIPOLAR JUNCTION TRANSISTORS
```

```
NAME Q1 Q2 Q3
MODEL BJT1 BJT1 BJT1

IB 2.93E-05 2.93E-05 3.68E-05
IC 2.90E-03 2.90E-03 3.65E-03
VBE 8.02E-01 8.02E-01 8.08E-01
VBC -1.20E+01 -6.12E+00 -5.88E+00
VCE 1.28E+01 6.92E+00 6.69E+00
BETADC 9.90E+01 9.90E+01 9.90E+01
GM 1.12E-01 1.12E-01 1.41E-01

V(70)/VI = 4.045E+00
INPUT RESISTANCE AT VI = 4.318E+04
OUTPUT RESISTANCE AT V(70) = 2.686E+01
```

CURRENT MIRRORS

A circuit that supplies a current of the same polarity and magnitude as its input current is a *current mirror*. A Widlar current mirror (after Bob Widlar – pronounced "Wide-ler") is shown below.



Widlar current mirror

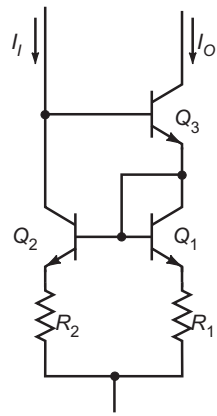
Input current I_I flows through the diode, creating a voltage that is also V_{BE} of the transistor. If the diode and *b-e* junctions are matched (that is, have the same *v-i* function), then the resulting emitter current equals I_I. Consequently, the output current I_O is equal to $\alpha \cdot I_I$. For a typical $\alpha \cong 1$, I_O is a replication of I_I. This circuit is useful, for example, in supplying I_O to a diff-amp. If the emitter and cathode are connected to $-V_{EE}$, the current can be set by a resistor from the input to ground. If the voltage across the resistor is much larger than the diode voltage, I_I is largely determined by the resistor value.

The basic Widlar current mirror can be improved by an additional transistor to compensate for base current lost to the transistor. If we take into account I_B in the Widlar circuit, then

$$I_O = \alpha \cdot (I_I - I_B) = \alpha \cdot \left(I_I - \frac{I_O}{\beta} \right)$$

Solving for the current gain gives

$$\frac{I_O}{I_I} = \frac{\beta}{\beta + 2} \cong \alpha, \quad \beta >> 1$$



Wilson current mirror

Three improvements have been made in the circuit shown here, invented by George Wilson. First, the diode has been made out of a similar transistor, Q_1, by connecting the base and collector. This kind of diode is often used in integrated circuits to achieve the best match of two *p-n* junctions. Second, the transistor Q_3 has been added to compensate for α loss, now occurring in Q_2. Third, to further reduce current-gain error, emitter resistors have been added.

The effect of Q_3 is to divert I_B amount of current from I_I. Q_3 emitter current is then $(\beta + 1) \cdot I_B$. From this current, I_B is diverted into the base of Q_2. This loss of base current to Q_2 was compensated by the diversion of Q_3 base current. Though it is better than the simple current mirror, the compensation is not

perfect, even with matched junctions. Assume that the *b-e* junctions are matched and $R_1 = R_2$. Because the bases of Q_1 and Q_2 are connected, the same voltage occurs across identical branches. Thus $I_{E1} = I_{E2}$ and

$$I_{E3} = \frac{I_{E2}}{\beta_2 + 1} + I_{E2} = I_{E2} \cdot \left(1 + \frac{1}{\beta_2 + 1}\right)$$

Also,

$$I_I = \alpha_2 \cdot I_{E2} + I_{B3}$$

Because $I_O = \beta_3 \cdot I_{B3}$, the current gain can be found from I_{E3} and I_I and is

$$\frac{I_O}{I_I} = \frac{\beta_3\beta_2 + 2\beta_3}{\beta_3\beta_2 + 2\beta_2 + 2}$$

For $\beta_1 = \beta_2 = \beta_3 = \beta$,

$$\frac{I_O}{I_I} = \frac{\beta^2 + 2\beta}{\beta^2 + 2\beta + 2}$$

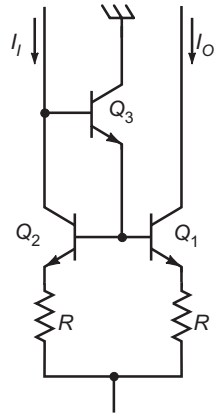
This current gain is a closer approximation to one than that for the Widlar source; it is tabulated here for various values of β.

β	I_O/I_I (Wilson)	$\beta/(\beta + 2)$ (Widlar)
1	0.60	0.33
2	0.80	0.50
10	0.98	0.83
50	0.999	0.96
100	0.9998	0.98

For $\beta = 100$, I_O/I_I is 100 times better than $\beta/(\beta + 2)$ and 40 times better at $\beta = 50$.

This analysis assumes perfect matching of Q_1 and Q_2. In practice, the effect of mismatch tends to be minimized by R_1 and R_2 if the voltage dropped across them is much greater than the *b-e* junction voltages of the transistors. Resistors can be matched much better than transistors and can be made very stable. The

emitter currents are thereby determined dominantly by the emitter resistors. In integrated current mirrors, transistor matching can be very good, and the additional voltage drop of the emitter resistors can be minimized, giving the circuitry connected to the mirror a wider voltage range.



IC current mirror

Another three-BJT mirror, well suited for integrated circuit (IC) layout, is shown here. For matched junctions, $I_{E1} = I_{E2}$ and

$$I_I - \frac{[I_{E2}/(\beta_2 + 1) + I_O/\beta_1]}{\beta_3 + 1} = \alpha_2 \cdot I_{E2}$$

where the numerator of the second term on the left is I_{E3}. Using the substitution $I_O/\alpha = I_{E2}$ yields the current gain

$$\frac{I_O}{I_I} = \frac{\beta_1\beta_2\beta_3 + \beta_1\beta_2 + \beta_1\beta_3 + \beta_1}{\beta_1\beta_2\beta_3 + \beta_1\beta_2 + \beta_2\beta_3 + \beta_1 + 2\beta_2 + 2}$$

For $\beta_1 = \beta_2 = \beta_3 = \beta$, then

$$\frac{I_O}{I_I} = \frac{\beta^2 + \beta}{\beta^2 + \beta + 2}$$

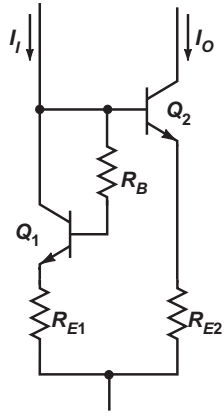
The terms in the numerator and denominator differ only by the constant term, resulting in accurate current mirroring.

The current gain versus β for several values is given in the following table, along with the gain values for the Wilson mirror. For $\beta \gg 1$, the Wilson and IC mirrors have almost identical current gains. Although for both, Q_3 recirculates their base currents, taken from I_I back to Q_2, only Q_3 of the IC source provides base current for Q_1 and Q_2 and carries no output current.

β	I_O/I_I (Wilson)	I_O/I_I (IC)
1	0.60	0.50
2	0.80	0.78
10	0.98	0.98
50	0.99923	0.99922
100	0.99980	0.99980

R_B-Compensated Current Mirror

A minimum-component current mirror is shown below. Input current, I_I is amplified as output I_O.



The loss of I_{B2} from I_I causes the voltage drop across Q_1 and R_{E1} to be low. This causes the voltage across Q_2 and R_{E2} to also be low, and consequently, I_{E2} is low. A second error is due to α_2. To compensate for these two current errors, R_B is added. It causes an increase in V_{B2} that compensates for both of the errors.

To analyze the circuit, first express emitter currents in terms of the input and output currents:

$$I_{E1} = I_I - I_O/\beta_2$$

$$I_{E2} = I_O/\alpha_2$$

Apply Kirchhoff's voltage law (KVL) around the input loop:

$$I_{E1}\cdot\left(R_{E1}+\frac{R_B}{\beta+1}\right)+V_T\cdot\ln\left(\frac{\alpha_1\cdot I_{E1}}{I_{S1}}\right)=I_{E2}\cdot R_{E2}+V_T\cdot\ln\left(\frac{\alpha_2\cdot I_{E2}}{I_{S2}}\right)$$

Assume Q_1 and Q_2 have matching β and I_S values. Substituting for I_{E1} and I_{E2} and solving for I_O:

$$I_O=\frac{R_{E1}+R_B/(\beta+1)}{\left(R_{E1}+\frac{R_B}{\beta+1}\right)\Big/\beta+R_{E2}/\alpha}\cdot I_I+\frac{V_{BE1}-V_{BE2}}{\left(R_{E1}+\frac{R_B}{\beta+1}\right)\Big/\beta+R_{E2}/\alpha}$$

or

$$I_O=\beta\cdot\left[\frac{R_{E1}+R_B/(\beta+1)}{R_{E1}+R_B/(\beta+1)+(\beta+1)\cdot R_{E2}}\cdot I_I+\frac{-\Delta V_{BE}}{R_{E1}+R_B/(\beta+1)+(\beta+1)\cdot R_{E2}}\right]$$

The bracketed expression is I_{B2}. The first term within it is I_{B2} due to I_I and the voltage-divider expression is the gain, I_O/I_I. The second term is an offset due to V_{BE} mismatch, which causes I_O to be low if the junction of Q_1 is too large relative to the area of Q_2.

In a good mirror design, the junctions are matched so that

$$\Delta V_{BE}=V_{BE2}-V_{BE1}=V_T\cdot\ln\left(\frac{\alpha_2\cdot I_{E2}}{\alpha_1\cdot I_{E1}}\cdot\frac{I_{S1}}{I_{S2}}\right)=0$$

For monolithic transistors, the α values will match and cancel. I_S is proportional to the geometric scaling of Q_1 and Q_2, allowing the ratio of emitter currents to be the inverse of the saturation current ratio, resulting in $\ln(1)$ and V_{BE} matching. Then the static offset term in I_O is removed and the current gain is

$$A_I=\frac{I_O}{I_I}=\beta\cdot\frac{R_{E1}+\frac{R_B}{\beta+1}}{\left(R_{E1}+\frac{R_B}{\beta+1}\right)+(\beta+1)\cdot R_{E2}}$$

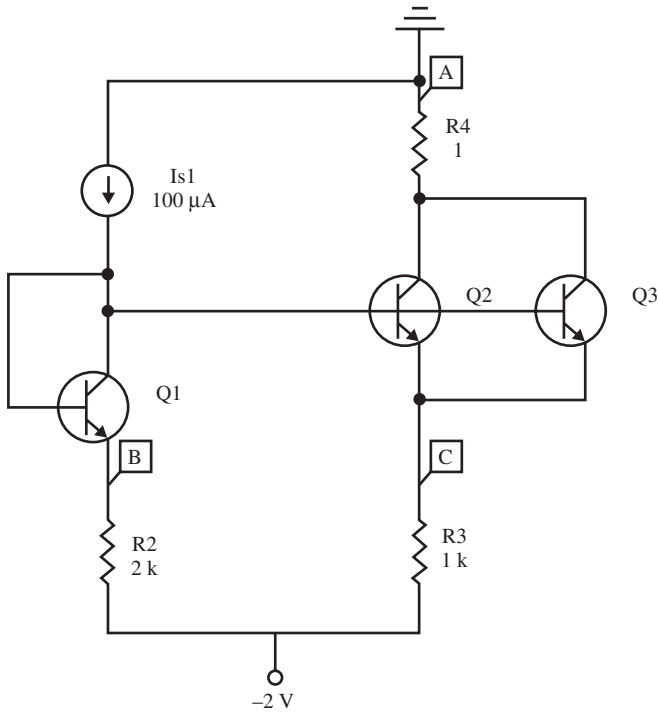
Furthermore, the mirror can be designed with the additional constraint that the ratio of emitter resistors equals the current gain, or

$$A_I = \frac{R_{E1}}{R_{E2}}$$

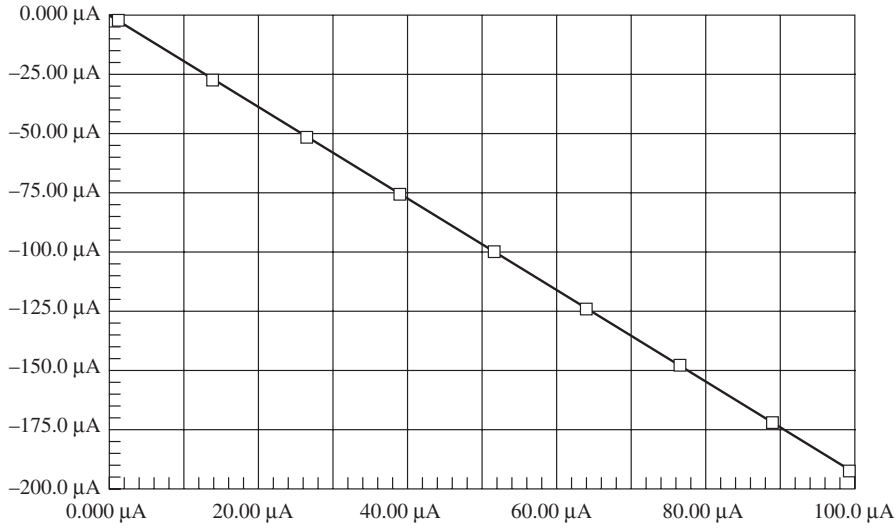
Substituting into the gain equation and solving for R_B,

$$R_B = \frac{(A_I + 1)}{\alpha \cdot \left(1 - \frac{A_I}{\beta}\right)} \cdot R_{E1}$$

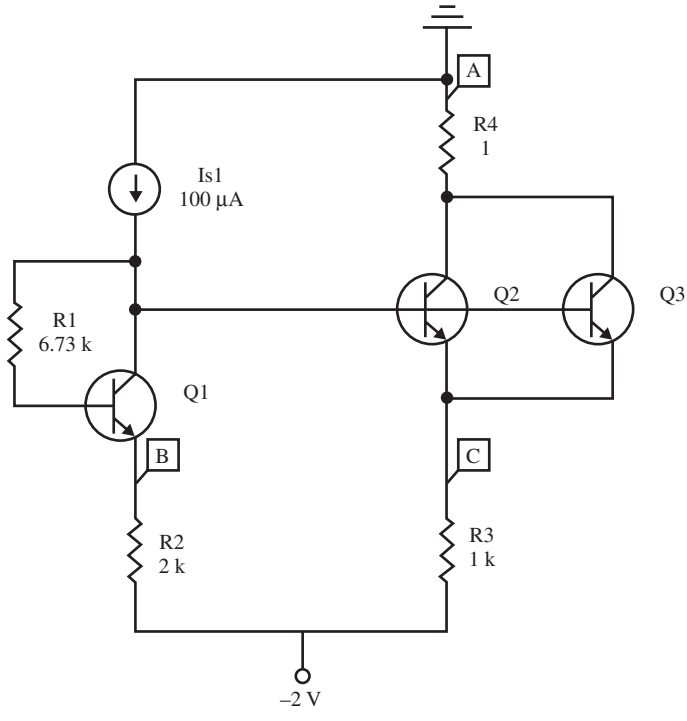
The compensating effect of R_B can be seen from circuit simulations of the circuit with and without it. For a × 2 current gain, the following circuit with matched transistors of $\beta = 100$ was simulated.



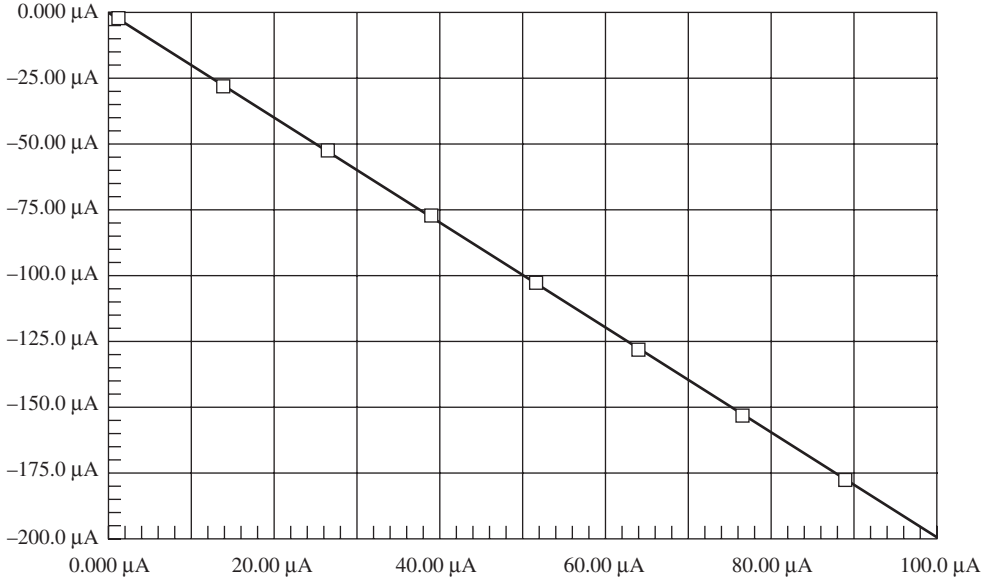
The resulting output current (plotted as a load voltage) is shown below.



Ideally, at 100 µA of input current, the output current should be –200 µA with a ×2 gain, but it is low in magnitude by about 3%. The R_B formula is now applied and R_B added to the circuit, as shown below.



The corresponding output plot is shown below.



The error is now much reduced, to about –0.1%.

Incremental Gain

The equation for I_O can be linearized for incremental (small-signal) analysis by letting

$$v_{be} = \frac{dv_{BE}}{di_E} \cdot di_E = \frac{V_T}{I_E} \cdot i_e = r_e \cdot i_e$$

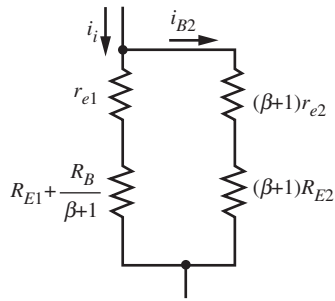
Substituting for v_{be}, the incremental expression for i_o results in

$$i_o = \beta \cdot \left[\frac{R_{E1} + R_B/(\beta+1)}{R_{E1} + R_B/(\beta+1) + (\beta+1) \cdot R_{E2}} \cdot i_i + \frac{r_{e1} \cdot i_{e1} - r_{e2} \cdot i_{e2}}{R_{E1} + R_B/(\beta+1) + (\beta+1) \cdot R_{E2}} \right]$$

When substitution is made for the emitter currents, the two terms join to result in a single incremental gain expression:

$$A_i = \frac{i_o}{i_i} = \beta \cdot \frac{r_{e1} + R_{E1} + R_B/(\beta+1)}{r_{e1} + R_{E1} + R_B/(\beta+1) + (\beta+1)\cdot(r_{e2} + R_{E2})}$$

The rational factor is the current divider, i_{b2}/i_i represented by an incremental equivalent circuit, shown below. This gain expression is valid for a given operating point, where r_{e1} and r_{e2} are dependent on the static current values.



Although this gain formula is easily derived from the equivalent circuit, it does not make the junction-voltage matching constraint explicit. But when $v_{be1} = v_{be2}$, then $r_{e1} \cdot i_{e1} = r_{e2} \cdot i_{e2}$ and when substituted into i_o in the previous equation, the resulting incremental current gain for matched junctions is the same as for the static current gain:

$$A_i = \frac{i_o}{i_i} = \beta \cdot \frac{R_{E1} + \dfrac{R_B}{\beta+1}}{\left(R_{E1} + \dfrac{R_B}{\beta+1}\right) + (\beta+1)\cdot R_{E2}}$$

When also subjected to the additional constraint,

$$A_i = \frac{R_{E1}}{R_{E2}}$$

and substituting into A_i and solving for R_B, the resulting expression is the same as for the static case:

$$R_B = \frac{(A_I+1)}{\alpha \cdot \left(1 - \dfrac{A_I}{\beta}\right)} \cdot R_{E1}$$

β Sensitivity of R_B-Compensated Current Mirror

R_B was chosen according to the previous R_E-ratio constraint, which simplifies the selection of emitter resistor values. The current gain is nevertheless sensitive to β variation, and R_B can instead be chosen for minimum β sensitivity, which is

$$\min \frac{\partial}{\partial \beta}\left(\frac{I_O}{I_I}\right)$$

For $\beta \gg 1$, I_O/I_I can be rewritten as

$$\frac{I_O}{I_I} \cong \beta \cdot \frac{\beta \cdot R_{E1} + R_B}{\beta \cdot R_{E1} + R_B + \beta^2 R_{E2}} = \beta \cdot \frac{a}{a + \beta^2 R_{E2}} = \beta \cdot f(\beta)$$

Then

$$\frac{\partial}{\partial \beta}\left(\frac{I_O}{I_I}\right) = \frac{\partial}{\partial \beta}\beta \cdot f(\beta) = f + \beta \cdot \frac{\partial f}{\partial \beta}$$

$$\frac{\partial f}{\partial \beta} = \frac{-a \cdot (R_{E1} + 2\beta \cdot R_{E2})}{(a + \beta^2 \cdot R_{E2})^2} + \frac{R_{E1}}{(a + \beta^2 \cdot R_{E2})}$$

Substituting yields

$$\frac{\partial f}{\partial \beta}\left(\frac{I_O}{I_I}\right) = \frac{a^2 + a\beta^2 R_{E2} - \beta a R_{E1} - 2a\beta^2 R_{E2} + a\beta R_{E1} + \beta^3 R_{E1} R_{E2}}{(a + \beta \cdot R_{E2})^2}$$

To obtain R_E at minimum sensitivity to β, the previous equation is set to zero and the numerator solved for R_B. The result is

$$R_B = -\beta \cdot \left(R_{E1} - \frac{\beta}{2} \cdot R_{E2}\right) \pm \frac{\beta}{2} \cdot \sqrt{\beta \cdot R_{E2}(\beta \cdot R_{E2} - 4 \cdot R_{E1})}$$

Under the previous constraint, the sensitivity of A_I with respect to β is

$$S_\beta^{A_I} = \frac{\partial A_I / A_I}{\partial \beta / \beta} = \left(\frac{\beta}{A_I}\right) \cdot \frac{\partial A_I}{\partial \beta}$$

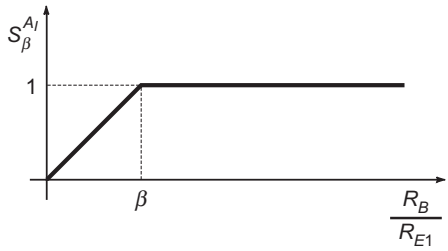
where A_I is given with R_{E2} expressed in terms of R_{E1} and A_I. With R_B given and substituting into S,

$$A_\beta^{A_I} = k^2 \cdot \frac{\beta}{(k-1)\cdot\beta^2 + [(k+1)\cdot(k-1)-1]\cdot\beta - (k+1)},$$

$$k = \frac{R_B}{R_{E1}}$$

$$\cong \frac{1}{\beta/k+1}, \quad \beta, k >> 1$$

The approximation to S is asymptotic with one, as graphed below.



For $k > \beta$, A_I varies directly with β. This result suggests that k be kept less than β to reduce gain sensitivity to β. This is not always possible when attempting to satisfy the R_E-ratio constraint. The circuit has gain limits for acceptable gain sensitivity to β that are set by β.

The dynamic current gain has a similar result to the static analysis given previously except that dynamic emitter resistances are included in series with R_{E1} and R_{E2}. Then the quasistatic base resistance is

$$R_B = (\beta+1)\cdot\left(\frac{A_I}{\alpha\cdot\left(1-\frac{A_I}{\beta}\right)}\cdot R_2 - R_1\right)$$

where

$$R_1 = r_{e1} + R_{E1}, \quad R_2 = r_{e2} + R_{E2}$$

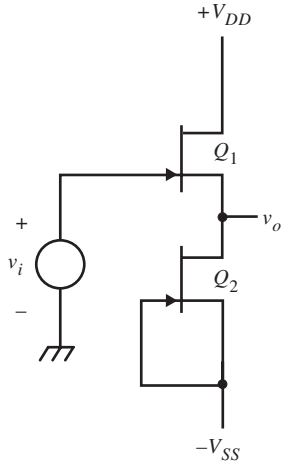
For $A_I = R_1/R_2$, then

$$R_B = \frac{A_I + 1}{\alpha \cdot \left(1 - \frac{A_I}{\beta}\right)} \cdot R_1$$

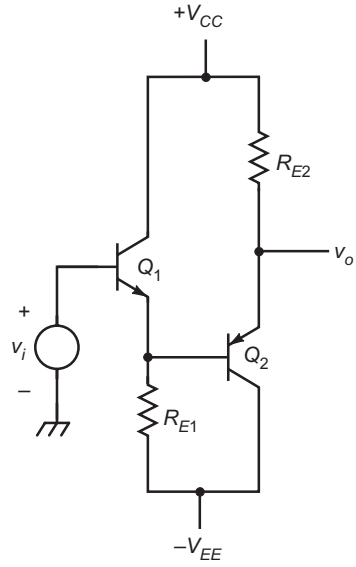
For a typical β of 99 and A_I of 1, $R_B \cong 2.04 \cdot R_1$.

Matched Transistor Buffers and Complementary Combinations

A simple but elegant circuit consists of a pair of matched junction field-effect transistors (JFETs). The lower JFET, Q_2, functions as a current source for the upper source follower. The beauty of this circuit is that the lower transistor sinks a particular amount of current (I_{DSS}) with $V_{GS} = 0$ and that, with negligible loss of current to the load, I_{DSS} also flows through the upper JFET, resulting in the same V_{GS} of zero volts (because they have matched characteristics). This voltage amplifier of unity gain (or ×1 *buffer*) consequently has zero voltage offset. This is desirable because the purpose of a buffer is to provide a voltage source at a much-reduced output resistance than the input voltage (from a higher-resistance source). A simple emitter- or source-follower would cause an offset due to an undetermined V_{BE} or V_{GS} and introduce a voltage error at the output.



Matched-FET buffer

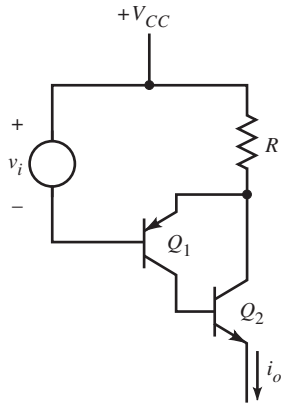


Complementary CC buffer

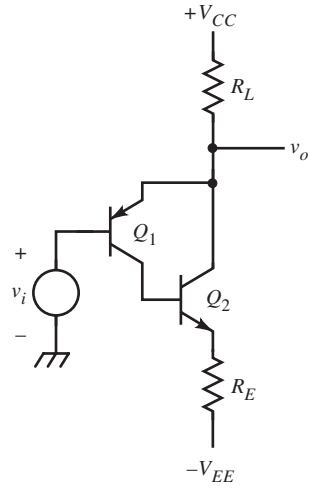
A BJT circuit based on the same general idea is shown here. The V_{BE} offsets of opposite polarity CCs cancel to the extent that their currents are equal (for matched junctions). This offset match is more difficult than with the JFET circuit in that the devices are of opposite polarity. This circuit is nevertheless quite useful for acceptable offsets of typically less than 50 mV.

The NPN/PNP pair shown below functions like a PNP BJT but with current gain from the output NPN. This circuit is commonly used in the output stage of amplifiers so that only power NPN transistors need be used (and is sometimes called a "quasi-complementary PNP" circuit). It can also be used, as shown below, to source current. Although the output is from an emitter, the base resistance is large (r_e of Q_1), resulting in an acceptably large output resistance to pass as a current source in applications in which the driven node is of relatively low resistance.

Some complementary pairs are regenerative and form latching circuits. *Thyristors* are a class of four-layer (PNPN or NPNP) devices that are used as high-power switches and also are formed as parasitic elements in ICs that have multiple n and p layers (such as complementary metal-oxide semiconductor (CMOS) or bipolor metal-oxide semiconductor circuits that have a tendency to latch if their

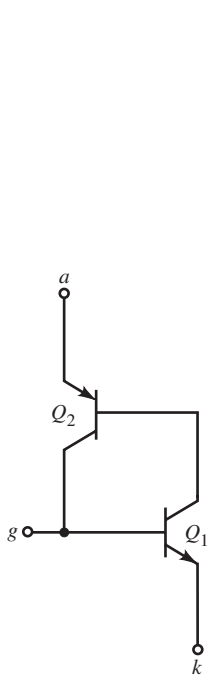


complementary current source

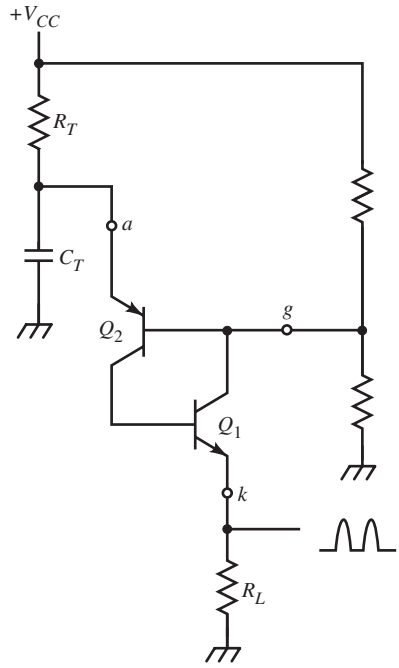


quasi-complementary CC

inputs exceed the supply voltages). A common thyristor, the *silicon controlled rectifier* (SCR), is shown below, along with a variant, the *programmable unijunction transistor* (PUT).



SCR



PUT circuit

For either device, transistor collector currents supply base current to the other transistor, causing regenerative action. SCRs cannot be turned off by the gate; the anode-to-cathode voltage must reverse, causing cessation of conduction. SCRs are used in power conversion, and PUTs are useful devices for constructing simple oscillators and programmable timers.

CLOSURE

We have examined a variety of amplifier circuits that have one to three transistors. More complicated "building blocks" will be introduced later. As additional transistors are added, complexity grows to the point that a multilevel or hierarchical organization is needed. Multiple circuits are combined to form complete subsystems, which in turn are combined with other subsystems to form the final system. We can manage complexity at these various levels in the same way. An op-amp (introduced in Chapter 3) contains many circuits but, like the transistor, can be modeled as a single device with a simple functional description. In this chapter, we developed a "library" of basic circuits that can be used to develop a library of basic subsystems, in the same way that commonly used computer routines can be joined to form more complicated routines.

This circuit discussion was based on simplifying assumptions that must now be examined. We assumed that a circuit input was independent of the output, that there was no *feedback*. Furthermore, although both static and quasistatic quantities were introduced, we omitted reactive components such as capacitors and inductors. These components require the use of complex numbers to describe their behavior and lead to discussion of transient and frequency response.

3

Amplifier Concepts

The Reduction Theorem

The β transform greatly simplifies open-loop amplifier circuit analysis and makes the transresistance method possible. We now examine circuits with more complex topologies. It is common for transistor amplifier stages to have a significant forward transmittance through r_o. This results in parallel forward paths. Parallel *c-e* or *c-b* resistance causes bilateral signal flow with a combination of feedback and multiple forward paths.

This leads to some network theorems that are useful for simplifying these circuits. Analytic techniques adaptable to intuitive use are based on powerful, general circuit theorems. The β transform is half of a more general theorem, the *reduction theorem*. It has two forms:

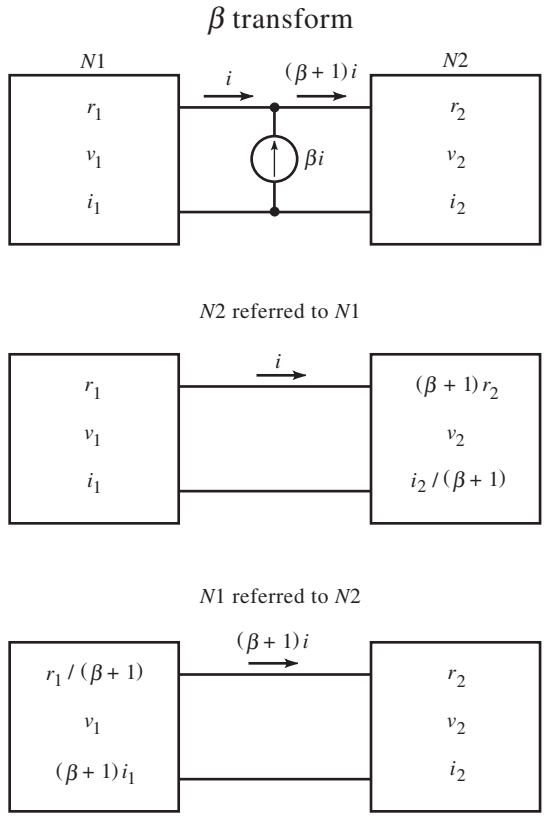
$$\text{current form} \Rightarrow \beta \text{ transform}$$

$$\text{voltage form} \Rightarrow \mu \text{ transform}$$

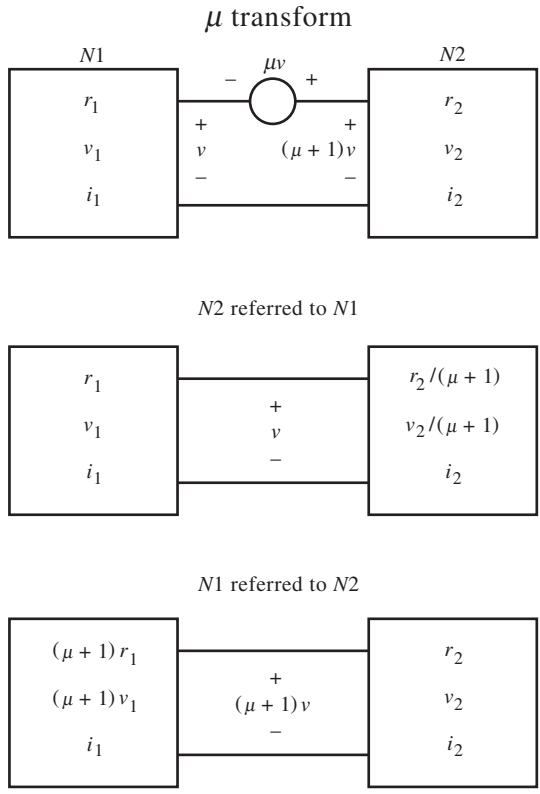
These forms are duals. The figure below portrays the β transform as a general network theorem. Two networks, represented by blocks, share a common port with a controlled source between them. In the current-source case, network $N1$ could be a bipolar junction transmitter (BJT) base circuit, in which i is the base current. Then network $N2$ is the emitter circuit, and the current source that shunts the common port is a BJT collector current source.

Wherever circuits are equivalent to the top network, two equivalent circuits (middle and bottom) are possible. These correspond, respectively, to equivalent

base and emitter circuits for a BJT. In the middle diagram, $N2$ is transformed using $\beta + 1$; in the bottom, $N1$ is transformed instead. All voltages, currents, and resistances in the transformed network are affected as shown.



The figure below displays the corresponding dual of the β transform, the μ transform. It applies to circuits with a voltage gain because μ is a voltage gain. This transform was used extensively in modeling vacuum-tube triodes and applies especially to field-effect transistors (FETs) because of their low drain resistance. It enables us to avoid use of feedback analysis in shunt-feedback circuits by transforming them into circuits most easily analyzed open-loop.



μ TRANSFORM OF BJT AND FET T MODELS

The μ transform cannot be applied directly to circuits using the T model because the transform is based on a controlled voltage source. The T model, shown below, is shunted by r_o. This familiar model is used later when feedback analysis is applied to multipath circuits. For now, it must be transformed into a model with a controlled voltage source. This can be done by first referring r_e to the base as r_π (using the β transform). Then r_o shunts the controlled current source and forms a Norton equivalent circuit with it. The Norton circuit can be converted to a Thevenin equivalent by noting that

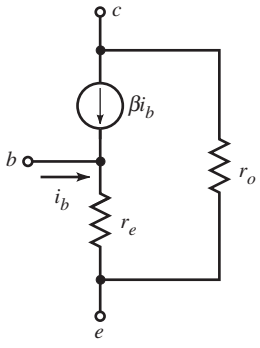
$$\beta \cdot i_b = \alpha \cdot i_e = \alpha \cdot \left(\frac{v_{be}}{r_e} \right) = \frac{v_{be}}{r_m}$$

This current is converted to a Thevenin voltage by multiplying by the series resistance r_o, resulting in

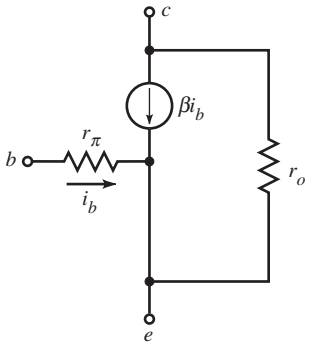
$$r_o \cdot \left(\frac{v_{be}}{r_m} \right) = \left(\frac{r_o}{r_m} \right) \cdot v_{be} = \mu \cdot v_{be}$$

More precisely, the definition of μ for the BJT model is

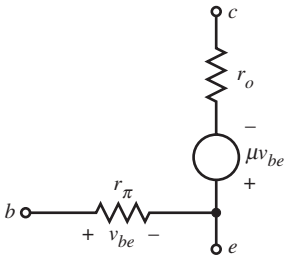
$$\mu \equiv -\left. \frac{v_{ce}}{v_{be}} \right|_{i_c = 0}$$



T model with r_o



Norton equivalent hybrid-π



Thevenin equivalent hybrid-π

The condition that i_c be zero allows v_{ce} to be the voltage of the controlled source alone, without additional drop across r_o. Furthermore,

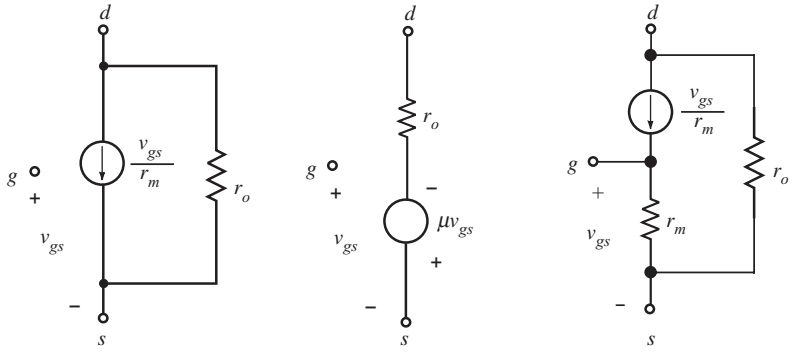
$$\left(\frac{\mu}{\mu+1}\right)=\frac{r_o}{r_o+r_m}=\frac{-v_{ce}/v_{be}\big|_{i_c=0}}{1-\left(v_{ce}/v_{be}\right)\big|_{i_c=0}}=\frac{-v_{ce}}{v_{be}-v_{ce}}\bigg|_{i_c=0}=\frac{v_{ce}}{v_{cb}}\bigg|_{i_c=0}=\frac{v_{ec}}{v_{bc}}\bigg|_{i_c=0}$$

For $i_c = 0$ (and finite collector resistance), then $v_c = 0$ and

$$\lambda=\left(\frac{\mu}{\mu+1}\right)=\frac{v_e}{v_b}$$

This relationship appears often in circuit analyses and is designated by λ, the counterpart of $\alpha = \beta/(\beta + 1)$.

A similar transformation can be applied to the quasistatic FET circuit model, shown below, where r_o is included.



The model immediately converts to the Thevenin equivalent form of the BJT model. An alternative equivalent model is also shown, in which the gate is connected to the current source and r_m is added. This is an FET T model. The gate current i_g remains zero because all i_g must flow through r_m. Its resulting voltage drop affects v_{gs}, and since the current source is controlled by v_{gs}, a change in drain current equal to i_g is injected into the gate node. In other words, since the voltage across r_m is v_{gs}, the current that must be flowing in r_m is v_{gs}/r_m. But this is the amount of current injected into the gate node by the drain current source. By Kirchhoff's current law (KCL), i_g must be zero.

The definition of μ applied to this FET model is substantially the same as the BJT model. The relationship between the BJT and FET models is simple: If r_e of the BJT model is replaced by r_m, the FET model results.

BJT to FET T-Model Conversion

$$r_e \Rightarrow r_m, \quad e \Rightarrow s, \quad b \Rightarrow g, \quad c \Rightarrow d, \quad v_{be} \Rightarrow v_{gs}$$

Applying this conversion to the BJT definition of μ results in the FET version of μ:

$$\text{FET } \mu \equiv -\left.\frac{v_{ds}}{v_{gs}}\right|_{i_d=0}$$

Because r_o for FETs is typically much lower than for BJTs, the use of transistor models that include r_o is more common for FETs.

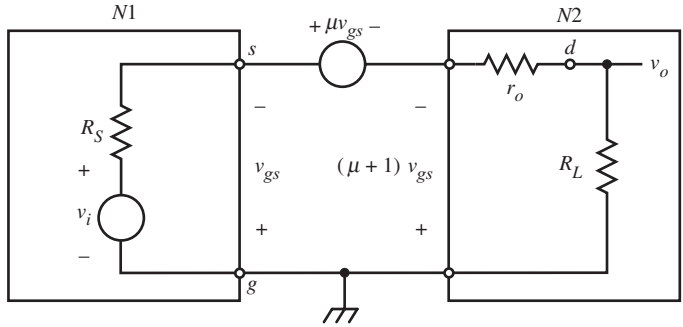
COMMON-GATE AMPLIFIER WITH r_o

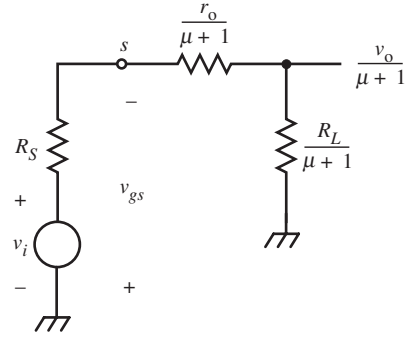
The figure below shows a common-gate (CG) amplifier, drawn so that the reduction theorem can be easily applied to it. The gate is at ground and is the common terminal of the two networks shown in boxes. Network $N1$ is the source circuit, and $N2$ is the drain circuit.

The FET model of the CG circuit is between $N1$ and $N2$. To make this circuit correspond to the μ-transform circuit, r_o must be included in $N2$. The result of transforming the drain circuit ($N2$) is shown in the lower equivalent circuit. The drain circuit has been referred to the source side. The output voltage v_o across R_L is also transformed to $v_o/(\mu + 1)$. This transformed circuit is now a voltage divider between input v_i and output $v_o/(\mu + 1)$:

$$\frac{v_o}{\mu+1} = \frac{R_L/(\mu+1)}{R_S + r_o/(\mu+1) + R_L/(\mu+1)} \cdot v_i$$

The voltage gain is thus





$$\text{CG } A_v = \frac{R_L}{R_S + (R_L + r_o)/(\mu + 1)}$$

This result is reminiscent of the transresistance method but uses the μ transform instead of the β transform. It demonstrates the voltage form of the transresistance method. The denominator of the expression for CG A_v can be interpreted as amplifier transresistance, r_M. The resistance in the drain contributes to r_M and appears smaller by $1/(\mu + 1)$ when referred to the source side of the FET. The β transform involves base and emitter networks; the μ transform involves the drain (or collector) and source (or emitter) circuits instead.

The input resistance r_{in} can be envisioned directly from the upper circuit to be

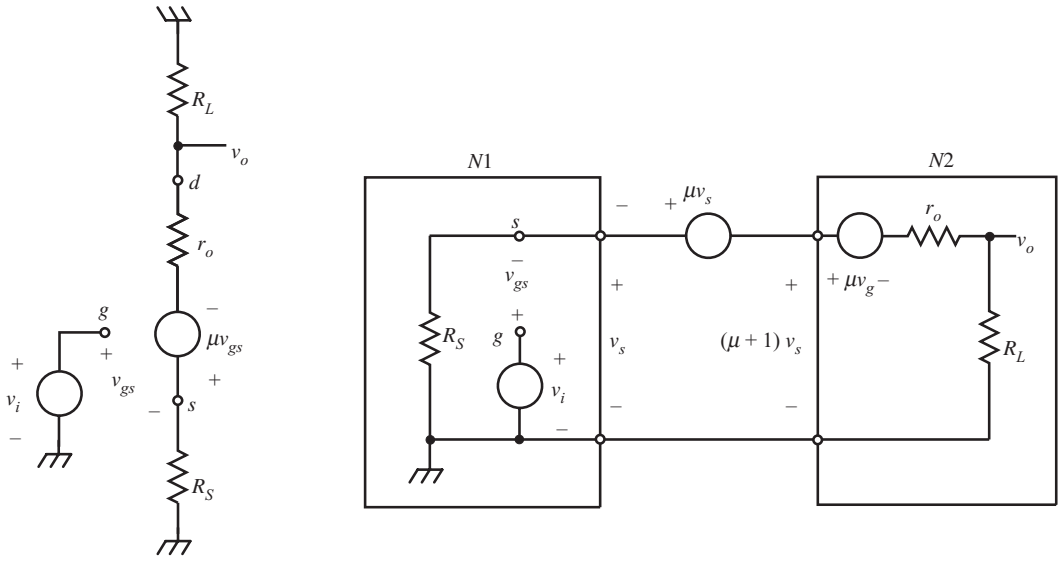
$$\text{CG } r_{in} = \frac{r_o + R_L}{\mu + 1} + R_S$$

The CG output resistance can be found by μ-transforming the source circuit. In this case, the resistance of the source referred to the drain is $(\mu + 1)$ times larger, so that

$$\text{CG } r_{out} = R_L \| [r_o + (\mu + 1) R_S]$$

COMMON-SOURCE AMPLIFIER WITH r_o

A common-source (CS) FET amplifier is shown below.



The voltage-source FET model makes Kirchhoff's voltage law (KVL) analysis easy with only one loop. The needed equations are

$$v_s = i_s \cdot R_S$$

$$v_o = -i_s \cdot R_L$$

$$i_s \cdot [R_S + r_o + R_L] = \mu \cdot v_{gs} = \mu \cdot v_g - \mu \cdot R_S \cdot i_s$$

Solving for A_v gives

$$\frac{v_o}{v_g} = -\frac{R_L}{r_m + (R_L/\mu) + ((\mu+1)/\mu)\cdot R_S}$$

Although this gain is a ratio of resistances, the terms in the denominator involving μ do not have a simple interpretation in terms of the μ transform and circuit topology. But by factoring $(\mu + 1)/\mu$ out of the denominator, we obtain two factors containing v_s:

$$\text{CS } A_v = -\left(\frac{\mu}{\mu+1}\right)\cdot\frac{R_L}{R_S + (r_o + R_L)/(\mu+1)}$$
$$\uparrow \qquad\qquad \uparrow$$
$$(v_s/v_g) \qquad\qquad (v_o/v_s)$$

The first factor, λ, is the gate-to-source gain. The second is the same as the CG A_v. Its denominator can be interpreted as r_M, keeping in mind that it is v_s (not v_g) across r_M that generates i_s. Consequently, the voltage form of the transresistance method is based on finding r_M across v_s and then (if needed) relating v_s to v_g through λ:

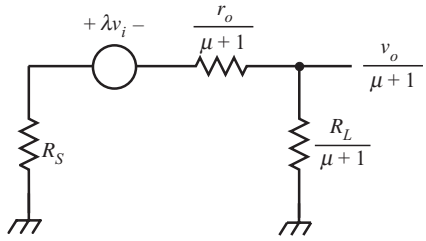
$$r_M = \frac{v_s}{i_s}$$

and

$$v_s = \left(\frac{\mu}{\mu+1}\right)\cdot v_g = \lambda\cdot v_g$$

The gain expression of v_o/v_g was found using basic circuit laws, not by applying the μ transform directly to the circuit topology. To do so for the CS is not as obvious as for the CG. The gate is not common to both source and drain circuits. In the circuit shown above, it is redrawn on the right so that application of the μ transform is explicit. Because the port voltage is chosen to be v_s, the drain voltage source $\mu\cdot v_{gs}$ is split into two sources so that the first is dependent upon v_s. The remaining source, $\mu\cdot v_g$, becomes part of the drain network and is

transformed along with it. When the μ transform is applied to the drain circuit, the circuit shown below results.



The voltage across the source-referred R_L is

$$\frac{v_o}{\mu+1} = -\lambda \cdot \frac{v_i}{r_M} \cdot \left(\frac{R_L}{\mu+1} \right)$$

Solving for the voltage gain gives

$$\text{CS } A_v = -\lambda \cdot \frac{R_L}{R_S + (r_o + R_L)/(\mu+1)} = -\lambda \cdot \frac{R_L}{R_S + r_s + R_L/(\mu+1)}$$

The expression $r_o/(\mu + 1)$ has been expressed as

$$r_s = \frac{r_o}{\mu+1} = \frac{r_o}{\mu} \cdot \left(\frac{\mu}{\mu+1} \right) = \lambda \cdot r_m$$

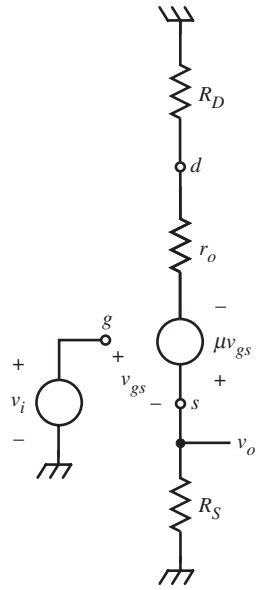
When r_o is referred to the source, it transforms to r_s, the FET analog of r_e, in that both are related to r_m by dual factors, α and λ. Although α expresses a current loss due to base current, λ expresses a voltage loss due to v_{gs}; μ and β are duals, as are $\lambda = \mu/(\mu + 1)$ and $\alpha = \beta/(\beta + 1)$.

μ Transform (Voltage)	β Transform (Current)
μ	β
$\lambda = \mu/(\mu + 1)$	$\alpha = \beta/(\beta + 1)$

The input resistance of the CS amplifier is infinite. The output resistance is the same as the CG; the source circuit referred to the drain is the same for both.

COMMON-DRAIN AMPLIFIER WITH r_o

The last of the three basic FET configurations is the common-drain (CD) or source-follower, shown below. Applying the voltage form of the transresistance method, r_M is found by determining the resistance across which the source voltage generates the source current i_s. The μ transform is required to refer the resistance on the drain side of the FET voltage source to the source side.



As before, it is

$$r_s + \frac{R_D}{\mu + 1}$$

This resistance, when referred to the source circuit, is in series with R_S. The total transresistance is thus

$$r_M = R_S + r_s + \frac{R_D}{\mu + 1}$$

The source current generated by v_s across r_M develops an output voltage across R_S. The voltage gain from gate to source must include the λ factor;

$$\text{CD } A_v = \lambda \cdot \frac{R_S}{R_S + r_s + R_D/(\mu + 1)}$$

This gain is more general than for a CD amplifier without a resistance in the drain, R_D.

The input resistance of the CD is infinite, and the output resistance is

$$\text{CD } r_{out} = R_S \left\| \left(r_s + \frac{R_D}{\mu + 1} \right) \right.$$

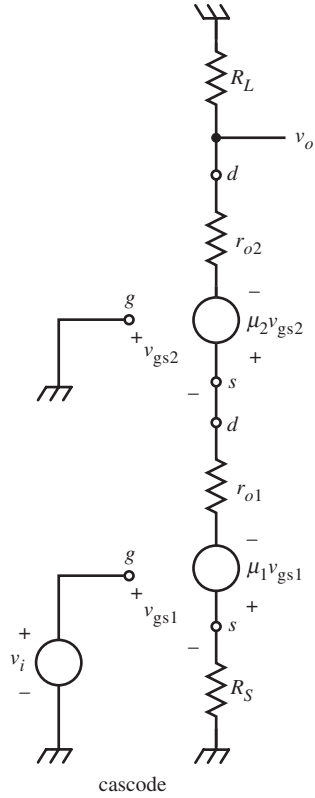
FET Cascode Amplifier with r_o

The voltage form of the transresistance method extends directly to multiple-transistor amplifier stages. The FET cascode amplifier model, shown below, has a voltage gain of

$$\text{cascode } A_v = -\lambda_1 \cdot \frac{R_L}{R_S + \dfrac{r_{o1} + \left((r_{o2} + R_L)/(\mu_2 + 1) \right)}{\mu_1 + 1}}$$

$$= -\lambda_1 \cdot \frac{R_L}{R_S + r_{s1} + r_{s2}/(\mu_1 + 1) + \dfrac{(R_L/(\mu_2 + 1))}{\mu_1 + 1}}$$

This can be interpreted (and also constructed) by inspection of the circuit diagram. The input voltage v_i at the gate of the CS produces v_s via λ_1. The CS r_M is R_S in series with the drain resistance, referred to the source. Drain resistance is r_{o1} in series with the CG drain circuit referred to its source, or $(r_{o2} + R_L)/(\mu_2 + 1)$. When these resistances are referred to the CS source, the denominator of the previous gain equations, r_M, results. The source current develops the output voltage, v_o, over R_L (in the numerator) and is an inverting output. A_v can be written as the lower equation just presented, using the definition of r_s, which is r_o referred to the source circuit.



cascode

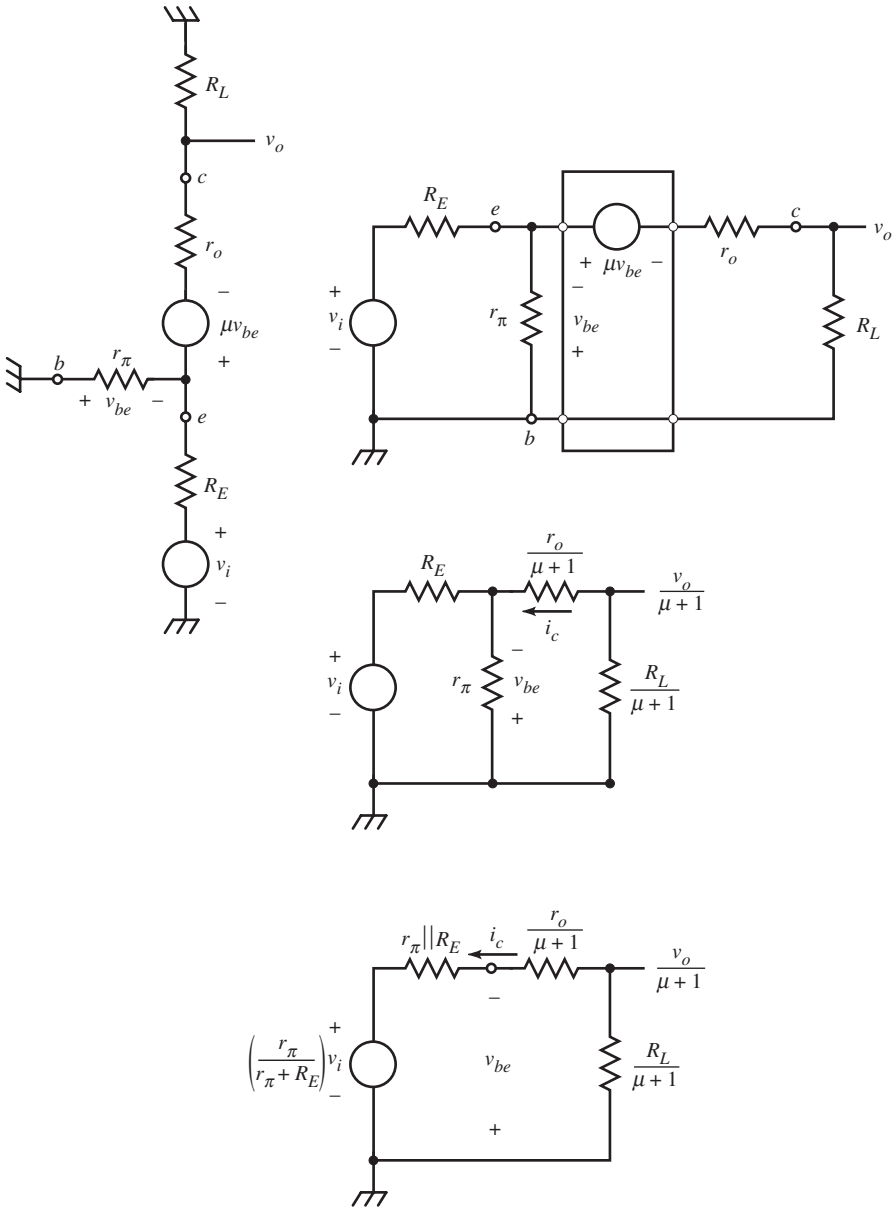
The output resistance of the cascode stage can be found using the same approach and is

$$\text{cascode } r_{out} = R_L \| \left[r_{o2} + (\mu_2 + 1) \cdot \left(r_{o1} + (\mu_1 + 1) \cdot R_S \right) \right]$$

To construct this expression for r_{out}, the μ transform is used to refer source resistances to the drain circuit.

COMMON-BASE AMPLIFIER WITH r_o

The application of the voltage form of the transresistance method to BJT amplifiers adds the complication of r_π. It forms an additional loop or node not present in the CG circuit. This complication does not significantly affect the approach.



The circuit model is redrawn (upper right) to make the application of the μ transform explicit. After the collector circuit is referred to the emitter side (middle), the divider formed by R_E and r_π is Thevenized (lower). The voltage gain can then be found by solving the voltage divider:

$$\text{CB } A_v = \left(\frac{r_\pi}{r_\pi + R_E}\right) \cdot \frac{R_L}{r_\pi \| R_E + (R_L + r_o)/(\mu + 1)}$$
$$\uparrow \qquad\qquad \uparrow$$
$$(v_e/v_i) \qquad (v_o/v_e)$$

This gain expression has two additional complications over that of the CG. At the emitter, R_E is now shunted by r_π. This affects r_M in the second factor of the common-base (CB) gain equation. The first factor accounts for the divider formed by r_π with R_E. An alternative formulation of A_v regards r_π and R_E as forming a current divider with a transmittance of (i_c/i_e):

$$\text{CB} \quad A_v = \left[\frac{r_\pi}{r_\pi + (r_o + R_L)/(\mu + 1)}\right] \cdot \frac{R_L}{R_E + r_\pi \| [(r_o + R_L)/(\mu + 1)]}$$
$$= \left(\frac{i_c}{i_e}\right) \cdot \left(\frac{i_e}{v_i}\right) \cdot \left(\frac{v_o}{i_c}\right) = \left(\frac{i_c}{i_e}\right) \cdot \left(\frac{1}{r_{in}}\right) \cdot (R_L)$$

For the CB gain equation, i_e is the common quantity of the transresistance method. The input v_i generates i_e across the input resistance r_{in}, which is r_M, the denominator of the second factor:

$$\text{CB} \quad r_{in} = R_E + r_\pi \left\| \left(\frac{r_o + R_L}{\mu + 1}\right)\right.$$

Some of i_e is lost to the base, leaving i_c, and is accounted for by the first factor of CB A_v. The output voltage is then developed across R_L by i_c. In this formulation, both voltage and current forms of the transresistance method are present. The μ transform refers the collector resistances to the emitter; the voltage form is applied. The (i_c/i_e) factor, however, is a circuit-dependent α characteristic of gain equations resulting from the current form. In contrast, the previous CB gain equation has a purely voltage-form interpretation. It is easier to apply only one form, and is preferred in most cases.

The CB output resistance is found by applying the μ transform to the emitter circuit:

$$\text{CB } r_{out} = R_L \| \left[r_o + (\mu + 1) \cdot (r_\pi \| R_E) \right] = R_L \| r_c$$

The μ-transformed expression for the collector resistance, r_c, has been derived before:

$$r_c = \frac{v_c}{i_c} = R_E \| R_B + r_o \left(1 + \beta \cdot \frac{R_E}{R_B + R_E} \right)$$

This equivalent formula was given a β-transform interpretation before. To derive r_c of CB r_{out} from r_c, substitute $\mu \cdot r_\pi$ for $\beta \cdot r_o$ and let $R_B = r_\pi$.

CC AND CE AMPLIFIERS WITH r_o

The common-collector (CC) (emitter-follower) is shown below, with a simplified equivalent circuit. This is a generalized CC amplifier in that collector resistance is included.

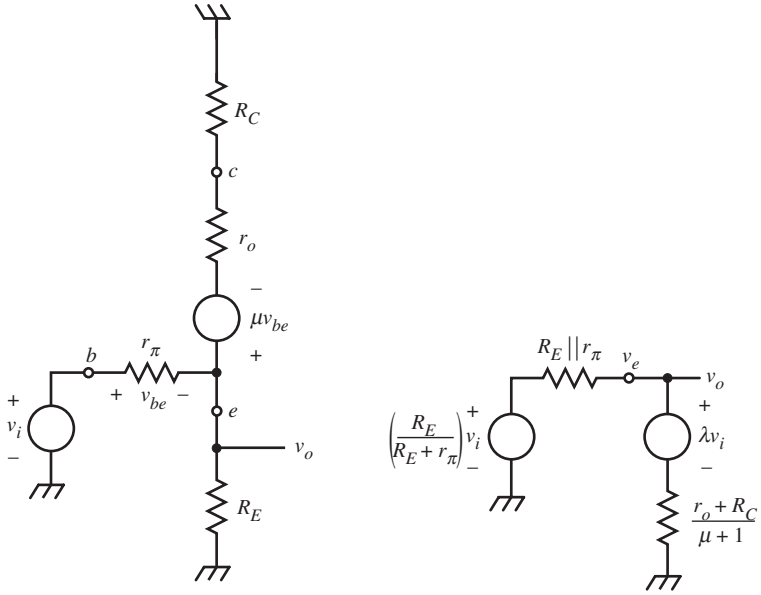
Following the approach used with the CS amplifier, the gain is

$$\text{CC } A_v = \left[\frac{R_E}{R_E + r_\pi} \right] \cdot \left[\frac{(R_C + r_o)/(\mu + 1)}{R_E \| r_\pi + (R_C + r_o)/(\mu + 1)} \right] \quad \text{passive path}$$
$$+ \lambda \cdot \left[\frac{R_E \| r_\pi}{R_E \| r_\pi + (R_C + r_o)/(\mu + 1)} \right] \quad \text{active path}$$

The CC has two gain paths: an active path due to the gain of the transistor, and a passive path due to a finite r_π. The first factor of both terms is (v_e/v_i). For the active path, the second factor is a ratio of load resistance $R_E \| r_\pi$ over r_M.

The second factor of the passive path term is a voltage-divider gain due to the drop across the collector resistance, referred to the emitter. This is a loaded divider with

$$R_E \left\| \frac{(r_o + R_C)}{(\mu + 1)} \right.$$



The passive path gain can be rewritten to make this explicit:

$$\frac{R_E \| [(r_o + R_C)/(\mu + 1)]}{R_E \| [(r_o + R_C)/(\mu + 1)] + r_\pi}$$

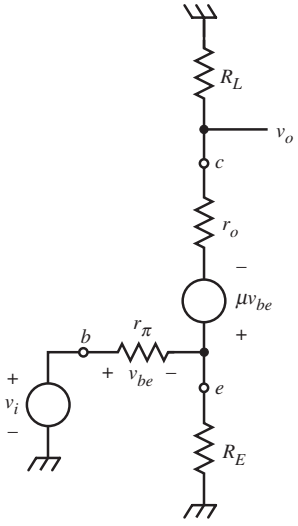
The figure below shows the common-emitter (CE) circuit model and its successive modifications leading to equivalent circuit (e).

Again, the μ transform reduces this circuit to a voltage divider. r_π creates a loaded divider (c) that is Thevenized in (d). The voltage source, $\lambda \cdot v_i$ is combined with v_i in (e), from which a voltage gain expression can be written as

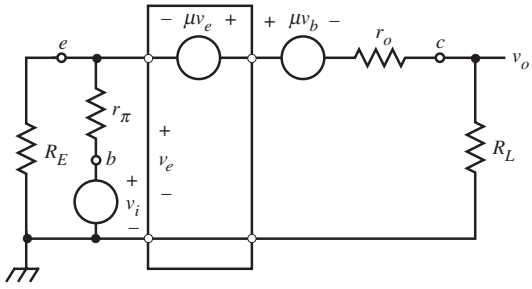
$$\text{CE } A_v = \underbrace{\left(-\lambda + \frac{R_E}{R_E + r_\pi}\right)}_{\text{active path}} \cdot \underbrace{\frac{R_L}{(R_L + r_o)/(\mu + 1) + r_\pi \| R_E}}_{\text{passive path}}$$

$$\uparrow \quad \uparrow \qquad\qquad \uparrow$$

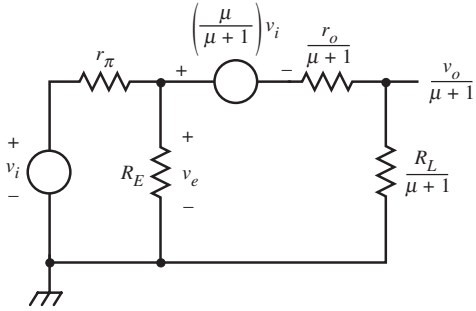
$$(v_e/v_i) \qquad\qquad (v_o/v_e)$$


R_L
v_o
c
r_o
μv_{be}
b
r_π
v_{be}
e
v_i
R_E

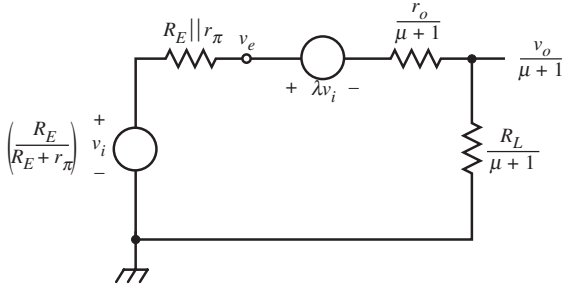
(a)


μv_e
μv_b
r_o
e
c
v_o
r_π
R_E
b
v_e
v_i
R_L

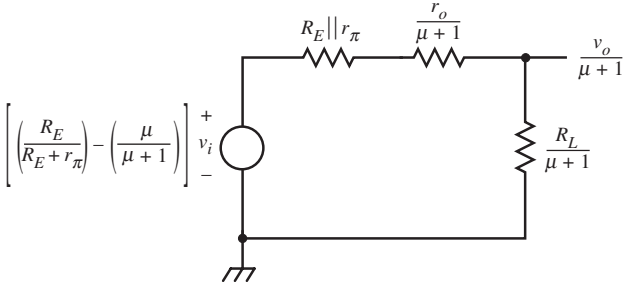
(b)


$\left(\frac{\mu}{\mu+1}\right)v_i$
r_π
$\frac{r_o}{\mu+1}$
$\frac{v_o}{\mu+1}$
v_i
R_E
v_e
$\frac{R_L}{\mu+1}$

(c)


$R_E||r_\pi$
v_e
$\frac{r_o}{\mu+1}$
$\frac{v_o}{\mu+1}$
λv_i
$\left(\frac{R_E}{R_E+r_\pi}\right)$
v_i
$\frac{R_L}{\mu+1}$

(d)


$R_E||r_\pi$
$\frac{r_o}{\mu+1}$
$\frac{v_o}{\mu+1}$
$\left[\left(\frac{R_E}{R_E+r_\pi}\right)-\left(\frac{\mu}{\mu+1}\right)\right]$
v_i
$\frac{R_L}{\mu+1}$

(e)

The second factor of the CE A_v is the load resistance over r_M, the same as for the CB amplifier. The novelty is in the first factor. The first term, $-\lambda$, is the μ-transform base-to-emitter voltage gain due to the active device μ amplification; it expresses the gain due to the active forward path. The CS A_v contained only this term. With the CE, the second term is added due (once again) to r_π. This term represents a voltage divider formed from r_π and R_E and expresses the gain of a passive path from input to output. This term gives the passive gain from v_i to v_e. The voltage component of v_e due to the passive path is then amplified along with the active path component by the second factor of the CE A_v. Because the passive-path gain is noninverting, it decreases the overall (inverting) gain somewhat.

The output resistance can be obtained by direct application of the μ transform to the input side of the circuit:

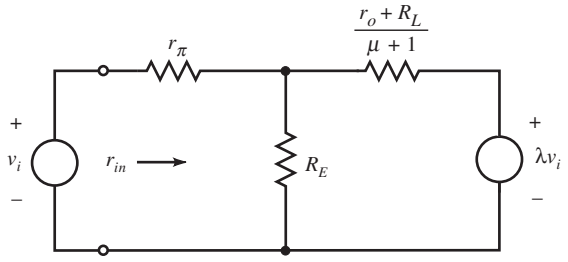
$$\text{CE } r_{out} = R_L \| \left[r_o + (\mu + 1) \cdot (r_\pi \| R_E) \right]$$

The input resistance r_{in} of the CE can be found by redrawing (c) as shown below. The right side is Thevenized in (b). The voltage source on the right is controlled by v_i and affects r_{in}. Resorting to basic circuit analysis, we can solve for the input resistance:

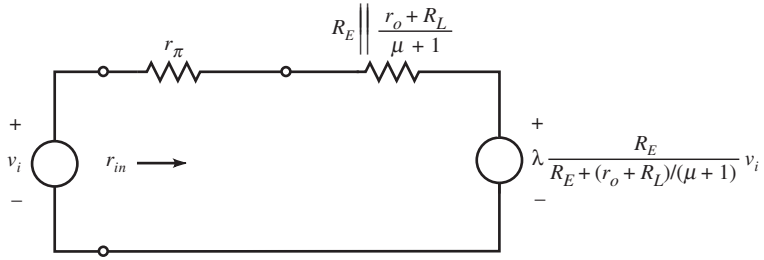
$$r_{in} = \frac{v_i}{i_i} \frac{v_i}{\dfrac{v_i - \lambda \cdot \left(\dfrac{R_E}{R_E + (R_L + r_o)/(\mu + 1)} \right) v_i}{r_\pi + R_E \| (R_L + r_o)/(\mu + 1)}}$$

$$\text{CE } r_{in} = r_\pi \left[\mu \cdot \left(\frac{R_E}{R_E + R_L + r_o} \right) + 1 \right] + R_E \| (R_L + r_o)$$

This expression is not immediately apparent from the circuit topology, as previous circuit expressions were and reveals limits to the extent a topology-oriented approach can take. Substituting $\beta \cdot r_o$ for $\mu \cdot r_\pi$ gives an alternative β-transform-like expression. It is left to the reader to find a topology-oriented explanation for these expressions of r_{in}.



(a)



(b)

Loaded Dividers, Source Shifting, and the Substitution Theorem

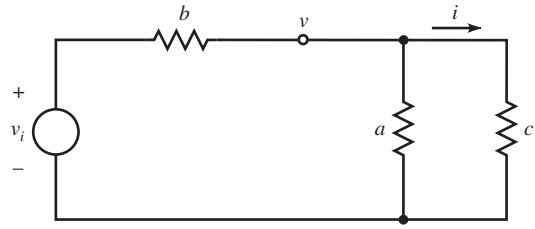
For circuits with more that three branch or loop equations, finding algebraic solutions can be tedious. In these situations, the following formulas are often useful. Thevenin and Norton circuits for loaded dividers are shown below. For the voltage divider in (a),

$$v = v_i \cdot \left(\frac{a \| c}{a \| c + b} \right), \quad i = \frac{v}{c} = \left(\frac{a \| c}{a \| c + b} \right) \cdot \left(\frac{1}{c} \right) \cdot v_i$$

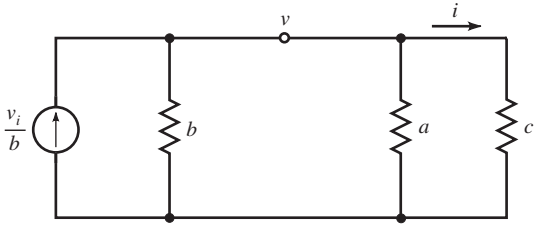
and for the current divider in (b),

$$i = \left(\frac{a \| b}{a \| b + c} \right) \cdot \left(\frac{v_i}{b} \right)$$

Loaded dividers often appear, and it is useful to be able to reverse the loading, as the following formulas allow:



(a)



(b)

$$a \| b + c = (a \| c + b) \cdot \left(\frac{a + c}{a + b} \right)$$

$$\frac{a \| c}{a \| c + b} = \frac{a \| b}{a \| b + c} \cdot \frac{c}{b}$$

It is also handy to note that

$$\frac{a \| b}{a} = \frac{b}{a + b}$$

The manipulation of expressions involving the ‖ operation are made easier by the following properties:

associative property of ‖: $(a \| b) \| c = a \| (b \| c)$

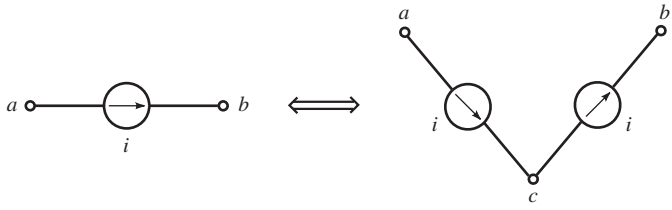
distributive property of × over ‖: $ab \| ac = a \cdot (b \| c)$

commutative property of ‖: $a \| b = b \| a$

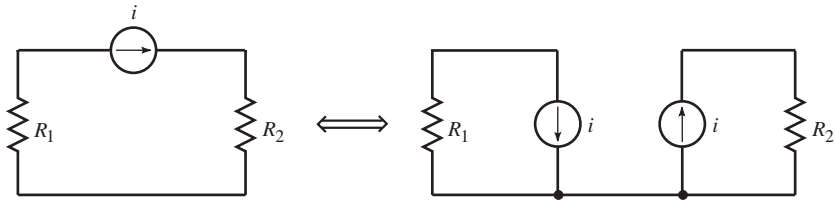
$$\frac{a \| b}{c \| d} = \frac{a \| b}{c} + \frac{a \| b}{d}$$

Source Shifting

An alternative to algebraic manipulation is the direct manipulation of circuit models. The *source-shifting* transformation can separate a circuit into two independent circuits, as shown below. A current source is replaced by two sources with the same current in series. This change introduces an additional node c between the two sources. This is useful, for example, in transforming a loop with a floating current source into two separate loops with ground-referenced current sources, as shown in (b).



(a)



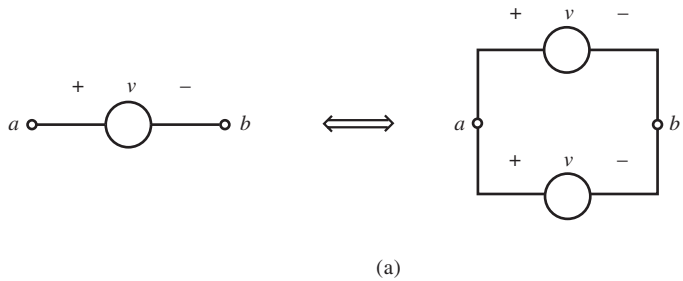
(b)

The voltage dual of this source-shifting transformation is shown below.

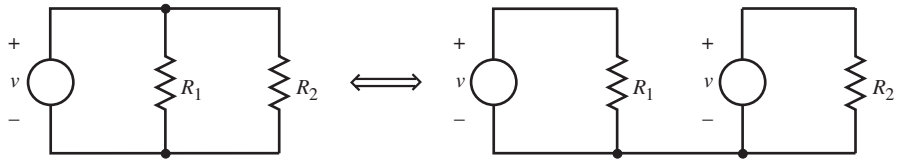
A voltage source is replaced by two parallel sources of the same voltage. This transformation is useful in separating two branches, giving each its own source, as in (b).

Substitution Theorem

The *substitution theorem* applies to controlled sources as shown below. It too has dual current and voltage forms. In (a), a voltage-controlled current source (VCCS) of current v/r has a terminal voltage of v. Because it is controlled by the voltage across its terminals, it behaves as a resistance of r. Similarly, the

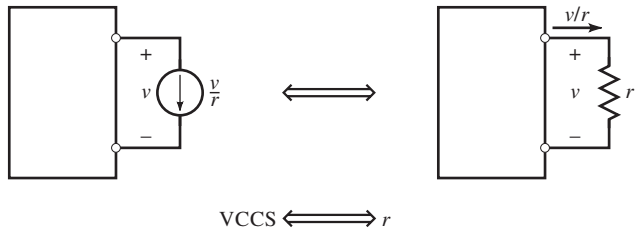


(a)

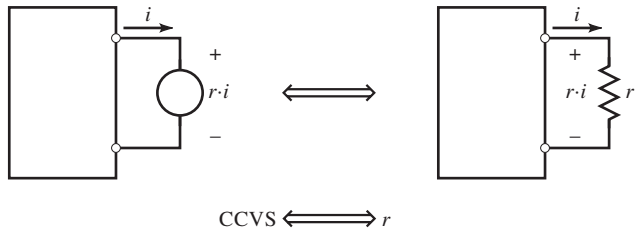


(b)

current-controlled voltage source (CCVS) in (b) has a terminal voltage of ri with current i, and is also equivalent to a resistance of r.

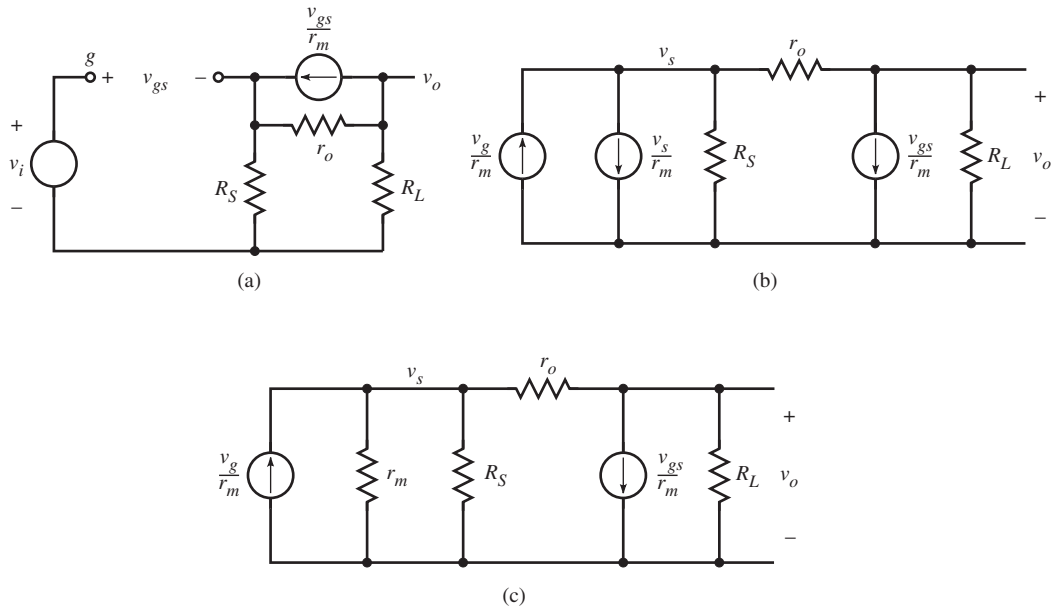


(a)



(b)

To demonstrate source shifting and the substitution theorem, the CS with r_o is modeled in (a) below.



Current-source shifting is applied, resulting in (b). This circuit is also modified by splitting v_{gs}/r_m into two sources, v_g/r_m and v_s/r_m. The current source, v_s/r_m is across v_s, and the substitution theorem can be applied, resulting in r_m in (c). Successive applications of Norton and Thevenin conversions then reduce the circuit to an equivalent form, from which the CS gain readily follows.

CLOSURE

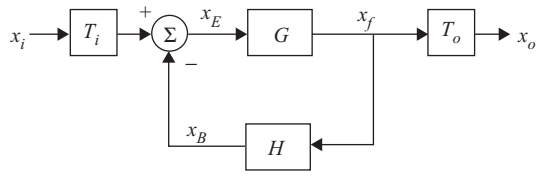
The dual forms of the reduction theorem, source-shifting, and the substitution theorem expand the power of circuit analysis, allowing the reduction of active amplifier stages to voltage and current dividers. These methods, however, are not sufficient in themselves. In the next chapter, feedback theory is developed – another analytic method that greatly simplifies circuit analysis. The methods of this chapter are based on transformations of networks that eliminate dependent sources and result in a single network. Feedback theory reduces signal paths (transmittances) instead, resulting in a single transmittance.

4

Feedback Amplifiers

Feedback Circuits Block Diagram

Circuits that combine some of their output with input are *feedback* circuits. The general case is shown as a block diagram below, where x quantities are voltages or currents.



Block diagrams do not represent circuit interconnections (*topology*) but instead describe the flow of electrical cause and effect. Each block has an input (cause) and an output (effect). The arrows represent causal constraints, pointing from the output of one block to the input of the next. The input multiplied by the *transmittance* written in the block is the output. For example, $x_f = G \cdot x_E$. The summing block, Σ, adds its inputs according to the sign by the arrowhead.

This block diagram is a graphic way of expressing the following algebraic equations:

$$x_f = G \cdot x_E$$

$$x_E = T_i \cdot x_i - H \cdot x_f$$

$$x_o = T_o \cdot x_f$$

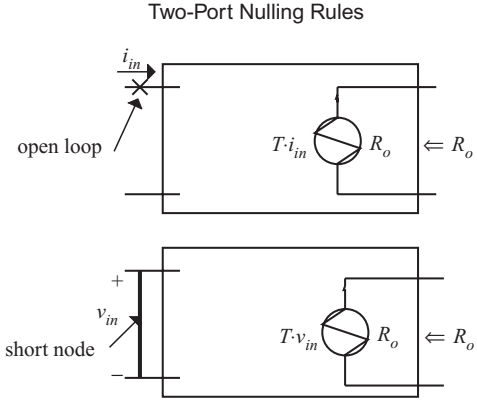
The first two equations describe the feedback loop itself. The loop is closed and consists of G, H, and Σ. T_i and T_o are outside the loop but are included because they commonly occur in feedback circuits. Solving for the overall *closed-loop* gain of the feedback amplifier, $T = x_o/x_i$ and is

$$T = \frac{x_o}{x_i} = T_i \cdot \left(\frac{G}{1 + GH} \right) \cdot T_o$$

The middle factor in parentheses is the transmittance, or gain, of the closed feedback loop itself. If the corresponding block transmittances of a circuit can be found, its closed-loop feedback gain can be calculated from the above general expression. Circuits are usually not obviously decomposable into the block transmittances. What is needed is a general procedure that derives the blocks from feedback circuits in equivalent circuit form so that circuit analysis can then be used to determine their transmittances.

Port Resistances with Dependent Sources

The resistance of a port, as represented by its Thevenin or Norton resistance, cannot be found by shorting a dependent Thevenin voltage source or by opening a dependent Norton current source. A dependent source can only be removed by causing its controlling variable to be set to zero – that is, by *nulling* it. The port resistance can then be found.

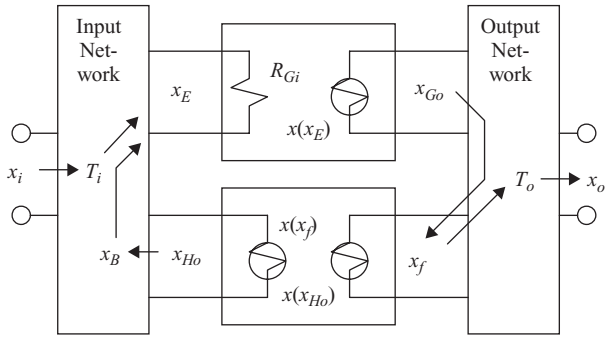


A dependent source can behave as a resistance if its controlling variable is the dual terminal (port) quantity. This is shown by the substitution theorem: Across an arbitrary network with a port having voltage v is a dependent current source of current v/r. This current source is equivalent to a resistance of r, by Ohm's law. The dual is a network with a port having current i flowing into a dependent voltage source of value $i \cdot r$. It too is equivalent to r. Any resistance associated with a dependent source must therefore be removed by nulling its controlling quantity so that the port resistance alone remains.

If a two-port network contains a source dependent on the voltage of the other port, then by shorting the other port, the resistance of the source's port can be found. Similarly, a source dependent on the current of the other port can be nulled by opening the other port. The controlling variable must be associated with the other port, or the network is not self-contained. The two-port nulling rules are shown above.

General Feedback Circuit

The feedback block diagram is brought closer to actual feedback circuit topology as a general feedback circuit, shown below.



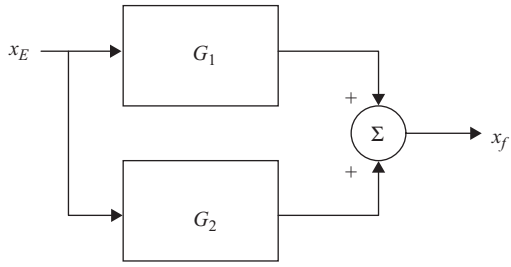
The x variables are generalized port quantities (current or voltage).

Relating this to the feedback block diagram:

- H starts at x_f of the lower two-port and extends from x_{Ho} through the input network to x_B.
- Σ is in the input network.

- T_i extends from x_i into the input network to sum with x_B to result in x_E.
- T_o extends from x_f to circuit output quantity x_o.
- G has two paths: an active path, G_1, and a passive path, G_2.
- G_1 starts at x_E of the upper two-port network and extends from x_{Go} through the output network to x_f.
- G_2 starts at x_E and extends through the input network to x_{Ho} and through the two-port network (in reverse of the H path) to x_f.

The upper and lower two-port networks in the general circuit diagram are not the G and H blocks of the block diagram. The upper two-port is the first transmittance of the G_1 forward path. The reverse source of the lower two-port, $x(x_{Ho})$, is the second transmittance of the G_2 path. Because the reverse transmittance through the active path (G_1) is usually insignificant, a reverse source at the x_E port is omitted. The block diagram with the two paths for G is shown below.



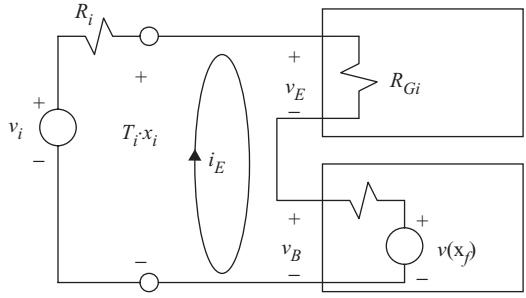
Input Network Summing

The summation symbol of the block diagram can be realized at the circuit level in the ways currents and voltages add (or subtract) using Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL):

- Currents sum at nodes.
- Voltages sum around loops.

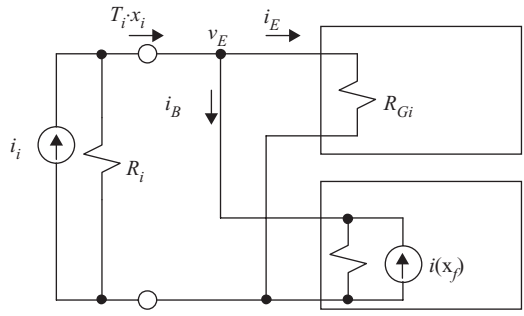
Input networks can be simplified to one of two basic topologies: series (common loop) or shunt (common node). The three port quantities of the input network combine as a sum of voltages around a loop for which loop

Input Network Series Topology



$$v_B = H \cdot x_f$$

Input Network Shunt Topology



$$i_B = H \cdot x_f$$

current is common or as a sum of currents at a node for which the node voltage is common. The input x_i is modified by T_i before it appears in the loop or at the node. The two input network topologies are shown above in generalized form.

In the series topology, the common input network quantity is the loop current, i_E. It is common to all three input-network ports and when set to zero, or *nulled* by opening the loop, nulls the G-path transmittances: $x(x_E)$ becomes nulled when i_E is nulled, thereby nulling the G_1 source, and nulling i_E also nulls the G_2-path source $x(x_{Ho}) = x(i_E)$. Similarly, both of the G-path transmittances, dependent on v_E in the shunt (parallel) topology, are nulled by shorting the common input node. By choosing the common input quantity as the error

quantity, both Kirchhoff's laws and Ohm's law (ΩL) are applied as follows. For an error voltage at the common node (shunt topology),

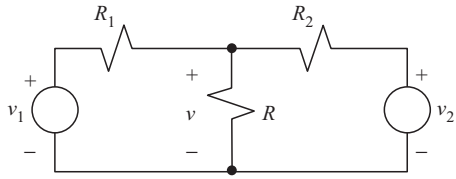
$$v_{node} = R \cdot \sum_{node} i_n$$

For an error current around the common loop (series topology),

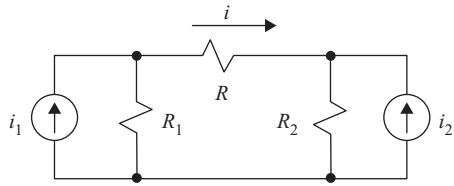
$$i_{loop} = \frac{1}{R} \cdot \sum_{loop} v_n$$

By choosing the error current in the shunt topology or the error voltage in the series topology, summation is by Kirchhoff's laws alone.

Another way of accounting for summing in circuits is by superposition. In linear systems, the contributions of sources independent of each other can be calculated and their individual contributions to circuit quantities added for the total quantities. The general principle is shown below for both voltage and current summing.



$$v = \sum_{node} T_n \cdot v_n = T_1 \cdot v_1 + T_2 \cdot v_2$$



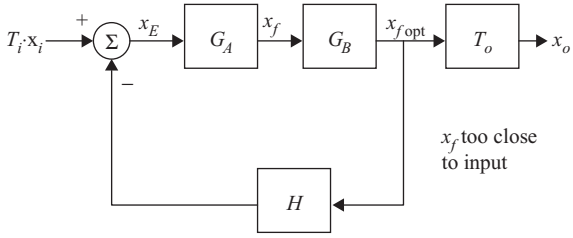
$$i = \sum_{branch} T_n \cdot i_n = T_1 \cdot i_1 + T_2 \cdot i_2$$

When superposition is used to effect error summing, the input network topology cannot be reduced to a single loop or node. Feedback analysis can still be done, but there is no common quantity which, when nulled, nulls the input to *G*. However, the input to *G* (upon which its controlled source depends) itself can be nulled.

CHOOSING x_E, x_f, AND THE INPUT NETWORK TOPOLOGY

Before transmittances can be found, x_E and x_f must be chosen. These choices are largely arbitrary and are usually not unique. However, some choices make the resulting feedback-circuit analysis easier than others. For a difficult analysis, choose a different circuit quantity for x_E or x_f, guided by the previously described input and output network considerations.

If x_f is chosen too close to the input, common factors appear in the expressions for H and T_o. To show this, let $G = G_A \cdot G_B$, where x_f is the output of G_A instead of G_B and is shown graphically below.



This block diagram results in feedback equations:

$$x_f = G_A \cdot x_E$$

$$x_E = T_i \cdot x_i - G_B \cdot H \cdot x_f$$

$$x_o = G_B \cdot T_o \cdot x_f$$

G_B is common to both the H term of x_E and T_o in x_o. By letting x_f be the output of G_B instead, G_B appears as a factor in the first equation and disappears from the others.

If x_f is instead chosen too close to the output, so that $T_o = T_{oA} \cdot T_{oB}$ and x_f is the output of T_{oA}, then

$$x_f = G \cdot T_{oA} \cdot x_E$$

$$x_E = T_i \cdot x_i - T_{oA}^{-1} \cdot H \cdot x_f$$

$$x_o = T_{oB} \cdot x_f$$

In this case, introducing factor T_{oA} into the third equation removes it from the first two.

If x_E is chosen too close to the output, common factors occur in the two terms of x_E. Let x_E be the input to G_B. Then

$$x_f = G_B \cdot x_E$$
$$x_E = G_A \cdot T_i \cdot x_i - G_A \cdot H \cdot x_f$$
$$x_o = T_o \cdot x_f$$

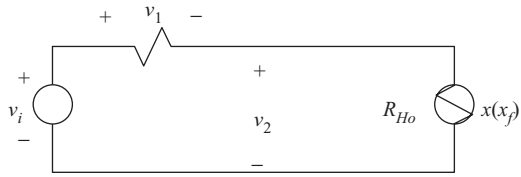
By letting $G = G_A \cdot G_B$, G_A becomes a factor in the first equation and is eliminated from the second.

The final case is that of choosing x_E too close to the input, as the input of T_{iB}. Then T_{iB} appears as a common factor with G and in the error term containing H.

$$x_f = G \cdot T_{iB} \cdot x_E$$
$$x_E = T_{iA} \cdot x_i - T_{iB}^{-1} \cdot H \cdot x_f$$
$$x_o = T_o \cdot x_f$$

By moving x_E to the output of T_{iB}, T_{iB} is eliminated from the first equation and from the H term of x_E. It becomes a factor in the first x_E term so that $T_i = T_{iA} \cdot T_{iB}$.

The form of input-network topology (series or shunt) is not generally determined by the circuit. But the choice of x_E affects the choice of input topology. This can be seen from the following input network.

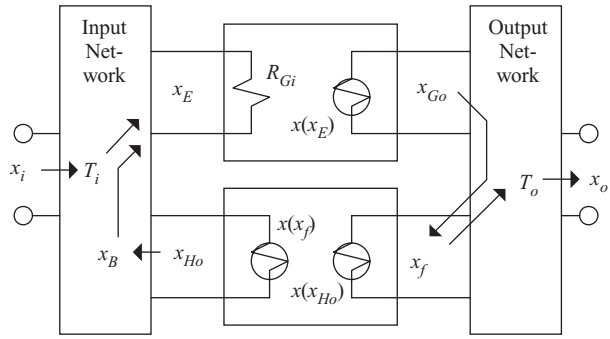


If v_1 is chosen as v_E, the H-path port (to the right) is made a Thevenin circuit and the input forms a loop – a series topology. If v_2 is chosen for v_E instead,

then converting the input and feedback ports to Norton equivalent circuits results in a common node with voltage v_E – a shunt topology.

TWO-PORT EQUIVALENT CIRCUITS

The transmittances of the general feedback block diagram are found by first finding the equivalent circuits of upper and lower two-port blocks shown in the general feedback circuit below.



Port resistances are found first by applying two-port nulling to dependent sources. Port resistances enter the calculation of transmittances by forming dividers in the input and output networks or by changing the gain of amplifier circuits. Other sources that contribute to the output-port quantity of the transmittance being found but that are not part of its path must be nulled. After nulling, transmittance is found by applying amplifier or divider analyses that include port resistances.

To find the two-port equivalent circuits, null the controlling variable of each port and find the port resistance. To find R_{Go}, null x_E and x_{Ho} to null the upper and lower two-port sources driving the output network. To null x_E, short the node of v_E or open the loop of i_E. Then inspect the x_{Go} and x_f ports for their resistances, using circuit analysis.

Nulling the common error quantity, $x_E = x_{Ho}$, of the input network nulls both output-network sources. For a series (summing loop) input topology, open the loop to null i_E. This nulls $v_E = R_{Gi} \cdot i_E$, thus nulling the G_1-path source $x(x_E)$, and

also nulls G_2-path source $x(x_{Ho})$, where $x_{Ho} = i_E$. For a shunt (summing node) input topology, short the node to null v_E. This nulls $i_E = v_E/R_{Gi}$.

To find R_{Ho}, null x_f: if v_f, short its node; for i_f, open its loop. This nulls the $x(x_f)$ source of the input-network feedback port, allowing its resistance alone to appear across the x_{Ho} port. Next, the transmittances are found.

To find T_i, null x_f. This both nulls the feedback contribution to x_E (which is x_B) and presents the feedback (x_{Ho}) port resistance, R_{Ho}, to the input network for calculation of T_i. Find the transmittance from x_i to x_E by circuit analysis.

To find G, both paths must be found. For the active (G_1) path, the effect of loading by the output network is chosen to be included in calculating transmittance. The G paths are in parallel from x_E to x_f and when the transmittance of each is calculated, the other must not be allowed to contribute to their common output. G_1 is through $x(x_E)$ and G_2 is through $x(x_{Ho})$. These sources are each nulled directly while transmittance of the other is being found. To null, open independent current sources and short independent voltage sources.

To find H, null the independent source x_i by shorting, if v_i, and opening, if i_i. Apply circuit analysis from x_f forward through the H path to x_B or to x_E. Then

$$H = \frac{x_B}{x_f} = -\frac{x_E}{x_f}\bigg|_{x_i=0}$$

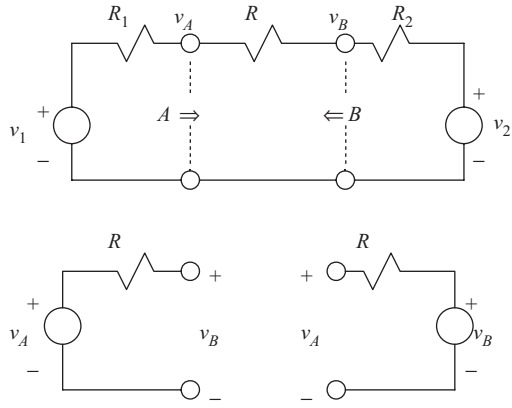
Finally, find $T_o = x_o/x_f$.

TWO-PORT LOADING THEOREM

Calculation of two-port equivalent circuits is simplified when two independent ports are connected via a common resistance, as shown below for the voltage case.

The two-port equivalent circuit is derived from the upper circuit by applying superposition at nodes having v_A and v_B:

$$v_A = \frac{(R_2 + R)\cdot v_1 + R_1 \cdot v_2}{R_1 + R_2 + R}$$



$$v_B = \frac{R_2 \cdot v_1 + (R_1 + R) \cdot v_2}{R_1 + R_2 + R}$$

These equations are equivalent to

$$v_A = \left(\frac{R}{R_1 + R} \right) \cdot (v_1 - v_B) + v_B$$

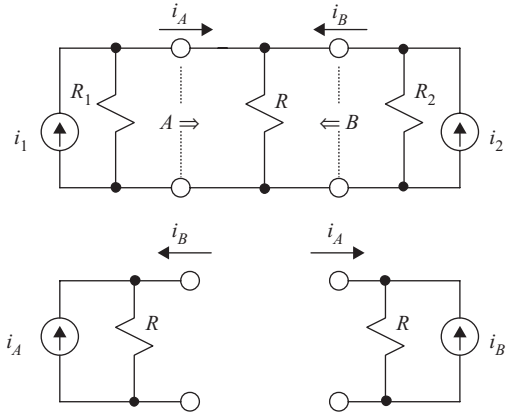
$$v_B = \left(\frac{R}{R_2 + R} \right) \cdot (v_2 - v_A) + v_A$$

where v_A and v_B are calculated assuming the other is given. In general, if v_A were found, including the loading by port B on the v_A node, then v_B can be found assuming v_A. What this derivation shows is that both v_A and v_B can be found assuming the other already has been.

The dual circuit for current is shown below. Port currents i_A and i_B are found assuming that the other is already determined. The corresponding equations are

$$i_A = \left(\frac{R_1}{R_1 + R} \right) \cdot (i_1 - i_B) + i_B$$

$$i_B = \left(\frac{R_2}{R_2 + R} \right) \cdot (i_2 - i_A) + i_A$$



This *loading theorem* is applicable, for instance, to the upper two-port block of a feedback circuit when x_f and x_E are connected by a resistance.

Feedback Analysis Procedure

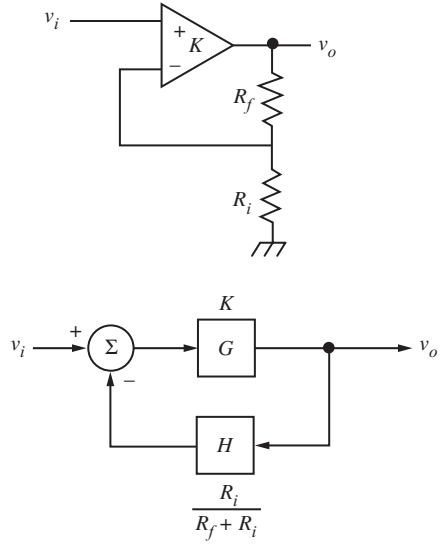
A general procedure can now be given for solving feedback circuits. Before the actual procedure is applied, simplify the circuit, if possible, using Thevenin and Norton equivalent circuits and then feedback-analyze the simpler circuit.

1. **Choose x_f.** x_f is dependent on x_E. For v_f, identify a node; for i_f, identify a loop.
2. **Choose x_E and identify the input network topology.** x_E is dependent on x_i and $x_B(x_f)$. Port voltages sum around a *loop*; port currents sum at a *node*.
   - For series (loop) topology, i_E is the common input-network quantity to both error and feedback ports; for shunt (node) topology, v_E is the common quantity. Either v_E or i_E can be chosen for x_E. Both output-network sources of G are nulled by choosing the common port quantity for x_E.
   - For error-summing by superposition, no common input-network nulling quantity exists; multiple loops or nodes exist.
3. **Find T_i.** T_i is found by nulling x_B by nulling x_f. Input-network feedback port resistance is found by nulling the output port, x_f, and determining R_{Ho}. If $x_f = v_f$, short the v_f node; if i_f, open the i_f loop. Then $T_i = x_E/x_i$ with $x_f = 0$.

4. **Find *G*.** $G_1 = x_f/x_E$ while nulling $x(x_{Ho})$. Output-network G_1-path port resistance, R_{Go}, and G_2-path port resistance, R_{Hi}, are found by nulling the input-network common error quantity (i_E for loop; v_E for node). Find $G_2 = x_f/x_E$ while nulling $x(x_E)$.
5. **Find H.** Null input source x_i. If $x_i = v_i$, short it; if i_i, open it. Then $H = x_B/x_f = -(x_E/x_f)$ with $x_i = 0$.
6. **Find T_o.** $T_o = x_o/x_f$.

NONINVERTING OP-AMP

Now that the general procedure for analysis of feedback amplifier circuits has been developed, it will be applied to specific amplifiers. The first example of its use is the *noninverting* operational amplifier (abbreviated as *op-amp*) configuration, shown here. The triangular amplifier symbol with + and − inputs (differential input) and single-ended (ground-referenced) output is the symbol of an op-amp. It has infinite input resistance, an ideal voltage-source output (zero output resistance), and infinite voltage gain. In practice, actual op-amps approach these conditions sufficiently so that use of the ideal op-amp model is



often justified. If the model is made slightly more realistic by assuming a finite voltage gain of K,

$$v_o = K \cdot (v_+ - v_-)$$

where v_+ is the voltage at the op-amp + (noninverting) input.

The voltage amplifier can be analyzed using the feedback analysis procedure:

1. Choose $x_f = v_o$. This choice is the only path back to the input from the amplifier output, through R_f. The amplifier output quantity is the same as the feedback quantity. The feedback node is the op-amp output.
2. Choose $x_E = v_E = v_+ - v_-$ and note that the input topology is a loop in which v_i, v_E, and v_B are in series. Because the op-amp input resistance (across v_E) is infinite, i_E is zero.
3. $T_i = 1$; v_i *adds directly to* v_E as v_+ in the error loop. This can be found by nulling $v_f = v_o$ by shorting the op-amp output. With $v_f = v_o$ shorted, $R_{Ho} = R_i \parallel R_f$. This results in $v_i = v_+$ and no input attenuation.
4. $G_1 = v_o/v_E = K$. This is found while nulling $x(x_{Ho}) = v(v_{Ho}) = v_f$ by shorting $v_{Ho} = v_-$. Because the op-amp output is an ideal voltage source, the output has no loading effect, no attenuation between the op-amp output (output of G_1) and the amplifier output, v_o. G_2-path port resistance, $R_{Hi} = R_f + R_i$. Similarly, $G_2 = 0$ because the ideal op-amp voltage-source output has no resistance across which to develop voltage from v_E through R_f.
5. With v_i shorted, $H = -(v_E/v_f)$. This is negative the attenuation of the voltage divider from $v_f = v_o$ to v_+ or

$$H = -\left(-\frac{R_i}{R_f + R_i}\right)$$

6. Because $x_o = v_o = x_f = v_f$, $T_o = 1$.

Now that all of the quantities of the feedback formula are known, the feedback-amplifier voltage gain (or *closed-loop gain*) can be found by substitution:

$$A_v = T_i \cdot \frac{(G_1 + G_2)}{1 + (G_1 + G_2) \cdot H} \cdot T_o$$

$$= \frac{K}{1 + K \cdot \left(\dfrac{R_i}{R_f + R_i} \right)}$$

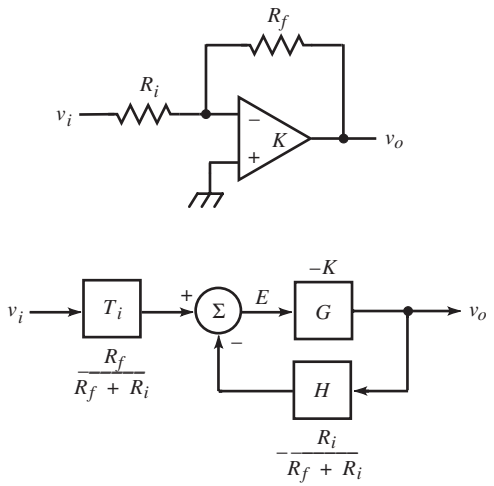
Multiplying numerator and denominator of the gain expression by $1/K$, then for large K (as K approaches infinity), $1/K$ approaches 0 and for the ideal non-inverting op-amp, the voltage gain formula is

$$A_v = \frac{R_f}{R_i} + 1$$

For example, if R_f = 10 kΩ and R_i = 1.0 kΩ, then the op-amp voltage gain is 11.

INVERTING OP-AMP

The other configuration, that of the inverting op-amp, is shown below.



To analyze this amplifier, apply the feedback analysis procedure:

1. Choose $x_f = v_o$. As with the noninverting op-amp, the only path back to the input is through R_f, which connects to v_o.
2. Choose $x_E = i_E$, the current flowing into the node of the inverting op-amp terminal. Error current is thereby summed at this node; it is dependent upon input current through R_i and feedback current through R_f. Both sources are Norton equivalents, in parallel across v_- (shunt topology). The common input-network node quantity is v_-. Shorting v_- nulls both forward paths of G.
3. Null $x_f = v_o$ by shorting the output (to ground). Then $R_{Ho} = R_f$. Also, $T_i = i_E/v_i$. No current flows into either input of the op-amp. Therefore, $i_E = i_i$ and $i_E/i_i = 1$. But from the Norton equivalent of the input current source, $i_i = v_i/R_i$, and $T_i = 1/R_i$.
4. For the G_1 path, v_- is the op-amp input quantity, not i_E. G_1 consists of two cascaded transmittances, (v_-/i_E) times (v_o/v_-). The second transmittance is $-K$, the voltage gain of the op-amp. The first transmittance is the resistance of the op-amp inverting-input node. The feedback is $x(x_f) = i_B$. This is the current source of the Norton equivalent feedback circuit, as shown, and it is nulled by setting $x_f = v_o$ to zero. Shorting v_o has the effect of grounding the output side of R_f, which results in an input-node resistance of $R_i \parallel R_f$. Then

$$G_1 = \left(\frac{v_-}{i_E}\right)\cdot\left(\frac{v_o}{v_-}\right) = \left(R_i \| R_f\right)\cdot(-K)$$

   For G_2, $x(x_E)$ must first be nulled before finding the passive path through R_f to v_o. But since $x(x_E) = x_{Go}$ determines v_o as an ideal voltage source, there is no feedback contribution through the passive path, and $G_2 = 0$. Consequently, $G = G_1$.
5. To find H, null i_i. This is the Norton input circuit, where $i_i = v_i/R_i$. To null it, open the current source, and $H = -(i_E/v_o)$. Then, i_E is only the current from the Norton feedback source, which is v_o/R_f. Substituting, $H = -1/R_f$.
6. Finally, $T_o = 1$.

Now that the transmittances have been found, the closed-loop voltage gain is

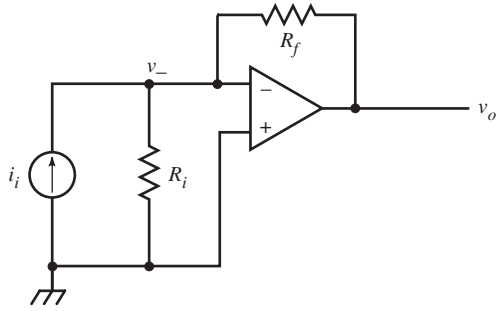
$$A_v = T_i \cdot \frac{G}{1 + G \cdot H} \cdot T_o$$

$$= \left(\frac{1}{R_i}\right) \cdot \frac{-\left(R_i \| R_f\right) \cdot K}{1 + \left[-\left(R_i \| R_f\right) \cdot K\right] \cdot \left[-1/R_f\right]} \cdot 1$$

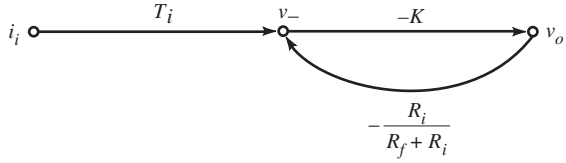
When K becomes infinite, this reduces to

$$A_v = -\frac{R_f}{R_i}$$

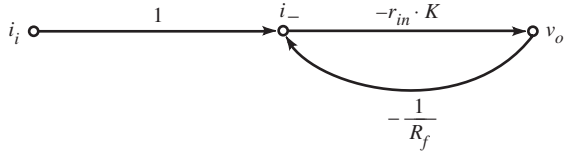
An alternative analysis demonstrates a different choice of x_E that uses superposition to sum error quantities.



(a)



(b)



(c)

1. Choose $x_f = v_o$. As with the noninverting op-amp, the only path back to the input is R_f which connects to v_o.
2. Choose $x_E = v_E = v_-$ and observe that x_B must be a voltage that sums in a series (loop) input topology. (x_B is the feedback quantity that sums directly with $T_i \cdot x_i$.) The input and feedback sources are a voltage source (v_i and v_o) in series with a resistance (R_i and R_f). Because the input port to G is across the op-amp input terminals, this port is in parallel with the input and feedback sources and no single series loop exists. Consequently, $x_E = v_E = v_-$ must be obtained by superposition. To null both G transmittances, v_E itself can be shorted.
3. $T_i = v_E/v_i$ with feedback nulled by shorting v_o. Then T_i is a voltage divider:

$$T_i = \frac{R_f}{R_f + R_i}$$

   The feedback port resistance R_{Ho} is found by nulling v_o, and it is R_f. R_{Go} and R_{Hi} are zero because of the ideal op-amp output. When v_E is nulled, the output node is still zero ohms.
4. $G_1 = v_o/v_E = v_o/v_- = K$. $G_2 = 0$ and $G = G_1$.
5. Null v_i by shorting it. Then the path from v_o to v_E is a divider – the same one as for T_i but in the reverse direction. Then

$$H = -\frac{R_i}{R_f + R_i}$$

   v_B is the Thevenin equivalent voltage at v_- due to v_o. Because of summing convention, it subtracts from v_E, and the – sign appears in H.
6. $T_o = 1$ because $v_{Go} = v_o$.

Substituting the previous transmittances into the feedback gain formula produces the same result as the previous analysis. Another choice of x_E, the voltage across R_i, is also workable, but much more difficult. In this case, the error quantity is chosen *too close* to the input and a redundant factor, R_f/R_i, appears in both G and H. (But it cancels, resulting in the same closed-loop gain as already derived.)

An example of an inverting op-amp: $R_f = 10\ \text{k}\Omega$ and $R_i = 1.0\ \text{k}\Omega$. Then the op-amp voltage gain is −10.

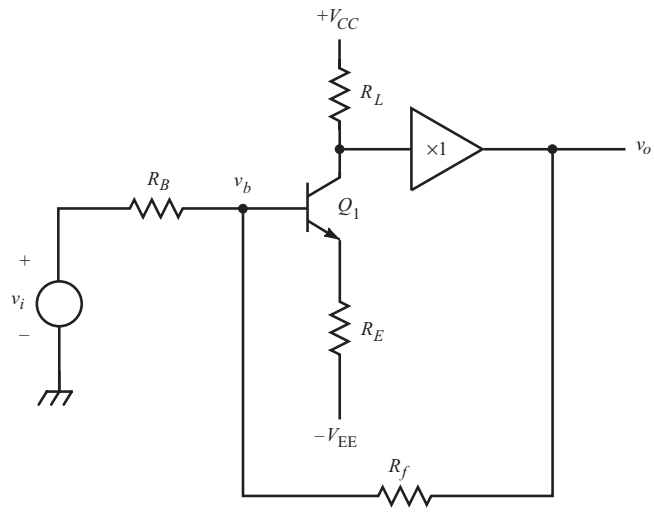
The topological difference between the inverting and noninverting op-amp configurations is where ground is connected. Without rewiring, if the inverting input voltage-source + terminal is grounded instead of its − terminal, the noninverting configuration results. The noninverting configuration has a gain of one more because the input source is in series with and adds to the op-amp output.

Inverting BJT Amplifier Examples

A discrete transistor feedback amplifier with idealized output buffer stage is shown below. To simplify analysis, the ideal unity-gain buffer has a voltage-source (zero resistance) output but otherwise could be an emitter-follower, as in the following example.

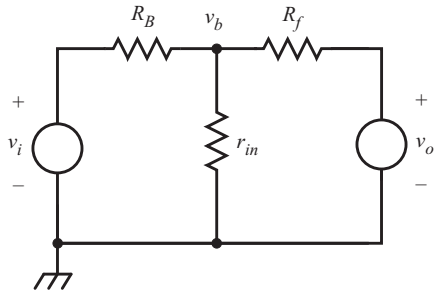
Applying the feedback-analysis procedure:

1. The output pickoff is simply $x_f = v_o$.
2. This circuit poses a challenge in identifying the input summing circuit, Σ. The input is v_i, a voltage, so the error quantity must also be a voltage (unless there is a quantity-transforming T_i). Voltages are summed around loops,



according to KVL. The error voltage must be in a loop that includes the feedback; then the loop containing R_f (the only component providing feedback) must include the error voltage.

It is appealing to let the error voltage be v_b because both v_i and v_o contribute to it, in the loop containing R_f and R_B. This error loop is shown below and is similar to that for the inverting op-amp configuration – two loops combine, resulting in error voltage v_b across the common branch of the loop.



What is not apparent is what to do with the input resistance of Q_1: $r_{in} = (\beta + 1) \cdot (r_e + R_E)$. It turns the two voltage sources with their series resistances into a loaded divider similar to that of the cascade amplifier. As in that case, a decision must be made about what to do with r_{in}. If it is included in calculation of v_b, then when the gain of G is found, the external base resistance is not included in the gain formula because it was taken into account in finding v_b.

3. We first analyze the circuit by letting r_{in} load the input. The contribution of v_i to v_b is through the divider formed by R_B in series with $r_{in} \| R_f$. This divider has a transfer function of $v_b / v_i = T_i$:

$$T_i = \frac{R_f \| r_{in}}{R_f \| r_{in} + R_B}$$

4. Because r_{in} is taken into account in T_i and H, v_b is the actual base voltage with external base resistance taken into account. Therefore, G does not include its effect and is

$$G = \frac{v_o}{v_b} = -\alpha \cdot \frac{R_L}{r_e + R_E}$$

5. A divider similar to T_i defines H:

$$H = -\left.\frac{v_o}{v_b}\right|_{v_i=0} = -\frac{R_B \| r_{in}}{R_B \| r_{in} + R_f}$$

6. There is no additional circuitry between the fed-back quantity and the output; both are v_o and therefore $T_o = 1$.

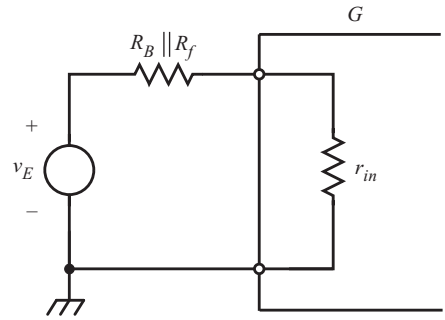
The closed-loop voltage gain is found by substituting the previous quantities into the feedback formula:

$$A_v = T_i \cdot \frac{G}{1+GH} = -\alpha \cdot \frac{R_L}{r_e + R_E + R_B/(\beta+1) + (r_e + R_E + \alpha \cdot R_L)(R_B/R_f)}$$

This gain expression is essentially a resistance ratio like that of nonfeedback amplifier stages but has no transresistance interpretation; it is merely a simplified form of the feedback formula in terms of component values and Q_1 parameters. It is useful for calculating the closed-loop gain but offers little of the insight into feedback characteristics of individual block transmittances.

An alternative gain derivation is based on incorporation of external base resistance into G. By solving for a Thevenin equivalent v_E in series with $R_B \| R_f$ (equivalent circuit shown below), this resistance is included in the gain formula of Q_1 as external base resistance. In this approach, v_E is different; the error voltage is no longer v_b but is the Thevenin equivalent voltage from the voltage divider formed by the two sources with the base of Q_1 open. The summing block is due to superposition of input and feedback voltages and is

$$v_E = \left(\frac{R_f}{R_f + R_B}\right) \cdot v_i + \left(\frac{R_B}{R_f + R_B}\right) \cdot v_o$$



This Thevenin error voltage is in series with a Thevenin resistance of $R_B \| R_f$ that forms a divider with r_{in}, as shown previously. Then,

$$v_b = \left(\frac{r_{in}}{r_{in} + R_B \| R_f} \right) \cdot v_E$$

It is apparent from the expression for v_E that T_i is different from the previous analysis and is

$$T_i = \left. \frac{v_E}{v_i} \right|_{v_o=0} = \frac{R_f}{R_f + R_B}$$

From the expression for v_E, H is

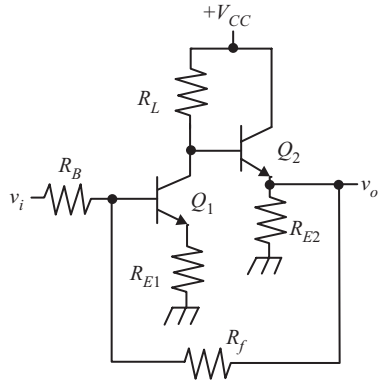
$$H = -\left. \frac{v_E}{v_o} \right|_{v_i=0} = -\frac{R_B}{R_f + R_B}$$

G is

$$G = \left. \frac{v_o}{v_E} \right|_{B=0} = -\alpha \cdot \frac{R_L}{r_e + R_E + \left(R_B \| R_f \right)/(\beta + 1)}$$

Choice of feedback quantity, v_o, and T_o remain the same.

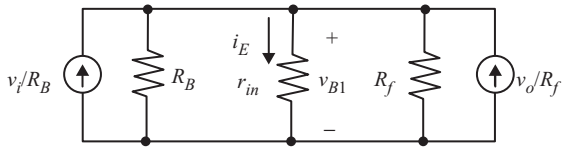
Combining the block transmittances in the feedback formula and rearranging gives the same result as before. This circuit demonstrates a multiplicity of valid choices for the error voltage. In the loaded-divider case, the error quantity is an actual node voltage v_b whereas in the second analysis it is the Thevenin voltage v_E. Both approaches produce the correct closed-loop gain.



The discrete-component BJT amplifier shown here resembles the inverting op-amp and replaces the buffer amplifier of the previous circuit with a common-collector (CC) stage. It also demonstrates use of the loading theorem.

Assuming that the transistors are biased for linear operation (so that the incremental BJT model is valid), the feedback analysis procedure is as follows:

1. Choose $x_f = v_o$. This is the output node.
2. Choose $x_E = i_E = i_{B1}$, the current into the base of Q_1. Components connect to this node from both the input and output nodes, in parallel. The shunt input-network topology is consequently the most easy to identify. The equivalent input network is shown below.



The Norton equivalent input and feedback circuits are in parallel with the input of G_1, and v_{B1} is the common input-port error-node quantity.

3. With output nulled ($v_o = 0$), $R_{Ho} = R_f$. T_i is the current divider formed by r_{in} in parallel with R_f and R_B. Nulling v_o (by shorting it) opens the corresponding current source in the equivalent circuit. Then

$$T_i = \frac{R_B \| R_f}{r_{in} + R_B \| R_f}$$

This is the fraction of input current, v_i/R_B, that flows through r_{in}, which is $i_{B1} = i_E$.

4. $G_1 = v_o/i_E = v_o/i_{B1}$. With the input-error node shorted, $R_{Hi} = R_f$. The gain of the first stage, neglecting the loading of Q_2, is $v_{C1}/i_{B1} = -\beta \cdot R_L$. R_{Hi} ($= R_f$) is in parallel with R_{E2}, as G_1 output loading. The total gain is the product of first- and second-stage gains, or

$$G_1 = (-\beta_1 \cdot R_L) \cdot \left(\frac{R_{E2} \| R_f}{R_{E2} \| R_f + r_{e2} + R_L/(\beta_2 + 1)} \right)$$

Both input and output loading of the G path is taken into account in G_1.

G_2 is the reverse path through the H block. It is

$$G_2 = \frac{x_f}{x_E} = \frac{v_o}{i_{B1}} = \frac{v_o}{v_{B1}/r_{in}} = r_{in} \cdot \frac{v_o}{v_{B1}}$$

and

$$r_{in} = (\beta_1 + 1) \cdot (r_{e1} + R_{E1})$$

While nulling the common input-port error-node quantity (the node voltage, v_{B1}), R_{Go} is the output resistance of the active path (at the emitter of Q_2). At the G output port, the β transform is applied to Q_2. The resistance is R_{E2} in parallel with r_{e2} plus the β-transformed base resistance, R_L, or $R_L/(\beta + 1)$.

$$R_{Go} = \left(r_{e2} + \frac{R_L}{\beta_2 + 1} \right) \Big\| R_{E2}$$

For the passive path G_2, R_f forms a voltage divider with R_{Go}. Then

$$G_2 = r_{in} \cdot \frac{R_{Go}}{R_{Go} + R_f}$$

By substituting component values into the previous equations, it will typically be the case that G_2 is much less than G_1 and can usually be ignored.

5. The feedback block, H, is found by nulling the input quantity, v_i, and calculating the fraction of current due to v_o that is i_{B1}. It is found by the current divider formula from the equivalent circuit:

$$i_{B1} = \left(\frac{R_f \| R_B}{R_f \| R_B + r_{in}} \right) \cdot \left(\frac{v_o}{R_f} \right)$$

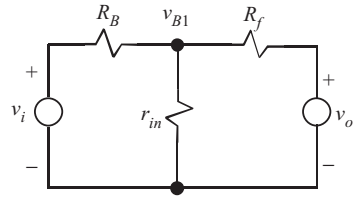
Then

$$H = -\frac{i_{B1}}{v_o}\bigg|_{v_i = 0} = -\frac{R_f \| R_B}{R_f \| R_B + r_{in}} \cdot \frac{1}{R_f}$$

6. $T_o = 1$, because the feedback quantity, $x_f = v_o$.

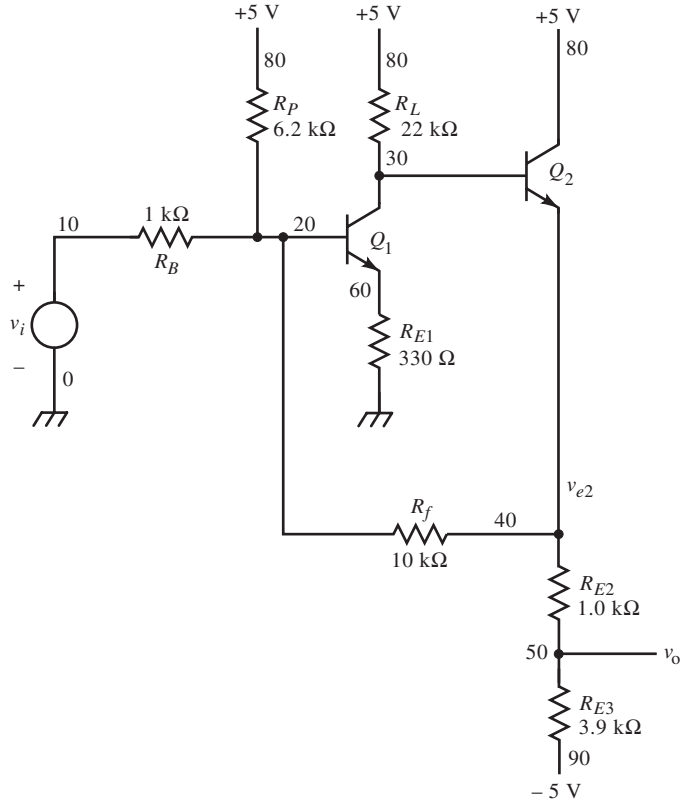
The previous quantities are substituted into the feedback equation to calculate the closed-loop gain.

For step 4, the loading theorem is applied to allow us to assume that v_o and v_{B1} are the actual (loaded) node voltages with R_f the connecting resistance. Then the effects of the input and output networks (due to loading) can be taken into account. Input summing would then be applied by superposition of v_i and v_o to the calculation of v_{B1}, from the equivalent circuit, shown below.



Two-BJT Inverting Feedback Amplifier

The schematic diagram shows an inverting feedback amplifier similar to that of the previous example. The SPICE program follows.



```
Inverting 2-BJT Feedback Amplifier

.OPT NOMOD OPTS NOPAGE
.OP
.DC VI -2V 2V 0.05V
.TF V(50) VI

VCC 80 0 DC 5V
VEE 90 0 DC -5V
VI 10 0 DC 0V

RB 10 20 1K
RP 80 20 6.2K
RL 30 80 22K
RF 20 40 10K
RE1 60 0 330
RE2 40 50 1K
RE3 50 90 3.9K
Q1 30 20 60 BJT1
Q2 80 30 40 BJT1

.MODEL BJT1 NPN (BF = 99)
.PROBE
.END
```

After static (direct current, or dc) analysis (and assuming $\beta = 99$, $I_S = 10^{-16}$ A), the dynamic emitter resistances are

$$r_{e1} = 230.93\,\Omega; \quad r_{e2} = 18.61\,\Omega$$

A few other incremental resistances are needed. The input can be Thevenized, combining R_B and R_P into r_s; the input voltage source has an attenuation of T_s:

$$r_s = R_B \| R_P = 861.11\,\Omega, \quad T_s = \frac{R_P}{R_P + R_B} = 0.8611$$

Let $E = v_b = v(20)$ and $x_f = v_{e2} = v(40)$. The input resistance of Q_1 is $r_{Gi} = (\beta_1 + 1) \cdot (r_{e1} + R_{E1}) = (100) \cdot (230.9\ \Omega + 330\ \Omega) = 56.093$ kΩ. Then,

$$T_i = T_s \cdot \frac{r_{Gi} \| R_f}{r_{Gi} \| R_f + r_s} = 0.78179$$

$$G = -\alpha_1 \cdot \frac{R_L}{r_{e1} + R_E} \cdot \frac{R_f \| (R_{E2} + R_{E3})}{R_f \| (R_{E2} + R_{E3}) + r_{e2} + R_L/(\beta_2 + 1)} = -36.201$$

The forward passive path through R_f to v_{e2} adds G_2 to G because it is in parallel with it:

$$G_2 = \frac{(R_{E2} + R_{E3}) \| (r_{e2} + R_L/(\beta_2 + 1))}{(R_{E2} + R_{E3}) \| (r_{e2} + R_L/(\beta_2 + 1) + R_f)} = 2.225 \times 10^{-2}$$

When added to G, $G = -36.179$, a difference of 0.06%.

The remaining transmittances are

$$H = -\frac{r_s \| r_{Gi}}{r_s \| r_{Gi} + R_f} = -7.8179 \times 10^{-2}$$

$$T_o = \frac{R_{E3}}{R_{E2} + R_{E3}} = 0.79592$$

Putting the transmittances together, we obtain a voltage gain of

$$\frac{v_o}{v_i} = T_i \cdot \frac{G}{1 + GH} \cdot T_o = -5.8802$$

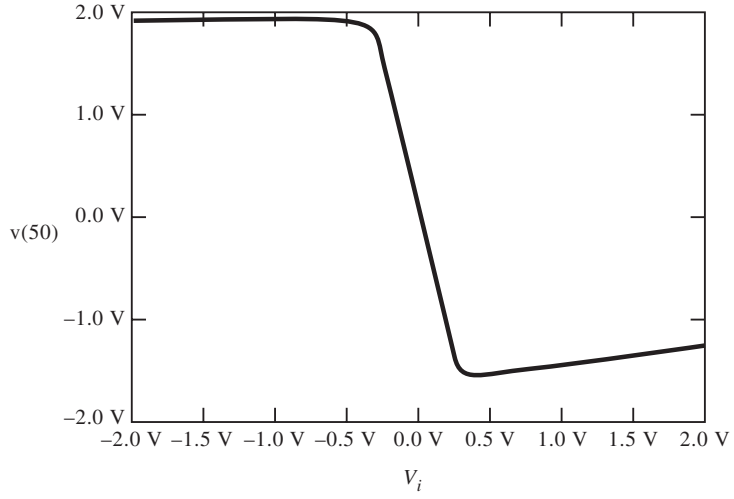
where $1 + GH = 3.8284$. The input and output resistances are

$$r_{in} = R_B + R_P \| r_{Gi} \left\| \frac{R_f}{1 + (-G)} \right. = 1.2566\,\text{k}\Omega$$

$$r_{out} = \left(\frac{(R_B \| R_P \| r_{Gi} + R_f) \| \lceil r_{e2} + R_L/(\beta_2 + 1) \rceil \| (R_{E2} + R_{E3})}{1 + GH} + R_{E2} \right) \Bigg\| R_{E3}$$
$$= 832.3\,\Omega$$

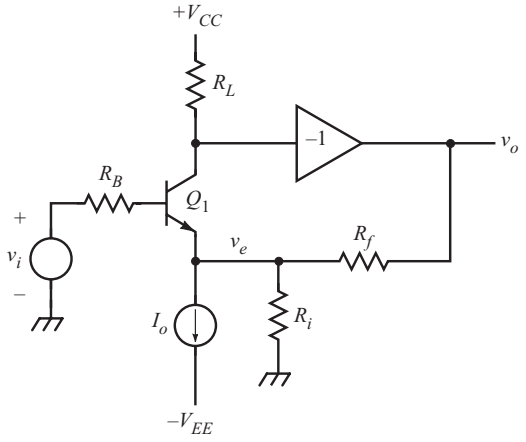
The SPICE simulation verifies these numbers to three digits, the simulation convergence accuracy.

The graph of $v_o = v(50)$ versus v_i (below) shows feedforward through R_f outside the linear range of the active path (through the transistor), when its gain is zero.



Noninverting Feedback Amplifier Examples

A noninverting BJT feedback amplifier is shown below.



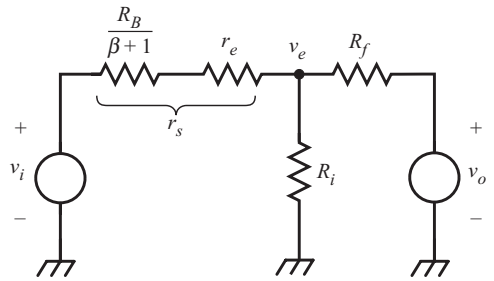
It feeds back via R_f to the emitter of the common-emitter (CE) input transistor Q_1. Both input and output quantities are voltages; this is a voltage amplifier. To simplify analysis, the output of the CE stage is buffered by an ideal $\times(-1)$ amplifier. (This could be implemented as a PNP CE stage.) The feedback quantity is v_o, and T_o is 1.

We analyze this circuit for three different choices of error, E. The first choice is preferred because it is simplest to reduce to gain and summing blocks.

From the schematic diagram, let

$$R_E = R_f \| R_i, \quad \text{and} \quad r_s = r_e + \frac{R_B}{\beta + 1}$$

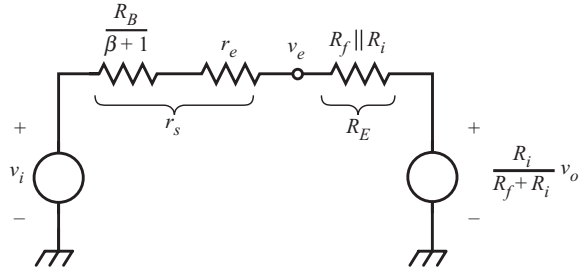
The summing loop must include both v_i and v_o terms, as shown below.



The feedback from v_o is Thevenized with a series resistance of R_E and Thevenin voltage source of

$$v_{fb} = \left(\frac{R_i}{R_f + R_i} \right) \cdot v_o$$

The output of the equivalent circuit is at the feedback node of v_e. The loop continues and includes r_s and v_i. The simplified Thevenin equivalent circuit is shown below.



The error is chosen to be

$$E = v_i - v_{fb}$$

The input quantity v_i adds directly to E so that $T_i = 1$.

The forward path is

$$G = \left.\frac{v_o}{E}\right|_{B=0} = \alpha \cdot \frac{R_L}{r_s + R_E}$$

From the choice of E,

$$H = -\left.\frac{E}{v_o}\right|_{v_i=0} = \left(\frac{R_i}{R_f + R_i}\right)$$

This results in a closed-loop voltage gain of

$$A_v = \frac{\alpha \cdot \left(R_L/(r_s + R_E)\right)}{1 + \left[\alpha \cdot \left(R_L/(r_s + R_E)\right)\right] \cdot \left[R_i/(R_f + R_i)\right]}$$

Now that the circuit has been analyzed by a straightforward analysis, we investigate alternatives, to show how the feedback circuit can be solved several ways. Examination of two alternatives should clarify some of the subtler aspects of the approach.

First, note that in the preceding solution of A_v, because v_{fb} was chosen as the term relating v_o to E, the gain of G was calculated to include R_E in the transresistance of Q_1. If v_e were chosen for E instead, R_E would have been taken into account in the expression for v_e, and G would have only r_s in its denominator. In this case, the transresistance of Q_1 would be around an input loop from v_i to v_e instead of from v_i to v_{fb}.

Taking this approach, we begin by noting that both v_i and v_o sum at the emitter of Q_1. The equivalent-circuit loop that contains E is shown above. An expression for the emitter voltage v_e can be constructed by superposition:

$$v_e = \left(\frac{R_E}{r_s + R_E}\right) \cdot v_i + \left(\frac{r_s}{r_s + R_E}\right) \cdot \left(\frac{R_i}{R_f + R_i}\right) \cdot v_o = T_i \cdot v_i + (-H) \cdot v_o$$

This time, let $E = v_e$. It qualifies because it contains terms for both v_i and v_o. The coefficients of these quantities are expressions for blocks T_i and $-H$, as designated. These blocks are

$$T_i = \frac{R_E}{r_s + R_E}, \quad H = -\left(\frac{r_s}{r_s + R_E}\right) \cdot \left(\frac{R_i}{R_f + R_i}\right)$$

Now, an unusual step must be taken to produce G. We need two equivalent expressions, which are

$$G = -\alpha \cdot \frac{R_L}{r_s} = \alpha \cdot \frac{R_L}{R_E}$$

These expressions for G are equal because the voltage developed across R_E is the same as that developed across r_s except inverted in polarity. If v_E increases, the voltage across R_E increases and I_E increases. This same voltage change across r_s causes a decrease in I_E out of the emitter of Q_1, inverting the polarity of the change in I_E; hence the minus sign for the first expression for G.

Combining the gain expressions gives the closed-loop gain as

$$A_v = \left(\frac{R_E}{r_s + R_E}\right) \cdot \frac{\alpha \cdot (R_L / R_E)}{1 + \alpha \cdot (R_L / r_s)\left[r_s / (r_s + R_E)\right] \cdot \left[R_i / (R_f + R_i)\right]}$$

This is equivalent to the previous A_v, after the T_i factor is multiplied by the numerator. Although the result is the same, the derivation is less obvious due to the need for two expressions for G.

The final approach uses a different choice for E in that v_e is used as the output-related term and is subtracted from the input v_i:

$$E = v_i - v_e$$

E can be expanded as in the previous case so that

$$E = v_i - \left[\left(\frac{R_E}{r_s + R_E}\right) \cdot v_i + \left(\frac{r_s}{r_s + R_E}\right) \cdot \left(\frac{R_i}{R_f + R_i}\right) \cdot v_o\right]$$

This reduces to

$$E = \left(\frac{r_s}{r_s + R_E} \right) \cdot \left(v_i - \left[\frac{R_i}{R_f + R_i} \right] \cdot v_o \right)$$

From the expression for E, T_i and H are

$$T_i = \left(\frac{r_s}{r_s + R_E} \right), \quad H = \left(\frac{r_s}{r_s + R_E} \right) \cdot \left(\frac{R_i}{R_f + R_i} \right)$$

With $H = -(v_i - v_e)/v_o$, solve the E equation for $-E/v_o$ with $v_i = 0$. This yields a negative expression that is then negated to produce a positive H of v_e/v_o.

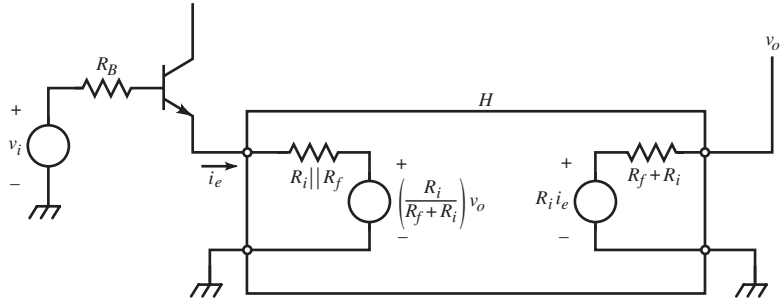
The gain of G is from E to v_o, and G is calculated starting from the difference voltage, E, which is

$$G = \alpha \cdot \frac{R_L}{r_s}$$

Combining block transmittances and rearranging gives the previous closed-loop A_v. This choice of E does not require two gain expressions for G and is somewhat simpler conceptually. In both solutions, E is expressed as a linear combination of v_i and v_o.

These solutions were constructed by inspection of the loops and nodes of the circuit without use of explicit two-port synthesis. The analysis took place at a higher level than basic circuit laws, however, since the transresistance method and identification of voltage or current dividers made it possible to write the block gains directly from inspection of the circuit. This approach is intuitively simple as long as the H loading is obvious and G and H are easy to identify.

Otherwise, the two-port approach would be applied, with a two-port H as shown below.



To set the error voltage to zero, the loop around which it is summed must be opened. The loop current is i_e; opening the loop at the emitter of Q_1 sets E to zero, and $i_e = 0$ nulls E. The H input source must be controlled by i_e. Also, since v_o is the feedback output of G, the H output source must be controlled by it. The H output is the result of Thevenizing R_i, R_f, and v_o. When v_o is shorted, the G input loading is $R_i \| R_f$. When i_e is nulled by opening the emitter, the G output loading is $R_f + R_i$. The H output source voltage is the voltage due to v_o with zero port current $(i_e = 0)$; it is

$$\left(\frac{R_i}{R_f + R_i}\right) \cdot v_o$$

because of the divider action of R_f and R_i. The H input source voltage appears at the H input when the H input port current is zero. When the port is opened (disconnecting it from v_o), the resulting voltage is due to i_e flowing through R_i, or $R_i \cdot i_e$.

As for the virtues of this feedback amplifier, two transistors provide a gain determined by R_f and R_i for large G. With two BJTs, G can be made larger by allowing the output inverting buffer to have a gain magnitude $\gg 1$. A typical achievable gain for G is 500. Then for a closed-loop gain $\ll 500$, the gain is set predominantly by the external resistors.

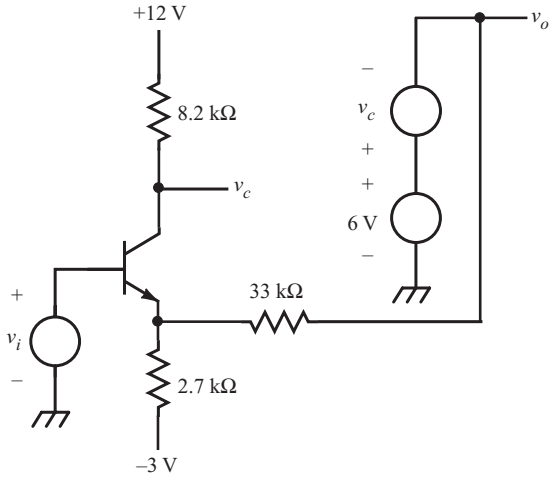
Example: Noninverting Feedback Amplifier

The noninverting feedback amplifier shown here has an idealized $\times(-1)$ buffer, similar to the one analyzed previously. Assuming the static analysis from the simulation data is available, $r_e = 32.73\ \Omega$. Then the three quantities of interest are

$$r_{out} = 0\,\Omega$$

$$r_{in} = (\beta + 1) \cdot \left(r_e + R_i \| R_f\right) \cdot (1 + GH) = 314.25\,\text{k}\Omega$$

$$\frac{v_o}{v_i} = \frac{G}{1 + GH} = \frac{3.2106}{1 + (3.2106)(7.5630) \times 10^{-2}} = 2.5833$$



The SPICE data are

$$\frac{v_o}{v_i} = 2.583; \quad r_{in} = 314.2\,\text{k}\Omega; \quad r_{out} = 0\,\Omega$$

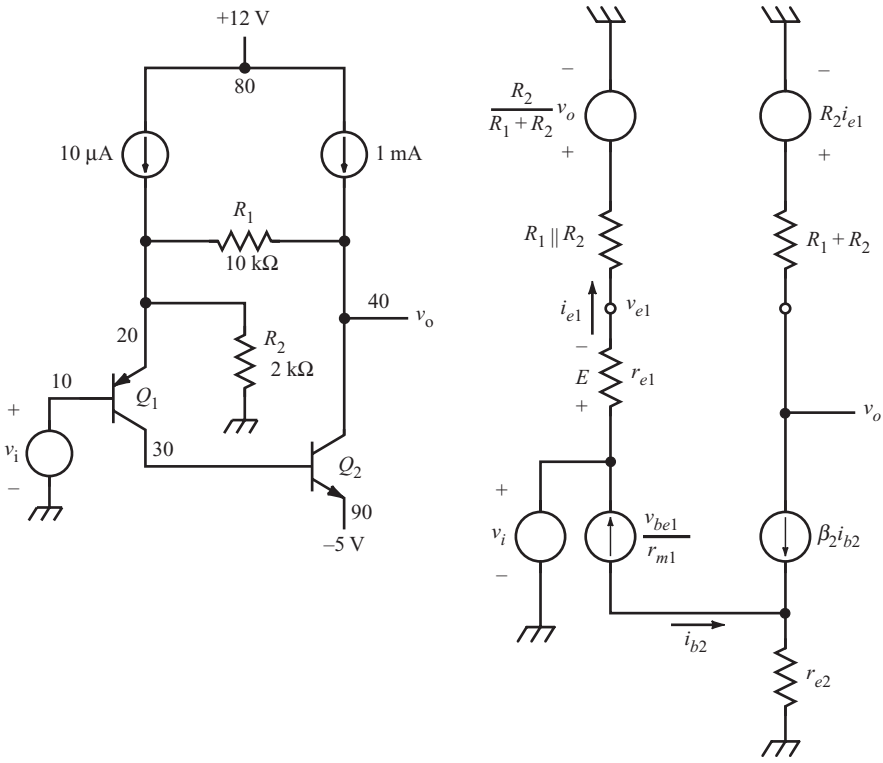
Also, from bias point calculation,

$$I_E = 0.7907\,\text{mA}; \quad V_{BE} = -0.7679\,\text{V}; \quad V_O = 0.4192\,\text{V}$$

Example: Noninverting BJT Feedback Amplifier

In this example, a two-port circuit for H is constructed, to simplify determination of the transmittances. The 2-BJT feedback amplifier is shown below. This amplifier is analyzed by first choosing the error E. Let $E = v_{be1}$. The choice of a Thevenin equivalent output port for H results in a single summing loop. The port current is i_{e1}. When the emitter is opened, $i_{e1} = 0$, and the H output voltage is

$$v_{fb} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot v_o = 0.167 \cdot v_o$$



From the emitter of Q_1, R_1 and R_2 are in parallel. The Thevenin resistance of the H output port is 2 kΩ‖10 kΩ = 1667 Ω.

The H input port samples the output voltage v_o. The reverse path through H is represented by a Thevenin equivalent circuit at the H input. The contribution to v_o through H is the voltage at v_o due to the path through the feedback network and is dependent on the H output port current i_{e1}. It is $i_{e1} \cdot R_2$. The resistance of the port is found by nulling i_{e1}. This is accomplished by opening the Q_1 emitter. The resistance is $R_1 + R_2$.

Next, reanalyze the circuit for a different choice of E:

$$E = v_i - v_{e1} = v_{be1}$$
$$= v_i - \left[\left(\frac{R_2}{R_1+R_2}\right)\cdot\left(\frac{r_{e1}}{r_{e1}+R_1\|R_2}\right)\cdot v_o + \left(\frac{R_1\|R_2}{r_{e1}+R_1\|R_2}\right)\cdot v_i\right]$$

and $v_o = G \cdot E$. When E is simplified, T_i and H are the coefficients of v_i and v_o, respectively. They are

$$T_i = \left(\frac{r_{e1}}{r_{e1} + R_1 \| R_2} \right)$$

$$H = -\frac{v_{be1}}{v_o}\bigg|_{G_{off}} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot \left(\frac{r_{e1}}{r_{e1} + R_1 \| R_2} \right)$$

G has two paths: the active path G_1 through the transistors, and the passive path G_2 through H. G is

$$G = G_1 + G_2 = \frac{v_o}{v_{be1}}\bigg|_{H, G_{2off}} + \frac{v_o}{v_{be1}}\bigg|_{H, G_{1off}} = \beta_2 \cdot \frac{R_1 + R_2}{r_{m1}} + \frac{R_2}{r_{e1}}$$

Finally, this is combined in

$$A_v = T_i \cdot \frac{G}{1 + GH}$$

for the closed-loop gain.

A numerical solution begins with a static solution, in which

$$I_{E1} = 6.87\,\mu\text{A}, \quad I_{E2} = 680\,\mu\text{A}$$

$$r_{e1} = 3.763\,\text{k}\Omega, \quad r_{m1} = 3.803\,\text{k}\Omega, \quad r_{e2} = 38.04\,\Omega$$

Substituting these and the circuit element values into the previous equations,

$$G_1 = 312.4, \quad G_2 = 0.5312, \quad G = 312.9$$

Also,

$$H = (0.1667) \cdot (0.6932) = 0.1155, \quad T_i = 0.6932$$

Putting it all together, the closed-loop gain is

$$A_v = 5.838$$

The circuit is solved more easily by choosing

$$E = v_i - v_{fb}$$

as shown below. This E is also a voltage, leaving the loading of G the same as we found before. But now

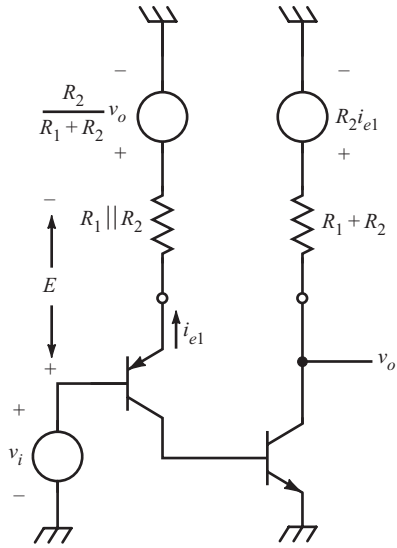
$$E = 1 \cdot v_i - \left(\frac{R_2}{R_1 + R_2} \right) \cdot v_o$$

and $T_i = 1$ whereas

$$H = \left(\frac{R_2}{R_1 + R_2} \right) = 0.1667$$

G has two paths, as before:

$$G_1 = \left. \frac{v_o}{E} \right|_{H, G_2 \text{ off}} = \alpha_1 \cdot \frac{R_1 + R_2}{r_{e1} + R_1 \| R_2} \cdot \beta_2 = 216.5$$



$$G_2 = \left.\frac{v_o}{E}\right|_{H,G_1\text{off}} = \frac{R_2 \cdot i_{e1}}{i_{e1}(r_{e1} + R_1 \| R_2)} = \frac{R_2}{r_{e1} + R_1 \| R_2} = 0.368$$

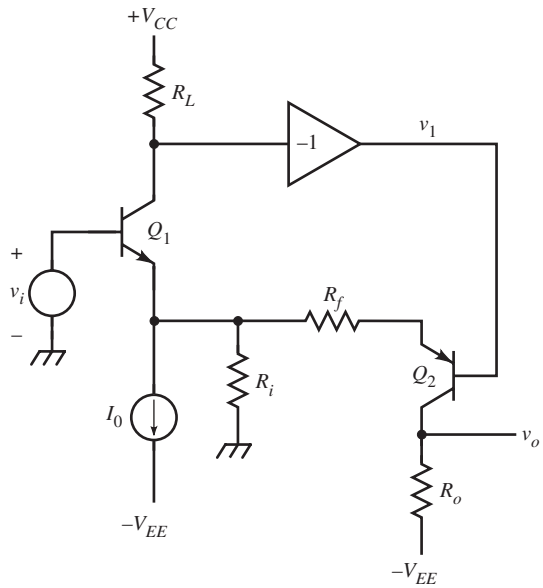
Then $G = G_1 + G_2 = 216.9$. The closed-loop gain is 5.838, the same as before. SPICE simulation confirms the result.

A Noninverting Feedback Amplifier with Output Block

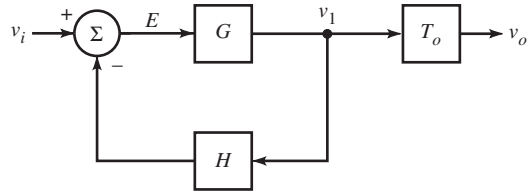
In previous examples, feedback was sampled directly at the output. This is not always the case, however. Another block between the sampling circuit and output must then be introduced, just as T_i was required for similar situations at the input. This block is T_o. Both T_i and T_o can be included in an expression for E:

$$E = T_i \cdot x_i - \frac{H \cdot x_o}{T_o}$$

The amplifier, shown below, is a modified version of a previous noninverting feedback amplifier, with a PNP BJT, Q_2, added.



Its block diagram is shown below.



Here, v_o/T_o, the feedback voltage, v_f is labeled v_1. Choosing feedback quantities is like choosing error quantities; within the loop, the choice is somewhat arbitrary. The sampled feedback quantity is preferably chosen as far forward in the signal flow-path as possible to minimize common expressions in T_o and H.

Choose the error quantity to be

$$E = v_i - v_{fb}$$

where v_{fb} is given in the complete error expression:

$$E = v_i - \left(\frac{R_i}{R_i + R_f + r_{e2}} \right) \cdot v_1$$

Again, a choice of summing is accomplished using superposition. Then for $r_{e2} \ll R_f + R_i$, the two resistors dominate the denominator of the coefficient of v_1, which is $-H$, and variations in r_{e2} are negligible. G is the same as in the previous non-Q_2 amplifier except that now $R_E = R_i \| (R_f + r_{e2})$ and

$$G = \alpha_1 \cdot \frac{R_L}{r_{e1} + R_i \| (r_{e2} + R_f)}$$

The final block to be determined is T_o:

$$T_o = \left. \frac{v_o}{v_1} \right|_{E=0} = -\alpha_2 \cdot \frac{R_o}{r_{e2} + R_f + R_i}$$

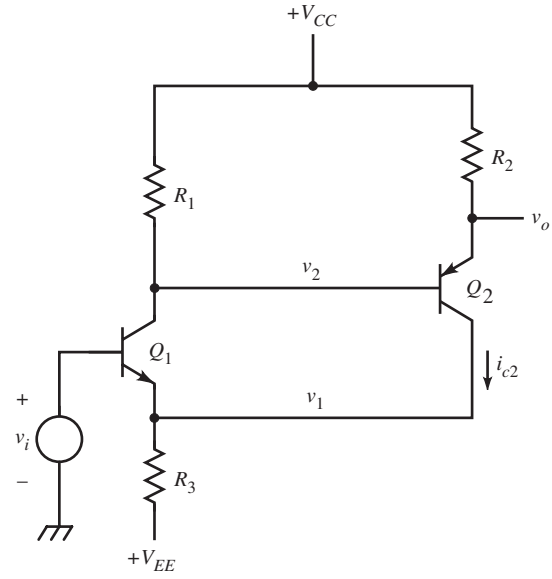
This is the gain of Q_2 from v_1 to v_o, with the loading of H included. (E is a voltage, so the summing loop is opened.)

The closed-loop gain can then be constructed by substituting the expressions for the blocks into

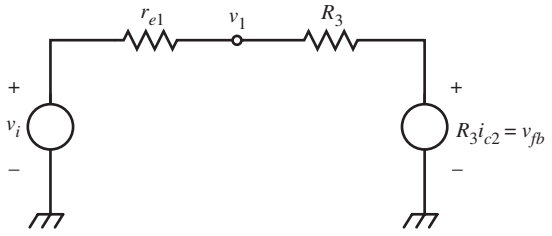
$$A_v = \frac{G}{1+GH} \cdot T_o$$

The resulting expression is unwieldy and not intuitively beneficial. For complicated feedback amplifiers, more insight is gained into amplifier behavior from the expressions for the blocks themselves.

An inverting voltage amplifier with the same block diagram is shown below (a), with equivalent input voltage error loop (b).



(a)



(b)

The error voltage is chosen to be

$$E = v_i - v_{fb} = v_i - R_3 \cdot i_{c2}$$

The collector current source of Q_2 is shunted by R_3. This Norton source can be transformed into a Thevenin source with a voltage of $R_3 \cdot i_{c2}$. The feedback quantity is i_{c2}. G is consequently a transconductance amplifier with a gain of

$$\begin{aligned} G &= \left.\frac{i_{c2}}{E}\right|_{v_{fb}=0} \\ &= -\alpha_1 \cdot \frac{R_1}{r_{e1} + R_3} \cdot \alpha_2 \frac{-1}{R_2 + r_{e2} + R_1/(\beta_2 + 1)} \\ &= \alpha_1 \cdot \frac{R_1}{r_{M1}} \cdot \alpha_2 \cdot \frac{1}{r_{M2}} \end{aligned}$$

H must be a transresistance and is

$$H = -\left.\frac{E}{i_{c2}}\right|_{v_i=0} = -(-R_3) = R_3$$

Finally, T_o is

$$T_o = \frac{v_o}{i_{c2}} = \frac{v_o}{i_{e2}} \cdot \frac{i_{e2}}{i_{c2}} = -R_2 \cdot \frac{1}{\alpha_2}$$

These blocks are combined for the closed-loop voltage gain:

$$A_v = -\frac{\alpha_1 \cdot (R_1/r_{M1}) \cdot (R_2/r_{M2})}{1 + \alpha_1 \cdot (R_1/r_{M1}) \cdot \alpha_2 \cdot (R_3/r_{M2})}$$

To demonstrate an alternative derivation, T_o can be eliminated from the feedback topology by choosing the feedback node to be v_o instead. For this choice,

the blocks are composed somewhat differently. With the same E, G and H become voltage amplifiers:

$$G = -\alpha_1 \cdot \frac{R_1}{r_{M1}} \cdot \frac{R_2}{r_{M2}}, \quad H = -\frac{R_3}{R_2} \cdot \alpha_2$$

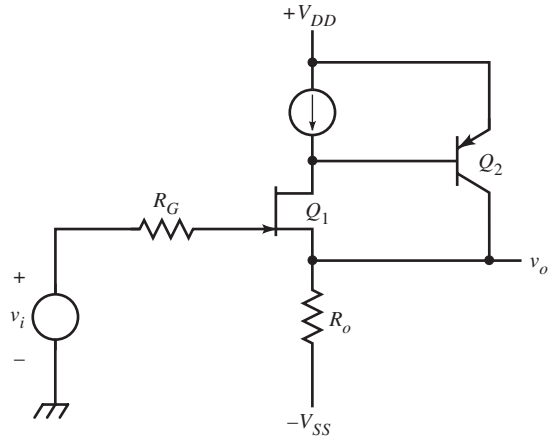
These expressions when substituted into the feedback formula yields A_v as given already. For this circuit, this latter choice of feedback quantity is better since it eliminates the redundant path of i_{c2} to v_o in H and T_o.

This two-transistor amplifier not only provides a gain determined by R_2 and R_3 but also has a low-resistance source at the output with voltage translation from the input. Unless R_3 is very small, the input resistance is large, approaching the ideal input-output requirements for a voltage amplifier. A limitation is that all of the voltage gain must be realized in the first stage (Q_1). Thus, R_1 must be large relative to R_3. R_1 is loaded by r_{in}(CC), which is large, so that a large gain can be realized from Q_1. This amplifier achieves much functional capability from its five components. Because V_{EE} constrains the value of R_3 for biasing, an additional resistor from the emitter of Q_1 to ground could be required.

FET Buffer Amplifier

Another amplifier similar to that above is a common buffer amplifier that uses feedback to reduce gain error. The goal is to achieve an accurate ×1 gain from an ideal voltage amplifier so that a high-resistance voltage source can be transformed into a low-resistance source capable of supplying varying amounts of load current at the input voltage. The circuit, shown below, provides this capability with few components.

The input transistor is chosen to be a junction field-effect transistor (JFET) for high resistance. A BJT could be used instead if the base current causes negligible static offset across R_G. The current source at the drain simplifies the analysis and can be implemented as a large resistor or a PNP collector current supply. Being constant, it enters into the dynamic analysis as an open circuit.



Choose the error voltage to be

$$E = v_i - v_o$$

Then $E = v_{gs}$. In the FET T model, v_{gs} develops i_d across r_m; then

$$G = \frac{v_o}{E}\bigg|_{B=0} = \frac{v_o}{v_{gs}} = \beta \cdot \frac{R_o}{r_m}$$

Both input and output directly contribute to E, and $H = 1$. To find G, set $B = 0$ as usual. This means that the feedback contribution to E must be eliminated to calculate forward path gain. In this case, set v_o to zero. But since v_o is the numerator of G, it must remain free to vary. Therefore, to conceptually satisfy the nulling requirement, another v_o source is separately connected to provide feedback, and it is set to zero. Think of it this way. If the loop is broken at the output and the feedback v_o is v_o', then v_o' is set to zero while calculating G.

For calculating H, $v_o' = v_o$. The closed-loop gain of this buffer is

$$A_v = \frac{R_o}{(r_m/\beta) + R_o}$$

For a typical JFET, $r_m = 100\ \Omega$. If $R_o = 1\ \text{k}\Omega$ and $\beta = 100$, then A_v is 0.9990 for approximately 10 bits of accuracy.

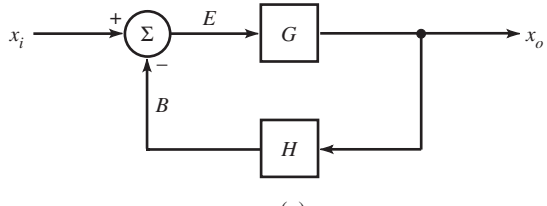
The previous examples illustrate the notion that feedback analysis is a very fluid activity in that a given circuit can be analyzed several ways, all consistent with the basic concept. Whenever a circuit can be cast in the feedback form, it can be viewed as having feedback. Even the common-drain (CD) (or CC) configuration can be analyzed from a feedback perspective. The FET buffer amplifier is a CD stage with an additional transistor that increases loop gain. But even without Q_2, feedback analysis can be applied. Let $E = v_i - v_o$ as before. Then $G = v_o/E = R_o/r_m$ and $H = 1$. The closed-loop gain is

$$A_v = \frac{v_o}{v_i} = \frac{G}{1+GH} = \frac{R_o/r_m}{1+(R_o/r_m)\cdot(1)} = \frac{R_o}{r_m + R_o}$$

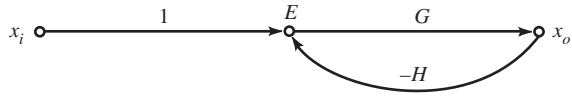
This result is what the transresistance method yields for the CC gain, with $R_B = 0$, $R_o = R_E$, and $r_e = r_m$.

Feedback Effects on Input and Output Resistance

The input resistance of a voltage-summing amplifier is increased and of a current-summing amplifier is decreased due to negative feedback. In the feedback-amplifier block diagram below, let x_i and E be voltages.

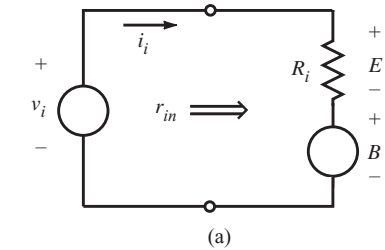


(a)

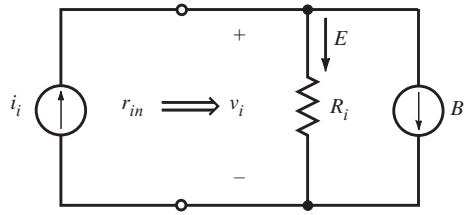


(b)

Let the circuit represented by this block diagram have an open-loop input resistance R_i, the input resistance of G shunting the output resistance of H, as shown below.



(a)



(b)

Without feedback in (a) above, $E = x_i = v_i$, and the input voltage is applied to R_i directly. The same input voltage is largely canceled by feedback, resulting in a much smaller error voltage applied to R_i. In effect, the input resistance is increased because the same input voltage, v_i produces a current in R_i that is E/R_i. The ratio of open-loop to closed-loop input resistance is the ratio E/x_i. This ratio can be derived from the basic feedback equations as follows:

$$\frac{E}{x_i} = 1 - H \cdot \frac{x_o}{x_i} = 1 - \frac{GH}{1+GH} = \frac{1}{1+GH}$$

The closed-loop voltage reduction across R_i makes the input resistance effectively larger by $(1 + GH)$ for voltage input:

$$r_{in}(cl) = \frac{v_i}{i_i} = \frac{E+B}{i_i} = \frac{E \cdot (1+GH)}{i_i} = R_i \cdot (1+GH)$$

For the noninverting op-amp circuit, the effective op-amp differential input resistance is made larger by feedback. The feedback voltage, v_- tracks v_+, and the differential voltage across the input terminals is the error voltage. This small voltage produces a much smaller current in the op-amp input resistance than v_i alone would produce (with v_- grounded). The op-amp input resistance is bootstrapped by the feedback voltage. Thus, the resistance across which the error voltage is developed is effectively larger with feedback.

A dual argument applies to an amplifier with an error current, from (b) above. In this case, input resistance is reduced by $(1 + GH)$. The input current (rather than voltage) is increased by feedback current. The same input current results in a reduced voltage across the input resistance due to feedback current cancellation. The resulting voltage across R_i produced by the error current is $1/(1 + GH)$ times smaller than the voltage that the input current alone would produce. A smaller voltage resulting from the same input current means that the effective resistance is smaller:

$$r_{in}(cl) = \frac{v_i}{i_i} = \frac{v_i}{E + B} = \frac{v_i}{E \cdot (1 + GH)} = \frac{R_i}{1 + GH}$$

The inverting feedback amplifier on page 113 has an open-loop input resistance of $R_i \| R_f$. With negative feedback, the resulting error current produces an error voltage of v_- that is much reduced from the voltage i_i alone would produce across r_{in} of the op-amp input. The closed-loop input resistance is effectively reduced to $(R_i \| R_f)/(1 + GH)$.

The effect of negative feedback on output resistance can be analyzed by representing the output as a Thevenin equivalent voltage source with internal voltage of v and open-loop output resistance of R_o.

An output voltage of v_o is produced by a current i_o through R_o toward the output node. With feedback, output voltage error due to the drop across R_o is corrected. The output voltage (with no input applied) is

$$\begin{aligned} v_o &= v + i_o \cdot R_o \\ &= -G \cdot H \cdot v_o + i_o \cdot R_o \\ &= \frac{i_o \cdot R_o}{1 + GH} \end{aligned}$$

The effective closed-loop output resistance for voltage output, v_o/i_o, is

$$r_{out}(cl) = \frac{R_o}{1+GH}$$

Similarly, for current output amplifiers, r_{out} is increased by feedback. If the output is represented by a Norton equivalent circuit with current source i, then internal shunt resistance R_o reduces output current. The output resistance can be found by applying a voltage v_o to the output. The resulting output current is

$$i_o = i + \frac{v_o}{R_o} = (-GH)\cdot i_o + \frac{v_o}{R_o} = \frac{v_o}{R_o\cdot(1+GH)}$$

The effective closed-loop output resistance is

$$r_{out}(cl) = R_o\cdot(1+GH)$$

Feedback has advantages for both input and output resistance. For the four cases considered, the effect of feedback is toward the ideal. We have considered the effect of input resistance at the error node or loop and output resistance at the feedback node or loop. For voltage summing, the resistance across the error voltage is increased; for current summing, the resistance through which the error current flows in decreased. If other gain blocks separate these nodes or loops from input or output, their effect on resistance must also be considered.

MILLER'S THEOREM

The inverting op-amp configuration has a resistor connected from output to inverting input. This is not uncommon for feedback amplifiers and can be generalized as shown below.

The inverting voltage amplifier has a gain of $-K$ with input quantities of v_i and i_i. The output voltage is v_o. The equivalent input resistance can be found as follows. First,

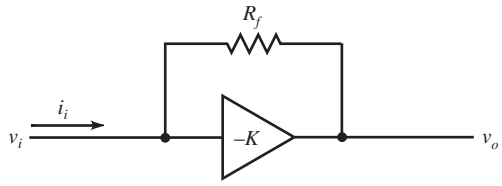
$$v_o = -K\cdot v_i$$

For the input node, applying KCL,

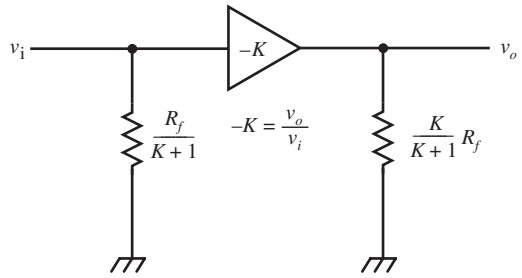
$$\frac{v_i - v_o}{R_f} - i_i = 0$$

Substituting for v_o gives

$$\frac{v_i - (-K \cdot v_i)}{R_f} = \frac{(1+K) \cdot v_i}{R_f} = i_i$$



(a)



(b)

Rearranging the previous equation for input resistance, we have

$$\text{Miller's theorem} \quad r_{in} = \frac{R_f}{1+K}$$

This result is in conformance with the current-input equation for closed-loop input resistance,

$$r_{in}(cl) = \frac{v_i}{i_i} = \frac{v_i}{E+B} = \frac{v_i}{E \cdot (1+GH)} = \frac{R_i}{1+GH}$$

and also follows from it as a special case. It is given separately here because it appears repeatedly when working with CE amplifier stages with collector-to-base feedback.

For an amplifier with output resistance, the equivalent shunt contribution due to R_f can be found similarly, or from the closed-loop output resistance equation for current output,

$$r_{out}(cl) = R_o \cdot (1 + GH)$$

It is

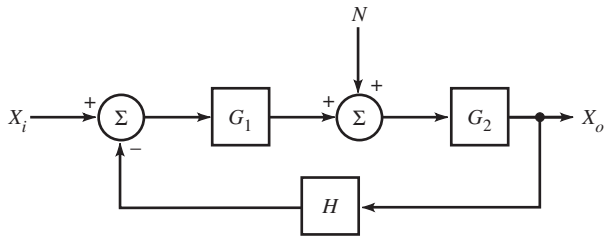
$$r_{out} = \frac{v_o}{-i_i} = \frac{(-K)\cdot v_i}{-v_i \cdot (1+K)/R_f} = \left(\frac{K}{1+K}\right)\cdot R_f$$

From the output, R_f appears to be slightly less than its actual value for large K. From the input, R_f appears to be $1/(1 + K)$ times its actual value, causing input resistance to be much reduced and providing a low-resistance path for i_i. For infinite K, the input node is a virtual ground, as is v_- for the inverting op-amp.

The equivalent circuit resulting from Miller's theorem is shown in fig. (b).

Noise Rejection by Feedback

Feedback increases the immunity of a circuit to *noise*, which is any undesirable electrical disturbance to the circuit. The figure below shows the classical feedback topology with the addition of noise N injected into the forward path.



Noise rejection ability will be defined as the *signal-to-noise ratio* (SNR), the ratio of signal to noise at the output, or

$$\text{SNR} = \frac{X_o/X_i}{X_o/N}$$

For the open-loop case,

$$\frac{X_o}{X_i} = G_1 \cdot G_2 = G, \quad \frac{X_o}{N} = G_2$$

Then open-loop SNR is

$$\text{SNR}_{ol} = \frac{G_1 \cdot G_2}{G_2} = G_1$$

With feedback, the ratios are

$$\frac{X_o}{X_i} = \frac{G}{1+GH}, \quad \frac{X_o}{N} = \frac{G_2}{1+GH}$$

The closed-loop SNR is

$$\text{SNR}_{cl} = G_1$$

The open- and closed-loop SNRs are the same, suggesting no advantage to feedback. However, for the same input, the open-loop output is much larger than the closed-loop output for G ≫ 1. Comparing X_o/X_i for open and closed loop, the open-loop gain is $(1 + GH)$ times larger. For the same X_o/X_i, the open-loop G must be $1/(1 + GH)$ that of the closed-loop amplifier, or

$$G_{ol} = \frac{G_{cl}}{1+G_{cl}H}$$

Normalizing open- and closed-loop gains, the ratio of open- to closed-loop SNRs shows the advantage of feedback:

$$\frac{\text{SNR}_{cl}}{\text{SNR}_{ol}} = \frac{G_{1cl}}{G_{1ol}} = \frac{G_{cl}/G_{2cl}}{G_{ol}/G_{2ol}} = \left(\frac{G_{2ol}}{G_{2cl}}\right)\cdot(1+G_{cl}H)$$

When $G_{2cl} = G_{2ol}$, the familiar $(1 + GH)$ factor reappears as the advantage of feedback on SNR. This feature of op-amp circuits leads to better rejection of noise from the power supply by stages following a sufficiently large G_1. The closer to the input that N is injected (that is, the smaller fraction of G that G_1 is), the less advantage feedback has. In the extreme, noise injected at the input summer is indistinguishable from signal. Noise at the output is rejected by a factor of $(1 + GH)$.

Reduction of Nonlinearity with Feedback

A further benefit of feedback is the linearization of nonlinear forward-path gain blocks. Assuming the classical feedback topology, let $G = K + \varepsilon$, where K is a fixed gain and ε represents the nonlinear terms of G; ε varies with E or x_o. The closed-loop gain T is then

$$T = \frac{(K+\varepsilon)}{1+(K+\varepsilon)\cdot H}$$

For $K \gg \varepsilon$, $1 + (K + \varepsilon) \cdot H \cong 1 + K \cdot H$, and T can be separated into linear and nonlinear terms:

$$T \cong \frac{K}{1+KH} + \frac{\varepsilon}{1+KH}, \quad K >> \varepsilon$$

The second term is the nonlinear closed-loop gain. The open-loop nonlinearity has been reduced by $(1 + KH)$.

In all of the improvements brought about by feedback, the improvement factor has been $(1 + GH)$. The improvements investigated in this and the previous sections were the sensitivity of the closed-loop gain to the open-loop gain, input and output resistances, noise rejection, and linearization of nonlinear open-loop forward-path gain.

CLOSURE

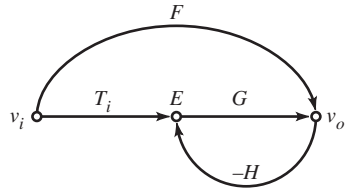
A major difficulty in analyzing feedback circuits is in relating flow-graph representation to their circuit diagrams. The major difficulty is in identifying error summing and feedback sampling. Furthermore, the G and H blocks load each other so that loading interactions must be accounted for. We approached loading by using two-port models of G and H to derive some general rules that are simple and intuitive. (No memorization of various kinds of two-port parameters was required.) Happily, as long as the summing and sampling quantities are chosen within the feedback loop, closed-loop analysis can be performed. This allows various choices for E and x_f. Various feedback amplifier examples were investigated with multiple derivations to cultivate the art of choosing them well.

5

Multiple-Path Feedback Amplifiers

Multipath Feedback Circuits

Feedback analysis can be extended to multipath topologies involving a feedback loop – in particular, topologies with a single *feedforward* path, *F*.

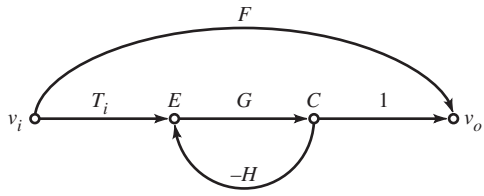


In this topology, the feedforward path injects its signal into the feedback loop at the output. When the flow graph is reduced, voltage gain and error are

$$\frac{v_o}{v_i} = \frac{T_i \cdot G + F}{1 + G \cdot H}$$

$$E = \frac{T_i - F \cdot H}{1 + G \cdot H} \cdot v_i$$

The next topology differs from the first in that it isolates the feedforward path from the feedback loop with a ×1 transmittance from *C* to v_o, functionally a ×1 buffer amplifier.



For this topology with an *isolated feedforward* path:

$$\frac{v_o}{v_i} = T_i \cdot \frac{G}{1 + G \cdot H} + F$$

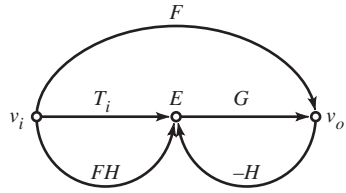
$$E = \frac{T_i}{1 + G \cdot H} \cdot v_i$$

F adds to the output without involving the loop, whereas in the nonisolated case, its output contributes to E via H, and the $F \cdot H$ term is also divided by $1 + G \cdot H$ of the loop.

The relationship between the two topologies can be made explicit by writing the isolated feedforward gain as

$$\frac{v_o}{v_i} = \frac{T_i \cdot G + F}{1 + G \cdot H} + \frac{(F \cdot H) \cdot G}{1 + G \cdot H}$$

where the first term is the nonisolated gain and the second adds transmittance $F \cdot H$ from v_i to E. This gives the equivalent topology of the flow graph shown below. Isolating F from C at v_o is equivalent to adding the $F \cdot H$ branch parallel to T_i. What this branch adds to E exactly cancels the contribution to E from the path from v_i through F and $-H$.



To find F from circuit topology, the G path must be nulled. Then F can be found from

$$F = \left.\frac{v_o}{v_i}\right|_{G \cdot E = 0}$$

If we set $G \cdot E$ to zero, then v_o is contributed by F alone. In the isolated-feedforward topology, C can be set to zero as an alternative to E, if the circuit

allows. Similarly, because the F branch contributes to v_o, finding G must have the additional condition (besides $B = 0$) that $F \cdot v_i = 0$. These conditions are necessary in envisioning the transmittances from the topology.

The general feedback analysis procedure can be applied only if forward and feedback paths are separately identifiable. Then their interactions can be reduced to loading and two-port source transmittances. If path interactions are not apparent, it is always possible to fall back on basic circuit laws: Kirchhoff's current law (KCL), Kirchhoff's voltage law (KVL), and Ohm's law (ΩL). The disadvantage to this is the difficulty of expressing equations so that the path transmittances are explicit.

Falling back to the basic laws is not always the end of intuitive elegance. More algebraic calculation is involved, but this too can be neatly minimized by an orderly procedure that builds flow graphs from circuit equations and then reduces the graphs:

1. Apply KVL, KCL, and ΩL to produce circuit equations.
2. Construct a flow graph from these equations.
3. Reduce the flow graph.

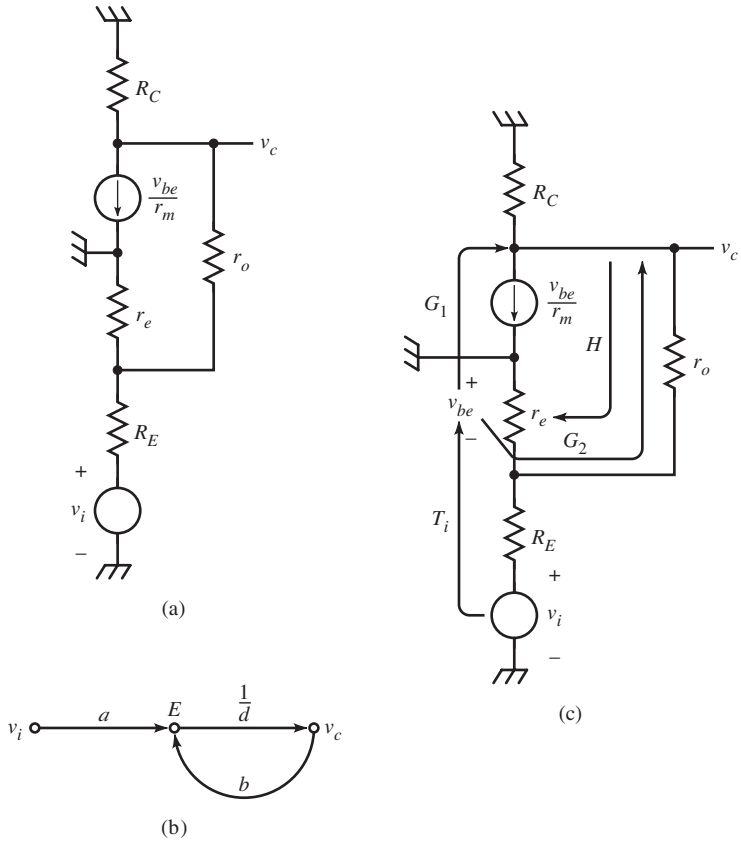
More generally, an approach to solving extremely difficult circuits is as follows:

1. Make simplifying assumptions and apply feedback analysis.
2. Apply flow-graph circuit analysis.
3. Use the results of step 1 to guide the formulation of equations from step 2 so that the flow paths become obvious.

An example of this approach is to solve a field-effect transistor (FET) equivalent of a given bipolar junction transistor (BJT) circuit and then attempt to construct the more complicated BJT expressions, guided by the FET results.

Common-Base Amplifier Feedback Analysis

We now apply both flow-graph and two-port feedback analysis to the BJT configurations with r_o since (as we shall see) the signal paths are not obvious. The corresponding FET circuits follow from the BJT-to-FET transformation. We begin with the simplest configuration, the CB amplifier, shown below.



Applying KCL to the emitter node *e*, we obtain

$$\text{at } e, \quad \frac{v_e - v_i}{R_E} + \frac{v_e}{r_e} + \frac{v_e - v_c}{r_o} = 0$$

This can be rewritten as

$$v_e = \left(\frac{r_e \| r_o}{r_e \| r_o + R_E} \right) \cdot v_i + \left(\frac{R_E \| r_e}{R_E \| r_e + r_o} \right) \cdot v_c$$

Similarly, for the collector node *c*, we obtain

$$\text{at } c, \quad \frac{v_c}{R_C} + \frac{v_c - v_e}{r_o} + \frac{v_{be}}{r_m} = 0$$

which reduces to

$$v_e = \left(\frac{r_m \| r_o}{R_C \| r_o} \right) \cdot v_c$$

The equations for v_e have the general form

$$\text{at } e, \quad E = a \cdot v_i + b \cdot v_c$$

$$\text{at } c, \quad E = d \cdot v_c \Rightarrow v_c = \left(\frac{1}{d} \right) \cdot E$$

where

$$E = v_e = -v_{be}$$

The form of these equations is similar to the basic feedback equations. These equations are represented as a flow graph in fig. (b).

From the flow graph, T_i, G, and H can be identified as

$$T_i = a \quad G = \left(\frac{1}{d} \right), \quad H = -b$$

When two-port loading rules from feedback analysis are applied, T_i is the voltage divider attenuation from v_i to v_e. R_E forms a divider with $r_e \| r_o$. From the expression for v_e, H is

$$-\left(\frac{R_E \| r_e}{R_E \| r_e + r_o} \right)$$

Finally, G requires further decomposition:

$$G = \frac{1}{d} = \frac{R_C \| r_o}{r_m \| r_o} = \frac{R_C \| r_o}{r_m} + \frac{R_C}{R_C + r_o} = G_1 + G_2$$

G can be interpreted as having two signal paths, G_1 and G_2. G_1 has a transresistance interpretation and is the gain from v_{be} to v_c. This is the active path. G_2 is a passive path attenuation from v_e to v_c, formed by divider resistances r_o

and R_C. Both paths are from E to v_o; therefore, these parallel paths both contribute to G.

In this analysis, the circuit equations led to the identification of four signal paths, shown in (c). From figure (b), these paths interact to form a familiar feedback topology. Combining T_i, G, and H into the feedback formula, the common-base (CB) closed-loop gain is

$$\text{CB} \quad A_v = T_i \cdot \frac{G}{1+GH} = \frac{a/d}{1-(b/d)}$$

The input and output resistances can be derived by making use of the gain calculations. The input resistance can be found by Nortonizing v_i and R_E. Then the closed-loop emitter resistance is

$$r_E = \frac{v_e}{i_i} = \frac{E}{i_i} = \frac{[R_i \cdot i_i/(1+GH)]}{i_i} = \frac{R_i}{1+GH} = \frac{r_e \| r_o \| R_E}{1+GH}$$

where

$$R_i = \left.\frac{v_e}{i_i}\right|_{B=0} = r_e \| r_o \| R_E$$

and $i_i = v_i/R_E$. Then r_{in} can be put in the following form:

$$\begin{aligned} \text{CB } r_{in} &= R_E + \left(\frac{r_e \| r_o}{1+GH}\right) \Bigg\| \left(\frac{R_E}{GH}\right), \quad R_E >> r_e \| r_o \\ &\cong R_E + \left(\frac{r_e \| r_o}{1+GH}\right) \end{aligned}$$

Feedback is ordinarily negative and $1 + GH > 1$. Here, $0 < 1 + GH < 1$; the feedback is positive but less than 1. This causes some reversals of effect. For r_{in}, R_E is in series with the closed-loop resistance of the emitter node. By inspection, the resistance there is due to r_e (grounded at the base) in parallel with the resistance to the output, r_o. The closed-loop resistance is $r_e \| r_o$, divided by $1 + GH$, or CB r_{in}. The exact expression includes R_E/GH in parallel with this resistance and is due to interaction of the feedback loop with R_E.

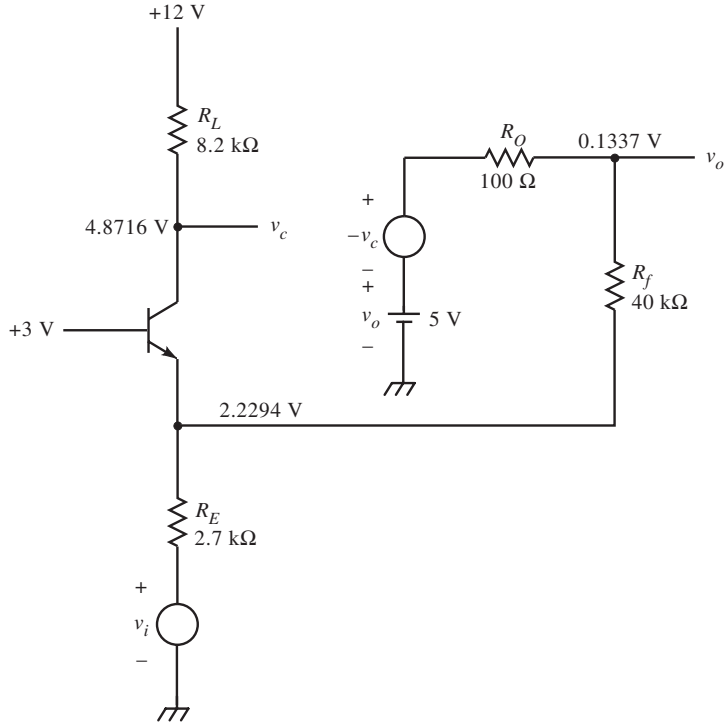
Easier to derive is r_{out}:

$$\text{CB} \quad r_{out} = \frac{R_C \| r_o}{1 + GH} = \frac{R_C \| r_o}{1 - b/d}$$

The loop gain for this amplifier is less that one because the feedback due to r_o is positive. Although typically $G > 1$, $-H \ll 1$ so that $|GH| < 1$, and the circuit does not oscillate. But the effect of positive feedback is to work against the benefits of negative feedback. In most circuits of this kind, where r_o is due to transistor base width or channel length modulation, r_o is much larger than external circuit resistances and has little effect on circuit performance. For accurate calculations, however, it is among the dominant second-order effects to be accounted for.

Example: Inverting CB Feedback Amplifier

The amplifier shown below is a CB stage with a ×(−1)-buffered output and with feedback through R_f.



(The default BJT model is being used here: $\beta = 99$, $I_S = 10^{-16}$ A.) From SPICE, we have the following data:

$$I_E = 0.8781\,\text{mA}, \quad V_E = 2.2294\,\text{V}, \quad V_C = 4.8716\,\text{V}, \quad V_O = 0.1337\,\text{V}$$

and

$$\frac{v_o}{v_i} = -2.470, \quad r_{in} = 2.724\,\text{k}\Omega, \quad r_{out} = 83.12\,\Omega$$

From these data, $r_e = 29.46\,\Omega$ and $r_e \| R_f = 29.44\,\Omega$. Then, applying feedback analysis with $E = v_e$, we have

$$G_1 = -\alpha \cdot \frac{R_L}{r_e} \cdot \left(\frac{R_f}{R_O + R_f} \right) = -274.78$$

$$G_2 = \frac{R_O}{R_O + R_f} = 2.4938 \times 10^{-3} \cong 0$$

Then $G = G_1 + G_2 = -274.78$. Furthermore,

$$H = -\frac{R_E \| r_e}{R_f + R_E \| r_e} = -7.2826 \times 10^{-4}$$

$$T_i = \frac{R_f \| r_e}{R_f \| r_e + R_E} = 1.0789 \times 10^{-2}$$

Combining these transmittances yields

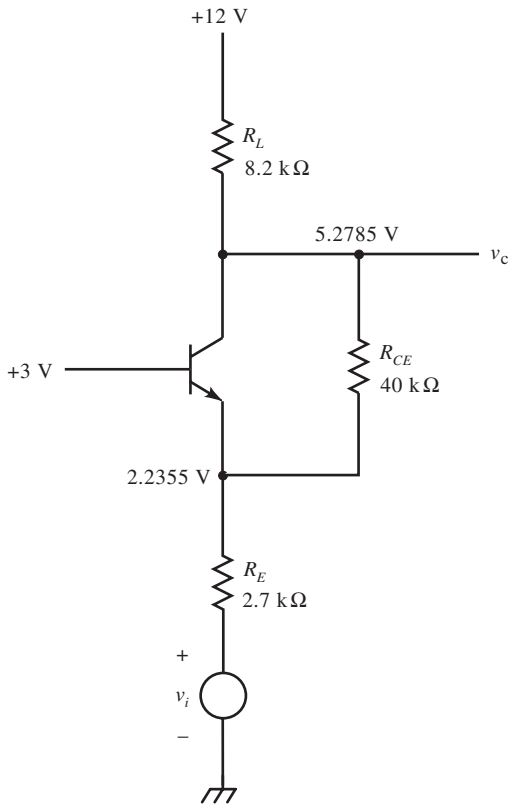
$$\frac{v_o}{v_i} = T_i \cdot \frac{G}{1 + GH} = -2.4703$$

$$r_{in} = R_E + r_e \left\| \frac{R_f}{1 + (-G)} \right. = 2.7245\,\text{k}\Omega$$

$$r_{out} = \frac{R_O \| R_f}{1 + GH} = 83.118\,\Omega$$

Example: CB BJT Amplifier with R_{CE}

The circuit shown below is a CB amplifier with a fixed external collector-emitter resistance R_{CE}.



This resistor is a simplified form of r_o since it is independent of I_E. To analyze this circuit, the approach given for difficult circuits will be taken. If we solve the same circuit without R_{CE}, then if R_{CE} has a minor effect on the circuit, the simplified analysis gives us an approximation by which to evaluate more complicated solutions. The static solution for the simplified circuit is

$$I_E = 0.826\,\text{mA}, \quad V_E = 2.231\,\text{V}, \quad V_C = 5.292\,\text{V}$$

(The default BJT model is being used here: $\beta = 99$, $I_S = 10^{-16}$ A.) Then the quasistatic solution follows:

$$r_e = 31.3\,\Omega, \quad v_c/v_i = 2.97, \quad r_{in} = 2.73\,\text{k}\Omega, \quad r_{out} = 8.2\,\text{k}\Omega$$

SPICE simulation results for the operating point are

$$I_E = 0.7511\,\text{mA}, \quad I_{RCE} = 76.075\,\mu\text{A}, \quad V_E = 2.2355\,\text{V}, \quad V_C = 5.2785\,\text{V}$$

The dynamic parameters are

$$r_e = 34.456\,\Omega, \quad r_m = 34.804\,\Omega, \quad r_\pi = 3.4456\,\text{k}\Omega, \quad \mu = 1149.3$$

The simulation results are

$$\frac{v_c}{v_i} = 2.955, \quad r_{in} = 2.741\,\text{k}\Omega, \quad r_{out} = 8.162\,\text{k}\Omega$$

With these data, we can now apply various methods to find the three parameters of interest and to verify the results. The first solution is based on the CB A_v equation, derived from KCL, and involves simple substitution:

$$\frac{v_o}{v_i} = \frac{a}{d-b} = \frac{1.2591\times10^{-2}}{-8.4982\times10^{-4} + 5.1100\times10^{-3}} = 2.9556$$

From this calculation, $1 + GH = 1 - b/d = 0.83370$. The resistances are

$$r_{in} \cong \frac{r_e \| r_o}{1+GH} + R_E = 2.7413\,\text{k}\Omega$$

$$r_{out} = \frac{R_C \| r_o}{1+GH} = \frac{6.8050\,\text{k}\Omega}{0.83370} = 8.1624\,\text{k}\Omega$$

Next, solve the circuit using the μ transform and equations from CB with r_o:

$$\frac{v_c}{v_i} = (0.56066)\cdot(5.2710) = 2.9552$$

$$r_{in} = 2.7\,\text{k}\Omega + 41.399\,\Omega = 2.7414\,\text{k}\Omega$$

$$r_{out} = 8.2\,\text{k}\Omega \| (40\,\text{k}\Omega + 1.7413\,\text{M}\Omega) = 8.1624\,\text{k}\Omega$$

For a third and final approach, use Miller's theorem to find r_{in}. First,

$$K = -\frac{v_c}{E} = -\frac{1}{d} = -195.69$$

then,

$$r_{in} = R_E + r_e \left\| \left(\frac{R_{CE}}{1+K} \right) \right.$$
$$= 2.7\,\text{k}\Omega + (34.46\,\Omega) \| (40\,\text{k}\Omega / (-194.7) = 2.7414\,\text{k}\Omega$$

Common-Emitter Amplifier Feedback Analysis

The common-emitter (CE) with r_o is shown in (a) below, with flow graph (b) and quasistatic circuit model (c).

KCL is applied at emitter and collector:

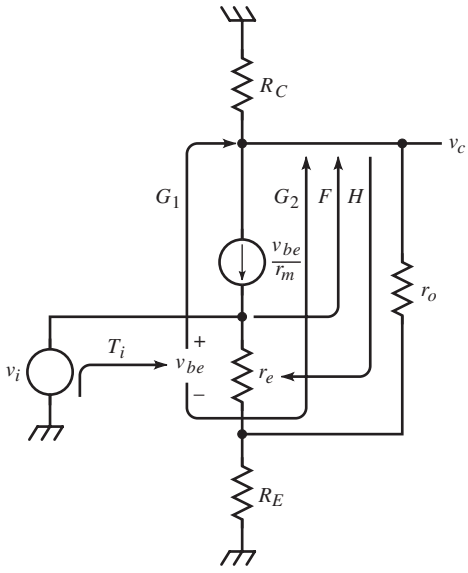
$$\text{at } e, \quad v_e = \left(\frac{R_E \| r_o}{R_e \| r_o + r_e} \right) \cdot v_i + \left(\frac{R_E \| r_e}{R_E \| r_e + r_o} \right) \cdot v_c$$

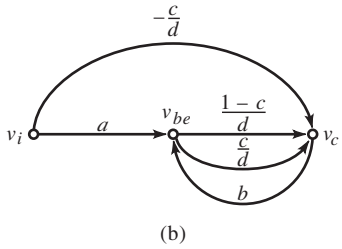
$$\text{at } c, \quad v_e = \left(\frac{r_o}{r_o + r_m} \right) \cdot v_b + \left(\frac{r_m \| r_o}{R_C \| r_o} \right) \cdot v_c$$

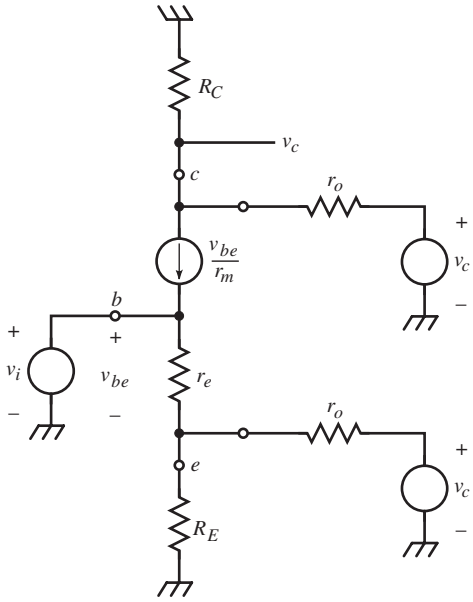
Let $E = v_{be}$. These equations can be made explicit in E by negating them and adding v_b. The result is

$$v_{be} = \left(\frac{r_e}{R_E \| r_o + r_e} \right) \cdot v_i - \left(\frac{R_E \| r_e}{R_E \| r_e + r_o} \right) \cdot v_c$$

$$v_{be} = \left(\frac{r_m}{r_o + r_m} \right) \cdot v_i - \left(\frac{r_m \| r_o}{R_C \| r_o} \right) \cdot v_c$$







The more general forms of these equations are, respectively,

$$E = a \cdot v_i + b \cdot v_c$$

$$E = c \cdot v_i + d \cdot v_c \Rightarrow v_c = \left(\frac{1}{d}\right) \cdot E - \left(\frac{c}{d}\right) \cdot v_i$$

Compared with the CB amplifier, the CE has the additional term with coefficient *c*. These equations are represented by the flow graph (b). The identifiable paths are

$$T_i = a, \quad H = -b, \quad G = \left(\frac{1}{d}\right), \quad F = -\left(\frac{c}{d}\right)$$

G is the same as for the CB but is negative, as expected for a CE. (The CE E is also the negated CB E.) G represents the same two paths, G_1 and G_2, as for the CB. H is a voltage divider from v_c to $v_e = -E$ (and is the negated CB H). T_i is the voltage divider attenuation from v_i to v_{be}. Because of the additional term in v_c, there is a feedforward path F from v_i to v_c in the CE that is absent in the CB. F can be expressed as

$$F = -\frac{c}{d} = \left(\frac{R_C \| r_o}{r_m \| r_o}\right) \cdot \left(\frac{r_m}{r_o + r_m}\right) = \left(\frac{R_C}{R_C + r_o}\right)$$

The path of F can be traced from this expression. It is the attenuation of the passive divider from emitter to collector through r_o. F is v_c/v_i, where v_i is at the base (not the emitter). To find F, as we did with G and H in multipath feedback analysis, the path through G must be nulled to allow only the signal through F to effect v_c. This can be accomplished in this case by setting E to zero, or

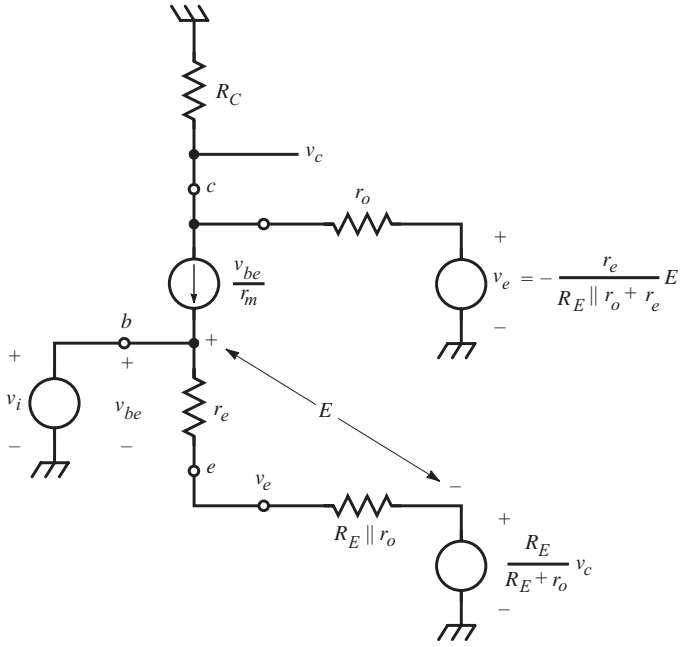
$$F = \left.\frac{v_c}{v_i}\right|_{E=0}$$

For $E = v_{be} = 0$, then $v_b = v_e$ and $F = v_c/v_e$, as in the previous expression for F above. Fig. (a) shows the five signal paths. The two additional paths, G_2 and F, are a result of the r_o branch to the collector. They differ in that G_2 is the path from v_{be} whereas F is from the input v_i. For F, an increase in base voltage causes an increase in emitter voltage. This increase is transmitted through the divider to the collector uninverted.

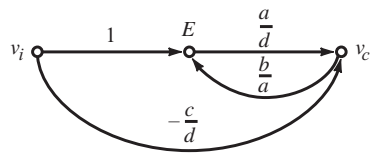
For $E = v_{be}$, an increasing E causes a decreasing v_e. Consequently, G_2 is inverted. E and v_i do not follow identical paths to the output since v_i (through F) can affect v_c even though the effect of G_2 on v_c is due to E.

A two-port equivalent of H is shown in (c). To meet the constraints on H (from two-port analysis), the H input source v_e must be expressed in terms of v_{be} (using a divider formula). Since this is possible, this approach also leads to

a solution. We now examine this aspect further. The choice of E is arbitrary within the feedback loop; an equivalent two-port circuit is shown below.



(a)



(b)

The choice of E is

$$E = v_i - \left(\frac{R_E}{R_E + r_o} \right) \cdot v_c$$

The two-port equivalent CE circuit is shown in (a). The flow graph (b) can be derived from the previous flow graph by moving a forward through the E node.

Paths out of E become multiplied by a and paths into E divided by a, as shown. The paths are identified as follows:

$$T_i = 1$$

$$G = \frac{a}{d} = \frac{(1-c)}{d} \cdot a + \frac{c}{d} \cdot a = G_1 + G_2$$

where

$$G_1 = -\alpha \cdot \frac{R_C \| r_o}{R_E \| r_o + r_e},$$

$$G_2 = -\left(\frac{r_e}{R_E \| r_o + r_e} \right) \cdot \left(\frac{R_C}{R_C + r_o} \right)$$

$$H = -\frac{b}{a} = \frac{R_E}{R_E + r_o}$$

$$F = -\frac{c}{d} = \left(\frac{R_C \| r_o}{r_m \| r_o} \right) \cdot \left[1 - \frac{r_o}{r_m + r_o} \right] = \frac{R_C}{R_C + r_o}$$

The choice of E places it across the transresistance r_M of the transistor forward path gain G_1. v_i is directly in the loop containing E, so that $T_i = 1$. Similarly, the divider formula of the Thevenin feedback source is H.

G_2 is the passive forward path through r_o and consists of two voltage divider factors in the G_2 expression. The first is the divider from E to v_e with output across v_{be}. The second is from v_e to v_c. To find G_2, then, G_1E, B and $F \cdot v_i$ must be nulled. The G_1 path is nulled by disconnecting the BJT collector current source. B is set to zero when the H output source is set to zero. Nulling the signal through F is not easy because it shares essentially the same path as G_2. In this case, instead of attempting to null Fv_i, subtract it from the derived expression for G_2. We must find

$$v_c = G_2 \cdot E + F \cdot v_i \big|_{B, G_1 = 0} = (G_2 \cdot T_i + F) \cdot v_i, \quad B, G_1 = 0$$

Then,

$$G_2 = \left.\frac{\left(\frac{v_c}{v_i}\right) - F}{T_i}\right|_{B,G_1=0} = \left(\frac{v_e}{v_i}\right)\cdot\left(\frac{v_c}{v_e}\right) - F$$

$$= \left(\frac{R_E \| r_o}{R_E \| r_o + r_e}\right)\cdot\left(\frac{R_C}{R_C + r_o}\right) - \left(\frac{R_C}{R_C + r_o}\right) = -\frac{r_e}{R_E \| r_o + r_e}\cdot\frac{R_C}{R_C + r_o}$$

This derivation of G_2 shows that when paths overlap significantly, isolating the path being found may require an approach other than nulling sources. Opening paths and subtracting the effects of intertwined paths are also options in meeting the transmittance constraints. Use of a different flow graph, such as the isolated feedforward topology instead of (a), can "untangle" paths. The basic idea when finding a path transmittance is to eliminate contributions from other paths to the output node of the path being found.

Example: CE Amplifier with R_{CE}

The figure shows a CE amplifier with the default BJT model:

$$\beta = 99, \quad I_S = 10^{-16}\ \text{A}$$

From SPICE, the static values are

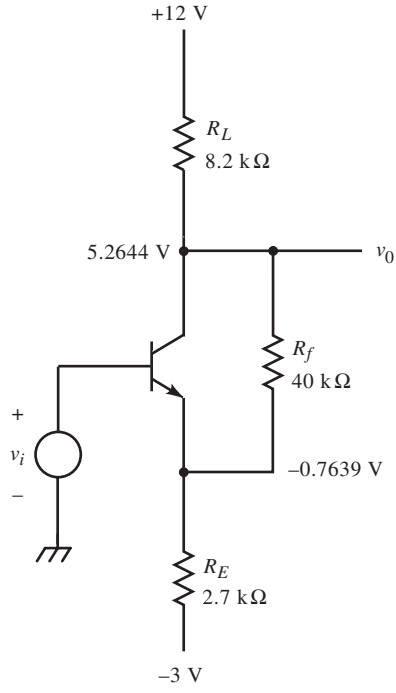
$$I_E = 0.6775\,\text{mA}, \quad V_E = -0.7639\,\text{V}, \quad V_C = 5.2644\,\text{V}$$

The simulated quasistatic solution is

$$\frac{v_c}{v_i} = -2.946, \quad r_{in} = 216.4\,\text{k}\Omega, \quad r_{out} = 8.160\,\text{k}\Omega$$

From these data, we can calculate the following:

$$r_e = 38.2\,\Omega, \quad r_m = 38.585\,\Omega, \quad r_\pi = 3.82\,\text{k}\Omega, \quad \mu + 1 = 1037.7$$



With $R_{CE} \to \infty$, $v_c/v_i \cong -2.96$. Applying Blackman's formula for r_{in}, we get $r_{in} = 216.40\ \text{k}\Omega$. Applying CE gain and resistance expressions gives:

$$\frac{v_c}{v_i} = (-0.58493)\cdot(5.0358) = -2.9456$$

$$r_{in} = (3.82\,\text{k}\Omega)\cdot(55.992) + (2.5568\,\text{k}\Omega) = 216.45\,\text{k}\Omega$$

$$r_{out} = 8.2\,\text{k}\Omega \| (1037.7)\cdot(1.5819\,\text{k}\Omega + 40\,\text{k}\Omega) = 8.1602\,\text{k}\Omega$$

Next, use feedback analysis to find a solution. Let $E = v_{be}$. Then the transmittances are

$$T_i = 1.4878\times10^{-2}, \quad G = -176.53, \quad H = 9.4079\times10^{-4}, \quad F = 0.17012$$

Then $1 + GH = 0.83392$ and

$$\frac{v_c}{v_i} = \frac{T_i G}{1+GH} + \frac{F}{1+GH} = -3.1496 + 0.20401 = -2.9456$$

$$r_{out} = \frac{R_C \| R_{CE}}{1+GH} = 8.1602\,\text{k}\Omega$$

Another feedback solution is based on a different choice of E,

$$E = v_i - \left(\frac{R_E}{R_E + R_{CE}} \right) \cdot v_c$$

The transmittances are

$$T_i = 1, \quad G = G_1 + G_2 = -2.6240 - 2.5312 \times 10^{-3}$$
$$= -2.6265$$
$$H = -6.3232 \times 10^{-2}, \quad F = 0.17012$$

Then $1 + GH = 0.83392$ and $v_c / v_i = -2.9456$. Furthermore,

$$r_{in} = (1+GH) \cdot (\beta + 1) \cdot \left(r_e + R_E \| \left(R_{CE} + R_C \right) \right) = 216.40\,\text{k}\Omega$$

$$r_{out} = \frac{R_C \| R_{CE}}{1+GH} = 8.1602\,\text{k}\Omega$$

These results are further confirmed by the μ transform:

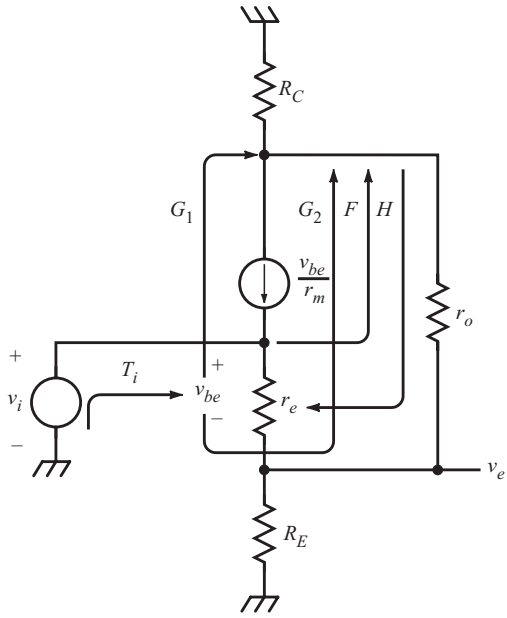
$$\frac{v_c}{v_i} = -2.9456,$$

$$r_{in} = 216.45\,\text{k}\Omega$$

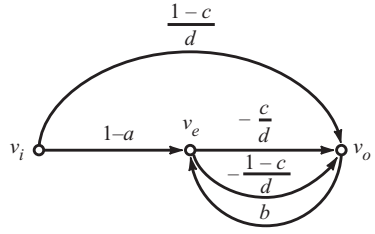
$$r_{out} = R_C \| \left[(\mu + 1) \cdot \left(r_\pi \| R_E \right) + R_{CE} \right] = 8.1602\,\text{k}\Omega$$

COMMON-COLLECTOR AMPLIFIER FEEDBACK ANALYSIS

The last BJT configuration is the common collector (CC), shown below. If we base the CC analysis on the results of the CE analysis, then E is chosen to be v_{be}. Combining



(a)



(b)

$$E = a \cdot v_i + b \cdot v_c$$

$$E = c \cdot v_i + d \cdot v_c \Rightarrow v_c = \left(\frac{1}{d}\right) \cdot E - \left(\frac{c}{d}\right) \cdot v_i$$

results in

$$E = v_{be} = a \cdot v_i + b \cdot \left[\left(\frac{1}{d}\right) \cdot E - \left(\frac{c}{d}\right) \cdot v_i\right] = \frac{a - (b \cdot c/d)}{1 - (b/d)} \cdot v_i$$

The CC output, v_e, is related to E by $v_e = v_i - E$. Dividing by v_i and substituting

$$\frac{v_e}{v_i} = 1 - \frac{E}{v_i} = 1 - \frac{a - (b \cdot c/d)}{1 - (b/d)} = \frac{(1-a) - (b/d)\cdot(1-c)}{1 - (b/d)}$$

The paths, identified on the topological model above correspond to paths of the CE flow graph. In (b), the v_e/v_i equation is represented as a flow graph with v_e made explicit. The paths are

$$1 - a = \frac{R_E \| r_o}{R_E \| r_o + r_e}; \quad -\frac{1}{d} = -\frac{c}{d} - \frac{(1-c)}{d} = \frac{R_C \| r_o}{r_m} + \frac{R_C}{R_C + r_o}$$

$$-b = \frac{R_E \| r_e}{R_E \| r_e + r_o}; \quad \frac{1-c}{d} = -\frac{R_C \| r_o}{r_m}$$

Inverting Op-Amp with Output Resistance

A feedback approach is now taken to the inverting operational amplifier (op-amp) with op-amp output resistance of R_o, shown below.

A nonzero R_o results in another forward path to v_o through R_f. Assume that the circuit can be represented by the flow graph in fig. (b), or as an equivalent block diagram in (c). Applying feedback analysis, let $E = v_-$. Then

$$v_1 = -K \cdot v_-$$

$$E = v_- = T_i \cdot v_i - H \cdot v_o$$

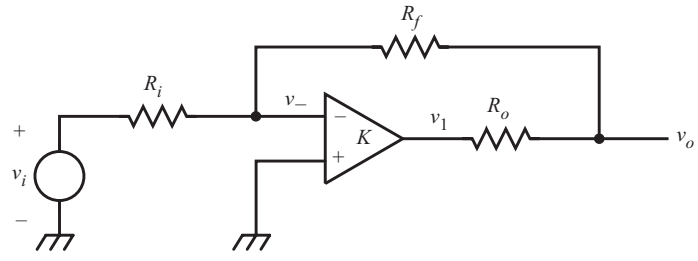
$$T_i = \frac{R_f + R_o}{R_f + R_i + R_o}$$

$$G = -K \cdot \left(\frac{R_f + R_i}{R_f + R_i + R_o} \right)$$

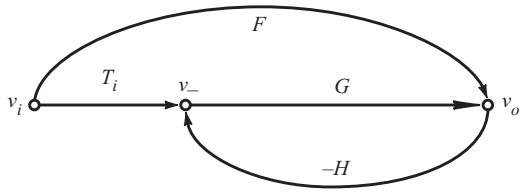
$$H = -\frac{R_i}{R_f + R_i}$$

The transfer function from fig. (b) is

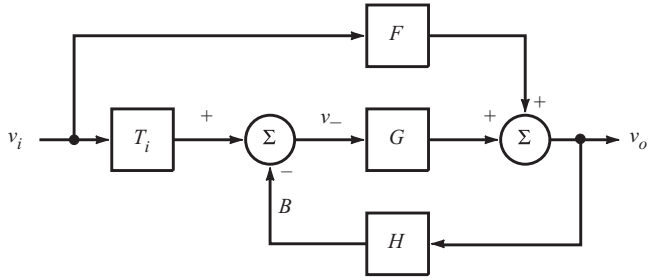
$$A_v = \frac{v_o}{v_i} = T_i \cdot \frac{G}{1 + GH} + F$$



(a)



(b)



(c)

The feedforward path F is determined as

$$F = \left.\frac{v_o}{v_i}\right|_{C=0} = \frac{R_o}{R_o + R_f + R_i}$$

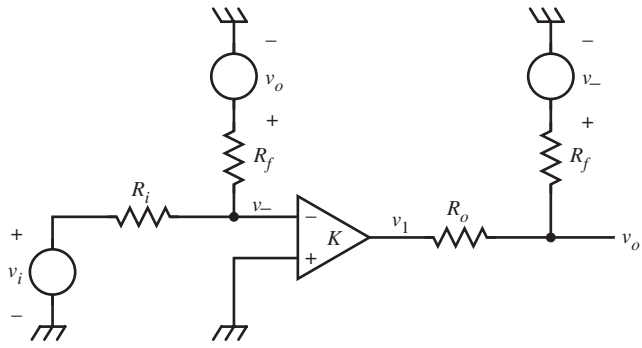
Substituting and solving, then simplifying, the closed-loop gain is

$$A_v = \left(\frac{R_f + R_i}{R_f + R_o + R_i}\right) \cdot \left(\frac{-K \cdot (R_f + R_o)}{R_f + R_o + (1+K) \cdot R_i}\right) + \frac{R_o}{R_o + R_f + R_i}$$

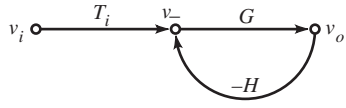
$$A_v = \frac{-K \cdot R_f + R_o}{R_f + R_o + (1+K) \cdot R_i}$$

Taking the limit of A_v as $K \to \infty$, the familiar $-R_f/R_i$ of the inverting op-amp results. This shows that R_o does not affect the closed-loop gain with sufficiently large K.

This op-amp circuit can also be used to demonstrate an alternative gain derivation based on a different flow graph, shown below, with its two-port equivalent circuit in (a) and flow graph in (b).



(a)



(b)

This choice of topology has no feedforward; instead, G has two parallel paths. H is configured as a two-port block, using the loading theorem. The transmittances, found by the feedback procedure, are

$$T_i = \frac{R_f}{R_f + R_i}$$

$$G = G_1 + G_2 = -K \cdot \left(\frac{R_f}{R_f + R_o} \right) + \left(\frac{R_o}{R_f + R_o} \right)$$

$$H = -\frac{R_i}{R_f + R_i}$$

Substituting these transmittances into the feedback formula,

$$\frac{v_o}{v_i} = T_i \cdot \frac{G}{1 + GH}$$

results in an equivalent equation for A_v. This flow graph gain expression can be verified by applying KCL at v_- and v_o. From the resulting equations, the three transmittances can be readily extracted.

We have examined in some detail the effects of r_o or similar shunt resistance around the active path of single transistors or amplifiers. Usually the contribution of the extra forward path is negligible, as for the preceding op-amp circuit. In precision amplifier stages such as the differential input stage of an op-amp, finite r_o can contribute significantly to imbalance in the collector (or drain) load resistance. A differential cascode input stage reduces this problem because the CB (or CG) output transistors have a maximum output resistance resulting from r_o.

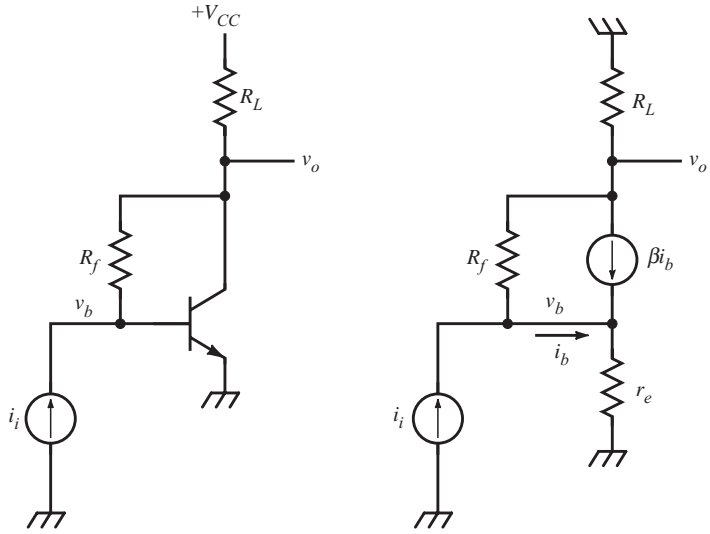
Another influence of a feedforward path is to cause the step response to begin with a momentary inversion before responding with the expected polarity of step. This is due to a faster passive forward path than the active inverting path. This dynamic phenomenon is called *preshoot*.

Feedback Analysis of the Shunt-Feedback Amplifier

The BJT shunt-feedback amplifier circuit is topologically similar to the inverting op-amp but has much less gain. It is shown below with its incremental model.

R_f shunts the transistor from collector to base. This is the third of three resistor shunting configurations for a transistor and is another basic kind of

amplifier stage. Unlike the base-emitter or collector-emitter (r_o) shunts, this one is quite useful. We analyze it from several perspectives.



This shunt-feedback amplifier is a transresistance amplifier. The input is a current i_i. Applying KCL at base and collector:

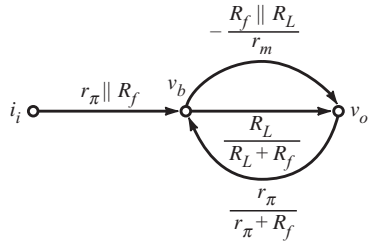
$$\text{at } b, \quad v_b = \left(r_\pi \| R_f\right)\cdot i_i + \left(\frac{r_\pi}{r_\pi + R_f}\right).v_o$$

$$at\ c, \quad v_o = \left[\frac{1}{R_f} - \frac{1}{r_m}\right]\cdot\left(R_L \| R_f\right)\cdot v_b = \left[\frac{R_L}{R_L + R_f} - \frac{R_f \| R_L}{r_m}\right]\cdot v_b$$

These equations can be expressed more compactly and directly as transmittances of the signal-flow diagram, shown below.

With $E = v_b$,

$$v_b = T_i \cdot i_i - H \cdot v_o, \quad v_o = \left(G_1 + G_2\right)\cdot v_b$$



The closed-loop gain is

$$\frac{v_o}{i_i} = T_i \cdot \frac{G}{1+GH} = -\frac{(r_\pi \| R_f)\cdot[R_L/(R_f+R_L)]\cdot[(R_f/r_m)-1]}{1+(r_\pi \| R_f)\cdot[R_L/(R_f+R_L)]\cdot[(R_f/r_m)-1]\cdot(1/R_f)}$$

This result can be simplified for a similar amplifier with R_i in shunt with i_i. This makes the input a Norton circuit, which can be converted to a Thevenin equivalent with

$$v_i = R_i \cdot i_i$$

resulting in a voltage amplifier. These equations are easily modified to account for R_i by replacing each occurrence of r_π with $r_\pi \| R_i$, since R_i shunts r_π.

Now consider some simplifications of the transresistance equation. If R_L is replaced by a current source,

$$\left.\frac{v_o}{i_i}\right|_{R_L \to \infty} = -\alpha \cdot R_f + r_e$$

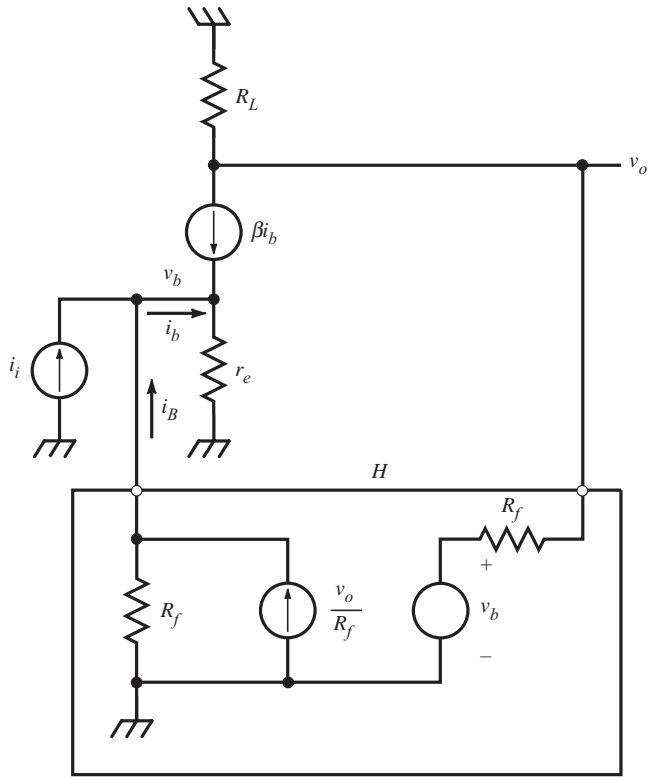
For BJT $\beta \to \infty$,

$$\left.\frac{v_o}{i_i}\right|_{\beta \to \infty} = (-R_f + r_e)\cdot\left(\frac{R_L}{R_L + r_e}\right) = -R_f \cdot \left(\frac{R_L}{R_L + r_e}\right) + r_e \| R_L$$

With both of the previous assumptions,

$$\left.\frac{v_o}{i_i}\right|_{R_L, \beta \to \infty} = -R_f + r_e$$

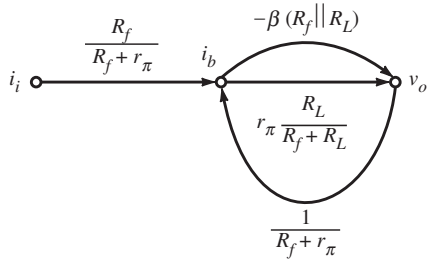
A two-port feedback analysis with error quantity $E = i_b$ is based on the following figure. H is identified as the two-port equivalent circuit shown as a block.



Here,

$$E = i_b = T_i \cdot i_i + \left(\frac{1}{R_f + r_\pi} \right) \cdot v_o$$

The transmittances can be derived from the previous calculations by shifting r_π forward, out of T_i in the above flow graph. The previous $E = v_b$ is then divided by r_π, making $E = i_b$. Then r_π multiplies G and divides H. The closed-loop gain is the same as the transresistance equation.



From inspection of the modified flow graph, T_i is now an input current divider:

$$T_i = \frac{R_f}{R_f + r_\pi} = \text{fraction of } i_i \text{ through } r_\pi\ (= i_b)$$

G has two paths, an active G_1 path and a passive G_2 path:

$$G = G_1 + G_2 = -\beta \cdot \left(R_f \| R_L\right) + r_\pi \cdot \left(\frac{R_L}{R_f + R_L}\right)$$

The first term is i_b, multiplied by β to become i_c. This current develops v_o across the collector resistance $R_f \| R_L$ and is negative. The second term is the passive path from the base through R_f to the collector. The H input source (see block diagram for H) is v_b. Now that E is i_b, then $i_b \cdot r_\pi$, or v_b, is divided by R_f and R_L. The second term G_2 is thus $(v_b/i_b) \cdot (v_o/v_b)$.

If $R_L << R_f$, then $G_2 \cong 0$ and, for $\beta \to \infty$,

$$\left.\frac{v_o}{i_i}\right|_{\beta \to \infty} \cong -R_f$$

This simple formula is the approximate transresistance for the single-BJT shunt-feedback amplifier for implementations in which R_f is large. The greatest error is usually due to finite β, causing r_π to excessively shunt R_f.

The input resistance is easy to find by using the feedback approach and is

$$r_{in} = \frac{v_b}{i_i} = \frac{r_\pi \| R_f}{1 + GH} = \frac{r_\pi \| R_f}{1 + \left[r_\pi / \left(r_\pi + R_f\right)\right] \cdot \left[R_L / \left(R_f + R_L\right)\right] \cdot \left[\left(R_f / r_m\right) - 1\right]}$$

This can be checked by resorting to the basic feedback equations for this circuit:

$$v_b = \left(r_\pi \| R_f\right) \cdot i_i - H \cdot v_o$$

$$v_o = G \cdot v_b$$

Then

$$v_b = \left(r_\pi \| R_f\right) \cdot i_i - G \cdot H \cdot v_b$$

Solving for v_b/i_i yields

$$r_{in} = \frac{r_\pi \| R_f}{1 + G \cdot H} = \frac{r_\pi \| R_f}{1 - G \cdot \left[r_\pi / \left(r_\pi + R_f\right)\right]} = r_\pi \left\| \left(\frac{R_f}{1 - G} \right) \right.$$

The last expression is cast in the form of Miller's theorem, showing that the theorem could have been applied to find r_{in}. R_f is across an amplifier of gain v_o/v_b and is reduced by $1/(1 - G)$ times its value. This effective resistance is shunted by r_π. If R_i is involved, r_{in} is easily modified by replacing r_π by the parallel combination, as for the complete transresistance equation.

A KCL solution for r_{out} follows from

$$i_o = \frac{v_o}{R_L} + \frac{v_o - v_b}{R_f} + \frac{v_b}{r_m}, \quad v_b = \left(\frac{r_\pi}{R_f + r_\pi} \right) \cdot v_o$$

and is v_o/i_o:

$$r_{out} = R_L \left\| \frac{R_f + r_\pi}{\beta + 1} \right.$$

For a current-source load, $R_L \to \infty$ and

$$r_{out}\big|_{R_L \to \infty} = \frac{R_f + r_\pi}{\beta + 1}$$

This result has a topological interpretation. A change in output voltage v_o causes a current in R_f that flows entirely into the base. This current is

$$i_b = \frac{v_o}{R_f + r_\pi}$$

The total current resulting from v_o is this current plus the collector current, or

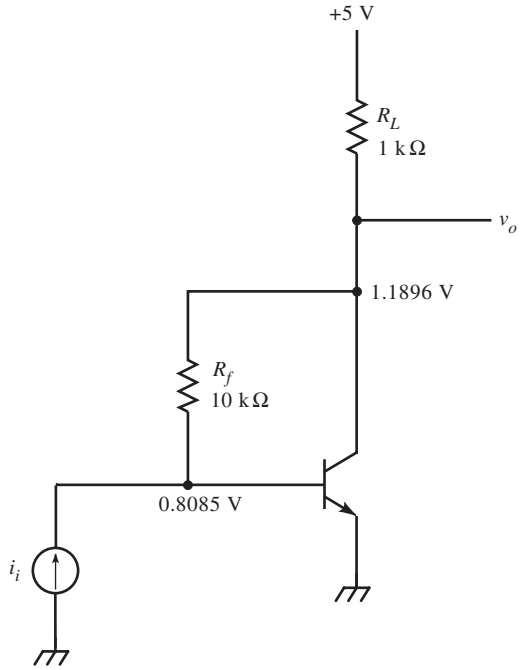
$$i_o = i_b + i_c = (\beta + 1) \cdot i_b = (\beta + 1) \cdot \frac{v_o}{R_f + r_\pi}$$

Solving for v_o/i_o, we again obtain the simplified equation for r_{out}.

Miller's theorem cannot be used to find r_{out} because v_o is not a voltage source; R_L is part of the internal resistance.

Example: Shunt-Feedback BJT Amplifier

The shunt-feedback amplifier shown below has $V_{BE} = 0.8085$ V, $I_E = 3.81$ mA, and $V_C = 1.1896$ V. (The default BJT model is used here: $\beta = 99$, $I_S = 10^{-16}$ A.)



Then $r_e = 6.79\,\Omega$, $r_m = 6.858\,\Omega$, and $r_\pi = 678.92\,\Omega$. Feedback analysis, with $E = v_{be}$, yields

$$G = -\frac{909.09\,\Omega}{6.858\,\Omega} + 9.0909 \times 10^{-2} = -132.47$$

$$H = -\frac{r_\pi}{r_\pi + R_f} = -6.3576 \times 10^{-2}$$

so that $1 + GH = 9.4220$ and, from the transresistance equation,

$$\frac{v_o}{i_i} = -8.9386\,\text{k}\Omega$$

The resistances are

$$r_{out} = \frac{R_L \| R_f}{1 + GH} = 96.486\,\Omega$$

Alternatively, from the simplified r_{out} equation,

$$r_{out} = R_L \left\| \left(\frac{R_f + r_\pi}{\beta + 1} \right) \right. = 96.486\,\Omega$$

Finally,

$$r_{in} = r_\pi \left\| \left(\frac{R_f}{1 - G} \right) \right. = 67.476\,\Omega$$

To check these results, the SPICE simulation produced

$$\frac{v_o}{i_i} = -8.939\,\text{k}\Omega, \quad r_{in} = 67.44\,\Omega, \quad r_{out} = 96.48\,\Omega$$

Shunt-Feedback Amplifier Substitution Theorem Analysis

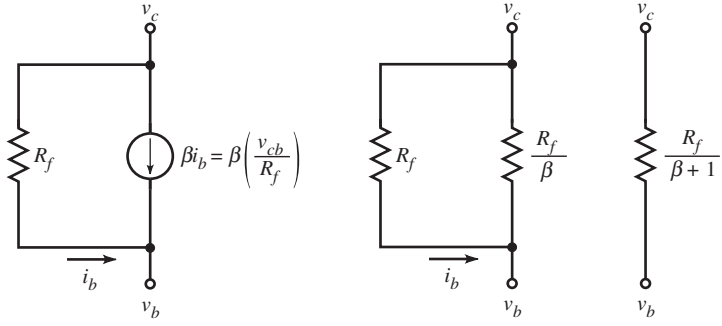
Another way to find shunt-feedback amplifier output resistance is to use the substitution theorem to find the equivalent resistance of the collector path. With

i_c controlled by v_o, it can be expressed as resistance r_c. Keeping in mind the μ transform,

$$r_c = \frac{r_m}{(v_b/v_o)} = \frac{r_m}{\left(r_\pi/(R_f + r_\pi)\right)} = \frac{R_f + r_\pi}{\beta}$$

The resistance shunting r_c through R_f is $R_f + r_\pi$. These parallel resistances combine to result in the r_{out} equation. In the previous equation, for feedback analysis, v_b/v_o is $-H$ instead of the reverse transmittance through G because i_i is nulled.

The substitution theorem can be applied in a more general way to the combination of R_f and the $\beta\, i_b$ current source, as shown below.



Starting from

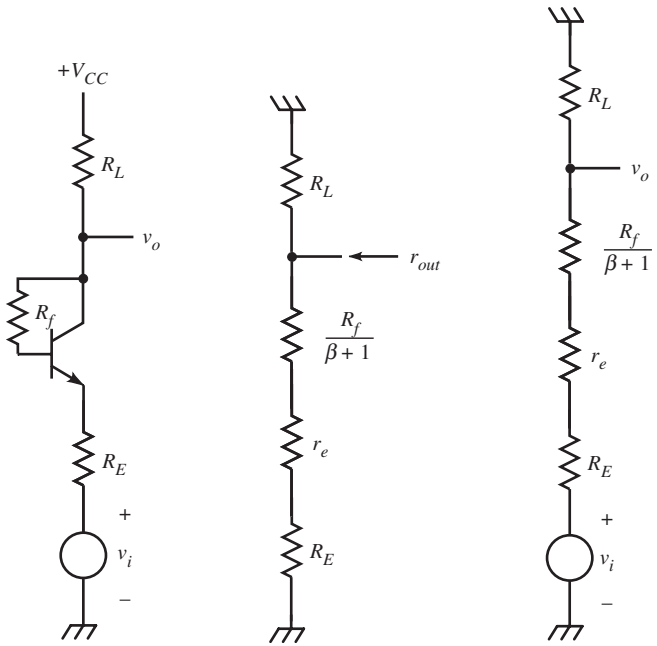
$$i_b = \frac{v_c - v_b}{R_f} = \frac{v_{cb}}{R_f}$$

the current source becomes $\beta \cdot v_{cb}/R_f$. The source is across v_{cb} and is also dependent on it, making the substitution theorem applicable. In the middle figure above, the current source is replaced by a resistance of R_f/β. When this is combined with R_f, the result is a single resistance of $R_f/(\beta + 1)$.

This equivalent circuit makes shunt-feedback amplifier resistance analysis much simpler than feedback analysis. The following figure shows the shunt-feedback amplifier equivalent circuit used to find r_{out}. The circuit has been generalized slightly by including external emitter resistance R_E. This resistance

always adds to r_e and can be lumped with it. With this equivalent circuit, output resistance reduces to divider formulas and parallel resistances:

$$r_{out} = R_L \left\| \left(\frac{R_f}{\beta + 1} + R_E + r_e \right) \right.$$



Input resistance cannot be found as easily because i_i is injected at the base. The resulting i_b is not from the R_f branch alone as assumed in the equivalent circuit.

Another direct application of the shunt-feedback equivalent circuit is to find r_{in} of the emitter-driven shunt-feedback amplifier, shown in the left figure above and modeled on the right. Here, $i_f = i_b$, and the shunt-feedback equivalent is exact. The input and output resistances and voltage gain of this amplifier are

$$r_{in} = R_L + \frac{R_f}{\beta + 1} + r_e + R_E$$

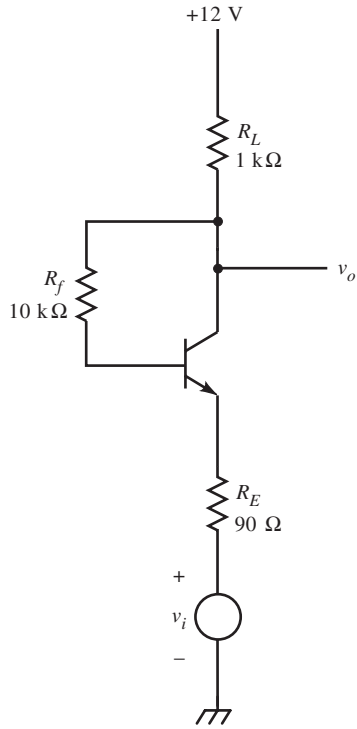
$$r_{out} = R_L \left\| \left[\frac{R_f}{\beta + 1} + r_e + R_E \right] \right.$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_f/(\beta + 1) + r_e + R_E}$$

This amplifier is of limited use for voltage amplification; it has a voltage gain of less than 1. It can be used as a voltage translator for meeting biasing conditions.

Example: CB Shunt-Feedback Amplifier

The example circuit is shown below.



For $\beta = 99$ and $I_S = 10^{-16}$ A, from SPICE,

$$V_E = 0.1671\,\text{V}, \quad V_B = 0.9571\,\text{V}, \quad V_C = 1.1428\,\text{V}, \quad I_E = 1.86\,\text{mA}$$

Then $r_e = 13.91\ \Omega$ and

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + R_f/(\beta+1) + r_e + R_E} = 0.83063$$

$$r_{in} = R_L + \frac{R_f}{\beta+1} + r_e + R_E = 1.2039\,\text{k}\Omega$$

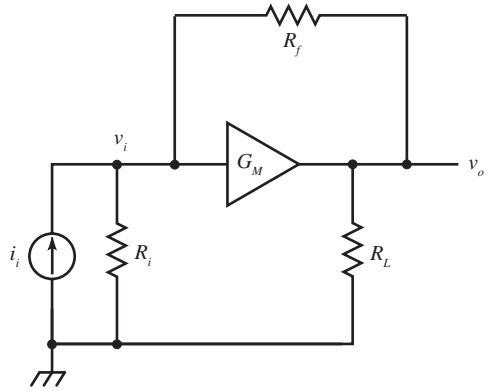
$$r_{out} = R_L \left\| \left[\frac{R_f}{\beta+1} + r_e + R_E \right] \right. = 169.37\,\Omega$$

The SPICE values for these parameters are

$$\frac{v_o}{v_i} = 0.8306, \quad r_{in} = 1.204\,\text{k}\Omega, \quad r_{out} = 169.4\,\Omega$$

IDEALIZED SHUNT-FEEDBACK AMPLIFIER

A more general form of shunt-feedback amplifier is shown here.

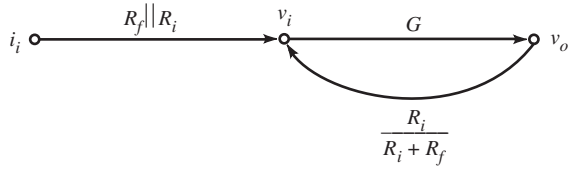


An ideal transconductance amplifier replaces the BJT, with source resistance R_i included. Solving for the usual incremental quantities using feedback analysis based on the flow graph, the resulting transmittances are

$$T_i = R_f \| R_i$$

$$G = \frac{R_f \| R_L}{(1/G_M)} + \frac{R_L}{R_f + R_L}$$

$$H = -\frac{R_i}{R_f + R_i}$$



Then if we combine these transmittances, the transresistance is

$$\frac{v_o}{i_i} = T_i \cdot \frac{G}{1 + GH} = (R_f \| R_i) \cdot \frac{\dfrac{R_f \| R_L}{R_f \| (1/G_M)}}{1 - \dfrac{R_f \| R_L}{R_f \| (1/G_M)} \cdot \dfrac{R_i}{R_f + R_i}}$$

Removing R_L simplifies this transresistance equation somewhat:

$$\left.\frac{v_o}{i_i}\right|_{R_L \to \infty} = (R_f \| R_i) \cdot \frac{\dfrac{R_f + 1/G_M}{1/G_M}}{1 - \dfrac{R_f + 1/G_M}{1/G_M} \cdot \dfrac{R_i}{R_f + R_i}}$$

If R_i is removed instead, then

$$\left.\frac{v_o}{i_i}\right|_{R_i \to \infty} = R_f \cdot \frac{\dfrac{R_f \| R_L}{R_f \| (1/G_M)}}{1 - \dfrac{R_f \| R_L}{R_f \| (1/G_M)}}$$

Finally, when both R_L and R_i are removed, the transresistance reduces to the simple form of

$$\left.\frac{v_o}{i_i}\right|_{R_i,R_L\to\infty} = -\left(R_f + \frac{1}{G_M}\right)$$

This expression is similar to the simplified BJT shunt-feedback transresistance. When R_i is set to r_π, $1/G_M$ to r_m, and the sign of G_M made negative, then the transconductance amplifier is equivalent to the BJT shunt-feedback amplifier.

The input resistance can be found from the feedback equations:

$$v_i = T_i \cdot i_i - H \cdot v_o, \quad v_o = G \cdot v_i$$

Substituting v_o from the second equation into the first and solving,

$$r_{in} = \frac{v_i}{i_i} = \frac{T_i}{1 + G \cdot H} = \frac{R_f \| R_i}{1 + G \cdot H}$$

An alternative solution that uses recursion begins with the first feedback equation. It is divided by i_i. Then,

$$\begin{aligned} r_{in} = \frac{v_i}{i_i} &= T_i - H \cdot \left(\frac{v_o}{i_i}\right) = T_i - H \cdot \frac{v_o}{v_i} \cdot \frac{v_i}{i_i} \\ &= T_i - G \cdot H \cdot r_{in} = \frac{T_i}{1 + G \cdot H} \end{aligned}$$

Writing H in terms of circuit component values, we can reformulate r_{in} as

$$r_{in} = \frac{R_f \| R_i}{1 - G \cdot \left[R_i / \left(R_f + R_i\right)\right]} = \frac{R_i \cdot R_f}{(1 - G) \cdot R_i + R_f} = R_i \left\| \left(\frac{R_f}{1 - G}\right)\right.$$

This result suggests that we can apply Miller's theorem as an alternative approach to finding r_{in}.

Output resistance is derived by applying KCL to the output node,

$$i_o = \frac{v_o}{R_L} + \frac{v_o - v_i}{R_f} - G_M \cdot v_i, \quad v_i = \left(\frac{R_i}{R_f + R_i}\right) \cdot v_o$$

Solving for v_o/i_o gives

$$r_{out} = R_L \left\| \frac{R_f + R_i}{1 - R_i/(1/G_M)} \right.$$

This form of r_{out} is similar to the simplified BJT equation when $R_i \to r_\pi$ and $1/G_M \to -r_m$. (G_M is positive as defined in the above figure of the idealized shunt-feedback amplifier.)

Because the feedback output quantity v_o is across r_{out}, feedback analysis applies directly, resulting in

$$r_{out} = \frac{R_f \| R_L}{1 + GH}$$

Finally, for a voltage-amplifier variation, the Norton equivalent circuit formed by i_i and R_i can be Thevenized so that $v_i = i_i R_i$. The voltage gain is then v_o/v_i. This transformation of the idealized amplifier is easy to make by changing T_i and the input node of the flow graph. The new transmittance is

$$T_i = \frac{R_i}{R_f + R_i}; \quad \text{input node is } v_i$$

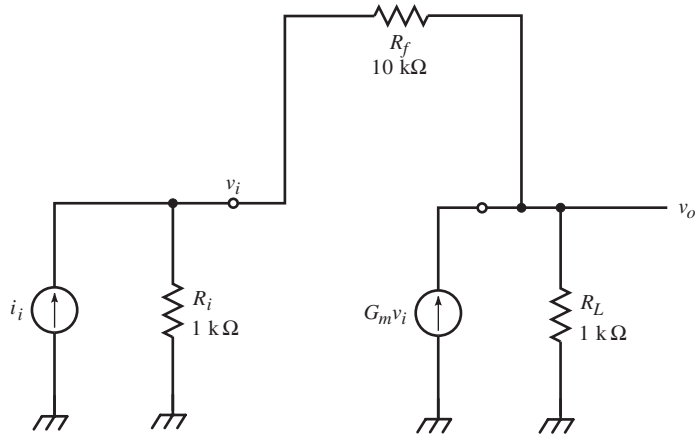
The voltage gain is

$$\frac{v_o}{v_i} = \frac{(v_o/i_i)_{ideal}}{R_i} = \frac{R_f}{R_f + R_i} \cdot \frac{\dfrac{R_f \| R_L}{R_f \| (1/G_M)}}{1 - \dfrac{R_f \| R_L}{R_f \| (1/G_M)} \cdot \dfrac{R_i}{R_f + R_i}}$$

where $(v_o/i_i)_{ideal}$ is the ideal shunt-feedback amplifier transconductance.

Example: Transconductance Amplifier

The figure shows a transconductance amplifier with a forward-path transconductance of



$$G_M = -10\,\text{mS} = -1/100\,\Omega$$

The SPICE circuit simulation results are

$$\frac{v_o}{v_i} = -4.500, \quad r_{in} = 500.0\,\Omega, \quad r_{out} = 500.0\,\Omega$$

As a curiosity, for $1/G_M = +100\ \Omega$ instead, this positive-feedback amplifier has

$$\frac{v_o}{v_i} = 50.50, \quad r_{in} = 5.500\,\text{k}\Omega, \quad r_{out} = 5.500\,\text{k}\Omega$$

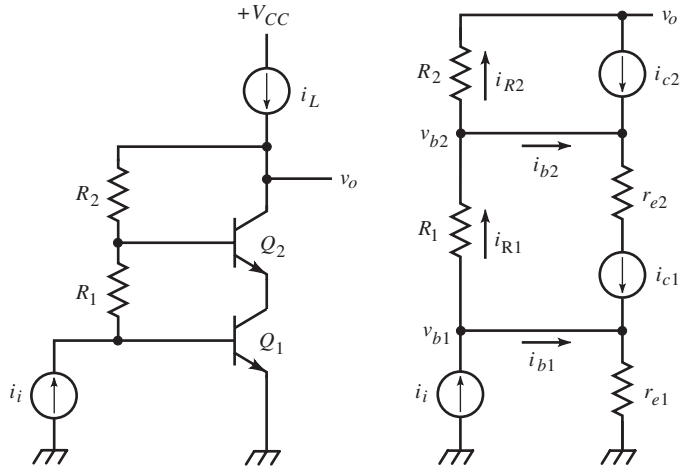
These results agree with those obtained from the previously derived equations for the ideal shunt-feedback amplifier.

Cascode and Differential Shunt-Feedback Amplifiers

The basic shunt-feedback amplifier can be combined with other elemental circuits such as the cascode or differential amplifiers. Shown below is a shunt-feedback cascode amplifier (a) with current-source load, modeled in (b).

First, i_{c2} must flow through R_2. Thus,

$$i_{R1} = i_{e2} = i_{c1}$$



and

$$i_i = i_{b1} + i_{R1} = i_{b1} + i_{c1} = (\beta_1 + 1)\cdot i_{b1} = (\beta_1 + 1)\cdot \frac{v_{b1}}{r_{\pi 1}}$$

Then

$$v_{b1} = r_{e1}\cdot i_i$$

The output voltage is

$$v_o = v_{b1} - i_{c1}\cdot R_1 - i_{c2}\cdot R_2$$

Substituting,

$$v_o = r_{e1}\cdot i_i - i_{c1}\cdot R_1 - \alpha_2\cdot i_{c1}\cdot R_2 = r_{e1}\cdot i_i - \alpha_1\cdot i_i\cdot\left(R_1 + \alpha_2\cdot R_2\right)$$

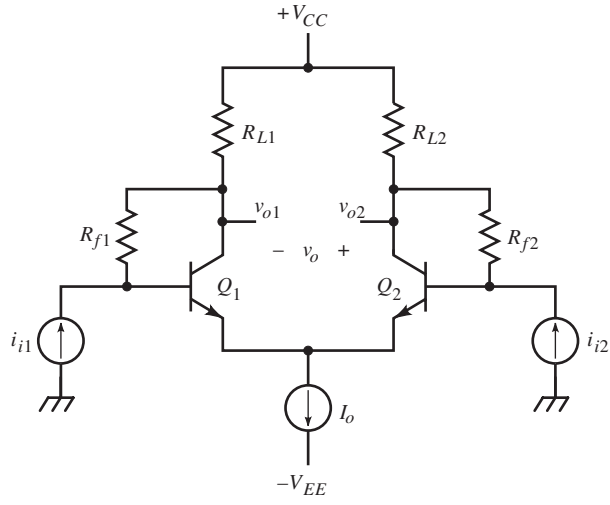
The transresistance is

$$\frac{v_o}{i_i} = -\alpha_1\cdot R_1 - \alpha_1\cdot\alpha_2\cdot R_2 + r_{e1}$$

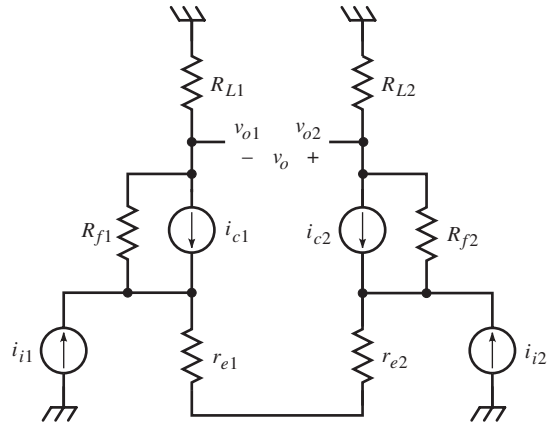
This result is similar to the single-BJT shunt-feedback amplifier transresistance, where R_f corresponds approximately to $R_1 + R_2$. Because i_{c2} loses current

to both Q_1 and Q_2 bases, both α_1 and α_2 are factors of R_2 in the previous transresistance expression.

A differential shunt-feedback amplifier and incremental model is shown below. The results of previous shunt-feedback input-resistance analysis can be applied in finding the equivalent emitter resistance of one side of the differential pair due to the other side. With one input open, the other input appears as a high resistance, but when both inputs are driven differentially, the emitter currents caused by each input are equal and opposite for a symmetrical circuit.



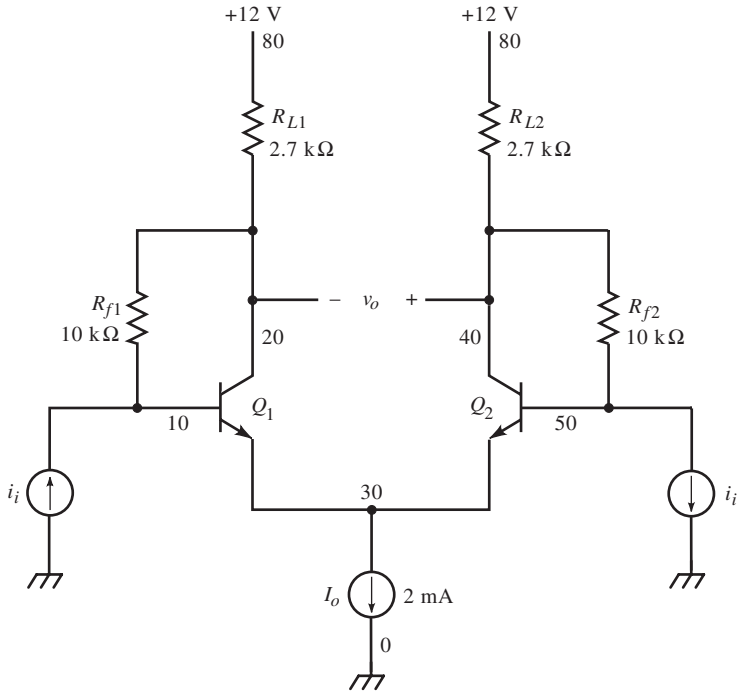
(a)



(b)

Example: Differential Shunt–Feedback Amplifier

Using the default BJT model, the amplifier shown below has the (reduced) SPICE program and output. The transmittance follows directly from substitution of circuit element values into the shunt-feedback gain equation and doubling the result to account for both sides of the circuit.




```
Shunt Differential Feedback Amplifier

.OPT NOMOD OPTS NOPAGE
.OP

.DC II -0.10mA 0.10mA 10uA
.TF V(40,20) II

VCC 80 0 DC 12V
IO 30 0 DC 2mA
II 50 10 DC 0A
RL1 80 20 2.7K
```

```
RL2  80  40  2.7K
RF1  20  10  10K
RF2  40  50  10K
Q1  20  10  30  BJT1
Q2  40  50  30  BJT1

.MODEL BJT1 NPN (BF=99)
.END

NODE VOLTAGE
(10) 9.2000 (20) 9.3000 (30) 8.4261
(40) 9.3000 (50) 9.2000

OPERATING POINT INFORMATION

TEMPERATURE = 27.000 DEG C

BIPOLAR JUNCTION TRANSISTORS
NAME Q1 Q2
MODEL BJT1 BJT1

IB 1.00E-05 1.00E-05
IC 9.90E-04 9.90E-04
VBE 7.74E-01 7.74E-01
VBC -1.00E-01 -1.00E-01
VCE 8.74E-01 8.74E-01

BETADC 9.90E+01 9.90E+01
GM 3.83E-02 3.83E-02

V(40,20)/II = 1.887E+04

INPUT RESISTANCE AT II = 2.325E+02
OUTPUT RESISTANCE AT V(40,20) = 2.405E+02
```

BLACKMAN'S RESISTANCE FORMULA

The effects of feedback on circuit resistances were introduced in "Feedback Effects on Input and Output Resistance." These resistance techniques are generally applicable to feedback circuits but have limitations. In previous sections, it was difficult to derive r_{in} for the CB, and no attempt was made for the CE and common collector (CC). Miller's theorem provides r_{out}, but under the condition that the amplifier output be a voltage source (no resistance). It has sometimes even been necessary to resort to Kirchhoff's laws. Feedback analysis, applied to

find transmittances, is not always useful in finding circuit resistances for several reasons.

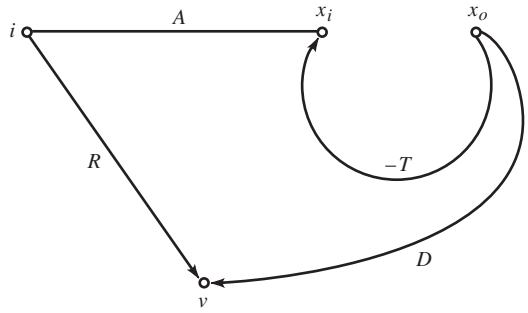
First, feedback results can be applied directly to determine the closed-loop resistance across an error voltage or in series with an error current. But these error quantities are often not associated with the resistances of interest. In the three basic BJT configurations, input and output resistances were not directly associated with the error quantity. For example, for the CE and CC, for $E = v_{be}$, we can immediately determine that r_e becomes effectively $r_e \cdot (1 + G \cdot H)$ with feedback. But r_{in} involves r_e in series with the rest of the emitter circuit. How is its resistance affected by feedback? It is usually not obvious.

Second, most of the equivalent-circuit methods used to find transmittances do not preserve circuit resistances. Two-port, Thevenin, and Norton equivalent circuits and divider formulas do not preserve resistances. For example, a voltage divider consisting of two 1 kΩ resistors has a transmittance of 0.5, $r_{in} = 2$ kΩ, and $r_{out} = 500$ Ω (when the input is driven by a voltage source). But $r_{out} \neq 0.5 \cdot r_{in}$; the transmittances do not apply to resistances as they do to voltages or currents.

The reduction theorem is resistance-preserving. Consequently, we are able to apply the β transform directly to circuit topology to find resistances. The same was done with the μ transform. The reduction theorem reduces circuits to a form that makes resistances available by topological inspection, by appealing to causal and topological reasoning, or intuition. Therefore, feedback analysis is usually not a good approach for finding resistance, whereas the reduction theorem is. Miller's theorem can also be used, but like feedback analysis, it is limited in its application. No single method is generally best; judgment is required, based on the particular circuit and what aspects of it are desired to be made explicit.

An early method for finding resistances was published by R. B. Blackman in 1943, but it lay dormant for decades and is found in few circuits textbooks. In more recent years, R. D. Middlebrook of Cal Tech and Sol Rosenstark of the New Jersey Institute of Technology have been reviving it. Blackman developed a simple formula for calculating resistances in feedback circuits that is also based on inspection of the topology. A feature of this method is that only loop gain is needed; no decisions about input or output feedback quantities are required. Another advantage is that it can use loop-gain results from feedback analysis. Its disadvantage is that it is not as intuitive to use as the reduction theorem

because the answer results from substituting these inspected values into a formula that, in itself, is not easily envisioned in terms of circuit topology. But it is easy to apply and minimizes calculation. Consequently, it is developed here.



The figure represents a feedback circuit with loop gain $-T$ and a port with terminal voltage v driven by a current source i. We want to find the closed-loop resistance at this port. Within the feedback loop, choose a convenient point where it can be opened so that two flow-graph nodes, x_i and x_o, are created. We have done this before when finding $G \cdot H$; the gain from E through G and H back to E again (or $-B/E = G \cdot H$) is represented here by $-T = -x_i/x_o$. To derive Blackman's formula, it is not necessary to choose E, but instead to pick a point within the loop where it can be opened so that loop gain under different conditions can be derived. The x quantities can be either voltages or currents somewhere in the loop. The simplicity of this approach is that G, H, and E need not be identified, only loop gains.

The flow graph can be expressed algebraically as

$$x_i = A \cdot i - T \cdot x_o$$

$$v = R \cdot i + D \cdot x_o$$

For a closed-loop amplifier, $x_o = x_i = x$, and

$$x = A \cdot i - T \cdot x = \frac{A}{1+T} \cdot i, \quad v = R \cdot i + D \cdot \left[\frac{A}{1+T} \right] \cdot i$$

Solving for closed-loop terminal resistance,

$$\left.\frac{v}{i}\right|_{cl} = \left.\frac{v}{i}\right|_{x_o=x_i} = R + \frac{D\cdot A}{1+T} = R\cdot\frac{1+\left[T+\left(D\cdot A/R\right)\right]}{1+T}$$

This is the resistance we are seeking, but to use it in this form requires that we know the transmittances it contains. This can be an onerous task. An ingenious simplification is made by finding the topological meaning of the sub-expressions in the closed-loop resistance.

Let us find the open-loop resistance. To obtain an open-loop circuit, set $x_o = 0$. Then from the equation for v,

$$r_{ol} = \left.\frac{v}{i}\right|_{ol} = \left.\frac{v}{i}\right|_{x_o=0} = R$$

From this, we know that R in the closed-loop resistance is the terminal resistance when the loop is opened.

Next, consider the expressions for loop gain that result from both open- and short-circuiting the port. For an open-circuited port, $i = 0$. Substituting for i in the flow-graph equations and solving for loop gain,

$$T_{oc} = \left.\frac{x_i}{x_o}\right|_{oc} = \left.\frac{x_i}{x_o}\right|_{i=0} = -T$$

The denominator of the closed-loop resistance expression can be expressed in terms of the open-circuit loop gain as $1 - T_{oc}$. Finally, for the short-circuit loop gain, set $v = 0$ and solve for T_{sc} from the flow-graph equations:

$$T_{sc} = \left.\frac{x_i}{x_o}\right|_{sc} = \left.\frac{x_i}{x_o}\right|_{v=0} = -\left[\frac{A\cdot D}{R} + T\right]$$

Interestingly enough, this matches the numerator of the closed-loop resistance, and can be expressed as $1 - T_{sc}$. When these expressions are substituted into the closed-loop resistance expression, *Blackman's resistance formula* results.

Blackman's Resistance Formula

$$r_{cl} = r_{ol} \cdot \frac{1 - T_{sc}}{1 - T_{oc}}$$

To find closed-loop resistance at an arbitrary port within the loop of a feedback amplifier:

1. Open the feedback loop and find the port resistance r_{ol}.
2. Open the port and find the closed-loop gain T_{oc}.
3. Short the port and find the closed-loop gain T_{sc}.
4. Substitute these results into Blackman's formula for r_{cl}.

Blackman's formula can be applied to feedback amplifiers to find input and output resistances. For the pathological cases of the CE and CC with r_o, r_{in} is easily found. From the input port, the topology of the CC is the same as the CE and r_{in} is identical:

$$r_{in} = (\beta + 1) \cdot \left[r_e + R_E \| (r_o + R_C) \right] \cdot \frac{1 - (b/d)}{1 - 0}$$

or

$$\text{CE, CC} \quad r_{in} = (\beta + 1) \cdot \left[r_e + R_E \| (r_o + R_C) \right] \cdot \left[1 - \left(\frac{R_E \| r_e}{R_E \| r_e + r_o} \right) \cdot \left(\frac{R_C \| r_o}{r_m \| r_o} \right) \right]$$

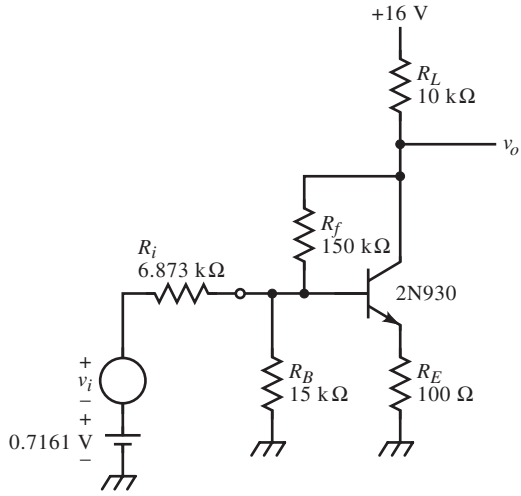
Finally, r_{in} for the CB with r_o is:

$$\text{CB} \quad r_{in} = \left[R_E + r_e \| (r_o + R_L) \right] \cdot \frac{1 - b/d}{1 - (1/d) \cdot \left[r_e / (r_e + r_o) \right]}$$

For the CB, neither T_{sc} nor T_{oc} is zero.

Example: Shunt-Feedback Voltage Amplifier

The analysis of the shunt-feedback amplifier shown below follows from shunt-feedback amplifier analysis and demonstrates the application of Blackman's formula. It is the output stage of a subsequent audiotape playback amplifier example.



The 2N930 BJT is used here with $\beta = 170$ and $I_S = 10^{-14}$ A. The static analysis yields

$$I_E = 411\,\mu\text{A}, \quad V_{BE} = 0.646\,\text{V}, \quad V_C = 8.4941\,\text{V}, \quad V_E = 70.3\,\text{mV}$$

The rather exacting value of R_i happens to be the output resistance of the previous stage, an inverting feedback amplifier. (In the audiotape playback amplifier, they are connected.) This R_i accounts for interstage loading. The feedback calculations are based on $E = v_b$, and the shunt-feedback amplifier formulas apply directly:

$$r_M = \frac{r_e + R_E}{\alpha} = r_m + \frac{R_E}{\alpha} = 37\,\Omega + \frac{100\,\Omega}{0.99415} = 137.63\,\Omega$$

$$G = -\left(\frac{10\,\text{k}\Omega}{160\,\text{k}\Omega}\right)\cdot\left(\frac{150\,\text{k}\Omega}{137.63\,\Omega} - 1\right) = -68.057$$

Let

$$r_s = R_i \| R_B \| (\beta + 1)\cdot(r_e + R_E) = 6.873\,\text{k}\Omega \| 9.1400\,\text{k}\Omega = 3.9230\,\text{k}\Omega$$

Then

$$H = -\frac{r_s}{r_s + R_f} = -\frac{3.9230\,\text{k}\Omega}{3.9230\,\text{k}\Omega + 150\,\text{k}\Omega} = -2.5487 \times 10^{-2}$$

From this, $1 + GH = 2.7346$ and $r_s \| R_f = 3.8230\ \text{k}\Omega$. Then,

$$\frac{v_o}{v_i} = \left(\frac{r_e \| R_f}{R_i}\right) \cdot \left(\frac{G}{1 + GH}\right) = \left(\frac{3.8230\,\text{k}\Omega}{6.873\,\text{k}\Omega}\right) \cdot (-24.888) = -13.843$$

Using Blackman's formula for r_{in},

$$r_{in} = \left[R_i + R_B \| (\beta + 1) \cdot (r_e + R_E) \| R_f\right] \cdot \left(\frac{1 + (GH)|_{sc}}{1 + (GH)|_{oc}}\right)$$

$$= 15.488\,\text{k}\Omega \cdot \left(\frac{2.7346}{4.9088}\right) = 8.6280\,\text{k}\Omega$$

For

$$(GH)|_{oc} = (GH)|_{i_i = 0}$$

G is the same as before, but H is

$$H|_{oc} = -\frac{9.1400\,\text{k}\Omega}{9.1400\,\text{k}\Omega + 150\,\text{k}\Omega} = -5.7434 \times 10^{-2}$$

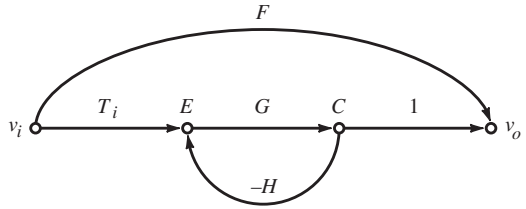
The output resistance is

$$r_{out} = \frac{R_L \| R_f}{1 + GH} = \frac{9.3750\,\text{k}\Omega}{2.7346} = 3.4283\,\text{k}\Omega$$

THE ASYMPTOTIC GAIN METHOD

In close connection with Blackman's resistance method is the *asymptotic gain method* for finding feedback-circuit gain. Middlebrook developed it extensively. Here the equivalence of the asymptotic gain method and the signal flow-graph feedback method we have been using are made clear.

Consider the isolated feedback topology, shown below, with feedforward to the output.



From previously developed multipath feedback theory,

$$\frac{x_o}{x_i} = \frac{T_i \cdot G}{1 + G \cdot H} + F$$

where the path transmittances are

$$T_i = \left.\frac{E}{x_i}\right|_{-H \cdot C = 0}, \quad G = \left.\frac{x_o}{E}\right|_{F \cdot x_i = 0}, \quad -H = \left.\frac{E}{C}\right|_{T_i \cdot x_i = 0}, \quad F = \left.\frac{x_o}{x_i}\right|_{C=0}$$

The asymptotic gain formula is

Asymptotic Gain Formula

$$\frac{x_o}{x_i} = G_\infty \cdot \frac{T}{1+T} + G_0 \cdot \frac{1}{1+T}$$

where

$$G_\infty = \left.\frac{T_i \cdot G}{1 + G \cdot H}\right|_{G \cdot H \to \infty} + F = \frac{T_i}{H} + F = \frac{T_i \cdot G}{T} + F$$

$$T = G \cdot H$$

$$G_0 = F$$

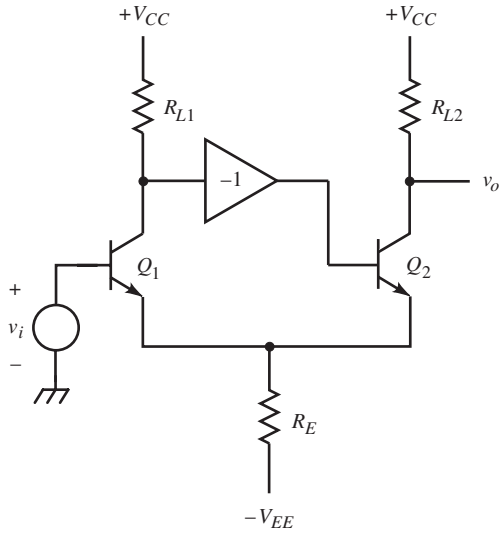
Substituting for G_∞ and G_0 into the asymptotic gain formula,

$$\frac{x_o}{x_i} = \left(\frac{T_i \cdot G}{T} + F\right) \cdot \frac{T}{1+T} + F \cdot \frac{1}{1+T} = \frac{T_i \cdot G}{1 + G \cdot H} + F$$

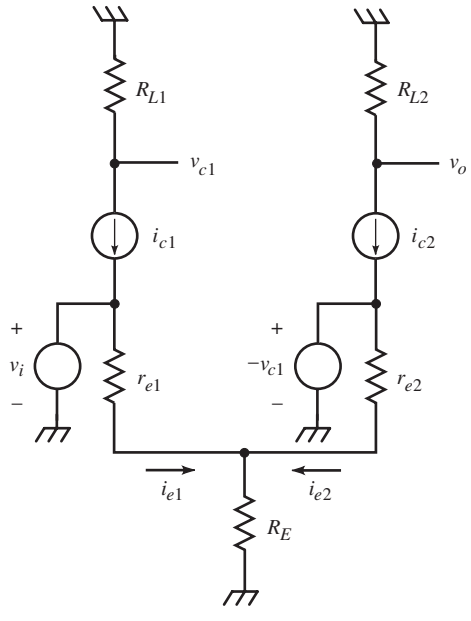
The significance of the asymptotic gain formula is that it reformulates the isolated feedforward gain in a form that makes another method explicit. By finding G_∞ and G_0 from circuit maneuvers, the results are substituted into the asymptotic gain formula. The method is similar to Blackman's formula: Find some circuit quantities by imposing constraints on the circuit, and then substitute these results into a simple formula. G_∞ is x_o/x_i with infinite loop gain. This is not unfamiliar; we analyzed what happens to op-amp circuits when op-amp gain becomes infinite. Discrete transistor amplifiers can be analyzed similarly; the result is the feedforward path added to $T_i \cdot (1/H)$. G_0 is x_o/x_i with zero loop gain, which is the feedforward path F.

EMITTER-COUPLED FEEDBACK AMPLIFIER

A common multipath topology involving both cascade and emitter coupling is shown below with small-signal equivalent circuit shown in (b), two-port equivalent for H in (c), and flow graph in (d).



(a)



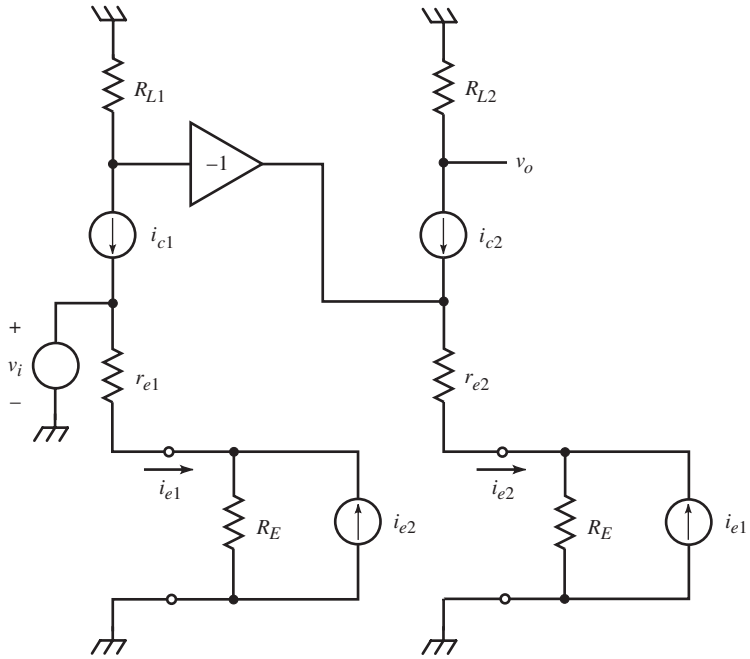
(b)

Let $E = i_{e1}$. The error-summing node is at the output port of H. $F = 0$, because all paths from v_i to v_o are along either G_1 (cascade path) or G_2 (emitter-coupled path). The transmittances are

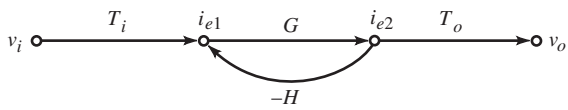
$$T_i = \left.\frac{i_{e1}}{v_i}\right|_{i_{e2}=0} = \frac{1}{r_{e1} + R_E}, \quad T_o = \frac{v_o}{i_{e2}} = -\alpha_2 \cdot R_{L2}$$

$$G = \left.\frac{i_{e2}}{i_{e1}}\right|_{Hi_{e2}} = G_1 + G_2 = \frac{\alpha_1 \cdot R_{L1}}{r_{e2} + R_E} - \frac{R_E}{r_{e2} + R_E}$$

$$H = -\left.\frac{i_{e1}}{i_{e2}}\right|_{v_i=0} = \frac{R_E}{r_{e1} + R_E}$$



(c)



(d)

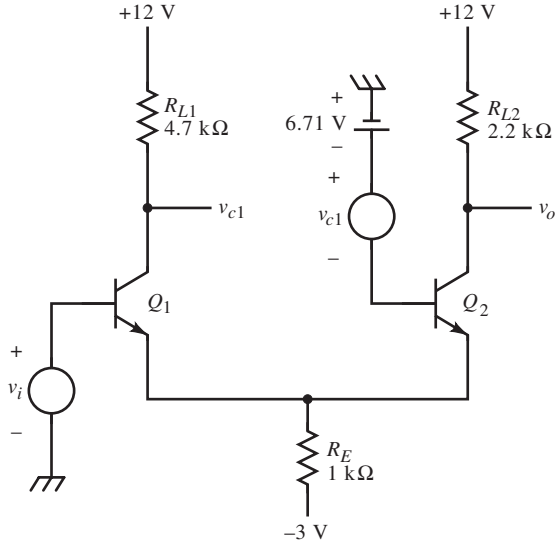
Because the input and output quantities are voltages and the error quantity is a current, T_i is a conductance and T_o is a resistance. Other choices for E are possible, of course. The voltage gain is

$$A_v = T_i \cdot \frac{G}{1+GH} \cdot T_o$$

This amplifier can be implemented by replacing the $\times(-1)$ amplifier with a PNP CE stage or NPN CE stage with negative voltage offset. Unlike previous amplifiers with multiple forward paths (for G), the passive path (G_2) has a significant transmittance and cannot be ignored.

EMITTER-COUPLED FEEDBACK AMPLIFIER EXAMPLE

Feedback analysis of the circuit in the figure below assumes $E = i_{e1}$.



Then static analysis produces the following values:

$$V_{C1} = 6.7112\,\text{V}, \quad V_E = -0.7773\,\text{V}, \quad V_O = 9.6345\,\text{V}, \quad V_{B2} = -0.0012\,\text{V}$$
$$I_{E1} = 1.14\,\text{mA}, \quad I_{E2} = 1.09\,\text{mA}$$

From the current values,

$$r_{e1} = 22.69\,\Omega, \quad r_{e2} = 23.73\,\Omega$$

The transmittances are

$$T_i = 9.7781 \times 10^{-4}\,\text{S}, \quad H = 0.97781, \quad G_1 = 4.5451, \quad G_2 = -0.97682$$
$$G = G_1 + G_2 = 3.5683, \quad T_o = -2.178\,\text{k}\Omega, \quad 1 + GH = 4.4892$$

Then

$$\frac{v_o}{v_i} = T_i \cdot \frac{G}{1+GH} \cdot T_o = -1.6928$$
$$r_{in} = (1+GH)\left[(\beta_1 + 1)\cdot(r_{e1} + R_E)\right] = 459.1\,\text{k}\Omega$$
$$r_{out} = 2.2\,\text{k}\Omega$$

Inverting Feedback Amplifier Example

The static bias solution is given in the SPICE simulation for the inverting feedback amplifier shown below, the first stage of an audiotape playback amplifier. The transistor parameters are for 2N930 transistors. From these values,

$$r_{e1} = 244\,\Omega, \quad r_{e2} = 26.6\,\Omega$$

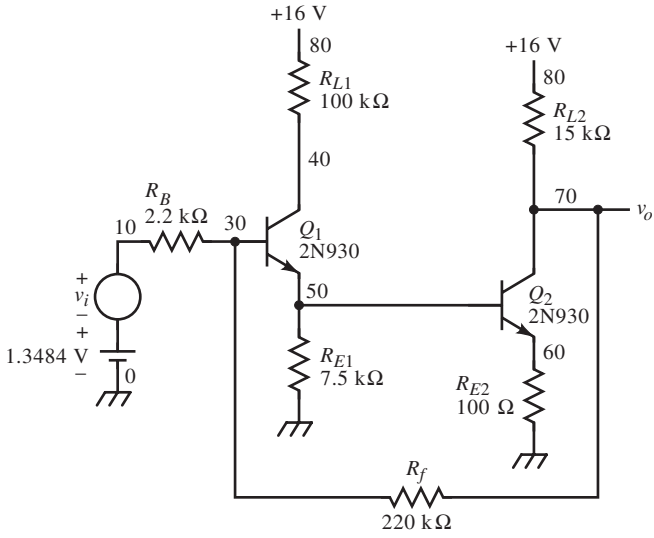
Furthermore, the base input resistance of Q_1 is

$$r_i = (\beta_1 + 1)\cdot\left[r_{e1} + R_{E1} \| (\beta_2 + 1)\cdot(r_{e2} + R_{E2})\right] = (171)\cdot(244\,\Omega + 5.5700\,\text{k}\Omega) = 994.2\,\text{k}\Omega$$

Additionally,

$$r_i \| R_f = 180.1\,\text{k}\Omega, \quad r_i \| 2.2\,\text{k}\Omega = 2.1951\,\text{k}\Omega$$

Let $E = v_{b1} = v(30)$. Then,




```
Inverting Feedback Amplifier

.OPT NOMOD OPTS NOPAGE
.OP
.DC VI -2V 2V 0.05V
.TF V(70) VI

VCC 80 0 DC 16V
VI 10 0 DC 1.3484V
RB 10 30 2.2K
RF 30 70 220K
RL1 80 40 100K
RE1 50 0 7.5K
RE2 60 0 100
RL2 80 70 15K
Q1 40 30 50 BJT1
Q2 70 50 60 BJT1
* 2N930

.MODEL BJT1 NPN (BF = 170 IS = 1E-14)
.END

NODE VOLTAGE
(10) 1.3484 (30) 1.3484 (40) 5.4728 (50) .7515
(60) .0973 (70) 1.4837

BIPOLAR JUNCTION TRANSISTORS
NAME Q1 Q2
```

```
MODEL BJT1 BJT1
IB 6.19E-07 5.69E-06
IC 1.05E-04 9.67E-04
VBE 5.97E-01 6.54E-01
VBC -4.12E+00 -7.32E-01
VCE 4.72E+00 1.39E+00
BETADC 1.70E+02 1.70E+02
GM 4.07E-03 3.74E-02

V(70)/VI = -5.106E+01
INPUT RESISTANCE AT VI = 4.260E+03
OUTPUT RESISTANCE AT V(70) = 6.873E+03
```

$$T_i = \frac{180.1\,\text{k}\Omega}{180.1\,\text{k}\Omega + 2.2\,\text{k}\Omega} = 0.9879$$

$$G_1 = \left(\frac{5.5700\,\text{k}\Omega}{5.570\,\text{k}\Omega + 244\,\Omega}\right)\cdot\left(-\frac{15\,\text{k}\Omega \| 220\,\text{k}\Omega}{26.6\,\Omega + 100\,\Omega}\cdot\frac{170}{171}\right) = (0.9580)\cdot(-110.3) = -105.7$$

$$G_2 = \frac{15\,\text{k}\Omega}{15\,\text{k}\Omega + 220\,\text{k}\Omega} = 6.3830\times10^{-2} \cong 0$$

$$G = -105.6$$

$$H = -9.8793\times10^{-3} = \frac{-1}{101.2}$$

$$1 + GH = 2.0433$$

Then

$$\frac{v_o}{v_i} = T_i\cdot\frac{G}{1+GH} = -51.057$$

$$r_{out} = \frac{15\,\text{k}\Omega \| 220\,\text{k}\Omega}{1+GH} = 6.8726\,\text{k}\Omega$$

$$r_{in} = [2.2\,\text{k}\Omega + 180.1\,\text{k}\Omega]\cdot\frac{1+(GH)|_{v_i=0}}{1+(GH)|_{i_i=0}}$$

$$= 182.3\,\text{k}\Omega\cdot\frac{2.0433}{87.466} = (182.3\,\text{k}\Omega)\cdot(2.3360\times10^{-2}) = 4.2586\,\text{k}\Omega$$

In using Blackman's formula to find r_{in}, G remains the same for both numerator and denominator, the numerator is the $(1 + GH)$ calculated previously, and what is different in the denominator is H:

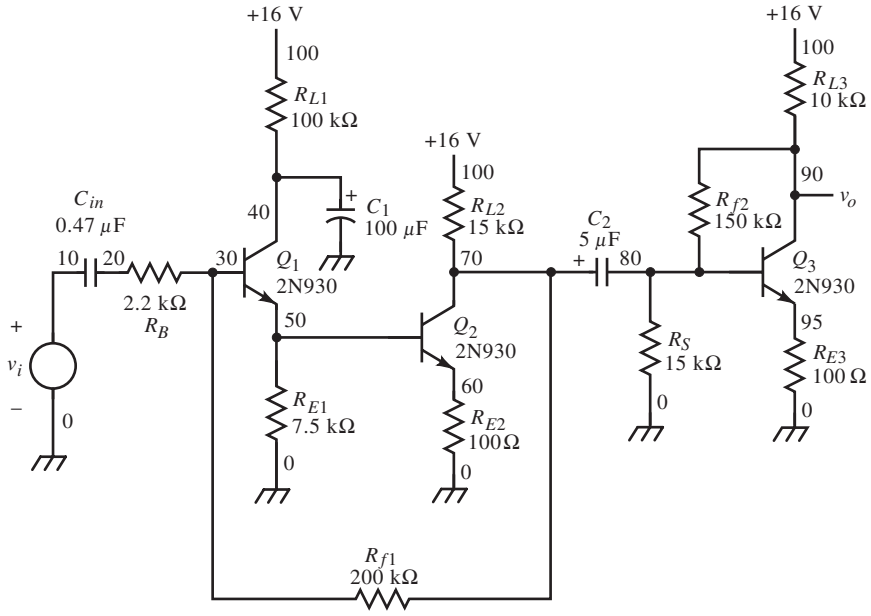
$$H|_{i_i=0} = \frac{r_i}{r_i + R_f} = -0.81881$$

The SPICE simulation results are

$$\frac{v_o}{v_i} = -51.06, \quad r_{in} = 4.260\,\text{k}\Omega, \quad r_{out} = 6.873\,\text{k}\Omega$$

AUDIOTAPE PLAYBACK AMPLIFIER EXAMPLE

The figure below combines the inverting feedback amplifier and the shunt-feedback voltage amplifier stages presented in previous examples into a discrete-BJT tape playback amplifier design.



From the prior analysis of these stages, the voltage gain is

$$A_{v1} \cdot A_{v2} = (-51.06) \cdot (-13.84) = 706.7$$

A SPICE ac simulation was performed on the amplifier. Although we have not considered frequency response in this volume, the quasistatic parameters are closely approximated by these results at high frequencies. At 100 kHz, $A_v = v(90)/v(10) = 706.8$, showing good agreement.

Additionally, from the simulation, $A_{v1} = v(70)/v(10) = 10.38$ and $r_{in} = v(10)/i(R_B) = 19.19\ \text{k}\Omega$ at 100 kHz. The discrepancy between A_v of the stand-alone inverting feedback amplifier and A_{v1} is due to how interstage loading is handled. In combining the two amplifier stages, the Thevenin equivalent of the first stage was used to drive the second instead of loading the output of the first by the second. The loaded first stage has a gain of A_{v1} and can be calculated using feedback analysis with a loaded

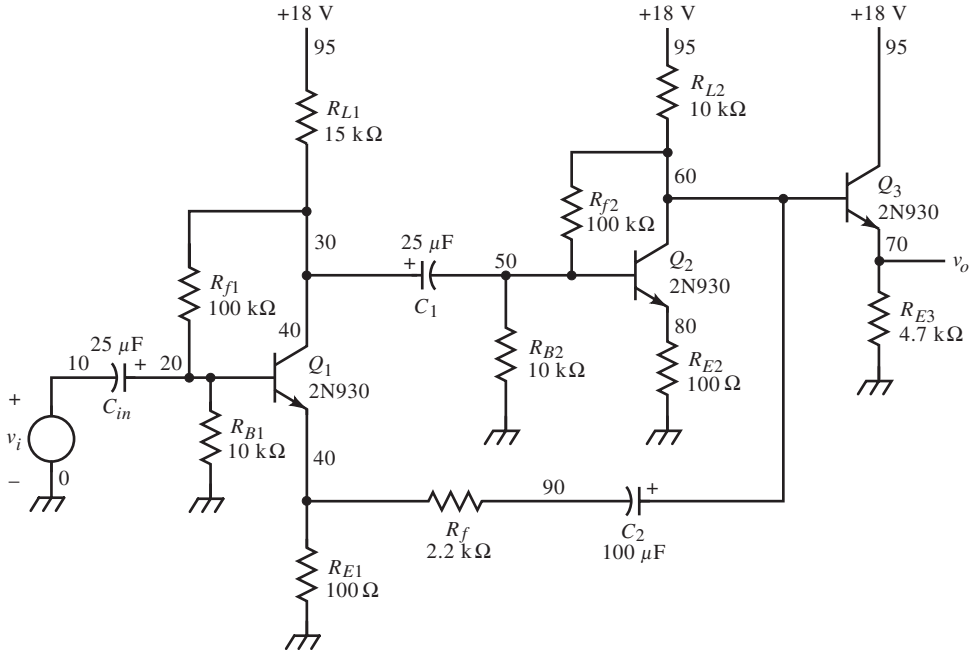
$$R_L = 15\,\text{k}\Omega \| r_{in}(Q_3) = 1.563\,\text{k}\Omega$$

Then the gain of the shunt-feedback output stage is its open-loop $G = -68.057$. By accounting for the loading of $r_{in}(Q_3)$ on the first stage, the actual output voltage of the first stage is derived, and this is the base voltage of the second (loading) stage. As v_{b3} is known (with Q_3-stage feedback taken into account in the loading), the remaining transmittance is from the base of Q_3 to its collector, or G. The overall gain is then

$$A_v = (-10.38) \cdot (-68.057) = 706.4$$

Example: Audio Preamplifier with Noninverting Feedback

The circuit shown below is an audio preamplifier design using similar gain stages as the previous one but employing a noninverting feedback amplifier.



It is followed by an emitter-follower to provide a low-resistance (voltage source) output. The amplifier has high input resistance due to feedback, as desired for a voltage amplifier. SPICE simulation produced the following voltage gains:

$$\frac{v(30)}{v(10)} = 1.512 \text{ at } 100\,\text{kHz}, \quad \frac{v(70)}{v(10)} = 21.09 \text{ at } 100\,\text{kHz}$$

CLOSURE

Multiple paths through amplifiers are common, and multiple methods are often applicable for finding a circuit quantity of interest. Multipath (or "composite") amplifiers appear again when frequency response is of primary interest, in *Designing Dynamic Circuit Response.*

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Index

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- Cascode
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- Emitter-coupled Stages
- Current Mirrors
- Darlington
- Reduction Theorem
- Effect of BJT and FET r_o

About the Author

D. Feucht has been involved in analog circuit design for over 40 years, fifteen of them with Tektronix and most of the rest in Innovatia. He has designed part or all of oscilloscopes, DMMs, function generators, pulse generators, medical ultrasonic scanners, phaco machines, and surgical lasers, microcomputer-based data acquisition and control systems, thermal energy meters, and various power conversion and motor-drive products.

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