GYROS

PLATFORMS

ACCELEROMETERS

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TECHNICAL INFORMATION

FOR THE ENGINEER

GYROS
PLATFORMS
ACCELEROMETERS

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HISTORY

In 1850 the eminent French physicist, Jean Foucault, first utilized the ability of a rotor's spin axis to remain fixed in space in seeking a method to demonstrate that the earth rotated. Foucault also coined the word by which we now designate devices based on the properties of a spinning rotor. He chose the word gyroscope from the Greek words "gyros," meaning rotation, and "skopein," meaning to view. Hence, the word gyroscope literally means to view rotation.

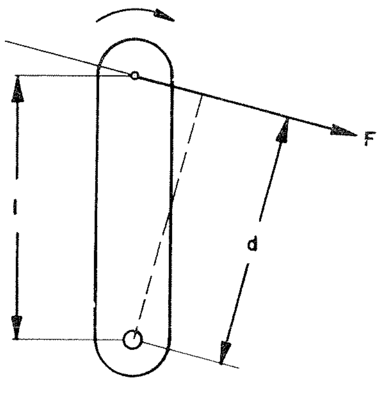
Today, gyroscopic devices remain the most convenient and practical reference instruments for attitude and rate sensing. However, their design characteristics and performance capabilities have traveled a long way from Foucault's laboratory device. This publication presents the underlying principles associated with gyroscopes, together with typical characteristics of such instruments.

THEORY OF THE GYROSCOPE

The gyroscope is basically a mechanical device, the essential element of which is a flywheel rotating at high angular velocity about an axis. The flywheel is mounted within gimbals which allow it one or two degrees of freedom.

Gyroscopes derive their basic applications from two inherent properties, namely precession and gyroscopic inertia. These properties are described below.

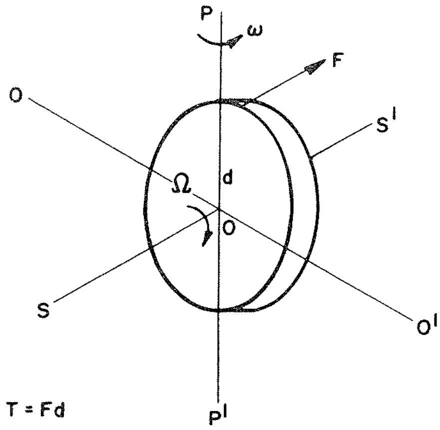
A force (F) is applied to a non-rotating body that is free to move about some axis O. If the line of force does not pass through that axis, rotation occurs. Rotation is a function of a torque (T) which is defined as the product of the force and the perpendicular distance between the axis of rotation of the force line of action. This is diagramed in Figure 1.



T = Fd

Figure 1

When a force (F) is applied to a gyroscope, rotation does not occur about axis O-O′ but in a direction normal to the applied torque. Applied torque causes the gyro spin axis S-S′ (the axis about which the rotating body spins) to move into the torque axis (O-O′) about axis P-P′ at a rate ω. (See Fig. 2). Skewing motion of the spin axis S-S′ is called precession, and axis P-P′ is called the precession axis.



T = Fd

Figure 2

This behavior is entirely consistent with Newton's Laws of motion. The moment of inertia (I) of a solid body with respect to a given axis is the sum of the products of the masses and the square of their distance from the given axis:

$$I = \int r^2 \, dm = \sum_{i=1}^{n} m_i r_i^2 \qquad (1)$$

The angular momentum (H) of a rotating body is the product of its moment of inertia and the angular velocity Ω of the rotating body when both refer to the same axis of rotation.

$$H = I\Omega \qquad (2)$$

A vector quantity has both magnitude and direction in contrast to a scalar quantity which has only magnitude. When a vector is represented by a line with an arrow, the line length represents magnitude while the arrow indicates the direction of the vector quantity. Graphically, vector length is proportional to the magnitude of the quantity being expressed.

Torque and angular momentum are vector quantities. Angular vector quantities can be represented by a right-hand screw rule. The vector is drawn in the direction that a right-handed screw would advance when rotated in the manner of the quantity being considered.

For a rotating body, Newton's Second Law is written as:

$$T = I\alpha \qquad (3)$$

where:

I = moment of inertia
α = angular acceleration
T = resultant torque

However, $\alpha = \dfrac{d\Omega}{dt}$, therefore $T = \dfrac{I\,d\Omega}{dt}$;

$$\text{but } I\frac{d\Omega}{dt} = \frac{dH}{dt} \quad \text{so } T = \frac{dH}{dt} \qquad (4)$$

Angular momentum of the rotating body in Fig. 3 is represented by H. When force F is applied to the rotating body, a torque T results which can be represented by ΔH_T. These two vectors have a resultant vector to which the spin axis aligns itself by rotating about axis P-P′ at a rate ω. This rotation is precessional motion. The relationship between $\bar{H}$, $\bar{T}$, and $\bar{\omega}$ is given as:

$$\bar{T} = \bar{\omega} \times \bar{H} \qquad (5)$$

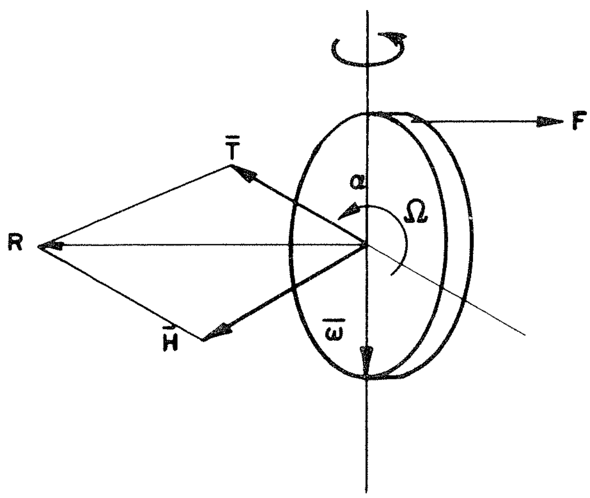


Figure 3

Torque is the vector cross product of $\bar{\omega} \times \bar{H}$. (The cross product of two vectors yields a vector whose direction is perpendicular to the plane of the two constituent vectors and so sensed that a right-handed screw, turned from the first to the second vector, advances in the direction of the resultant vector.) The magnitude of the resultant vector equals the product of the magnitudes of the constituent vectors and the sine of the angle between them. Precession is always in such direction as to align the direction of rotor rotation with the direction of applied torque rotation.

Though this publication deals principally with gyroscopic instruments and their application, one should not overlook the fact that gyroscopic phenomena are not concentrated only in special instruments. Wherever rotation and torque are simultaneously present, gyroscopic action may occur. For example, our spinning earth itself may be considered to be a massive gyroscope, and is therefore subject to precession. In Figure 4, the moon exerts a slightly greater gravitational attraction on the earth's point A than on point B because the distance between A and the moon is a little shorter than that between the moon and point B. Hence, a torque is generated which attempts to pull the earth's equator into the plane of the ecliptic, thereby acting to erect the earth's axis. Because the earth spins, it reacts to the torque by precessing. This precessional "wobble," though, is very slow, requiring a period of 25,725 years to complete a single cycle.

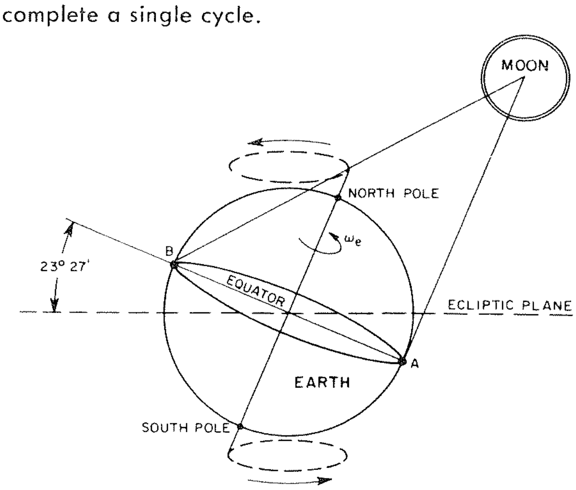


Figure 4

The precession law is a reversible one. Just as a torque input results in an angular velocity output, an angular velocity input results in a torque output along the corresponding axis.

The expression for ω for torque input T is:

$$\omega = \frac{\bar{H} \times \bar{T}}{H^2} \qquad (6)$$

A gyroscope has three mutually associated axes. These axes, illustrated in Fig. 5, are used in describing inputs and outputs of the gyroscope.

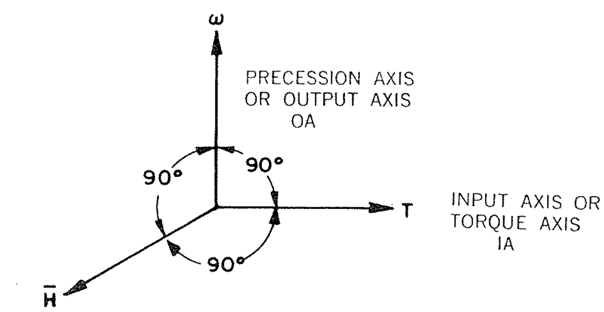


Figure 5

Gyroscopic inertia is a fundamental property of a gyroscope. The gyroscope spin axis tends to remain fixed in space, making the gyroscope an inertial reference. Corrective torques must be applied to a gyroscopic instrument if it is to be used as an earth reference. "Inertial space" simply means a system of coordinates which is unaccelerated with respect to "fixed stars."

Having very briefly considered the fundamental principles relating to gyroscopic instruments, attention may now be directed to the various restrictions and modifications applied to the simple rotating body to convert it to a useable reference instrument. Gyro design begins by controlling the restraints acting upon the rotating body or rotor. These restraints are expressed either as torques or limiting angular freedom of the spin axis, and are provided in the form of precession torque input and gimballing.

Gyros are classified in this publication by the degrees of freedom which they possess and by their use. Mathematically, *degrees of freedom* are determined by the number of coordinates required to define the position of a point or body. In gyroscopes, degrees of freedom refer to the restraints imposed upon the gyro rotor spin axis.

A gyro has a maximum of two degrees of freedom when two angular coordinates can specify its exact orientation. These angles are usually given by the angular positions of the supporting gimbals. Hence, a gyro supported by two gimbals has two degrees of freedom. A gyroscope supported by one gimbal only has the position of its spin axis (momentum vector) determined by one angle and is therefore called a single-degree-of-freedom gyro. It is customary to consider this type of gyro as a special case of the general two-degree-of-freedom configuration.

It is useful to know that inputs about the precession axis as well as those about the input axis cause a precession axis output. Three such cases are considered here:

1. Torque Applied at Input Axis

The basic gyro equation is reversible. An input rate produces a torque and an input torque produces an angular rate precession of the float about the output axis. Thus, a gyro is a rate-to-torque and torque-to-rate converter:

$$T = H\dot{\phi} \tag{7}$$

$$\dot{\phi} = \frac{T}{H} \tag{8}$$

2. Torque Applied at Output Axis

A torque can be deliberately applied about the output axis by using a torquer. This torque can also result from mass unbalance, anisoelastic deflection under vibration, spring restraint associated with pigtails, pickoff reaction, etc. As a result of the torque, the gyro precesses about the input axis. Such precession is opposed by the pivots that bear against the bearings.

A reaction torque is developed about the input axis.

Although this is an induced torque, it affects precession about the output axis, where $\dot{\theta} = \omega$.

$$\dot{\theta} = \frac{T}{H} \tag{9}$$

Thus a torque applied at the output axis produces a precession about the output axis.

3. Angular Rate Applied at Output Axis

An angular rate applied at the output axis induces a torque T_i about the input axis due to reactions at the bearings:

$$T_i = H\dot{\phi} \tag{10}$$

The induced torque develops a precession $\dot{\theta}$ about the output axis:

$$\dot{\theta} = \frac{T}{H} \tag{11}$$

Equating expressions (10) and (11):

$$\dot{\theta} = \frac{H\dot{\phi}}{H} \tag{12}$$

$$\text{or } \dot{\theta} = \dot{\phi} \tag{13}$$

Thus, the rotor and frame have the same velocity, and the gyro follows the input rate $\dot{\phi}$.

GIMBAL ERRORS

Gimbals are used as means of isolating a device from motion by providing various degrees of rotational freedom. A number of geometrical phenomena associated with gimbal systems exist which are called gimbal effects. The gyro designer and user are most concerned with the effects known as gimballing errors. *Gimbal errors* occur when the angular motions of gimbals do not correspond to the actual motion occurring about their reference axes. When a gimbal axis transducer is used, its output measures relative motion between gimbals, which is not necessarily the actual angular motion of the base.

The gimballing error encountered in a directional gyro is known as *inter-cardinal tilt error*. When such a gyro is tilted about the outer axis and then turned in yaw, the azimuth indication will be in error. In a stable platform, the desired gimbal sequence from innermost to outermost is azimuth, pitch, and roll. Altering this sequence can lead to a variety of gimballing errors.

Gimballing errors can be analyzed by means of spherical trigonometry, and by reduction to kinematically equivalent mechanisms.

NUTATION

A two-degree-of-freedom gyro is also susceptible to a phenomenon known as nutation, which is simply a wobbling of the rotor spin axis. It is in effect a self-sustaining oscillation which physically represents a transfer of energy from one degree of freedom to another and back again. In contrast to precessional motion, nutation needs no external torques to sustain it.

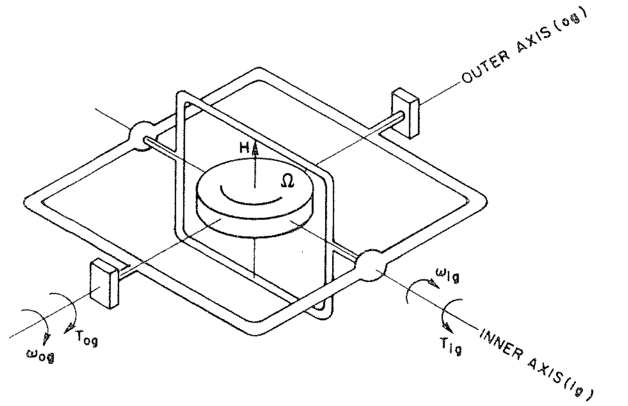


Figure 6

An input torque, T_{ig}, will cause gyro precession, ω_{ig}, causing a reaction torque, T_{og}, which in turn produces a precession ω_{og}. Setting up the equations as follows will provide an expression for the natural frequency of this interaction between torques and reaction torques which causes the spin axis to sweep out an elliptical path.

By the gyro precession law:

$$T_{ig} = H\,\omega_{og} \qquad (14)$$

$$-T_{og} = H\,\omega_{ig} \qquad (15)$$

While by Newton's Second Law:

$$T_{ig} = I_{ig}\,\alpha_{ig} = I_{ig}\frac{d\,(\omega_{ig})}{dt} \qquad (16)$$

$$T_{og} = I_{og}\,\alpha_{og} = I_{og}\frac{d\,(\omega_{og})}{dt} \qquad (17)$$

Equating (14) to (16) and (15) to (17) yields

$$T_{ig} = H\,\omega_{og} = I_{ig}\,\frac{d\,(\omega_{ig})}{dt} \qquad (18)$$

$$T_{og} = -H\,\omega_{ig} = I_{og}\,\frac{d\,(\omega_{og})}{dt} \qquad (19)$$

Solving for ω_{ig}

$$\omega_{ig} = \frac{-I_{og}}{H}\cdot\frac{d(\omega_{og})}{dt} \qquad (20)$$

Differentiating (20) yields

$$\frac{d(\omega_{ig})}{dt} = \frac{-I_{og}}{H}\cdot\frac{d^2(\omega_{og})}{dt^2} \qquad (21)$$

Substituting (21) into (18)

$$H\,\omega_{og} = -I_{ig}\,\frac{I_{og}}{H}\cdot\frac{d^2(\omega_{og})}{dt^2} \qquad (22)$$

Solving for ω_n, where ω_n = natural frequency

$$\left(D^2 + \frac{H^2}{I_{ig}\,I_{og}}\right)\omega_{og} = 0 \qquad (23)$$

$$\omega_n = \frac{H}{\sqrt{I_{og}\,I_{ig}}} \qquad (24)$$

Avoiding or minimizing the occurrence of nutation is of concern to the gyro designer and user. In a frictionless system, nutation would persist indefinitely. In reality, however, gimbal bearing friction serves to damp out nutation. In an oscillating system having friction, energy dissipation varies proportionately to oscillation frequency.

Hence, it is desirable to have as large an angular momentum as possible in conjunction with gimbals having low moments of inertia.

GIMBAL LOCK and TUMBLING

The two-degree-of-freedom gyro cannot have complete 360 degrees of freedom about both its inner and outer gimbal axes because of the phenomenon known as *gimbal lock*. When the gyro spin axis coincides with the outer gimbal axis, the gyro no longer has two degrees of freedom. If a yaw input is then applied, the outer gimbal begins to spin. Once such spinning has begun, the spin axis remains locked to the outer gimbal regardless of the attitude assumed by the gyro thereafter. To prevent gimbal lock, mechanical stops are incorporated into the gyro design to limit motion. Usually, stops permitting ±85 degrees of motion are placed about the inner axis. (Gimbal lock can also occur in three-gyro, three-gimbal stable platforms if stops are not provided).

The use of stops, however, causes another problem. When the inner gimbal strikes one of the pitch axis stops, the outer gimbal turns through 180 degrees about its gimbal axis. This outer gimbal motion is called tumbling.

ADDITIONAL REFERENCES

Preceding comments on gyroscope theory have been purposely simplified to ease the novice's introduction to the subject. Those seeking a more definitive mathematical approach should refer to publications listed in the bibliography. In doing so, the reader will note that different authors prefer different mathematical expressions to describe the same physical phenomena. Hence, gyroscopic behavior is described in terms of Cartesian coordinates, Lagrangian equations, vectors, Laplace transforms, Eulerian angles, and matrices, depending on a particular author's preference.

RELATIVE MERITS OF SINGLE-DEGREE AND TWO-DEGREE OF FREEDOM GYROS

Single Degree of Freedom Gyros

Three gyros needed to establish triad reference system.
Gyro not subject to nutation.
Constant angular momentum essential.
Not subject to gimbal lock.
Caging for only one degree of freedom required.
Case reference may be restored by in-flight caging.
Errors occur due to coupling for large gimbal deflections.

Two Degree of Freedom Gyros

Two gyros needed to establish triad reference system.
Gyro subject to nutation.
Angular momentum (rotor speed) tolerance loose.
Subject to gimbal lock; unsatisfactory performance near gimbal lock position.
Caging for two axes is required.
Lost reference cannot be regained without external reference source.
Striking mechanical stops causes "tumbling" and loss of reference.
Balancing and production problems many times greater than for single degree of freedom gyro.
Gyro rotor speed less critical.

RATE GYROS

A rate gyroscope is constrained to one degree of freedom, and its displacement about the output axis is proportional to the angular rate input to the input axis. The rate gyro utilizes the precession phenomenon in which an angular velocity input produces an output torque. A rate gyro is so called because it is used to measure angular rates of motion about a selected axis. It also provides a means for introducing artificial damping into a vehicle's control system. The exact amount of damping is a function of the vehicle's aerodynamic characteristics.

A mathematical analysis of the behavior of the rate gyro is made by using differential equations. The time solution is found by using either classical methods or LaPlace Transforms. The latter are used here.

To reduce vibration about the output axis, the gyro is provided with a damping mechanism of some sort. Damping may be provided by hydraulic dashpots, eddy current devices, or viscous shear of a fluid.

The torque $H\dot{\phi}$ induced by precession must maintain the angle Θ, accelerate the gimbal assembly about the torque axis, and overcome the axis damping moment. This is expressed by:

$$J\frac{d^2\Theta}{dt^2} + B\frac{d\Theta}{dt} + K\Theta = H\dot{\phi} \qquad \dot{\phi} = \omega \tag{25}$$

$$\text{or} \quad \frac{d^2\Theta}{dt^2} + \frac{B}{J}\frac{d\Theta}{dt} + \frac{K\Theta}{J} = \frac{H}{J}\dot{\phi} \tag{26}$$

Using the LaPlace Transform, equation (25) can be re-written as:

$$\frac{\Theta(s)}{\dot{\phi}(s)} = \frac{H}{Js^2 + Bs + K} \tag{27}$$

$$\text{or} \quad \Theta(s) = \frac{\frac{H}{K}\dot{\phi}(s)}{\frac{Js^2}{K} + \frac{Bs}{K} + 1} \tag{28}$$

To investigate the response of $\Theta(s)$, a step input velocity $\frac{1}{s}$ for $\dot{\phi}(s)$ is applied. This step input represents a constant velocity since a small transient disturbance would have little effect.

$$\Theta(s) = \left[\frac{\frac{H}{K}\cdot\frac{1}{s}}{s^2\frac{J}{K} + \frac{Bs}{K} + 1}\right]\dot{\phi}(s) \tag{29}$$

To find the steady state response, the Final Value Theorem $\left(\lim_{s=0} s\,F(s) = \lim_{t\to\infty} f(t)\right)$ is applied to equation (29) and the following is obtained:

$$\Theta(s) = \frac{H}{K}\dot{\phi}(s) \tag{30}$$

$$\text{or} \qquad \Theta = \frac{H}{K}\dot{\phi} \tag{31}$$

which shows that if the input to a rate gyroscope is an angular velocity $\dot{\phi}$, the output is an angle Θ which is directly proportional to $\dot{\phi}$.

A time solution of equation (25) yields

$$\Theta(t) = \frac{H}{K}\dot{\phi}\left[\frac{1}{\sqrt{\frac{J}{K} - \left(\frac{B}{2K}\right)^2}} e^{-B/2J\cdot t} \quad \sin\sqrt{\frac{K}{J} - \left(\frac{B}{2J}\right)^2}\; t\right] \tag{32}$$

For rapid damping $\frac{B}{2J}$ should be large. Hence design practice is to maintain a high B/J ratio where possible.

$\left(\frac{B}{2J}\right)^2$ is usually negligible in comparison to $\frac{K}{J}$; hence the natural frequency of the rate gyro is essentially

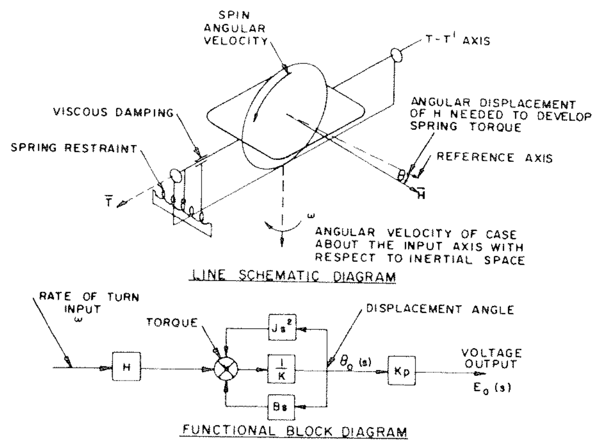
$$\sqrt{\frac{K}{J}} \tag{33}$$

In designing a rate gyro, certain parameters can be controlled and specified. These parameters are:

- I = Rotor moment of Inertia
- ω_s = Rotor angular velocity
- J = Moment of inertia about precession axis
- K = Spring constant of restraint element
- Θ_m = Maximum allowable precession angle
- B = Damping coefficient about precession axis

They are the controllable parameters on which the derived parameters depend.

RATE GYRO



J = MOMENT OF INERTIA GM-CM²
K = SPRING CONSTANT-DYNE-CM/RADIAN
Kp = TAKE OFF SENSITIVTY - VOLTS/RADIAN
H = WHEEL ANGULAR MOMENTUM ABOUT SPIN AXIS $\frac{\text{DYNE-CM-SEC}}{\text{RADIAN}}$
B = VISCOUS FRICTION $\frac{\text{DYNE-CM-SEC}}{\text{RADIAN}}$
ω = INPUT VELOCITY - $\frac{\text{RADIAN}}{\text{SECOND}}$
E_0 = VOLTS OUTPUT
s = LaPLACE TRANSFORM
θ = ANGULAR DISPLACEMENT OF H ABOUT TORQUE AXIS

Figure 7

SPRING RESTRAINED RATE GYROS

Two types of rate gyros are in extensive use. One type provides a voltage output proportional to the input rate and the gimbal displacement. The other type acts as a switch which closes a circuit when the rate reaches a predetermined value. The latter is called a rate switch. Both units may be mounted in any position which correctly orients the input axis.

APPLICATION

Rate-measuring gyros are widely used to provide voltage outputs proportional to rates of turn in the following applications:

Aircraft autopilots	Oscillation dampers
Fire control systems	Missile stabilization
Telemetering systems	

Gyro rate switches are used to cut out erection in vertical gyros and stable platforms during turns, or to cut out slaving voltages in directional gyros during turns. It has been shown that the output of a rate gyro is a voltage proportional to the derivative of the input angle. Hence, any drift of this angle is of little importance.

CONSTRUCTION

There are many types of rate gyro configurations. Most commonly, rate gyros consist chiefly of a motor supported on ball bearings or torsion bars, a pickoff or switch, and a viscous damping device. Typical construction features of Kearfott gyros are described below.

Gyro Motor The gyro motor has a rotor which is dynamically balanced. This rotor is driven at synchronous speeds of either 12,000 or 24,000 rpm on permanently lubricated precision ball bearings mounted in a rigid housing. A maximum inertia-to-weight ratio is achieved in the rotor design, and various 400 cycle, two or three-phase excitation voltages are used.

Pickoff Mounted on one end of the rate-measuring gyro assembly is an AC induction or resistance potentiometer pickoff that furnishes a voltage signal proportional to the angular precession of the motor assembly. The induction potentiometer is most widely used where high accuracy and linearity are desired. In the induction potentiometer, better resolution is achieved with no restraining friction, for there are no wiper contacts. Voltage outputs as high as 4.5 volts per mechanical degree, with a linearity of better than 0.25 percent at three degrees displacement, are obtained with the induction pickoff.

Switch The gyro rate switch utilizes a single-pole, double-throw switch in place of the pickoff to close a circuit at an input rate as low as 12 degrees per minute. Switch contacts can safely handle 1/2 ampere at 28 volts DC.

Damping Device In the rate measuring gyro the damping device and the rate switch consist of a cylinder attached to both the motor assembly and a machined cylindrical surface on the supporting frame. The gap between the rotor and the machined surface on the frame is filled with silicone fluid of a viscosity sufficient to provide damping ranging from 20 to 70 percent of critical, depending upon requirements. Damping is varied by varying fluid viscosity. Provision for compensating viscosity change as a result of temperature is required and can be achieved by special damper geometry or external heaters.

Springs Restraining springs are attached to the motor assembly and to the frame in which the motor precesses. Manufacture and handling of these springs (which are made from low hysteresis materials) require extreme care to withstand the high "g" vibration accelerations experienced in certain missile and aircraft applications. Springs may be linear or torsional. The torsion bar spring provides both gimbal restraint and gimbal suspension.

Bearings Bearings supporting the motor assembly in the frame are selected for minimum friction to provide high accuracy and maximum resolution. These bearings retain their low friction characteristics even after the unit has been subjected to shocks up to 80 g's and vibration accelerations up to 15 g's. Torsion bars at one or both ends are also used as suspension.

Stops Stops located approximately ±3° from the null position minimize cross coupling errors inevitably caused by abnormal input rates about the normal spin axis line when excessive precession occurs.

Case A hermetically sealed cylindrical case filled with dry inert gas encloses the gyro, effectively preventing corrosion and contamination and providing more uniform heat distribution. Floated rate gyros are fluid-filled and hermetically sealed.

Electrical Connections All internal circuits are connected through multi-terminal, hermetically sealed terminals at one end of the case assembly. Connectors are available as an option.

Mounting The unit is suitable for "bracket" or bulkhead mounting. Three or four tapped holes in the mounting surface, which is parallel to the spin axis, are provided. Flanges or clamp mounts are also suitable.

TESTING

Mass unbalance is tested by observing the pickoff null value while rotating the gyro unit through 360 degrees about the output axis.

Natural Frequency, Phase, and Amplitude Response Tests are performed on a frequency response analyzer which applies oscillations of various frequencies about the gyro input axis. The phase angle and the amplitude of the gyro pickoff outputs are compared to the phase angle and amplitude of the input oscillations to establish phase and amplitude characteristics. Natural frequency and damping are sometimes measured by recording the output voltage of the pickoff with a step function applied about the sensitive axis.

Accuracy and Linearity of Output Voltage Versus Rate Input A rate table and voltage-comparing device are used to obtain output versus rate information. Output voltages for increasing, decreasing, counterclockwise, and clockwise rates are observed, and deviations from the average and theoretical values are calculated. The resolution or minimum recognizable rate change is also observed. Data are obtained by mounting the rate gyro with its input axis parallel to the rate table axis of rotation and plotting the gyro output as the rate table input is varied. The plot of this response is linear within the gyro's range to the accuracy specified for the unit.

RATE SWITCH

Inspection Tests

These tests include:

Time for switch to close at a prescribed rate.

Time for switch to open after a prescribed rate.

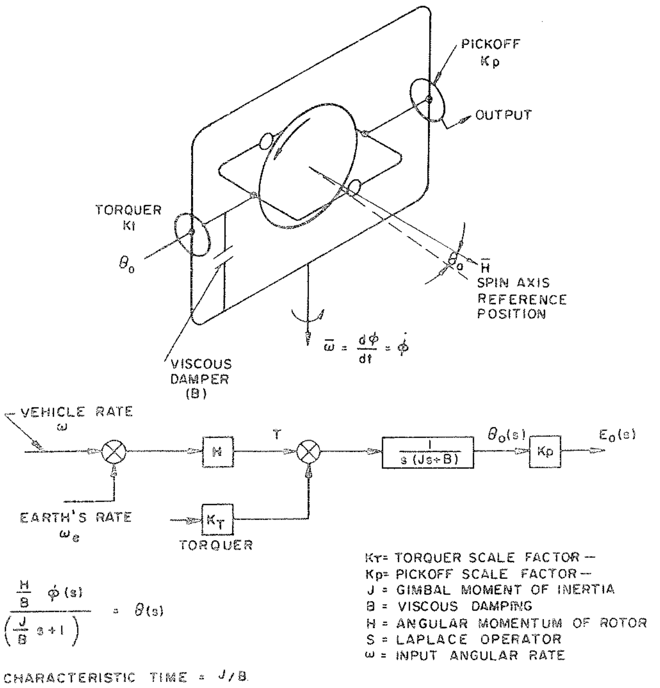
Oscillation about sensitive axis.

(This test is performed to insure that the switch does not close during normal low frequency in-flight aircraft oscillations.)

Rate to close switch.

RATE INTEGRATING GYROS

A rate integrating gyroscope is constrained to one degree of freedom, and its displacement about the axis is proportional to the integral of the angular rate input. This type of gyro is an application of the precession principle.



RATE INTEGRATING GYRO

Figure 8

Rate gyros and rate integrating gyros are similar in their theory of operation. The difference between the two lies in the nature of their respective precession axis restraints. In rate gyros, damping restraint is kept as low as practicable, whereas in rate integrating gyros, spring restraint is as small as possible.

The differential equation of motion common to both is

$$\frac{Jd^2\Theta}{dt^2} + \frac{Bd\Theta}{dt} + K\Theta = H\omega \qquad (34)$$

where Θ is precession axis motion and ω is the velocity rate input $\frac{d\phi}{dt}$.

For a rate integrating gyro, the spring restraint term K is set equal to zero, hence equation (34) becomes:

$$\frac{Jd^2\Theta}{dt^2} + \frac{Bd\Theta}{dt} = H\omega \qquad (35)$$

Using the LaPlace Transform, equation (35) becomes:

$$\omega(s)\,H = (Js + B)\,\dot{\Theta}(s) \qquad (36)$$

Rearranging:

$$\dot{\Theta}(s) = \frac{H}{(Js + B)}\,\omega(s) \qquad (37)$$

Apply an input rate of $\frac{\omega}{s}$ for $\omega(s)$ and observe the response $\dot{\Theta}(s)$. The step input represents a constant velocity ω. Also rearrange the form of (37):

$$\dot{\Theta}(s) = \frac{\frac{H}{B}\omega}{s\left(\frac{sJ}{B} + 1\right)} \qquad (38)$$

The inverse transform of (38) yields the expression:

$$\dot{\Theta} = \frac{H}{B}\,\omega\left(1 - e^{\frac{-tB}{J}}\right) \qquad (39)$$

However, as time (t) approaches infinity, equation (39) becomes:

$$\dot{\Theta} = \frac{H}{B}\,\omega \qquad (40)$$

By integrating both sides of equation (40):

$$\Theta = \frac{H}{B}\int \omega dt \quad \text{or} \quad \Theta = \frac{H}{B}\,\phi \qquad (41)$$

Two points are of interest. Since the $\int \omega\, dt$ represents a total angle, the precession angle output Θ is proportional to the total input angle ϕ. The parameter determining the behavior of an integrating gyro is the time constant J/B. This time constant is called the characteristic time of the gyro. As was the case for the rate gyro, a large value of B/J is desirable for good transient response.

Rate integrating gyros are very often built in a design version known as a floated rate integrating gyro, which is used where exceptionally high-level performance is required. Should the rotor mass of a gyro wheel having an angular momentum of 1.0×10^6 gm. cm.^{2}/sec, shift by as little as one micro-inch, a drift of 0.1 degree per hour could result. The difficulty in achieving ultra-high precision and accuracy in this respect is therefore apparent, but the necessity for obtaining low drift gyros is of paramount importance.

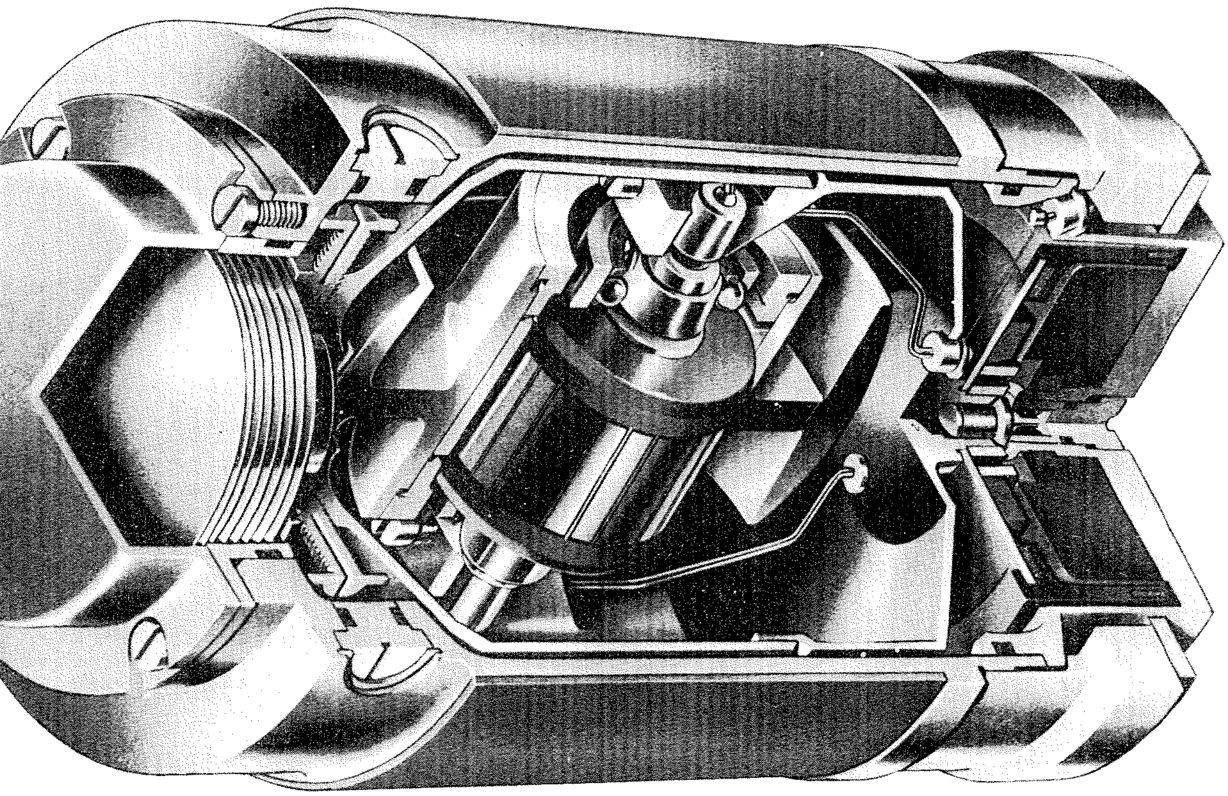
FLOATED RATE INTEGRATING GYROS

Over two thousand years ago Archimedes discovered that a body immersed in a fluid is resisted by a buoyant force equal to the weight of the displaced fluid. This principle, utilized by floated gyroscopes, is implemented by enclosing a gyro's rotor in a can, which in turn is immersed, or "floated," in another container filled with a fluid of such density that the resultant weight of the rotor and the can is effectively reduced to zero. Under such an ideal theoretical condition, the gyro rotor exerts no load on precession axis supports. Consequently, frictional torque — the product of normal load, coefficient of friction, and the moment arm of the support — is theoretically zero too, since the normal load itself is zero. In actual practice, however, it is never possible to reduce the load to an absolute zero, so some friction torque, though slight, is always present. By utilizing the damping restraint of the flotation fluid, an integrating floated gyro may be produced.

DESIGN FEATURES

The floated rate integrating gyro most commonly used consists of a gyro motor, induction pickoff, torque motor, gimbal assembly, flotation fluid, and case assembly. In this type of unit the gyro motor is sealed in a cylindrical float. The microsyn pickoff rotor is located at one end of the assembly while the torque motor rotor is located at the other. Density of the gimbal assembly is the same as the surrounding fluid in the instrument case. With this arrangement, very small diameter pivots and jewels are used to locate radially and axially the gimbal assembly in its surrounding case. The buoyant effect of the fluid supporting the gimbal assembly not only reduces friction to a minimum, but also provides effective damping.

The fluid-floated rate integrating gyro's accuracy is responsible for its widespread use as an inertial guidance system sensor. In earth-referenced systems and similar applications, corrective signals must be fed to the precession axis torquer of such a gyro. Recalling that $T = \omega \times H$, it is essential that gyro rotor angular momentum remain constant, for should the same torque be applied to a rotor having varying motor speed, gyro response will also differ, thus compromising computational accuracy. For this reason, precise floated rate integrating gyros employ synchronous hysteresis motors to spin the gyro wheel at a constant rate. Such a motor has three elements consisting of (a) a wound pole stator that is integral with the gyro float, (b) the gyro wheel itself which serves as the hysteresis motor rotor, and (c) a magnetic hysteresis ring attached to the gyro wheel.



TYPICAL FLOATED RATE INTEGRATING GYRO

Motor speed is expressed as:

$$N = \frac{2\phi}{p} f\ (60) \qquad (42)$$

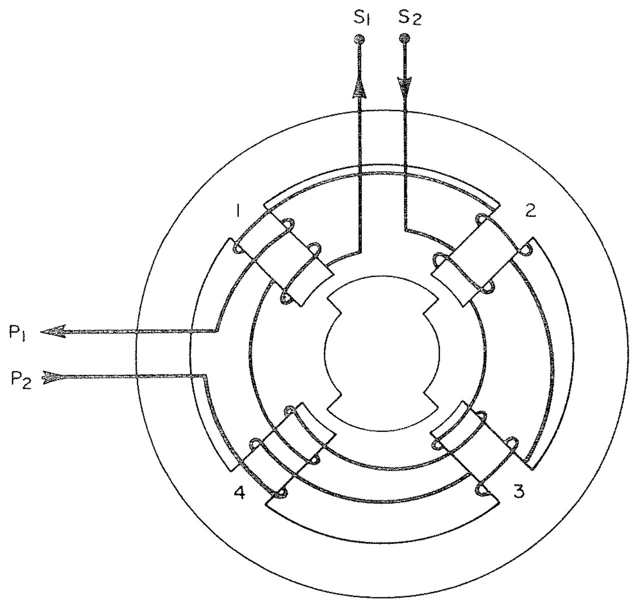
where ϕ = number of phases
p = total number of poles
f = excitation of frequency in cps
N = revolutions in cycles per minute

Unless specified otherwise, gyro motors used by Kearfott for floated rate integrating gyros are three-phase, six-pole components which rotate at 24,000 rpm when excited from a 400-cycle source. Wheel speed depends solely on excitation frequency. In situations where beat frequencies must be avoided, a motor may be operated at a lower frequency. With sufficient power supplied, for example, a motor may be operated at 350 cps instead of 400 cps, in which case angular momentum is reduced by a proportion of $\frac{350}{400}$.

Another attractive feature of the synchronous hysteresis motor lies in its ability to maintain at synchronous speed any load which it can accelerate initially from a dead stop. The gyro wheel incurs only those loads owing to windage and bearing friction. Synchronous hysteresis motor efficiency is about 50%, which is the ratio of load to input power.

Mounted on one end of the gyro is the pickoff. The output axis signal generator is an induction type pickoff or microsyn device providing a signal proportional to the angular precession of the motor assembly. These pickoffs are designed for minimum reaction torque and a large signal-to-noise ratio, and can be used over a considerable excitation frequency range. Resolution is better than one second of arc with a minimum range of $\pm 5°$.

Though a variety of devices are used on the gyro precession axis for signal pickoff and torquing purposes, a special type, known as a microsyn, is widely used. This is an electromagnetic device having an unwound rotor and a wound stator whose function is determined by the manner in which its windings are connected. A microsyn producing a voltage proportional to the rotational displacement of the rotor from its zero position is a *signal generator*. One producing a torque as a function of currents flowing through its primary and secondary windings is a *torque generator*. Another, producing magnetic forces causing the rotor to remain at the stator bore's geometric center both axially and radially, is known as a *magnetic suspension*.



P_1, P_2 = PRIMARY WINDING

S_1, S_2 = SECONDARY WINDING

Figure 9

Figure 9 shows a stator having four wound poles and a two-pole rotor with a slight air gap between them. A low-voltage alternating current passing through stator pole input windings when the rotor is at the null position induces voltages in stator poles 1 and 3 that are equal in magnitude, but 180° out of phase with voltages induced in poles 2 and 4. Any rotor rotation generates a greater voltage in one pair of poles than that in the other pair, the difference in voltages being the signal output of the microsyn. The voltage change is caused when the moving rotor presents greater pole area to one stator pole pair than to the other. This change in areas creates a corresponding alteration in magnetic flux lines between the poles which results in altered voltage output.

Except for the number of turns in stator pole windings, and the substitution of the terms reference and control windings for stator and rotor windings, a microsyn torque generator is the same in construction as a signal generator. When current is applied to the reference winding alone, no torque results because the four poles exert equal magnetic forces on the rotor. Control windings are so wound that when they are energized, the field of one pair of poles is strengthened while that of the other pair is weakened, thus producing a torque that is proportional to the product of reference and control winding currents, and which is in a direction depending on current phases. A microsyn torque generator characteristic is measured in dyne cm/ma^2. Note that while a microsyn signal generator is an AC instrument only, a microsyn torque generator may be excited by either direct or alternating current. Signal generator and torque generator functions can be built into a single device, called a dualsyn, which employs an eight-pole stator and a four-pole rotor. To prevent interaction between torquer and pickoff functions, the torque generator is excited by direct current, while the signal generator operates on alternating current.

In floated rate integrating gyros, damping is provided by a small gap between the cylindrical float and the inner case, which is filled with fluorolube and temperature-stabilized within ± 1°F. Temperature control is essential because flotation fluid density varies with temperature.

Pivots and jewels are used which are strong enough to support the entire gimbal weight during assembly prior to filling with fluid. After fluid flotation to better than 99%, and as a result of fluid lubricity, friction is reduced to less than 0.05 dyne-cm.

Operation of the DC torque generator is similar to the operation of a D'Arsonval meter movement. A permanent magnet is fixed rigidly on the gyro housing so that all torque couples have their axes coincident with the precession axis of the gyro. The return path of the flux is part of the float and is made of steel chosen for minimum hysteresis. A coil form rigidly fastened to the outer housing has its windings placed in an air gap with extremely uniform flux distribution. Any passage of current through the coil results in a torque about the precession axis of the gyro.

The precession angle is usually limited to $\pm 2°$ by spring stops which reduce cross coupling errors induced by rates about the normal spin axis line that become significantly greater with an increased precession angle. Pigtail distortion is also prevented by the stops.

Having its functional parts sealed together with the floating and damping fluid in an inner case, with a bellows to allow for fluid expansion, the floated rate integrating gyro is equipped with a sufficiently heavy outer case to guard against transmission of any significant ambient pressure differentials to the bellows. This outer case is hermetically sealed, and all internal circuits are connected through hermetically sealed terminals at one end of the case assembly. The flotation fluid is an organic-halogen fluid.

A heater and temperature sensing elements are also provided to assure stable ambient temperature. To energize heater elements, an external amplifier is required.

Mounting is accomplished through use of a flange located either at the base or center of the unit. An accurately machined diameter face and an indexing notch provide proper orientation of the input axis. Trunnion mounting is also available.

SELF-TEST FEATURE

In many situations, it is desirable to be sure that the gyro rotor is running properly. To meet this need, a self-test feature has been built into some models of gyros made by Kearfott. This feature permits a check to be made quite simply. In the initial construction of the gyro rotor, a magnetized bar is inserted near the rim. A conductor is built into the gyro case in such a manner that the rotor rotation causes a voltage to be induced in the conductor. An external inductive test probe can measure the induced voltage and relate it to gyro rotor behavior.

TESTING

Performance characteristics of floated rate integrating gyros are determined by performing drift rate, torquer linearity, damping, transfer function, characteristic time, mass unbalance, and null tests.

Drift rate is measured when the unit is mounted with the input axis vertical on a servoed table which is free to rotate about a vertical axis. A servo motor and a pickoff are attached to the servo table, and the servo table itself is mounted on an accurate rotary table. When an amplified signal from the gyro excites the servo motor, the servo table turns at some rate depending on the signal. Input signal to the servo motor depends on the earth's rate and upon any unbalances or restraints within the gyro. Drift rate is then determined by turning the rotary table through a definite angle (after the table has turned through null), and measuring the time interval required.

The same test arrangement is used in determining torquer linearity, in the following manner: the torquer is excited with enough current to maintain the gyro at null, and this current is then increased and subsequently decreased by the same amount. Turn rates of the servo table are measured for both the increase and decrease in current.

Damping values are determined by observing the input case angles for definite outputs, and transfer functions are obtained by observing outputs for definite input case angles.

APPLICATIONS

Floated rate integrating gyros are widely used as basic sensors of angular rate and angular position.

Some of their applications include:

- Flight control systems for reference stabilization
- Flight control systems for navigation
- Aircraft fire control systems (stabilization and tracking)
- Shipboard fire control systems
- Rate measurement

A rate integrating gyroscope is always used as the sensing element in a closed loop servo which is devised as a null seeking loop whose error signal is provided by the gyro precession axis output. Accordingly, rate integrating gyros are customarily built so that very little angular motion takes place about their precession axes. In fact, the amount of permissible angular motion about the precession axis of such a gyro serves to categorize the application of rate integrating gyros. In their conventional use as sensors for a gimballed stable element, precession angle motion rarely exceeds three degrees for a non-floated rate integrating gyro nor more than ± 2 degrees in floated rate integrating gyroscopes.

However, a newer method, called "strap-down system" or "gimbal-less platform," in which gyros are mounted directly to a flight vehicle's frame instead of being stabilized in space by gimbals, is coming into wider use. Since a flight vehicle's response to input perturbations is slower than the response of a gimballed platform, error angles of several degrees can occur before the necessary flight corrections are made. This means that the gyros must have greater angular freedom about their output axes than gyros used on conventional gimballed platforms. This requirement has led to the development of rate integrating gyros known as "wide angle" gyros. Since such gyros are required for guidance they are almost always floated gyros with very low drift rates. Wide angle gyros usually have an angular freedom of ± 10 degrees or more.

When very accurate measurements of angular rate are required, a rate integrating gyro can be used as a rate gyro. This is done by incorporating the gyro in a null seeking servo loop which is closed upon itself. An input to the gyro's sensitive axis causes a precession motion about the output axis which is measured by a pickoff, the signal from which is fed to an amplifier whose output is applied to a torquer physically located on the gyro's precession axis. Torquer current required to drive the gyro pickoff signal to null is a direct measure of the input angular rate. Characteristics of the loop are a function of both the gyro and the associated amplifier, which permits the user a reasonable degree of design flexibility.

RATE MODE: The differential equation describing the integrating gyro in a rate mode is expressed as:

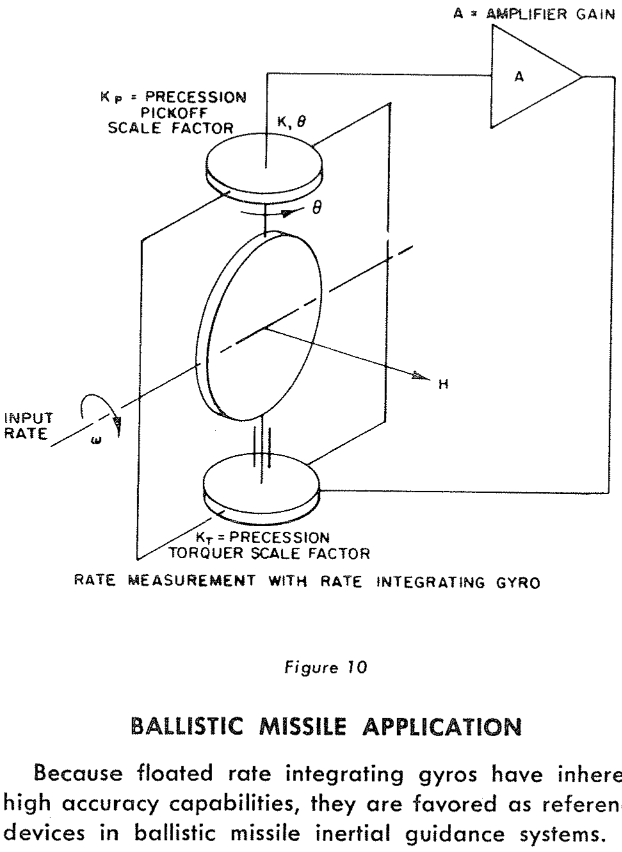
$$J\frac{d^2\Theta}{dt^2} + B\frac{d\Theta}{dt} + AK_p K_T \Theta = H\omega; \quad \omega = \dot{\phi} \quad (43)$$

Using the LaPlace Transform, for zero conditions at time zero, the equation becomes:

$$Js^2\Theta(s) + Bs\Theta(s) + AK_p K_T \Theta(s) = H\omega(s) \quad (44)$$

or $$\Theta(s) = \frac{H}{Js^2 + Bs + AK_pK_T}\omega(s) \quad (45)$$

Note that equation (45) is identical to equation (27) on page 5 if AK_pK_T is made equal to K. In effect, by closing a rate integrating gyro's loop on itself, an electrical rather than a mechanical spring is produced. Further analysis of gyro rate mode characteristics is analogous to that for a conventional rate gyro. By using an integrating gyro in a rate mode, lower input threshold sensitivity, wider dynamic range, and greater accuracy is obtained than is possible with a conventional rate gyro.



RATE MEASUREMENT WITH RATE INTEGRATING GYRO

Figure 10

BALLISTIC MISSILE APPLICATION

Because floated rate integrating gyros have inherent high accuracy capabilities, they are favored as reference devices in ballistic missile inertial guidance systems. A rocket-propelled ballistic missile develops high-g accelerations, high vibration, and much acoustical noise, so any meaningful inertial guidance system error analysis must consider gyroscope behavior under a high-g environment. A mathematical error model representing a typical gyroscope in such an environment appears below, in which R is a fixed drift rate term, U represents mass unbalance terms, and S indicates compliance effects like aniso-elasticity, all of which are referred to as error coefficients.

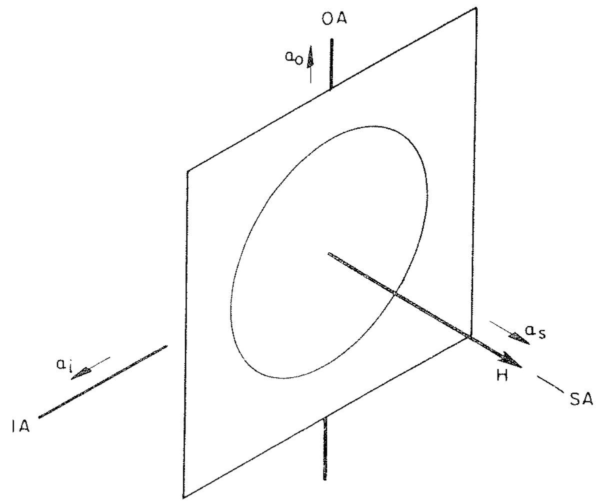


Figure 11

$$\omega_i = R + U_s a_i + U_i a_s + S a_i a_s \quad (46)$$

ω = gyro drift rate about input axis

a_i = thrust acceleration component along input axis

a_s = thrust acceleration component along spin axis

It is readily apparent that resultant accuracy depends on the magnitude of both error coefficients and applied acceleration, the latter being a function of missile trajectory. Although the gyro designer initially minimizes each error coefficient, the gyro user can achieve optimum gyro accuracy by orienting the gyro to a position in which it is least susceptible to the adverse effects of trajectory acceleration. Preferred orientation depends on the relative magnitude of individual error coefficients.

Gyro drift rate depends largely on the physical orientation of gyro axes relative to the flight trajectory. For example, drift caused by compliance coefficients, S, can be eliminated by orienting the gyro such that no acceleration acts upon either input or spin axes, a condition which can be approximated by positioning the gyro so that its output axis lies in the trajectory plane. Mass unbalance coefficients, U, may also be eliminated through gyro orientation refinement. By orienting the gyro, with its output axis still within the trajectory plane, a position will be reached at which mass unbalance coefficients either disappear or become minimal. This is the position that is best for minimum effect from both S and U coefficients.

In a ballistic missile inertial platform using three single-degree-of-freedom rate integrating gyros, the proper orientation of each gyro relative to the others guarantees maximum accuracy. To achieve this, the input axis of each of the three gyros must be normal to each other, forming an orthogonal triad. Two of these input axes should lie in the trajectory plane, with the third perpendicular to it.

FLOATED RATE INTEGRATING GYRO PERFORMANCE

EVALUATION AND TEST PROCEDURES

Because performance evaluations and test procedures on floated gyros are often conducted differently by one manufacturer as opposed to another, it is useful to know the exact methods by which gyro operating characteristics are established. Applications in which floated rate integrating gyros are used demand precise performance levels throughout the life of these instruments. Consequently, a test and evaluation method must be employed which can faithfully reflect a component's operating characteristics under conditions identical to those which will be encountered in actual use.

Generally, two fundamental methods are used in testing gyro performance, namely a "closed loop test" and an "open loop test." In the former, the gyro's torquer is slaved to the signal generator and holds the gimbal at null. Under this condition, gyro performance is reflected by the amount of torquer current required to maintain the gimbal null position. In open loop tests a servoed test table is slaved to the gyro signal generator and holds the gimbal at null by rotating the gyro about its input axis. Gyro performance under these conditions is reflected by the table rates required to maintain the gimbal null position. Actual performance of a gyro used in a stable platform is simulated by the latter test method.

Testing a gyro thoroughly in each of six positions has an advantage in that detrimental effects caused by undetected flaws are readily discovered, for gyro failure or poor performance generally become more apparent in some orientations than in others. Hence, the six-position mode of testing usually exposes any such defects early in the gyro's life.

SIX-POSITION SERVO TEST PROCEDURE

A servoed test table mounted on a rotary tilt table is a combination which not only simulates actual platform operation but also permits a gyro placed in the servoed table to be oriented in any of six positions desired. As the gyro drifts freely through a precise angle, it is timed. To obtain five consecutive time recordings, the gyro is torqued back to its original position and allowed to drift through the same precise angle five times. This procedure is repeated for each of six orientations, and each group of six-position tests, is, in turn, repeated five times. Between each series of six-position tests, the gyro is inoperative for the length of time required for cooling.

Mass unbalance along spin and input axes, restraint levels in six positions, and day-to-day repeatability of these parameters are the data obtained. Short term repeatability is also taken at each position.

SIX-POSITION SERVO TEST ANALYSIS

The time required for the gyro to drift completely through the precise angle mentioned earlier is termed the "servo table rate" and is a measure of the sum of the torques acting about the gyro's precession axis. These torques, schematically illustrated in Figure 12, are the result of (1) a component of earth's rate, E_R, along the input axis, (2) a mass unbalance, M_U, of the motor float assembly about the precession axis, and (3) a constant spring restraint, R, about the precession axis.

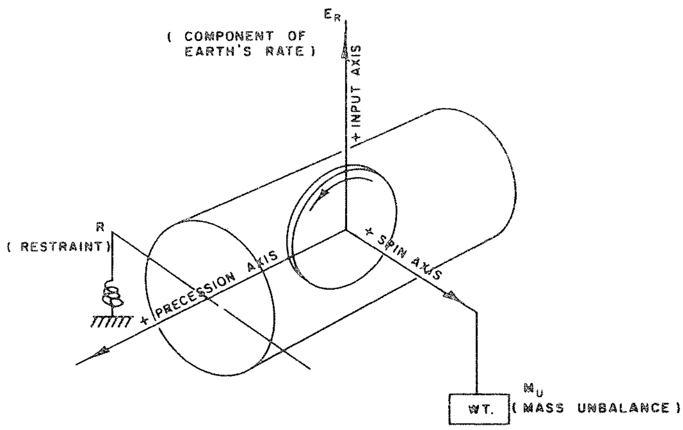


Figure 12

FORMULA DERIVATIONS

In the derivations shown on page 14 the following is assumed:

9.843°/hr = component of earth's rate along a vertical axis, at a latitude of 41.9°N.

11.372°/hr = component of earth's rate along a horizontal north-south axis, at a latitude of 41.9°N.

Earth's rate, E_R, is positive when it causes a positive input to the gyro.

Mass unbalance along the spin reference axis, $M_{U(SRA)}$, is positive when the positive spin reference axis is heavy. Mass unbalance along the input axis, $M_{U(IA)}$, is positive when the positive input axis is heavy.

Restraints are positive when acting in a counterclockwise direction about the positive precession axis.

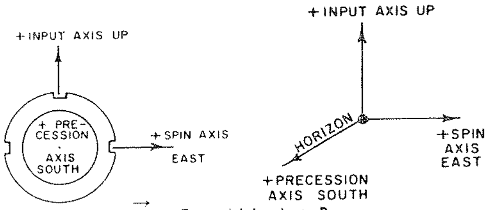
The component of earth's rate along the gyro input axis is greater than the algebraic sum of mass unbalance and restraints.

ω_1 = table rate in position 1
ω_2 = table rate in position 2
R_2 = table rate in position 3
R_1 = table rate in position 4
V_1 = table rate in position 5
V_2 = table rate in position 6
R_ω = restraints in positions 1 and 2
R_{R1} = restraints in position 4
R_{R2} = restraints in position 3
R_V = restraints in positions 5 and 6

GYRO ORIENTATIONS FOR SIX POSITION SERVO TEST

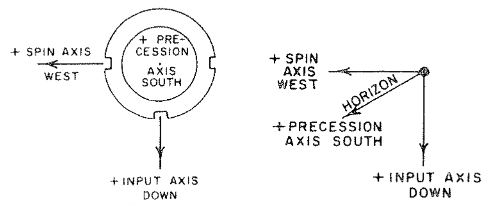
Six Position Servo Test Derivations

POSITION 1



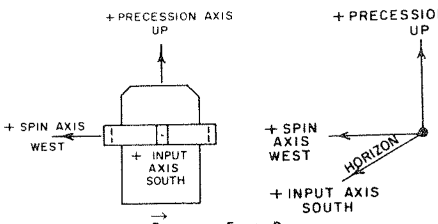
$$\vec{\omega}_1 = E_R - M_u(SRA) + R_\omega$$
$$\vec{\omega}_2 = -E_R + M_u(SRA) + R_\omega$$
$$\vec{\omega}_1 - \vec{\omega}_2 = 2E_R - 2M_u(SRA)$$
$$M_u(SRA) = E_R - \frac{\vec{\omega}_1 - \vec{\omega}_2}{2}$$
so
$$M_u(SRA) = E_R - \left[\frac{|\omega_1| + |\omega_2|}{2}\right]$$

POSITION 2



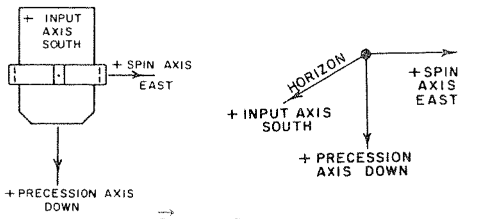
$$\vec{\omega}_1 = E_R - M_u(SRA) + R_\omega$$
$$\vec{\omega}_2 = -E_R + M_u(SRA) + R_\omega$$
$$\vec{\omega}_1 + \vec{\omega}_2 = 2R_\omega$$
$$R_\omega = \frac{\vec{\omega}_1 + \vec{\omega}_2}{2}$$
so
$$R_\omega = \frac{|\omega_1| - |\omega_2|}{2}$$

POSITION 3



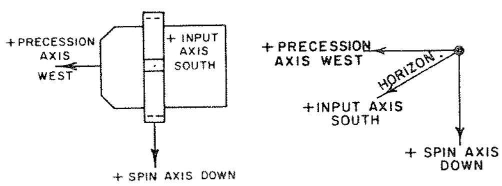
$$\vec{R}_2 = -E_R + R_{R2}$$
$$R_{R2} = E_R + \vec{R}_2$$
so
$$R_{R2} = E_R - |R_2|$$

POSITION 4



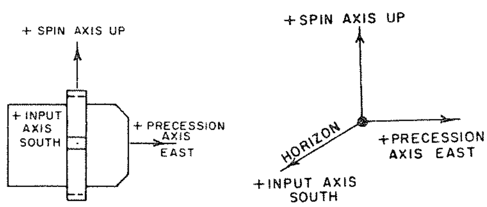
$$\vec{R}_1 = -E_R + R_{R1}$$
$$R_{R1} = E_R + \vec{R}_1$$
so
$$R_{R1} = E_R - |R_1|$$

POSITION 5



$$\vec{V}_1 = -E_R - M_u(IA) + R_v$$
$$\vec{V}_2 = -E_R + M_u(IA) + R_v$$
$$\vec{V}_2 - \vec{V}_1 = +2M_u(IA)$$
$$M_u(IA) = \frac{\vec{V}_2 - \vec{V}_1}{2}$$
so
$$M_u(IA) = \frac{|V_1| - |V_2|}{2}$$

POSITION 6



$$\vec{V}_1 = -E_R - M_u(IA) + R_v$$
$$\vec{V}_2 = -E_R + M_u(IA) + R_v$$
$$\vec{V}_1 + \vec{V}_2 = -2E_R + 2R_v$$
$$R_v = E_R + \frac{\vec{V}_1 + \vec{V}_2}{2}$$
so
$$R_v = E_R - \left[\frac{|V_1| + |V_2|}{2}\right]$$

Figure 13

SIX-POSITION RATE TEST

The six-position rate test method of floated rate integrating gyro evaluation most closely simulates the actual conditions the gyro will encounter in operation. A gyro that performs satisfactorily under the conditions imposed by the six-position rate test will perform satisfactorily under virtually any test conditions. The rate test is also vastly superior to the servo test in that performance can be monitored and recorded instantaneously, and the most minute discrepancies noted immediately. In addition, complete evaluation of the gyro can be accomplished through the use of relatively few test orientations.

The possible number of orientations in which a gyro may be tested are many: with Output Axis Up the Input Axis could point North, East, West, or South; with Output Axis Down there are also four positions for Input Axis (North, East, West, or South); with Output Axis South, Input Axis can then be oriented Up, Down, East, or West; for Output Axis North, Input Axis can be Down, Up, East, or West also. Altogether there are 24 positions, and similarly there can be another 24 orientations with respect to Earth's polar axis. For example, with Output Axis parallel to Earth's axis, Input Axis can be East, West, 49° from vertical, or 131° from vertical.

Using all of these combinations would result in unnecessarily excessive testing. Experience has shown that a relatively few of these can be used for complete evaluation and analysis.

If each axis is oriented in more than one orthogonal position and in both headings in each position, the effects of magnetic and gravitational fields in any conceivable position can be determined. Design criteria predicated on operating characteristics obtained from such tests then apply to any position in which the gyro is placed in actual use. Because these gyros have three adjustments (input axis mass unbalance, spin axis mass unbalance, and restraints) at least four positions must be tested to prevent calibration from obscuring drift. Although either the six position servo or rate test satisfies these requirements, the rate test is more feasible because it provides a practical solution to the problem of designing test equipment for today's high performance gyros.

For example, a gyro with a short term drift of 0.005°/hr. would precess through a one-degree angle with Input Axis Up in a matter of approximately six minutes time, corresponding to an Earth's rate component of 9.846°/hr. If the test equipment employed in a servo test is to be an order of magnitude better than the gyro tested, it must measure time to 15 milliseconds, and the one degree of arc must be repeatable to 0.18 seconds of arc, the latter being a primary standard level of accuracy. It is obviously impractical to test this way.

In rate or closed loop testing, 0.01% power supplies are feasible and available. The gyro is fixed on a test stand and the current in the torquer required to capture the gyro is proportionate to its drift rate. Knowing the torquer scale factor and Earth's rate component, the error can then be computed.

The principal objection to rate testing has previously arisen because of the reaction torque resulting from torquer current. However, in Kearfott gyros, air core pickoff and torquer rotors operating on several noninteracting frequencies are used, and this virtually eliminates reaction torques. These high-performance gyros have provided identical results whether evaluated by either rate or servo testing. These instruments, trimmed at one location for one type of test and subjected to completely different environments or tests at other locations, have performed equally well regardless of test conditions.

DESCRIPTION OF SIX-POSITION RATE TEST

The gyro is captured by torquer current in each of the following positions:

(1) OA North, IA West, SA Down
(2) OA North, IA East, SA Up
(3) OA North, IA Up, SA West
(4) OA North, IA Down, SA East
(5) OA Up, IA North, SA East
(6) OA Up, IA South, SA West.

SPIN VECTORS FOR SIX-POSITION TEST

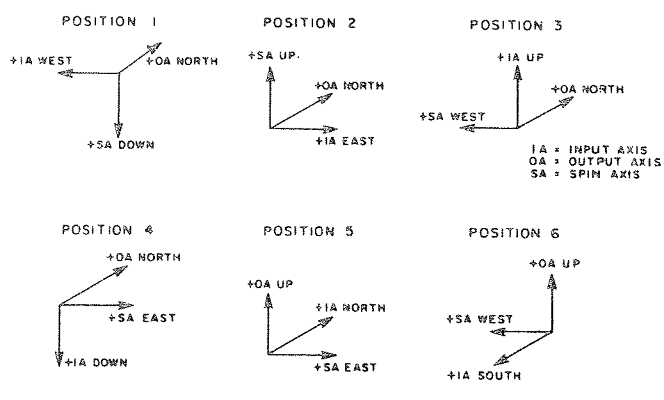


Figure 14

The relation between drift rate and torquer current depends on the relation between gyro torque and precession rate ($T = H\omega$) and between torque and electrical current in a permanent magnet field. The relation is measured by reference to the simultaneous equations in positions five and six.

The simultaneous equations are:

$$KJ_1 = -MU_{IA} + R_{SAV} \quad (47)$$
$$KJ_2 = +MU_{IA} + R_{SAV} \quad (48)$$
$$KJ_3 = -MU_{SA} + R_{IAV} + \omega_e \sin lat \quad (49)$$
$$KJ_4 = +MU_{SA} + R_{IAV} - \omega_e \sin lat \quad (50)$$
$$KJ_5 = +R_{OAV} + \omega_e \cos lat \quad (51)$$
$$KJ_6 = +R_{OAV} - \omega_e \cos lat \quad (52)$$

where —

ω_i (°/hr.) = inertial drift rate for gyro in position "i".

K (°/hr./ma) = torquer scale factor.

J_i (ma) = capture current to hold gyro at null in position "i".

$\omega_e \sin\lambda$ or $\omega_e \cos\lambda$ (°/hr.) = Earth's rate component along gyro input axis.

MU_{SA} (°/hr.) = Mass unbalance along the spin axis (input axis vertical) positive when positive spin reference axis is heavy.

MU_{IA} (°/hr.) = Mass unbalance along the input axis (spin axis vertical) positive when positive input axis is heavy. R_{IAV}, R_{SAV}, and R_{OAV} (°/hr.) = Restraints (non-g-sensitive) drift as determined from data in position indicated positive when its vector is directed along the positive output axis.

Combining equations (51) and (52) yields torquer scale factor, K, and restraints (non-gravity-sensitive drift rate), R_{OAV}, as follows:

$$K = \frac{2\omega_e \cos\lambda}{J_5 - J_6} \qquad R_{OAV} = K\frac{J_5 + J_6}{2} \qquad (53)$$

With K thus determined and capture currents J_i measured, all gravity-sensitive (MU) and non-gravity-sensitive (R) drift rates can be separated by working equations (49) with(50) and (51) with (52).

$$MU_{IA} = K\frac{J_2 - J_1}{2} \qquad MU_{SA} = K\frac{J_4 - J_3}{2} + \omega_e \sin\lambda \qquad (54)$$

$$R_{SAV} = K\frac{J_1 + J_2}{2} \qquad R_{IAV} = K\frac{J_3 + J_4}{2} \qquad (55)$$

Short-term variation is easily determined from an instantaneous type test. A sample of points from one-hour runs is taken and the corresponding rates used to calculate short term variations. Sample traces are shown in Figure 15.

Typical plots in Figures 15 through 17 demonstrate that variations from day-to-day and position-to-position are quite small. Obviously, controlled position determinations can be made quite accurately in almost any type of inertial reference system.

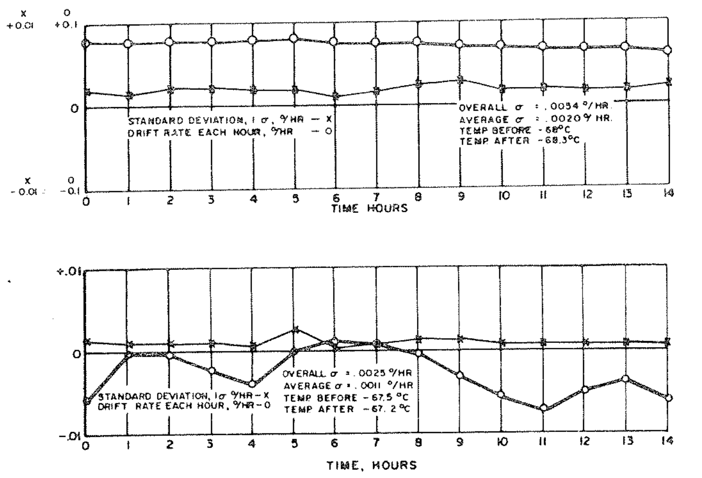


Figure 15

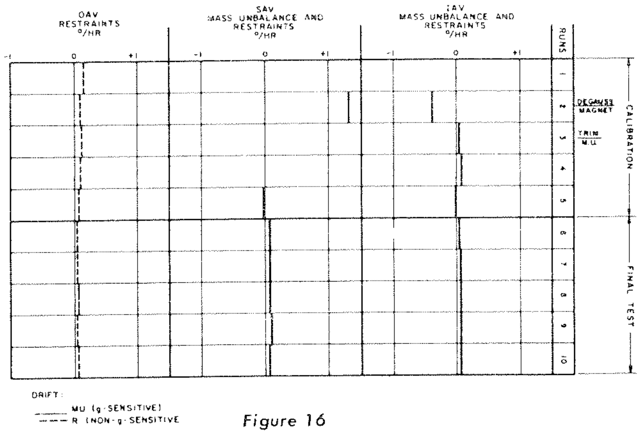


Figure 16

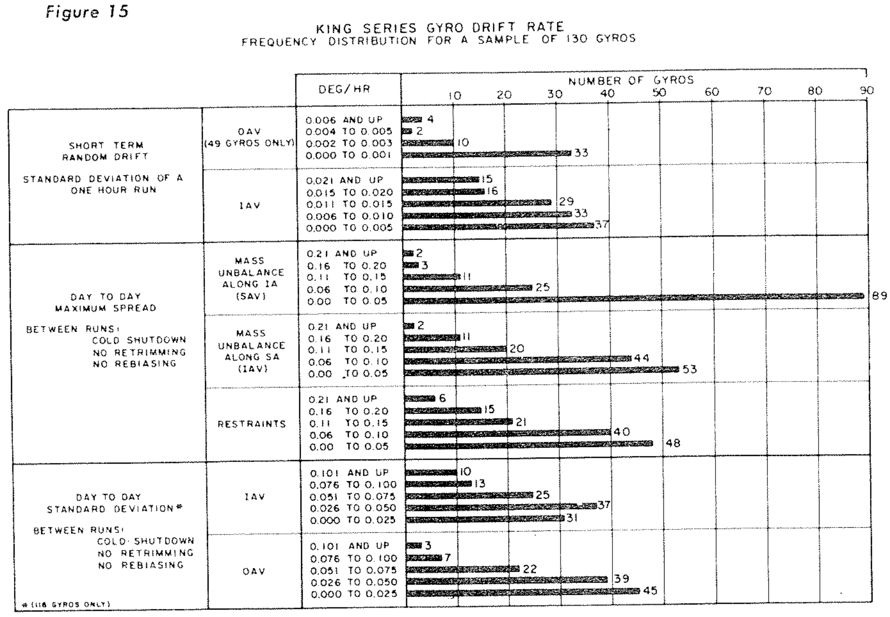


Figure 17

TUMBLE TEST

The tumble test is a quick and simple means for determining gyro drift coefficients.

In addition it also indicates adjustments necessary to enable the gyro to meet rigid specifications. Optimum flotation temperature, mass unbalance, and constraints are established simply and directly, providing a necessary service at nominal cost.

The test table is oriented such that its axis is parallel to the polar axis of the earth (see Figure 18). The gyro undergoing test is mounted with its output axis parallel to the table axis. Thus, no component of earth's rate can be sensed by the gyro. Hence, any gyro drift is due to the effects of gravity and constraints alone.

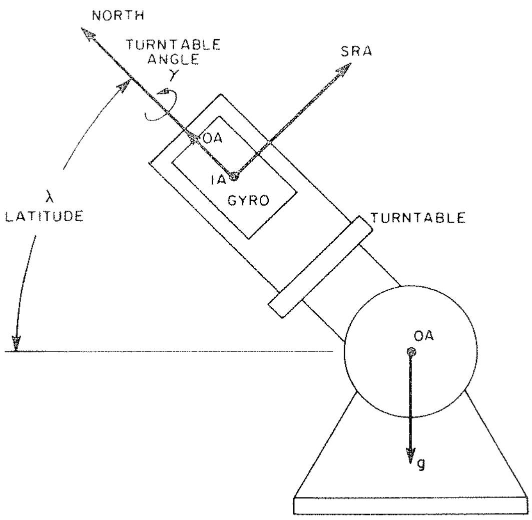


Figure 18

Unbalance resulting from a tumbling test may be expressed:

$$U_T = R + gU_{SRA}\cos\lambda\sin\gamma + gU_{IA}\cos\lambda\cos\gamma - Kg^2\cos^2\lambda \quad (56)$$

where:

- R = fixed restraints
- U_{SRA} = mass unbalance about spin reference axis
- U_{IA} = mass unbalance about input axis
- λ = latitude
- γ = turntable angle
- K = g^2 sensitive coefficient
- g = local gravity.

The gyro is captured by amplifying the pickoff signal and feeding it to the gyro torquer. Torquer current is fed to a recorder to which the gyro scale factor is applied. The table is driven at a constant rate usually equal to ten times earth's rate.

INFORMATION REVEALED BY THE TUMBLE TEST

Mass Unbalance — equal to the amplitude of the first harmonic. For example, when full scale is 2°/hr., drift rate due to mass unbalance is 0.2°/hr. Balance weights are adjusted to reduce this effect.

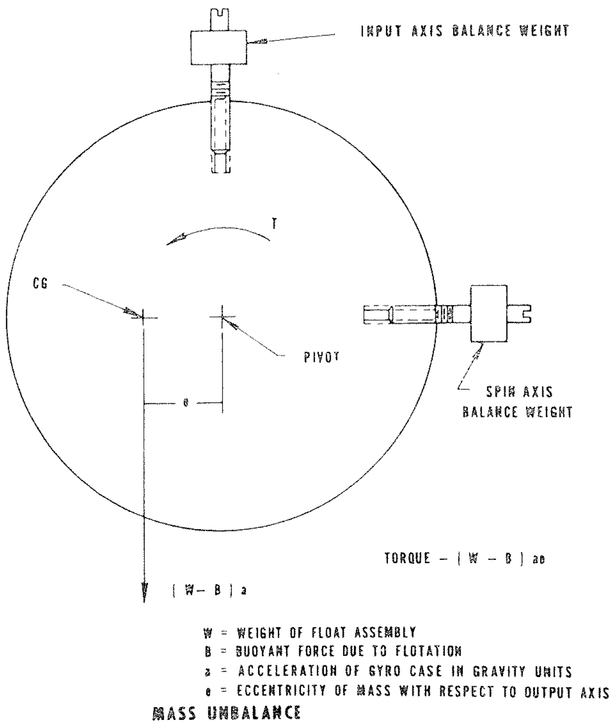


Figure 19

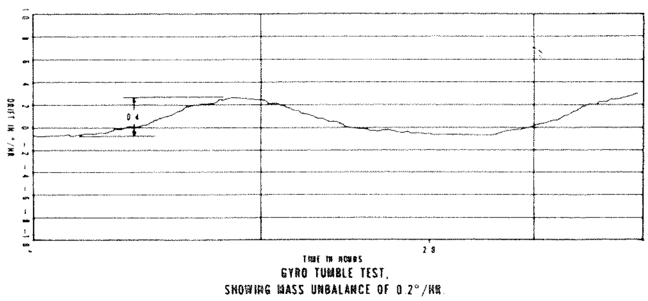


Figure 20

Constraints — equal to the displacement of the trace zero axis from that of the chart. A shift of 0.2 indicates a drift rate of 0.2°/hr. due to constraints. Flex leads can be adjusted to reduce this effect.

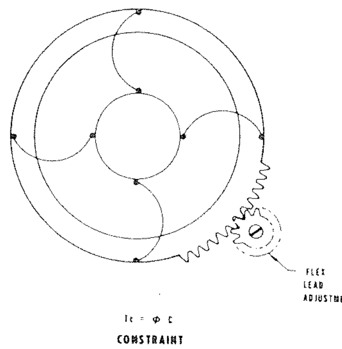
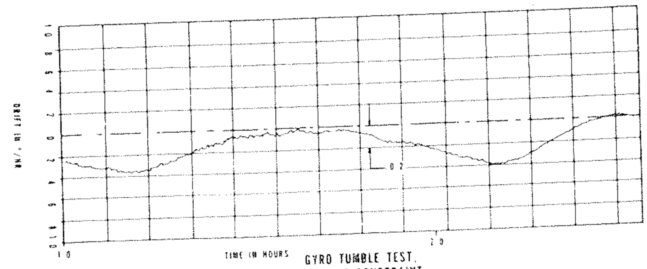


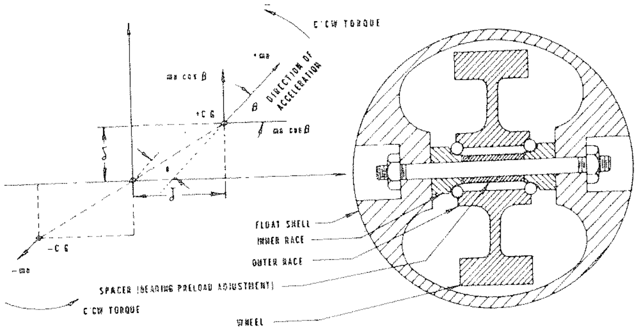
Figure 21



GYRO TUMBLE TEST.
SHOWING CONSTRAINT
OF 0.2 °/HR.

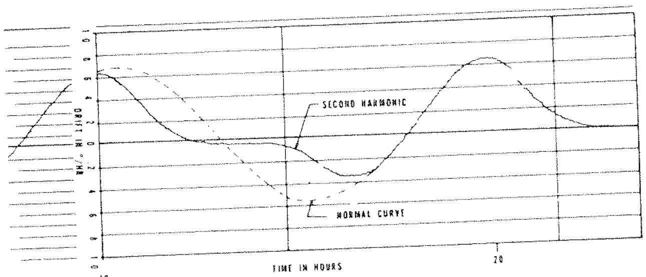
Figure 22

Anisoelasticity — evidenced by the appearance of the second harmonic. The effect is due to anisoelasticity associated with the bearings and float.



ANISOELASTICITY

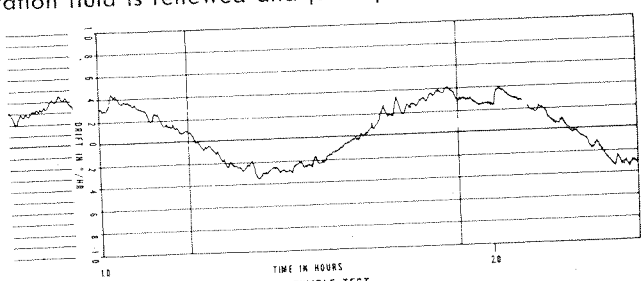
Figure 23



GYRO TUMBLE TEST.
SHOWING EFFECT OF
ANISOELASTICITY

Figure 24

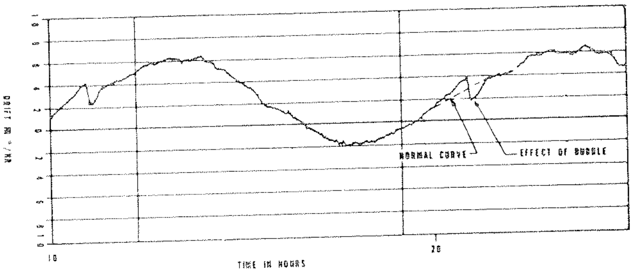
Friction and Binding — evidenced by somewhat erratic drift-rate pattern for short time. When this occurs many times during a one-hour cycle, it is attributed to dirt. Flotation fluid is renewed and jewel pivots are cleaned.



GYRO TUMBLE TEST.
SHOWING FRICTION
AND BINDING

Figure 25

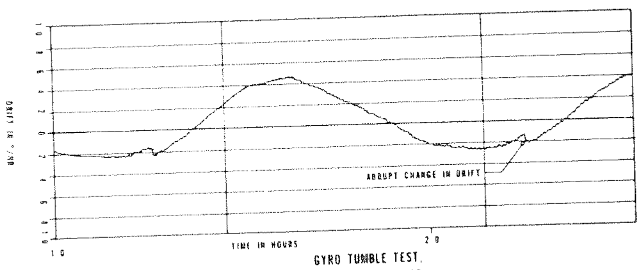
Gas Bubble — indicated by a single cyclic variation whose period is much shorter than one hour. This condition is manifested when the bubble, rising to the top of the fluid, must pass through the narrow opening offered by gyro stops or other discontinuity. Fluid is drained and replaced as a remedy.



GYRO TUMBLE TEST.
SHOWING EFFECT OF
GAS BUBBLE

Figure 26

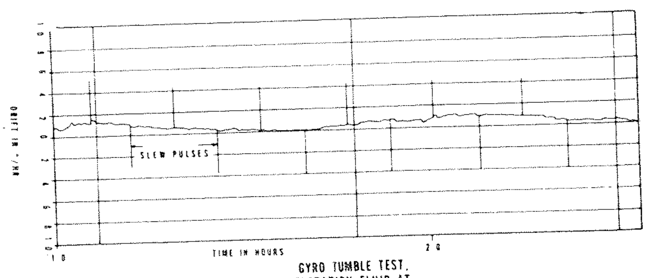
Mass Shift — displayed as an abrupt change in drift rate. The abrupt shift, which occurs once per cycle, indicates that a part may be loose or that bearings are not sufficiently tightened. The gyro is opened, inspected, and the cause remedied.



GYRO TUMBLE TEST.
SHOWING EFFECT OF
MASS SHIFT

Figure 27

Optimum Flotation Temperature — determined when the gyro drift rate follows the normal curve under the disturbing influence of periodic slewing signals. The ability to follow the normal curve indicates that flotation fluid is at the optimum temperature. Thermostat adjustment is locked at this setting.



GYRO TUMBLE TEST.
FLOTATION FLUID AT
OPTIMUM TEMPERATURE

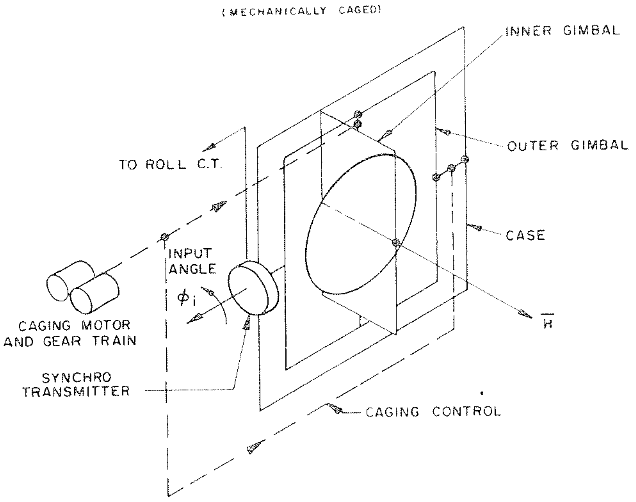
Figure 28

FREE GYROS

A free gyro is a two-degree-of-freedom gyro whose gimbal displacements about the output axes are a measure of input angular motion from a predetermined reference about corresponding axes, i.e., the roll axis motion is a measure of roll input. By application of the principle of gyroscopic inertia, the free gyro tends to maintain the orientation of the spin axis fixed in space. This instrument is so named because the spin axis may be set to any desired position rather than referenced only to earth's gravity or north. An inherent requirement for such use is provision for a caging mechanism which locks the rotor to the case until released.

This type of gyro is usually caged initially so that the spin axis, the inner gimbal axis, and the outer gimbal axis are mutually orthogonal. It has 360 degrees of angular motion about the outer gimbal axis and at least ±85 degrees of angular motion about the inner gimbal axis. To prevent "gimbal lock" which occurs at a 90-degree position of the inner gimbal, mechanical stops are utilized to restrict angular motion about the inner gimbal axis. However, if the rotor strikes the mechanical stops, "tumbling" results and reference is lost. Therefore, careful attention must be directed to initial orientation of the spin axis reference.

FREE GYRO
(MECHANICALLY CAGED)



LINE SCHEMATIC DIAGRAM
(ORIENTED TO MEASURE ROLL)

Figure 29

APPLICATIONS

Free gyros are used in those applications which require information indicating displacement from some specific reference attitude. The gyro is uncaged when the desired initial orientation is attained. Thereafter, deviation from the initial orientation is indicated by pickoff devices on the gimbal axes. Two common methods are used to reference the free gyro initially. The first method uses gimbal axis torquers to drive the gimbals to the proper orientation while the second cages the gyro by mechanical or electrical means and then uncages it for use.

To use a free gyro properly, two tenets must be recognized. First, a free gyro is a case-referenced device whose reference is not reset once the unit is uncaged. Secondly, the free gyro is a displacement device which indicates amount of input motion rather than rates.

Basically a short term device, a free gyro usually functions accurately for less than five minutes because of relatively high drift rates. A typical free gyro drift rate is 0.5 degree/minute. The instrument is commonly used in air-to-surface tactical missiles to avoid deviation from a predetermined flight path. It can also be used as a stabilization device which is able to sense deviation from desired positions.

DESIGN FEATURES

A typical Kearfott free gyro consists of a motor mounted in two gimbals supported by ball bearings, a single synchro pickoff on the outer axis, and a caging mechanism. This gyro has no torquer. Major components are described below:

Gyro Motor The gyro motor consists of a squirrel-cage motor excited by a 400-cycle, 3-phase source. Motors are available for either 26 or 115 volts and attain an operating speed of approximately 22,300 rpm in 30 seconds. The rotor is made from Mallory metal and is designed to achieve desired momentum while maintaining a favorable weight-to-inertia ratio. The rotor is mounted on permanently lubricated precision bearings mounted in a rigid housing.

Gimbals Gimbals are precision-cast units designed to give maximum stiffness and rigidity to the gyro so that it can withstand the high degree of vibration and shock to which it is subjected. Gimbals can be floated or non-floated.

Pickoffs Pickoffs are usually three-phase synchros located on the outer axis of the gyro, and have an output proportional to the sine of the displacement angle of the case about the axis. Excited from a 26 volt 400 cps single-phase source, various outputs can be obtained by slight modifications. Typical maximum open-circuit stator voltage is 11.8 volts, with a voltage gradient around a null of 0.208 volts per degree. Synchros can be supplied as control transmitters or control transformers.

Caging and Uncaging Mechanism The gyro can be returned to its original position with respect to its case by means of the caging mechanism. Excited by a 26 volt 400 cps single-phase source or a 28 VDC source, the remotely controlled mechanism can cage the gyro from any position within 30 seconds. The axes are caught and held by lever arms in a gear train which assures return of the gimbals to their original position within very close limits.

Means are also provided to uncage the gyro remotely for operation. Such uncaging is accomplished in 0.1 second and does not cause displacement of the outer axis by more than 0.1 degree or of the inner axis by more than 0.5 degree. The uncaging cycle may be energized from an external 28 volt DC source.

Incorporated into the caging train are two switches which provide a means for indicating externally whether the unit is caged, uncaged, or at mid-cycle. Torquer caging is also used. This replaces mechanical complexity with interacting electromagnetic fields.

Bearings Both gyro motor and gimbals are supported on bearings chosen to give minimum friction, resulting in low static drift rates. These bearings are designed to maintain their low friction level even after subjection to shocks as high as 50 g's and vibration up to 10 g's. A design assuring minimum anisoelasticity is used.

Case A cylindrically shaped cover, consisting of two parts, is soldered to the gyro case and provides a hermetic seal. The unit, containing one atmosphere of a dry inert gas, is unaffected by moisture, dust, etc., increasing life and reliability.

Electrical Electrical leads, none of which are common, and all of which are insulated from the case, are brought through the cover by means of a hermetically sealed multi-terminal header.

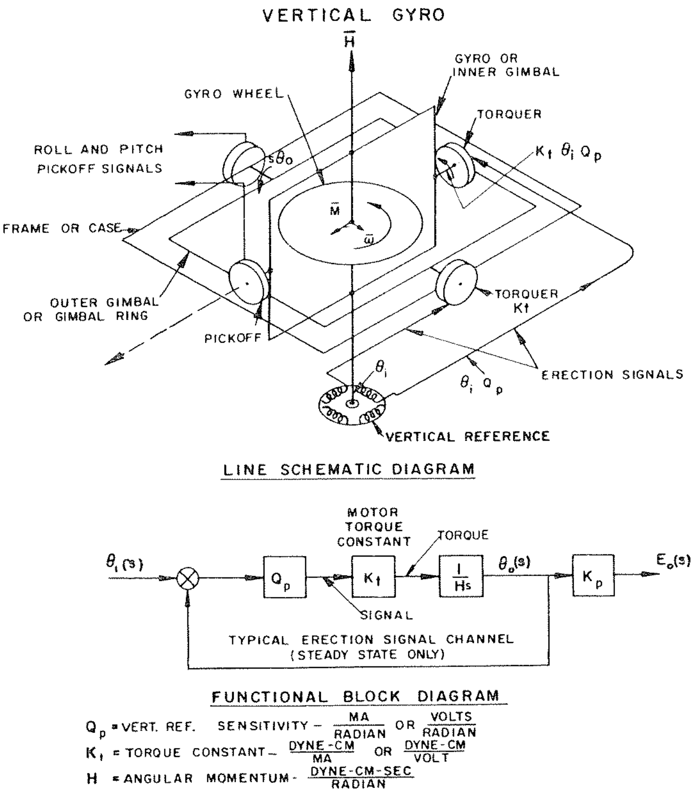
Mounting A typical mounting arrangement uses a mounting flange, located approximately at the center of gravity, with four holes for insertion of mounting screws, and a reference surface perpendicular to the spin axis. Special mounting conditions can accommodate a custom design.

VERTICAL GYROS

DESCRIPTION

A vertical gyro is a two-degree-of-freedom instrument whose gimbal displacements about each output axis constitute a measure of angular deviation from the local vertical axis. The rotor spin axis is maintained parallel to local gravity vertical by a gravity sensing device. The term *vertical gyro* derives from the fact that it is used to measure displacement from the vertical reference. A vertical gyro effectively serves the same purpose as a pendulum, with the advantage that a maneuver does not cause it to oscillate. This advantage is particularly desirable since attitude information is most needed during maneuvers.

A vertical gyro usually has 360 degrees of angular motion about the outer gimbal axis and ±85 degrees of angular motion about the inner gimbal axis. Restriction of motion about the inner gimbal axis is accomplished by mechanical stops which are necessary since ±90 degrees of displacement of the inner gimbal with respect to the outer gimbal causes "gimbal lock." Since roll motions of 360 degrees are more common than pitch motions of ±85 degrees, the conventional practice is to use the inner gimbal axis for measuring pitch and the outer gimbal for measuring roll. At the zero reference position, the spin axis, inner gimbal axis, and outer roll axis are mutually orthogonal to one another.



Q_p = VERT. REF. SENSITIVITY – $\frac{MA}{RADIAN}$ OR $\frac{VOLTS}{RADIAN}$

K_t = TORQUE CONSTANT – $\frac{DYNE\text{-}CM}{MA}$ OR $\frac{DYNE\text{-}CM}{VOLT}$

H = ANGULAR MOMENTUM – $\frac{DYNE\text{-}CM\text{-}SEC}{RADIAN}$

K_p = TAKE OFF SENSITIVITY – VOLTS / RADIAN

θ_i = INPUT ANGLE CAUSED BY VERT. REF. SWING – RADIANS

θ_o = OUTPUT ANGLE CAUSED BY GIMBAL DISPLACEMENT – RADIANS

Figure 30

APPLICATION

Vertical gyros have found numerous applications in the aviation, marine, or ordnance fields, where position relative to a gravity reference is essential to precise and accurate operation of many indication, control, and stabilization systems. They are also employed as control units for numerous panel-mounted indicating instruments which display roll and pitch attitudes. These control units may be integral parts of indication instruments or may be remotely positioned if the immediate space limitations are severe. Vertical gyros have found extensive use in antenna platform stabilization where a continuous vertical reference is necessary to provide accurate search and height-finding data and also where precise gunfire control is a necessity. Where an antenna stabilization system is involved, the vertical gyro may be mounted either directly on the stabilized element or located remotely. The vertical gyro has also been extensively used in airborne camera mounts, which must be precise during high-altitude operation.

When a vertical gyro is used in conjunction with a directional gyro in various autopilot control units, heading or course, roll attitude, and pitch attitude information are all available to make airborne autopilot operation possible. Vertical gyros are now being used as the primary control unit for many types of autopilots, and have been incorporated into the control systems used in helicopters.

The vertical gyro, relatively simple in construction, can be inexpensively produced in large quantities, and is sufficiently accurate to perform many operations. However, extreme accuracy requires a stable platform.

DESIGN FEATURES

In normal operation, three principal gyro axes are aligned as follows: the roll axis is aligned parallel to the vehicle's fore-aft axis; the pitch axis is parallel to the transverse axis, and the gyro spin axis is parallel to the vertical axis.

In Kearfott's vertical gyros the gravity-sensitive element is rigidly attached to the inner gimbal. Mounted along the roll axis and the pitch axis are transmitters which are sources of the intelligence obtained from the instrument.

Also mounted on the pitch and roll axes are torque motors which align the spin axis to gravity vertical on initial starting and which maintain this spin axis position during operation. Provision may be made for external torquing.

The gyro motor consists of a housing, flywheel (or rotor), and motor stator. Common design practice incorporates three desirable features into a gyroscope motor. These include low power consumption coupled with an angular momentum that is as high as possible within the limits of weight standards. The latter two features are often expressed together as a momentum-to-weight ratio (which should be as high as possible). Motors may be of several types (induction, synchronous, etc.). The synchronous, hysteresis type used in most Kearfott vertical gyros has a flywheel that is either symmetrical or umbrella-shaped, depending on the overall requirements and size and weight limitations.

The gravity-sensing element, or vertical reference mechanism, is a pendulous, liquid-damped device which may operate by electromagnetic or electrolytic means. Both types are equally reliable, but each differs basically in concept and in the accuracy obtainable. The electromagnetic type is the more accurate of the two and consists of an E-bridge circuit and a pendulous mass. The electromagnetic vertical reference mechanism requires voltage and power amplification to facilitate overall gyroscope performance.

An electrolytic vertical reference has a pendulous mass which is suspended in an electrolyte, making overall operation possible without external amplification.

The process whereby a vertical gyro is initially aligned to gravity is termed *erection,* and usually employs a gravity-sensitive device such as an electrolytic switch.

A vertical gyro is susceptible to the accelerations induced by turns of the aircraft in which it is mounted. Roll reference would be subjected to a centrifugal force of:

$$F = mV \frac{dR}{dt} \qquad (57)$$

where

m = vertical sensor reference mass

V = aircraft velocity

$\frac{dR}{dt}$ = rate of turn

Since a vertical sensor cannot distinguish between gravity and acceleration, errors in verticality will result unless some means for correction is provided. Turn error compensation can take two forms. The first is simply to provide a switch which disconnects the vertical reference during a turn. This is known as *erection cutout.* The other form of compensation maintains the vertical erection circuit but modifies the erection signal. One such scheme is called *pitch bank compensation.* It is based on the fact that it is usually only the roll reference which is affected during a turn. Accordingly when the turn is detected, the roll reference is disconnected and the roll erection system is controlled by the output of the pitch reference. Still another means of turn compensation is accomplished by initially building into the gyro an intentional inclination of the pitch reference, which in turn causes a corresponding inclination of the gyro spin axis. This compensation is accurate for only one bank angle.

Transmitters mounted on the pitch and roll axes to provide the intelligence output are of two basic types, and are selected according to the definite form of information required. Either synchro transmitters or resolvers are used. Synchro transmitters furnish an electrical AC signal output voltage proportional to the sine of the displacement angle, whereas resolvers provide dual output voltages — one proportional to the sine of the displacement angle and the other proportional to the cosine. Special applications may require a linear AC signal output or even a linear or functional DC output signal proportional to relative displacement angle. In the latter case, precision potentiometers are incorporated into the gyro design. Potentiometer devices, however, introduce friction torque caused by the sliding of a wiper arm in contact with a resistive element. Motors mounted along the pitch and roll axes to drive the gimbal elements to reference vertical are referred to as "torque" or "precession" motors. These components are essentially two-phase, squirrel-cage motors. One phase is excited with a fixed voltage of a certain phase and magnitude (the fixed phase) and the other phase is excited by the output signal from the vertical reference mechanism (the control or variable phase). Control phase excitation should be in quadrature with the fixed-phase excitation voltage.

Construction of the frame, outer gimbal, and inner gimbal should be such that maximum strength, rigidity, and thermal stability are provided.

In practice, Kearfott vertical gyros incorporate precision-cast inner and outer gimbals, and either die-cast aluminum frames or precision-cast steel frames. A complete, positive hermetic seal of the entire gyroscope is an advantageous feature which assures accurate performance under extreme environmental conditions.

Originally, rigid caging of gimbal elements was thought desirable, but experience has shown that with such a system the disadvantages outweigh the advantages. The complexity of such a caging system usually results in diminishing performance and reliability. Further, rigid caging may also be the cause of impressing large shock loads on ball bearings and other elements. The completely electrical erection system utilized in Kearfott vertical gyros has proved to be more than adequate as a replacement for a mechanical caging system.

TESTING

Certain performance features of vertical gyroscopes, when correctly determined and evaluated are definite and conclusive indications of gyro characteristics and the suitability of particular gyros to perform specific functions. These features are defined, and determination methods are described in the following paragraphs.

Free Drift is the term defining the rate at which the spin axis departs from its reference position when all erecting voltages are removed. Commonly expressed in degrees per minute, degrees per five minutes, or degrees per hour, free drift results indicate characteristics of friction, and static and dynamic balance of a gyro. To obtain these characteristics, a gyro moving in an oscillatory manner simulating aircraft or marine vessel motion (with allowances made for earth's rotation) is operated for a predetermined length of time, but without torque motor excitation. At the conclusion of a prescribed time interval, excursion from the gyro's initial reference position is observed, usually by noting roll and pitch transmitter output signals.

Vertical repeatability of a gyro is one of the most important features to be determined and is discovered by first establishing a vertical reference and then displacing the gimbal elements by electrical means. Precession is provided by torque motors. Gimbal elements are then allowed to re-erect under control of the vertical reference mechanism. Precession and erection are accomplished about roll and pitch axes and a reference position, together with four other erected positions, is noted for both roll and pitch axes. The extreme positions observed define the maximum limits of a cone. Gyro-established vertical is now assumed to be midway between the extremes, and the repeatability error is defined in terms of half-cone angle — the error being half the spread between the extreme settling positions. A test to determine repeatability characteristics is performed with the instrument subjected to an oscillatory motion simulating vehicle movements. Overall frictional and balance characteristics, and the accuracy designed and built into the vertical reference mechanism may all be determined by the test described above.

"Normal" and "fast" erection rates may be determined by measuring the length of time required for the pitch and roll elements to traverse a definite angular distance while known voltages are applied to the torque motors. Erection rates are usually expressed in degrees per minute, and serve to determine friction and balance characteristics and also to re-establish the torque motor features as they affect overall gyro performance.

Though a high erection rate results in good repeatability, it makes the gyro more susceptible to those dynamic errors which accompany aircraft accelerations. For example, during turns the pendulous erection device senses not only gravity, but centrifugal forces as well, and assumes a position which is no longer parallel to the gravity vector. In this case, the higher the erection rate, the more rapidly will the gyro be driven to this undesired "dynamic vertical." Thus, the selection of an erection rate in any particular application is a compromise between repeatability and dynamic error.

Naturally, any testing carried out to determine vertical gyro acceptance includes numerous additional tests. These consist of continuity and insulation resistance checks, and tests to establish that the electrical components are functioning properly and are providing the desired output information in the correct form.

DIRECTIONAL GYROS

A directional gyro is a two-degree-of-freedom gyro which indicates the motion of a flight vehicle in azimuth measured from a heading reference. This heading reference is established by the initial orientation of the gyro spin axis which is maintained level in a plane parallel to a tangent to the earth's surface and usually aligned to point to a north, magnetic or otherwise.

Gimballing serves the dual purpose of keeping the spin axis horizontal and aligned to north and indicating angular motion of the flight vehicle in azimuth as measured from the spin axis reference heading. A directional gyro has 360 degrees of angular motion about the outer gimbal axis and ±85° angular motion about the inner gimbal axis. The restriction of motion about the inner gimbal axis is accomplished by mechanical stops. These stops are necessary because ±90-degree displacement of the inner gimbal with respect to the outer gimbal causes "gimbal lock." Since gimbal stops are provided on the inner gimbal, the outer gimbal axis is used to sense yaw. At a zero reference position, the spin axis, inner gimbal axis, and outer roll axis are mutually orthogonal to one another.

A line perpendicular to a plane tangent to the earth's surface constitutes a vertical reference (disregarding earth's ellipticity and gravity anomalies). Hence a conventional vertical reference gravity sensor can be used to provide initial erection of the directional gyro. Torquers mounted on the gimbal axis provide a means for initial erection when controlled by the vertical reference, electrical caging when fed by a caging current, and directional slaving when fed by an external reference signal.

A pickoff is used on the outer gimbal axis to indicate azimuth and is usually an inductive type of component rather than a potentiometer, which causes undesired friction torque due to the contact of the wiper on the resistive element.

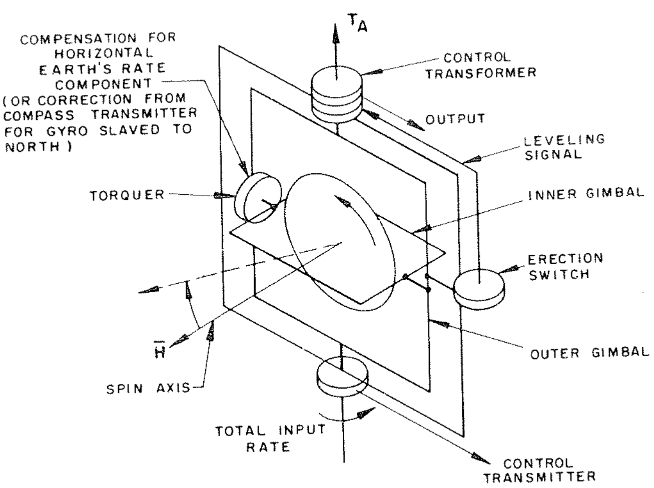
The three most common operating modes of a directional gyro are identified as *free*, *latitude corrected*, and *slaved*. A *free directional gyro* does not have torquers and is not slaved to any sensing device. Its spin axis accordingly acts as an inertial reference rather than an earth reference; (conversion to an earth reference device entails the use of external correction signals). This operating mode is used for navigation in polar regions where magnetic headings are likely to be erroneous. Input signals are usually inserted manually by the pilot.

The *latitude corrected mode* permits the gyro to be precessed by an amount sufficient to cancel out the effects of earth's rotation, resulting in a gyro whose spin axis is stationary with respect to the earth. Any constant heading then describes a great circle course.

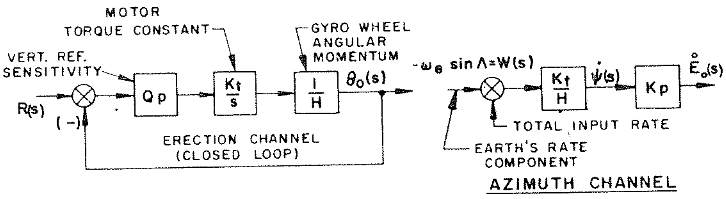
The *slaved mode* of operation uses an external reference such as an aircraft flux gate transmitter to provide a continuous azimuth torquing signal. This arrangement keeps the gyro spin axis aligned with the magnetic meridian. Specifically, this is accomplished by using the flux gate signal to torque the inner gimbal axis which precesses the gyro about the outer gimbal axis. Similarly, the erection system maintains a horizontal spin axis by torquing the outer gimbal axis which precesses the inner gimbal until the spin axis is horizontal to the earth. The gimbal axis pickoff measures the angular displacement between the gyro spin axis and the roll axis of the flight vehicle. In the slaved mode, the spin axis is aligned to magnetic north and the roll axis indicates aircraft heading. The pickoff signal indicates the heading of the aircraft with respect to magnetic north.

Directional gyros can be made which have interchangeable modes of operation. For instance, a disconnect switch can remove the torquer input signals to a slaved directional gyro which would then behave as a free directional gyro.

DIRECTIONAL GYRO



LINE SCHEMATIC DIAGRAM
(TWO AXIS GYROSCOPIC UNIT)



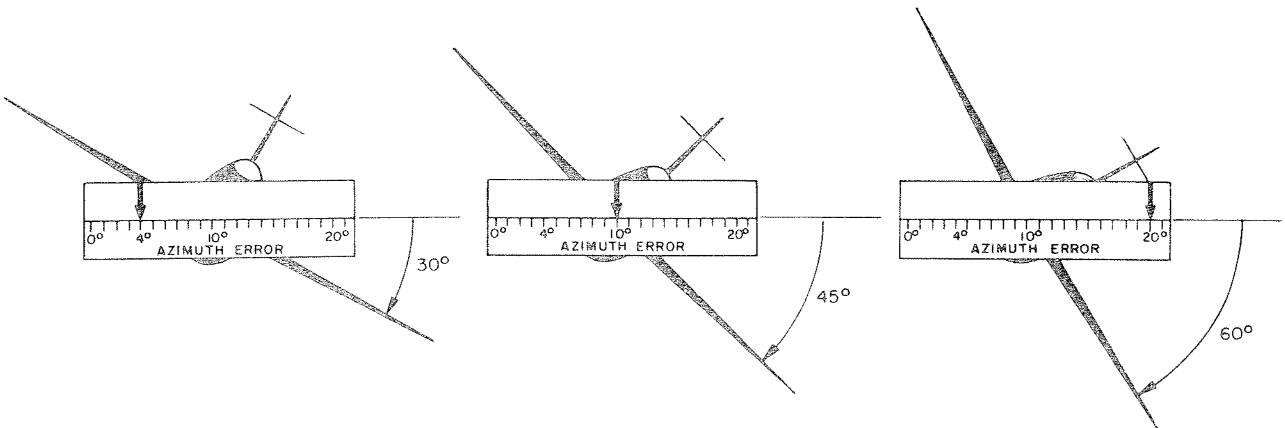
FUNCTIONAL BLOCK DIAGRAMS

R(s) = INPUT FUNCTION
Qp = VERT. REF. PICKOFF SENSITIVITY - MA/RADIAN OR $\frac{\text{VOLTS}}{\text{RADIAN}}$
Kt = TORQUER SENSITIVITY - DYNE-CM/MA OR $\frac{\text{DYNE-CM}}{\text{VOLT}}$
H = GYRO ANGULAR MOMENTUM - $\frac{\text{DYNE-CM-SEC}}{\text{RADIAN}}$
Kp = PICKOFF SENSITIVITY - VOLTS/RADIAN
W(s) = MA/RADIAN - COMPUTED SIGNAL SOURCE
$\mathring{E}_o(s)$ = OUTPUT RATE - CORRESPONDS TO VEHICLE'S RATE

Figure 31

ROLL STABILIZED DIRECTIONAL GYROS

Directional gyros may also be provided in a special configuration known as a roll stabilized directional gyro, which is designed to circumvent the form of gimbal error described earlier on page 3. Gimbal error is most severe when a conventional directional gyro's spin axis tilts at ±45° from the flight vehicle's roll axis. During roll of an aircraft using a conventional directional gyro, gimbal error results in an apparent change in azimuth which increases as the roll angle increases. Such apparent azimuth error reduces to zero only where the roll angle becomes zero. When integrated into navigational equipment, any change in azimuth resulting from gimbal error represents an undesirable permanent navigational error. Figure 32 shows the magnitude of azimuth error in a conventional directional gyro having its spin axis 45° from the aircraft's longitudinal axis for roll angles of 30°, 45°, and 60°. Figure 33 is a plot of azimuth errors relative to spin axis orientation versus roll angles.



MAXIMUM AZIMUTH ERROR DUE TO ROLL ANGLE IN A TYPICAL TWO GIMBAL DIRECTIONAL GYRO
SPIN AXIS ORIENTED AT APPROXIMATELY 45° TO THE LONGITUDINAL AXIS OF AIRCRAFT.

Figure 32

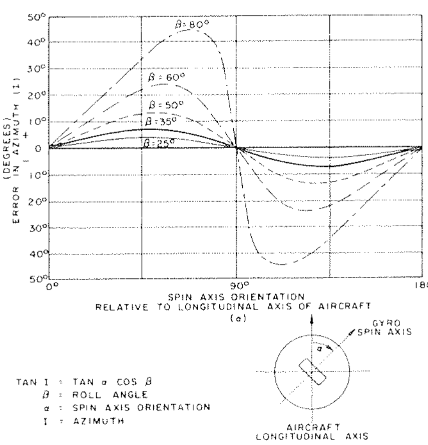
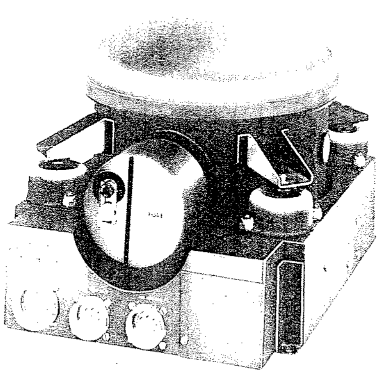


Figure 33

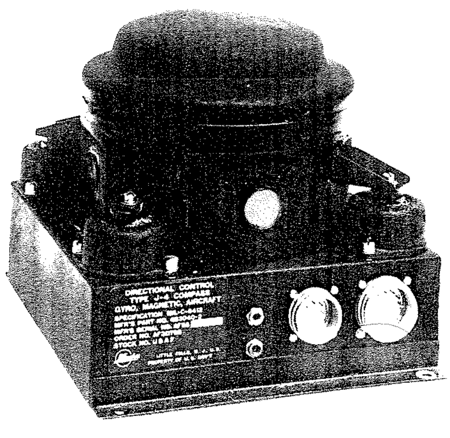
To eliminate roll-produced gimbal errors and consequent apparent azimuth errors, the roll-stabilized directional gyroscope proves most useful. This type of gyro is essentially a conventional directional gyro mounted within an additional outer gimbal that is free to rotate about its roll axis. Usually controlled by a position servo slaved to the roll signal transmitted by a vertical gyro, the extra gimbal maintains the directional gyro's azimuth axis such that it is always parallel to local vertical through 360° of roll rotation.

The USAF J-4 Directional Gyroscope, a typical Kearfott-designed and built instrument, is provided in both conventional and roll stabilized versions.

ROLL-STABILIZED DIRECTIONAL GYRO



J-4 DIRECTIONAL GYRO



DRIFT PHENOMENA

Gyroscopes considered in this publication are instruments used to provide references for navigational, guidance, or stabilization purposes. A reference must be either inherently invariable or predictably variable. Hypothetically, an ideal gyro remains immobile in inertial space. However, actual gyroscopes do not maintain absolute immobility but deviate from their initial fixed position. The rate of deviation is a measure of gyro performance: the lower the rate the better the gyro. "Gyro drift rate" is the best and most important single figure of merit used to describe the performance of a gyroscope. The discussion which follows has two distinct facets, namely (1) nature and source of gyro drift and (2) description of gyro drift.

Since all motion is relative, displacement must always be referred to some initial point which is assumed to be fixed. Although the earth is assumed to be the fixed reference for most conventional purposes, this assumption cannot be accepted for a gyroscope. A gyro used as a space reference and viewed by an observer on earth appears to be drifting, although in reality it is the earth that is rotating about its axis. This particular form of drift is known as Earth's Rate Drift.

Since the earth rotates about its axis 360 degrees every day, its rate of rotation is 15 degrees an hour. However, the gyroscope is an inertial device, and it is therefore necessary to consider the rotation of the earth with respect to inertial space rather than in relation to the sun. Any given point on the earth faces the sun 365 times a year. But the earth also revolves once about the sun in the same time, which means that the point faces the fixed stars of inertial space 366 times. This motion, known as the sidereal rate, is slightly greater than one turn per day, being 1.00274 turns in each 24-hour period, or 15.0411 degrees per hour. It is this value for sidereal earth's rate which should be used in gyroscopic computations. Usually a gyro is located in a plane parallel to a plane that is tangent to the earth's surface. Therefore, depending on the orientation of its spin and input axes, a gyro senses various components of the earth's rate as an angular input as shown in Figure 34.

This angular input appears when the gyro case is fastened to a bench and rotates with the earth.

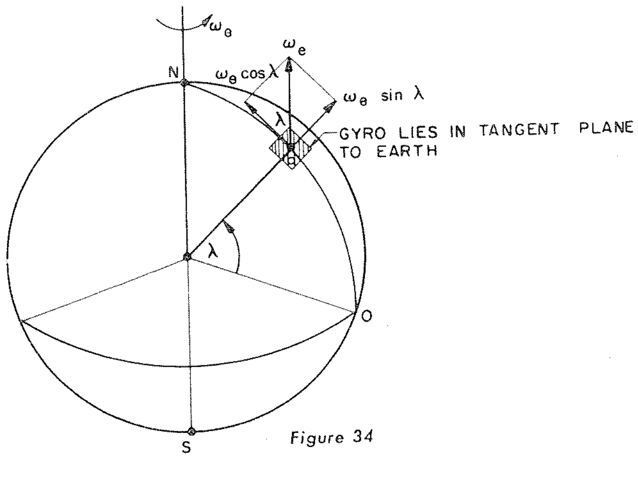


Figure 34

For the azimuth-gyro orientation, the azimuth earth rate component (refer to Fig. 34) is the input.

In Figure 34

- ω_e = earth's rotation rate
- λ = latitude of gyro location
- ϕ = heading angle measured clockwise from North
- $\dot\phi_a$ = input rate-azimuth = $\omega_e \sin\lambda$
- $\dot\phi_r$ = input rate-roll = $\omega_e \cos\lambda \cos\phi$ (spin axis north for $\phi = 0$)
- $\dot\phi_P$ = input rate-pitch = $\omega_e \cos\lambda \sin\phi$ (spin axis east for $\phi = 0$)
- θ = precession motion

Using rate integrating gyroscopes for vertical gyro orientation with the input axis pointing north, the gyro senses the earth's rate component in the tangent plane. Then:

$$\theta_V = \frac{H}{D}\,\omega_e \cos\lambda \sin\phi \qquad (58)$$

For azimuth gyro orientation, the gyro senses earth's rate in the tangent plane as:

$$\theta_a = \frac{H}{D}\,\omega_e \sin\lambda \qquad (59)$$

When the gyro is mounted in a pivoted fixture such that it is not forced to rotate with the earth, it remains stationary with respect to inertial space, and appears to be rotating with respect to the earth. This "apparent" drift is equal to the component of earth's rate along the gyro input axis.

Note the difference. When the gyro is fixed to the earth, it precesses at a rate equal to $\frac{H}{D}$ times the component of earth's rate. When it is mounted in a pivoted fixture the gyro is free, and *appears* to be rotating at a rate equal to the component of earth's rate, while the pick-off signal is zero. Both of these effects are defined as drift. The first induces gyro precession, and the second appears when the gyro functions as an inertial reference.

GYROCOMPASSING

The gyrocompass is a gyroscopic instrument which senses the earth's rotation and utilizes that information to precess the gyro until its spin axis lies in the earth's axial plane. Such an instrument is distinctive in its ability to indicate true geographic north independent of the earth's magnetic field.

A north reference device can utilize the earth's apparent drift, for when a single-degree-of-freedom gyro is oriented so that its input axis lies in a plane tangent to the earth's surface, its apparent drift varies with heading. When the spin axis is colinear with the horizontal earth's rate component, drift is zero. When the input axis is colinear with the earth's rate component, drift is equivalent to full earth's rate. Hence, zero drift occurs when the gyro's spin axis points north.

The input rate $\omega_i = \omega_e \cos\lambda \sin\phi$ is the same for both single-degree-of-freedom and two-degree-of-freedom gyros.

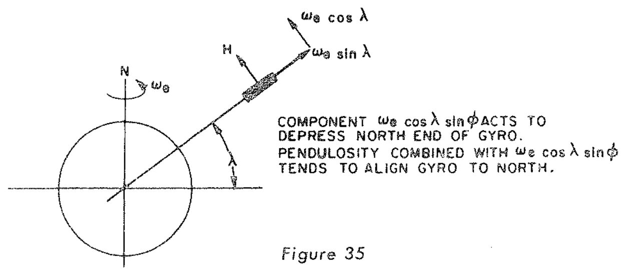


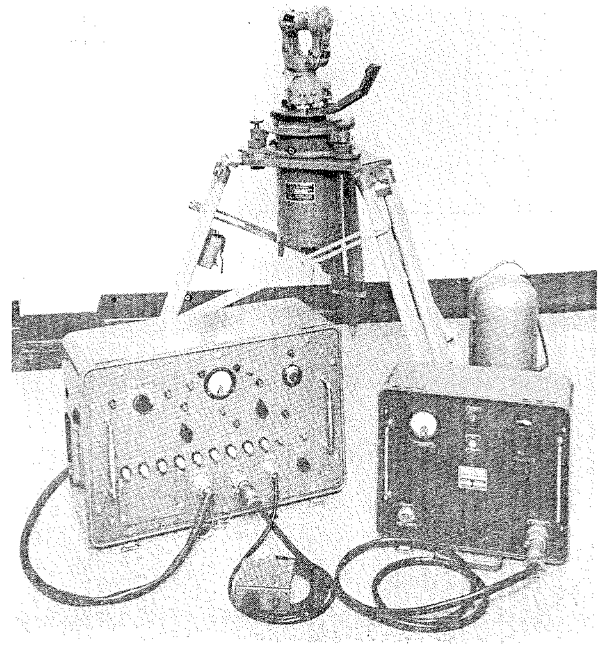
Figure 35

Hence, it is apparent that a gyrocompass may be instrumented in a variety of forms, all of which employ the same principles. The pendulous gyrocompass was invented by Dr. Hermann Anschütz-Kaempfe in 1906. Since then the term "gyrocompass" has been expanded to include a variety of north-indicating devices. The pendulous gyrocompass consists of a spinning rotor and a mass unbalance such that the equilibrium position of the gyro spin axis under its combined effects of gravity and earth's rate tends to be horizontal, pointing to geographical north. To produce a gyrocompass, an additional mass is added to the bottom of a directional gyro's rotor so that it effectively becomes pendulous. While the spin axis remains horizontal, the unbalance has no effect, but any spin axis deviation from that plane moves the weight, which in turn causes a torque about the tilt axis. Earth's rate input causes spin axis motion, which is then corrected by the pendulous mass to keep the gyro in a north-seeking position. Mathematical analysis shows that pendulous gyros oscillate. Therefore, damping must be introduced into actual designs.

A more sophisticated method utilizes the gyro as part of a null-seeking servo loop in which the gyro's drift produces the error signal controlling the loop. A more detailed description of this technique will be found on page 45 of this book in the section dealing with stable platforms. Gyrocompasses require an accurate knowledge of northerly velocity and are therefore usually limited to use in vehicles traveling at relatively low speeds, such as those associated with surface vehicles and ships. Gyros having exceptionally low drifts are required for in-flight gyrocompassing.

Portable gyrocompassing instruments have been developed which can determine north with a degree of accuracy comparable to that obtained by means of star observations, and are commonly used for establishing base lines and heading data for mobile missile units. Kearfott's GYTAR (Gyro Theodolite Azimuth Reference) system is such a device, and can rapidly orient to north with an accuracy of less than 15 arc seconds.

An accurate gyro, mounted beneath a theodolite such that its input axis is horizontal and lying in the same vertical plane as the theodolite telescope axis, detects a component of earth's rate motion. Both gyroscope and theodolite are coupled to rotate together. The earth's rate component is proportional to cos λ sin ϕ, where λ is latitude angle and ϕ is the angle between the gyro input axis and an east-west line. When the input rate is zero, the gyro's input axis lies on an east-west heading.



GYTAR

DRIFT INPUTS

In addition to the apparent drift caused by earth's rate, drift rate is a consequence of spurious and undesired torques which cause motion. Systems using gyros cannot distinguish between precession which results from a motion of the flight vehicle and that from an unbalance moment, anisoelastic moment, or flight trajectory.

Certain errors, which must be recognized and compensated for when using a gyroscopic reference, are caused by apparent motion resulting because a gyro is an inertial reference on or above an earth that is moving continuously in relation to inertial space. Since these types of error are independent of gyro design or construction, they would be present even if a theoretically perfect gyro were used. Vertical gyros and stable platforms are particularly susceptible to such errors because they employ vertical sensors which are inherently incapable of distinguishing between gravity and acceleration. Indeed, the development of an instrument which could distinguish between gravity and acceleration would revolutionize navigational instrumentation, to say nothing of the effect that such a discovery would have on our knowledge of physics.

Turning errors, discussed earlier under "Vertical Gyros," are centrifugal and Coriolis force phenomena for which compensation must also be made when navigation relative to the earth is considered. The latter, named for the French engineer and mathematician Gaspard Gustave de Coriolis, can be defined as an acceleration affecting any particle moving over a rotating surface.

Since the earth rotates, an object on its surface is always subjected to a centrifugal force whose magnitude may be expressed as:

$$F = m\omega_e^2 R \cos\lambda \qquad (60)$$

where R = earth's radius
m = plumb mass
ω_e = earth's rotation
λ = latitude

A vertical established by a stationary plumb bob is really a resultant of this centrifugal force and the earth's mass attraction. However, should the plumb bob move over the earth's surface, the effect is that of a change in the earth's rotational velocity, for which a correction must now be applied. Referring to Figure 36, in which the earth's axis passes through the geographical poles, resultant motion of the plumb mass, m, is referenced to the east-west north-south grid established by latitude and longitude lines. In a rhumb-line flight from west to east, the resultant angular velocity of the plumb bob about the earth's axis relative to inertial space becomes:

$$\omega_e + \frac{V_E}{R \cos \lambda} \qquad (V_E = V \sin \phi) \qquad (61)$$

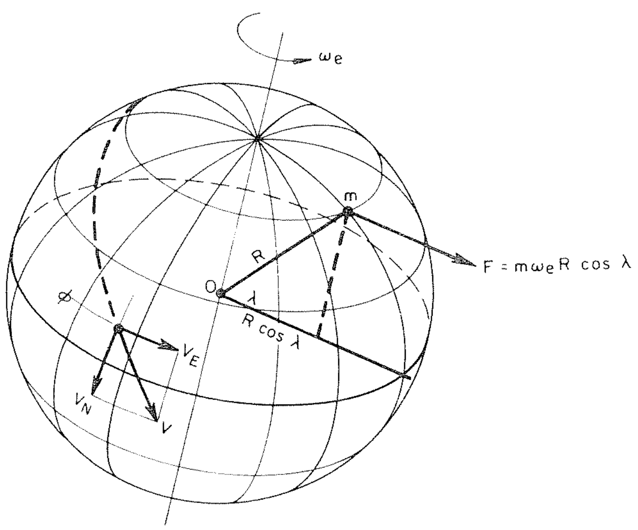
A rhumb-line path is one which crosses all meridians at a constant angle. Referring to equation (60), centrifugal force now becomes:

$$F = m\omega_e^2 R \cos \lambda + 2m\omega_e V_E + \frac{mV_E^2}{R \cos \lambda} \qquad (62)$$

where the first term is earth's rotational centrifugal force cited in equation (60), the second term is Coriolis force, and the third is additional centrifugal force introduced by plumb bob velocity. The Coriolis effect is a consequence of the earth's motion relative to the flight vehicle when both are referred to inertial space.

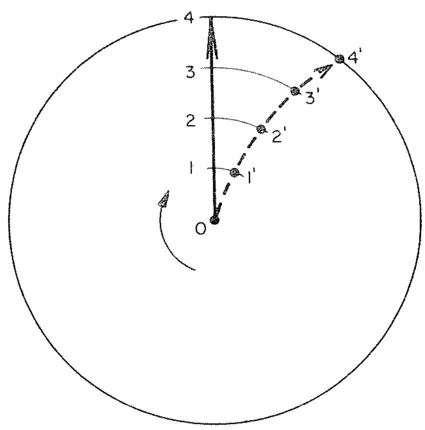
If the earth did not rotate, a vehicle flying a straight ground course would also fly a straight-line path in space, and no Coriolis correction would be required. But since the earth turns constantly, the flight vehicle must continuously change its velocity and heading if it is to maintain a straight ground track on the earth's surface. Because any change in velocity is an acceleration, and the product of acceleration and mass is a force, the flight vehicle is subjected to the apparent force described earlier as the Coriolis effect. When an accurate vertical reference is desired, or whenever a navigation computer is used in conjunction with a stable element, correction for this Coriolis effect must be made. Since Coriolis effect is manifested as an acceleration, it is customary to provide the correction to the vertical reference of a vertical gyro, and to the accelerometers of a stable platform.

A simple example serves to illustrate clearly just how important the Coriolis effect is. Suppose a vehicle having no Coriolis correction in its guidance system were launched at an average velocity of 5300 ft/second from the North Pole and initially aimed toward New York. During the vehicle's flight of about one hour, the earth will have rotated about 15 degrees (a distance of approximately 900 miles at the latitude of New York). Consequently, since no Coriolis correction was made during the course of the flight, the vehicle would strike the Chicago area rather than the New York target originally selected.



CORIOLIS EFFECT AND CENTRIFUGAL FORCE

Figure 36



DOTTED LINE IS TRAJECTORY REQUIRED TO CORRECT FOR CORIOLIS IN TRAVELING FROM 1 TO 4.

Figure 37

DESCRIBING DRIFT

Gyro drift is usually measured in terms of drift rate, which is expressed as a measure of displacement per unit time.

The most widely used expression for gyro drift is degrees per hour, although the MIT Instrumentation Lab has devised an alternate unit of drift called a "meru." This is a milli-earth's rate unit or 0.015 degree/hour. Since rate is displacement per unit time, the interval of time over which drift is measured assumes rather great importance. To describe gyro drift rate adequately, a corresponding time interval should be stated. The average rate of a gyro over some specified interval is sought. An example is shown by the curve in Fig. 38 which represents the drift performance of a gyro. The time intervals ab, a'b' and a"b" are all the same, yet there are three different rates; for a'b' the rate is zero; for a"b" and ab, the gyro would not conform to specification. For a large time interval t, the displacement is only d" from the initial position and is equal to the displacement c". Yet for the same displacement d", the two different times, a"b" and t give vastly different rates. There is no single best way to describe gyro drift. Instead a variety of methods are considered which permit a realistic appraisal of gyro performance.

Since time is the critical parameter, one means of categorizing drift is by establishing "short term drift rates" and "long term drift rates." "Short term drift" is defined as drift measurements made for a one-hour interval of continuous operation. This is the time interval for the test run and is independent of the manner in which the data is presented. That is, the data may be the standard deviation drift rate, maximum drift, or integrated drift.

Similarly, "long term drift rate" is defined as drift measurements made for an eight-hour interval of continuous operation.

Since attention must also be directed to the change in drift between successive tests, a gyro remains inoperative for the length of time required for cooling before it is restarted. Then if a change in drift occurs, it is usually a change in mean drift level rather than a change in random error. Two methods are employed to describe this change. One specifies that the total spread of values for a stated number of runs is included within a stated range. A more useful method defines "day-to-day drift rates" as the change in mean level of "long term drift rates" taken on successive days with a cooling period in between.

The above standardized but arbitrary time-interval definitions of gyro performance do not account for unusual situations which may require distinctive tests. For instance, in the case of a missile with an abbreviated flight time, drift rate is often specified over the anticipated duration of flight.

Having established the time interval for tests, the manner of describing performance remains. One method produces the integrated drift, which is the total drift angle for the test interval, i.e., the final position minus the initial position (excluding earth's rate). This value represents the integral of instantaneous drift rate with respect to time. Dividing the integrated drift by the test interval gives the "integrated drift rate." If the random drift is equally distributed about the mean during the test interval, the integrated drift rate equals the mean drift rate due to the averaging of the random portion. Fig. 39 indicates that the integrated drift is very sensitive to the time chosen. For time t', integrated drift is d', five times as great as d, which yields an integrated drift rate 7.5

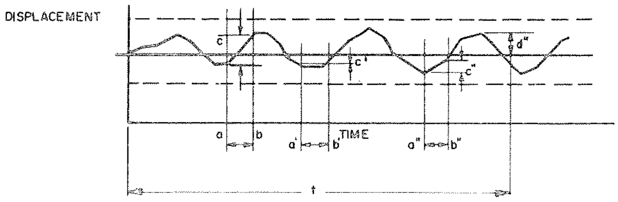


Figure 38

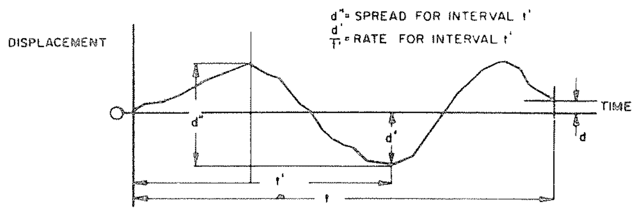


Figure 39

times as large. The random behavior of a gyro causes such an apparent variation. The mean drift rate is usually a result of mass unbalance and constraints. The mean or bias level is controlled by an appropriate mechanical or electrical counter-bias.

A mathematical technique, less sensitive to infrequent variation and more representative of the drift performance of the gyro, is needed.

It has already been stated that the drift performance of a gyro is compounded of two constituent elements, a bias level, and a portion due to random inputs. The random effects arise from a number of sources acting singly or in combination. Random inputs may be caused by non-uniformity of bearing surface, ellipticity of balls, minor local bearing contamination, thermal stresses, external disturbances, non-linear flex lead response, fluctuation of damping fluid viscosity, etc.

Random behavior is a common occurrence in nature. The most obvious characteristic of random behavior is that it is non-periodic. A sinusoidal phenomenon is described completely at any instant by knowledge of its past behavior. However, knowledge of past behavior of random phenomena can be used only to predict the relative probability of behavior, but it cannot predict the exact behavior at any given instant. Paradoxically, random behavior is not erratic but observes definite mathematical rules. These mathematical tenets are applicable to random behavior in general and as such can be applied to the analysis of gyro drift rate data.

Among the most common parameters used to describe random behavior is the random deviation identified by the Greek letter σ (sigma). Standard deviation is best defined in terms of variance, which is a measure of the spread or dispersion about the mean value. The standard deviation is defined as the square root of the variance, also known as the root mean square.

The standard deviation is a measure of the dispersion of the random drift rate about the mean, μ, and is expressed as the root mean square of the deviation of all the readings about the mean. The standard deviation is given by:

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{i=n} (x_i - \mu)^2} \qquad (63)$$

where:

n = number of readings or points on curve used for this computation

x_i = drift rate at point i

μ = mean value of drift rate

σ = standard deviation (rms) from the mean in a set of data having Gaussian distribution

The term *standard deviation* implies that random deviations from the mean follow Gaussian distribution. Examination of actual drift data reveals that this is not quite the case. However, a fair approximation can be made.

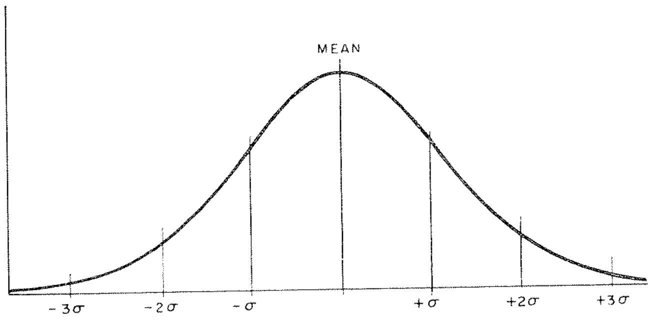


Figure 40

The curve in Fig. 40 represents the probable density of readings or points following normal or Gaussian distribution. Called the normal density function, the area under any portion of the curve represents the probability that a given reading will lie between the two limits bounding the area. The theoretical curve has a maximum spread of infinity, while 99.7% of the area lies between $\pm 3\sigma$ from the mean. Since the maximum spread for reliable gyros lies between 2σ and 3σ the 0.3% outside the 3σ limits is considered negligible. (The assumption is made that a Gaussian distribution is in error by much more than 0.3%). The significance of standard deviation σ, is that 68.3% of a large number of drift rate readings (or points on a curve) will not exceed this value, 95.4% will not exceed 2σ, and 99.7% (actually 100% since selection and rejection of gyros eliminates the possibility of exceeding this value) will not exceed 3σ. 50% of the readings will fall in the range of $\pm .674\sigma$.

If a gyro is permitted to drift freely, its drift rate should be equal to the earth's rate component parallel to the component. A deviation from this rate is attributable to gyro errors. An analysis of such data is expressed in terms of a mean value and standard deviation about the mean. A maximum spread value is also used which also will not exceed 3σ. The significance of maximum spread is dependent upon the use to which the gyro is put. By definition, maximum spread value is reached infrequently and for short intervals, and therefore the standard deviation value is usually employed to specify gyro drift performance.

Root mean square has been defined earlier. Another expression used to describe a number of errors taken in combination is called the root sum square, which is expressed:

$$\sigma_{rss} = \sqrt{\sum_{i=1}^{i=n} \sigma_i^2} \qquad (64)$$

Since deviation of samples from a mean is required when describing a component parameter, it is possible that when errors from various sources are combined, one may be so small that it does not tend to reduce the effect of a large error source. Hence, it is the probable sum of the effects which is sought. In making this calculation, care should be exercised to assure that constituent errors are compatible, i.e., all are 1σ or 3σ, and that their distributions are Gaussian.

Another technique is utilized to describe random behavior. This technique, known as autocorrelation, varies with time and is quite suited for analysis of gyro drift data.

Gyro drift has been discussed heretofore only in terms of amplitude, but it may also be considered in terms of frequency, a point of view which is less common, but one which is growing more important wherever it is necessary to discover gyro drift effects on system performance. Determination of the time required to measure gyro drift so that its steady-state value may be discovered depends upon frequency analysis.

In considering a typical graph of gyro drift amplitude versus time, an analysis of its frequency components may be made through appropriate application of Fourier analysis. Such a graph may be re-plotted to show drift amplitude as a function of frequency, but a more common presentation shows the square of the amplitude plotted for each frequency, a graph that is known as a power spectral density curve and which is widely used in communications work.

Fourier analysis of an actual gyro drift record is a tedious undertaking, particularly since the LaPlace transform of a power spectral density curve, called the autocorrelation function, is simpler to obtain. Although it contains the same information as a power spectral density curve, this function presents it in terms of period instead of frequency. A power spectral density curve frequently contains terms of the form $\frac{1}{1 + ts}$, (where s is the LaPlace operator), because time constants are usually found in either the gyro or the recording system. Their presence leads to terms of the form in the autocorrelation plot, so that such diagrams appear typically as decaying exponentials. Time constants associated with these exponentials are termed autocorrelation times. In addition, a power spectral density curve may well show a predominant frequency which changes to a sine wave modulation of the exponential autocorrelation plot. Typical curves are illustrated by Figures 41 and 42.

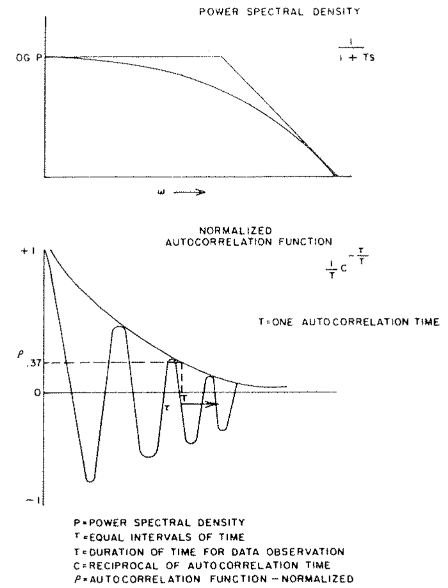


Figure 41

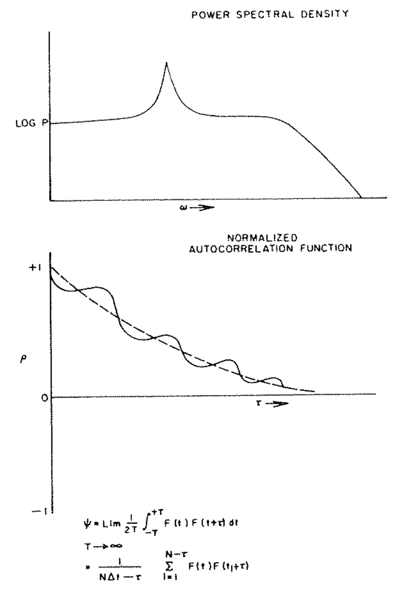


Figure 42

Using a digital computer to determine the autocorrelation function, a gyro drift record is plotted in the form of discrete drift amplitudes versus time. Gyro drift angle readings may be taken, for example, at convenient one-minute intervals and recorded on punched cards. The value on each card is then multiplied by that appearing on the card immediately succeeding it, after which an average of all products is determined to yield an autocorrelation function for a one-minute period of time. To establish the next point in the diagram, the value on each card is multiplied by that taken for each successive two-minute interval. Repeating this operation for three-minute, four-minute, five-minute, etc., intervals establishes additional points to complete the curve. Any moderate-speed digital computing machine can perform these operations rapidly, once the cards have been produced, and the equation describing them may be expressed as

$$\psi = \frac{1}{n\Delta t - \tau} \sum_{i=1}^{i=n-\tau} f(t)f(T_i + \tau) \qquad (65)$$

where

τ = correlation interval

and

t = total elapsed time during which ψ is evaluated.

Autocorrelation time, which should be as short as possible, varies in different gyros from as little as one minute to as much as four hours, but a value of about twenty minutes is most typical.

Use is made of the autocorrelation function, for example, whenever a gyro is an integral part of a missile guidance system. Commonly, the characteristics of such a gyro are such that gyro biasing is necessary before each flight, and the principal factor which must be determined is the length of time during which gyro drift must be measured to arrive at a reliable bias value. Plainly, average drift must be measured over a period of time sufficient to minimize errors caused by random drift fluctuations, and it is equally apparent that when gyro autocorrelation time is short, contributory long-period drift components are correspondingly low, while drift averaging time is short. 68.3% of random drift fluctuations is eliminated through averaging when drift is measured over one autocorrelation time. Should drift measurements be taken over two autocorrelation times, the percentage increases to 95.4%. Measurements made over additional autocorrelation times increase the percentage of accuracy still further.

Averaging of gyros in inertial guidance systems originally required as much as thirty six hours. This time has steadily decreased to a present practical averaging time on the order of ten minutes for most gyros in production today. Some of the more modern gyros equipped with gas-lubricated spin axes will undoubtedly display exceedingly short correlation times, thus permitting very rapid drift measurement and gyro biasing.

DRIFT ERRORS

Drift errors are undesired drift motions caused by design limitations or constructional deficiencies within a gyro. Such errors are in contrast to the apparent drifts described earlier, which would appear even if a theoretically perfect gyro were used.

Drift errors in single-degree-of-freedom gyros arise from many sources. The most common ones are categorized as those acting about the precession axis and those acting about the input axis. As discussed elsewhere in this publication, precession-axis and input-axis sources produce the same apparent effect.

Precession-axis drift sources are those which are inherent parts of the gyro design, i.e., the precession axis supports, electrical flex leads, mass unbalance, and electrical reaction torques. Each of these is considered in succession.

Ideally, the gyro precession axis should be supported by a frictionless device. Practical necessity dictates acceptance of a torque that is as low as possible; therefore, specially designed instrument-type ball bearings are the supports most commonly used. Bearing torque levels of less than 10 dyne-cm are readily obtainable. Bearing friction is complex in nature and depends upon many factors. Relative motion takes place between the inner race, outer race, retainer, and balls, all of which contribute to the overall friction level. The relative smoothness of rubbing parts determines to a large extent the amount of friction. Other contributing factors include lubricant viscosity drag, lack of perfect symmetry of balls and races, type of retainer and contamination of bearings by foreign matter.

In addition to bearings, taut wires find use as suspension devices, but care must be taken to eliminate any torsional bias in the wire. A residual torsional bias manifests itself as a directional torque, i.e., drift in one direction is greater than that in the other.

In gyros employing fluid flotation suspension, jeweled pivots are used to assure low level friction torques. Jewels are usually synthetic sapphires, although other jewels, including diamonds, have found use in special designs. Flotation fluids may be either liquid or pressurized gases.

A gyro must be very accurately balanced about its precession axis. If it is unbalanced, the product of the unbalanced mass and the perpendicular distance from the precession axis causes a torque about the precession axis which manifests itself as drift. Mass unbalance drift is particularly important in cases where the gyro precession axis lies in the horizontal plane. However, mass unbalance may be overcome by use of appropriate balance weights. Balancing accuracy is limited by the measuring instruments and the normal static friction threshold. When the precession axis is vertical, the mass unbalance moment cannot act either about the precession axis or the input axis, so no error results. However, if the precession axis is tilted from vertical, and there is a mass unbalance, an error results which is a function of the tilt angle deviation from vertical.

$$\omega_T = \omega_H \sin \Theta \qquad (66)$$

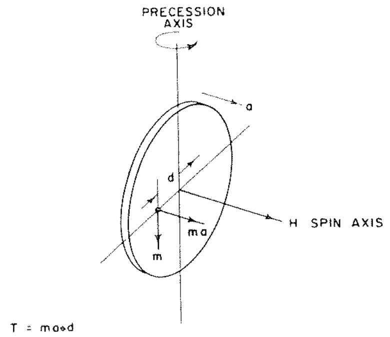
where:

ω_H = maximum unbalance torque when precession axis is horizontal.

Θ = tilt angle of precession axis from vertical in plane of rotor.

Preceding references to mass-unbalance effects have implied that the gyro was subjected only to gravity. If the gyro is now subjected to an acceleration, according to Newton's second law, the acting mass-unbalance moment is increased proportionately. Should the acceleration be 5g, then the mass unbalance is increased fivefold. When a gyro's precession axis is vertical under 1g conditions, mass unbalance is not critical.

Gyro behavior when its precession axis is vertical assumes greater importance when acceleration effects are considered. While a gyro may have no mass-unbalance error when unaccelerated, acceleration along the spin axis will cause a torque about the precession axis. (Refer to Figure 43.) Mass unbalance error is an acceleration-sensitive error and varies directly with acceleration. Complete specification of a gyro's performance usually includes a measure of mass unbalance expressed in terms of degrees/hour drift per "g" of acceleration. When a gyro's precession axis is horizontal, mass unbalance is critical even at 1g. Hence, all gyros have a higher drift rate when used in a horizontal position.



- T = TORQUER PRECESSION AXIS
- a = ACCELERATION ALONG SPIN AXIS
- m = MASS UNBALANCE
- d = MOMENT ARM OF MASS UNBALANCE FROM PRECESSION AXIS

Figure 43

Special provisions must be made to carry electrical power between two elements that are moving relative to one another. Where complete freedom of rotation is required, the conventional solution employs brushes and slip rings. These are used on the gimbal axes of two-degree-of-freedom gyros and on stable platform gimbals. In single-degree-of-freedom gyros, precession axis motion is usually restricted to a small angle. It is therefore possible to use flexible leads, which are in the form of dead soft uninsulated conductive metal strips and are mounted radially and symmetrically about the axis of rotation. Although the material is made dead soft to be as free of spring restraint as possible, there is always some residual spring effect. Balancing whatever restraint may be present for clockwise and counterclockwise motion is important. Total flex lead restraint is a function of the geometry and number of leads, but is usually less than 0.1 dyne-cm.

The precession axis of a single-degree-of-freedom gyro is usually fitted with pickoff devices generally electrical in nature. Inductive types are preferred to capacitive or potentiometer devices and both AC and DC types find use. Reaction torques may result when the point of zero torque does not coincide with the signal null point, an inherent characteristic in the design of vane type devices (including microsyns) attributable to their elastic-restraint characteristic approximating a hyperbolic sine function. Magnitude of reaction torque varies with unit size, but in small precise units it can be held to less than 1 dyne-cm.

Drift can also arise from phenomena which are not immediately apparent. Two of these are anisoelasticity effect and torque rectification.

ANISOELASTIC EFFECT ON GYRO DRIFT

When the torque input to a gyro is spurious in nature, a drift rate error results.

Anisoelasticity of a structure is defined as non-uniform elastic deformation which varies with the angle at which a load is applied. This anisoelastic effect produces a torque which can be sensed by a gyro, causing a drift error.

Consider the effect of anisoelasticity in a gyro rotor bearing.

A bearing is anisoelastic if its radial and axial yield rates are not equal. When an external force, F, is applied to a rotor mass at an angle Θ, the displacement of the rotor's center of gravity depends upon the relative values of axial and radial yield rates. The condition resulting when anisoelastic bearings are used is shown in Figure 44.

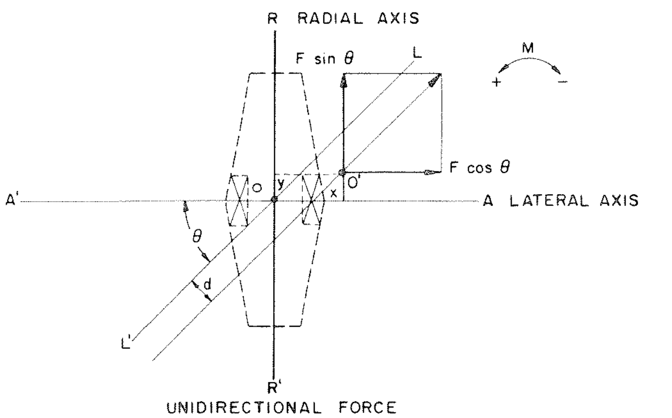


Figure 44

O is the input axis about which torques can be sensed. A-A' is the lateral axis of bearings and the spin axis of the gyro rotor.

R-R' is the radial axis of bearings and the axis around which the spin axis precesses. External force, F, has caused the center of gravity originally at O to move to O' which does not lie on L-L', the line along which force F was applied. Taking moments about O:

$$M_o = Fd = F \sin \Theta(x) - F \cos \Theta(y) \tag{67}$$

$$M_o = F [\sin \Theta(x) - \cos \Theta(y)] \tag{68}$$

Representing the axial yield rate as $1/K_x$ and $1/K_y$ as the radial yield rate of the bearing, the displacements are the product of the force and the yield rates about the respective axes.

$$x = F \cos \Theta \left(\frac{1}{K_x}\right) \tag{69}$$

$$y = F \sin \Theta \left(\frac{1}{K_y}\right) \tag{70}$$

Substituting (69) and (70) into (67), the following expression can be written:

$$M_o = F\left[\left(F \sin \Theta \cos \Theta \frac{1}{K_x}\right)\left(\cos \Theta \; F \sin \Theta \frac{1}{K_y}\right)\right] \tag{71}$$

$$M_o = F^2 (\sin \Theta \cos \Theta)\left(\frac{1}{K_x} - \frac{1}{K_y}\right) \tag{72}$$

$$\sin \Theta \cos \Theta = \frac{\sin 2\Theta}{2}$$

$$M_o = m^2 a^2 \frac{(\sin 2\Theta)}{2}\left(\frac{1}{K_x} - \frac{1}{K_y}\right) \qquad F = ma \tag{73}$$

Equations (71), (72), and (73) show that when the axial yield rate $1/K_x$ is made equal to the radial yield rate $1/K_y$, no moment results and isoelasticity is attained. It also shows that when the angle at which external force is applied equals 0° or 90°, no moment occurs. At 45°, maximum moment occurs.

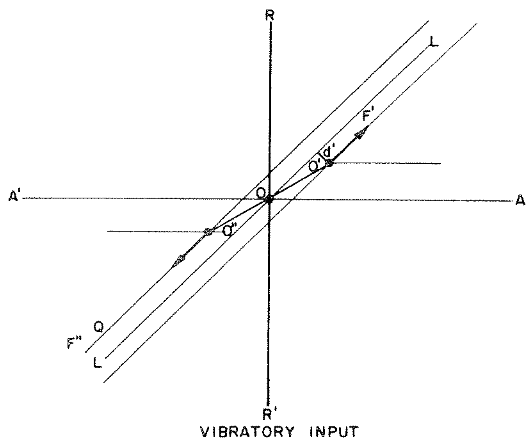


Figure 45

The discussion above assumed a unidirectionally applied force F. However, an oscillating vibratory force input will also cause a moment to occur as a consequence of anisoelasticity. Equation (73) indicated the moment resulting from a constant acceleration. Assuming that a sinusoidally varying input is applied, the resulting torque is also a function of the input frequency, ω, and amplitude, A, taking the form:

$$M_o = \frac{(K_x - K_y)}{2}\left[\frac{A^2 \sin \Theta \cos \Theta}{\left(\frac{K_x}{m} - \omega^2\right)\left(\frac{K_y}{m} - \omega^2\right)}\right] \sin^2 \omega t \tag{74}$$

Other forms vary with the nature of the input, e.g. such as with an aperiodic input. Convenience usually dictates the taking of an average torque value when calculating anisoelastic effects.

Attaining an isoelastic bearing involves proper balancing of a number of variables. However, there are two variables which offer the most fruitful means for achieving isoelasticity — preloading and initial contact angle. Preloading is defined as the application of a thrust load to a set of mounted bearings, and is a procedure used to eliminate or minimize undesired play. Contact angle of a bearing is defined as the angle between a line drawn through two contact points of a given ball with its races and a plane perpendicular to the bearing axis, under thrust load.

An increase in preloading lowers both axial and radial yield rates at the expense of increased friction torque. Increasing the contact angle decreases the axial yield, increases the radial yield and also reduces frictional torque caused by preloading. High contact-angle bearings are essential for isoelasticity. Angles in excess of 30° are becoming more and more common.

Remember, however, that bearing quality can be negated if the supporting structures are themselves anisoelastic. Gimbals and shafts should also be designed for isoelastic behavior in the overall design of a gyro structure.

Anisoelastic effect in gyro bearings has already been demonstrated in the illustrative example given earlier. In fluid floated gyroscopes a similar effect appears in the float suspension system, and is shown schematically in Figure 46.

An analysis made for the float suspension system of Figure 46 is similar to that performed for the case of the **bearings on page 33.** The resulting expression (75) is a function of accelerations along the spin and input axes and a coefficient C_1 which depends on system damping. To determine drift, the expression $M_o = \omega H$ is used. When constructing gyro error models, resultant drift is expressed as $\omega = S a_i a_s$, where S is a value representing all factors contributing to anisoelastic response. A typical value for S in a high-quality gyro is less than 0.01 degree/hour.

TORQUE RECTIFICATION DRIFT

In a gimballed structure, such as a stable platform using single-degree-of-freedom gyros, an important source of drift emanates from kinematic rectification or torque rectification. In aircraft or missile applications, the vibration environment generates oscillatory disturbing torques on the platform. An oscillatory precession of the gyro momentum vector results such that platform motions are sensed about axes other than those under control, e.g., a roll gyro would then sense a pitch input.

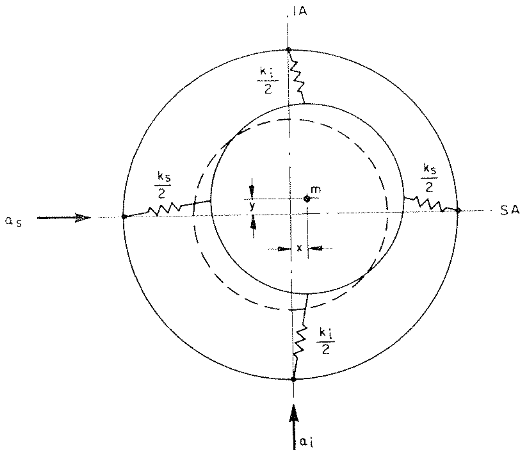
Rectification arises when gyro angular momentum vector, H, while subject to sinusoidal input ϕ_X, precesses back and forth across the Y axis so that it senses a component of angular motion $\Theta\phi_Y$, about the Y axis as well as its X axis angular input $\dot{\phi}_X$. Summing the gyro inputs includes a coarse torque component, $H\Theta\dot{\phi}_Y$, as well as its primary input $H\dot{\phi}_X$.

$$I_Z\ddot{\Theta} + H\dot{\phi}_X + H\Theta\dot{\phi}_Y \cong 0 \qquad (76)$$

The expression for rectification drift of an undamped single axis gyro has the form:

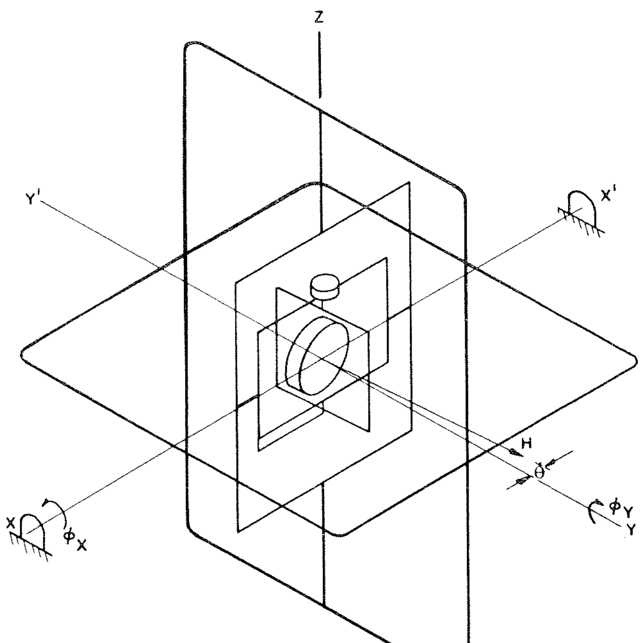
$$(\dot{\phi}_X)\text{ avg.} = 1/2\ \Theta\text{ peak }\phi_Y\text{ peak }\cos(\Theta - \phi) \qquad (77)$$

ANISOELASTIC ERROR MODEL



m = MASS OF MOTOR
a_i = ACCELERATION ALONG INPUT AXIS
a_s = ACCELERATION ALONG SPIN AXIS

Figure 46



TORQUE RECTIFICATION

Figure 47

$$M_o = \frac{m^2 a_s a_i}{2}(k_i - k_s)[C_1] \qquad (75)$$

ADVANCED GYRO DESIGNS

Devising means by which undesired restraints in gyro rotor suspension may be eliminated is a major task for the gyro designer. In the search for new suspension methods, a wide variety of designs have been used or investigated, among which are the common ones utilizing ball bearings, torsion wires, flexure strips, or fluid flotation. A number of newer methods have been developed, however, and these are described in detail below.

GAS BEARING GYROSCOPES

A gas bearing rotor support which is experiencing wider application consists essentially of bearings using gas for support and lubrication. Like all designs, gas bearings exhibit advantages as well as disadvantages.

Gas bearing designs fall into two major categories. These are designated as (1) hydrostatic gas bearings and (2) hydrodynamic gas bearings. The former requires an external pressure source in the form of a small pump which provides a metered flow of air at the desired pressure and have found widespread application in gyros used in ballistic missiles. Whenever hydrostatic gas bearings are employed, however, contamination of the air supply must be prevented. Commonly, the gyros used in ballistic missiles are single-degree-of-freedom types in which pressurized gas serves the same function as flotation fluids in floated gyros. In these instruments gas bearings support the precession axis, while the gyro rotor is supported conventionally by ball bearings. Hydrostatic pressures used for gyro support range from as low as 10 psia to 75 psia.

Hydrodynamic gas bearings do not require an external supply of pressurized gas. The geometry of both the journal and bearing is such that relative motion between the two pressurizes the gas in much the same manner as oil is pressurized in a conventional oil-lubricated journal bearing. A hydrodynamic gas bearing's load-carrying capacity depends on gas viscosity, bearing projected area, and ambient gas pressure. Because an efficient design requires extremely small gaps between moving parts, it is not uncommon to see air gaps as small as 0.00010 inch. Surface finishes and dimensions must also be held to extremely critical tolerances to assure successful fabrication of hydrodynamic gas bearings. When used for radial loads, lift force and load directions form an angle between them, a phenomenon producing an effect similar to the anisoelasticity of a conventional ball bearing. Hence, much effort in gas bearing design is directed toward minimizing this angle. A disadvantage inherent in a hydrodynamic gas bearing arrangement lies in the fact that contact between rotor and stator is maintained until the rotor reaches a speed sufficient to provide the resultant gas pressure needed to separate the two elements. This lift speed, in some designs, is about 200 rpm. A fixed supply of clean gas (usually helium or a similar inert gas) is assured, since the rotor assembly is totally enclosed within a hermetically sealed pressurized container. Obviously, the primary advantage in using hydrodynamic gas bearings as opposed to hydrostatic types lies in the elimination of a special pump. Further, hydrodynamic gas bearings find application as gyro rotor bearings because high rotational speed is required. In a gyro having a rotor angular momentum of 5×10^5 gm cm^2/sec., a hydrodynamic pressure of 10 psia of projected bearing area is typical.

The greatest single advantage inherent in gas bearings is the absence of wear. Theoretically, such a bearing should have unlimited life in contrast to conventional lubricated ball bearings which suffer wear and loading damage. Most often the limiting factor in gyro life is the life of the gyro rotor bearing-lubricant combination. Other advantages of a gas bearing include relative insensitivity to temperature variation, lower restraint levels, cleaner output signals due to the absence of mechanical noise, and immunity to radiation.

Gyroscopes utilizing hydrostatic gas bearings both for support and their precession axes can be used in place of comparable fluid floated gyros. Since a gas bearing is less sensitive to temperature variation than flotation fluid, associated temperature control circuitry is simplified. However, alignment stabilization circuitry becomes more complex.

Hydrostatic gas bearings require a constant gas supply from a pump or storage tank, and filtering is essential. Though the air gap between parts in a gyro equipped with hydrostatic gas bearings can be many times larger than that permissible in a hydrodynamic type, extremely close tolerances are still mandatory to assure symmetrical gas flow within the bearing. Typical supply pressures for a hydrostatic gyro bearing range from 15 to 30 psi, while gas consumption ranges from 0.5 to 2 cubic feet per minute. Air or nitrogen are most commonly used.

In operation, gas passes through a filter to enter a chamber between the gas-supported rotor and the case, passing through small nozzles into supporting radial and axial gaps. The gas supply sustains gas flow through the nozzles at a constant pressure. Should the centered rotor float move, an imbalance results in the gas flow pattern, thus narrowing the gap on one side while widening it on the other. Gas pressure therefore increases at the smaller gap and decreases at the wider one, creating a pressure difference which re-establishes an equilibrium with the applied load.

A typical hybrid rate integrating gyro design utilizes a hydrodynamic gas bearing in its rotor, but the float assembly remains in a flotation fluid which requires precise temperature control. This type of gyro is used whenever the need for instant warm-up exists, a feature which is most easily accomplished by keeping the gyro's rotor running continuously. Once started, there is no limit on rotor bearing life, for gas cannot wear. The number of starts anticipated for a gyro is the limiting factor in hydrodynamic gas bearing design, not life or load carrying capacity. It would be wasteful to use such a gyro in applications demanding thousands of starts. Hydrodynamic gas bearings are best suited for applications where extended periods of continuous operation are required. One such rapidly growing application is in space flight where guidance must be continuously operative during flight times measured in days or even weeks. A more earthbound need occurs in missiles and manned aircraft requiring quick response to an alarm. To eliminate warm-up time delay, a gas bearing type gyro rotor may be kept spinning continuously. Because extremely hard metals tend to gall when rubbed together, ceramic materials such as carbides and alumina, having a hardness greater than 9 on the Mohs Scale, are used for hydrodynamic gas bearing surfaces.

A particularly useful gyroscopic device in which a hydrodynamic gas bearing is used incorporates a two-axis free rotor in its design. Spherical in shape, this bearing has essentially three degrees of freedom, but senses input only about two axes. The rotor has complete and unlimited freedom of motion about its spin axis, but motion about the two orthogonal sensing axes is restricted. Since the spherical bearing provides all the support required, a separate float assembly, whether gas or liquid, is not needed. This type of gyro generates an output to its alignment loop by effectively remaining fixed in space while its case moves in relation to it.

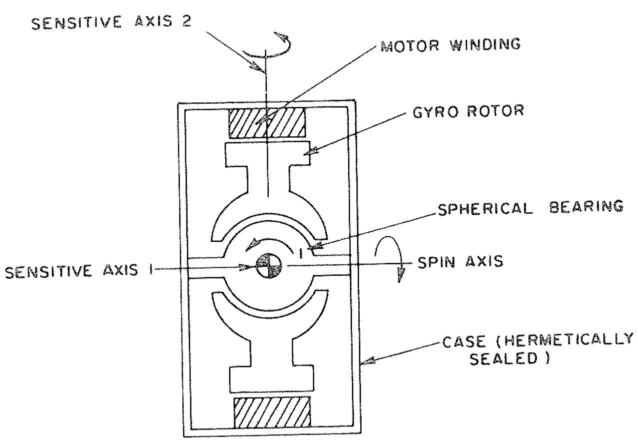
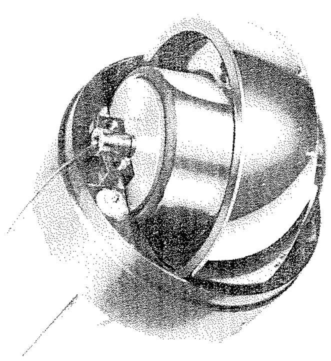


Figure 48

Gas bearings contribute anisoviscous effects in much the same manner that anisoelastic effects occur in mechanical structures. Since no limit exists for the life of a gas bearing, it is natural to assume that a gyro rotor equipped with such bearings would operate at extremely high speeds, and that a large angular momentum could be achieved in a small package. Unfortunately, however, such assumptions are incorrect because at high speeds the power needed to overcome rotor windage losses in a gaseous medium are prohibitive. Consequently, a typical efficient operating speed for a hydrodynamic gas bearing must be held to about 12,000 rpm.



SINGLE AXIS HYDRODYNAMIC BEARING ROTOR

CRYOGENIC GYROS

Cryogenic gyros are devices whose basic performance capabilities derive from the superconductivity property of matter that occurs at temperatures near absolute zero, and in which condition electrical resistance totally disappears. Very large currents can flow under such circumstances, but practical applications of superconductivity are based not so much on current flow as on the magnetic fields generated thereby.

A cryogenic gyro, in effect, is a body floating in a magnetic environment, and is rotated to function as a gyro rotor in which superconductive magnetic bearings replace conventional gimbals. Practical aspects of such a design are quite formidable, for not all metals become superconductive at extremely low temperatures. Hence, special elements like niobium must be used. Since superconductive materials are very sensitive to temperature and stray magnetic fields, effective thermal insulation and magnetic shielding must be incorporated into any good design. When depressed temperatures must be maintained for a long period of time, a cryostat becomes necessary, a feature which increases the size of the system to such an extent that its use is limited to applications aboard ships and submarines. One method which might permit the use of cryogenic gyros in a missile whose anticipated flight duration is short, entails the charging of the gyro assembly with liquid helium which can boil off during the limited operating interval. The anticipated performance of such a device would justify the design complexity.

ELECTROSTATIC GYROS

While some researchers use magnetic fields to suspend a gyro rotor, as in the cryogenic gyro above, others have developed designs based on the use of electrostatic suspensions. Here, the rotor is suspended by electrodes creating a powerful electrostatic field. The advantage of such an approach over that utilizing cryogenics is the elimination of very low temperatures. However, to create electrostatic fields of adequate strength, it is necessary to use very high voltages operating in a very low vacuum. Effective functioning of such a scheme depends greatly on extreme uniformity of fields. One of the practical problems that has arisen concerns the obtainability of rotors having a very high degree of sphericity. At present, the bulk of the associated electronic equipment required for use with the electrostatic gyro limits its use to ships and submarines.

PARTICLE GYROS

The ultimate in gyro design is the complete elimination of a spinning rotor and gimbal combination, and its replacement by rotating atomic particles. Basically, this idea involves the use of the inherent angular momentum of atomic particles that are spatially oriented by means of magnetic or electric fields. Such particles have both angular and magnetic moments. One scheme is based on detecting the change in precession frequency while another approach to a particle gyro envisions the use of molecular spin state rigidity.

A basic problem which has prevented the effective implementation of all such approaches is detection, for the resultant changes caused by externally applied inputs are microscopically small. Put quite simply, the problem is one of obtaining a useable signal-to-noise ratio. However, much work is being done on the problem and in associated areas so that the next decade may begin to see conventional gyros replaced by solid state devices.

LASER GYRO

Experimental efforts are also being directed toward the development of a laser, or optical gyro.

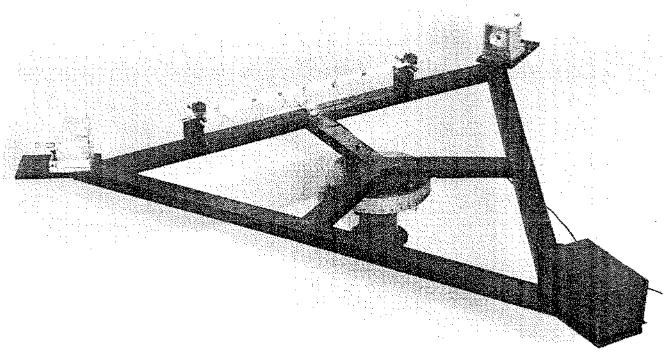
In normal laser operation when a single atom spontaneously drops to the ground state, it emits a short burst of light. Part of this light, traveling along the axis of a gas-filled tube, strikes an end mirror, reflects back, strikes another end mirror, and keeps repeating this motion. Each time the light traverses through the gas, it stimulates more atoms to emit light of exactly the same frequency and phase. Thus, light intensity builds up rapidly, the frequency of which is such that an exact integral number of wavelengths exist between the two mirrors. If the mirrors are of good quality and are kept at a constant separation, this frequency can be well controlled.

In one experimental arrangement, three mirrors are positioned so that light coming from one end of the laser bounces around the corners of a triangle and re-enters at the other end. The optical path length determines laser frequency. Light, coming from both ends of the laser, travels around the same optical path in opposite directions. If the device is stationary, the two beams will have the same frequency. If it rotates, the effective optical paths in the clockwise and counterclockwise directions will be different because it takes light a longer time to go one way than the other, depending on the direction of rotation. Therefore, the laser will operate at two slightly different frequencies. The frequency difference is directly proportional to the angular velocity of the device. By sampling both the clockwise and counterclockwise beams in a photomultiplier tube, the frequency difference can be detected. The photomultiplier is a square law device that gives an output proportional to the sum and difference frequencies of the incident light. Interest is in the difference frequency, which is proportional to the angular velocity of the apparatus.

Photosensitive semiconductors are being considered in place of photomultipliers to measure the optical beat frequencies.

The crucial problem in the construction of a useful "optical gyro" is the degree of coupling between the two closely spaced oscillation modes which are simultaneously present in the laser. If significant energy transfer between the two modes occurs, the effect will be analogous to a "drag" tending to rotate the standing wave pattern relative to the inertial frame of reference. The magnitude of this effect will determine the basic measurement threshold of the "optical gyro."

The ability of a laser "optical gyro" to detect angular rates of the order of 100 degrees an hour has been demonstrated and has been shown to be five orders of magnitude poorer than believed fundamentally possible. This is three to four orders of magnitude poorer than the best present-day floated rate integrating gyro. Thus, although the fundamental physical feasibility of a laser "optical gyro" is established, a considerable amount of research and development will be required before a practical instrument can be built.



STABLE PLATFORMS

When a point is reached at which accuracy capabilities of simple displacement type gyros are exceeded, it becomes necessary to use a more complex gyroscopic device termed a stable platform.

Stable platforms are gyro instuments which provide accurate azimuth, pitch, and roll attitude information. In addition to serving as reference elements, stable platforms are utilized to stabilize accelerometers, star trackers, or similar devices in space.

Stable platforms are the instruments which generate signals that are in turn manipulated according to a computational scheme built into an analog or digital computer. The computer may be programed in a great variety of ways to achieve the desired output information which may be attitude, position, heading, verticality, or a combination of these. In inertial guidance systems, a variety of platform configurations are possible which in turn determine the mode of computing in the computer and provide for different types of coordinate reference schemes. These platform configurations are identified as follows:

(a) *Geometric:* Gyros are fixed in space while the accelerometers follow local vertical. The advantage of this scheme is that the gyros need not be torqued with Earth's rate or with the flight vehicle rate. Disadvantages lie in the fact that the gyros tumble and the entire gimballing assembly becomes rather complex to mechanize.

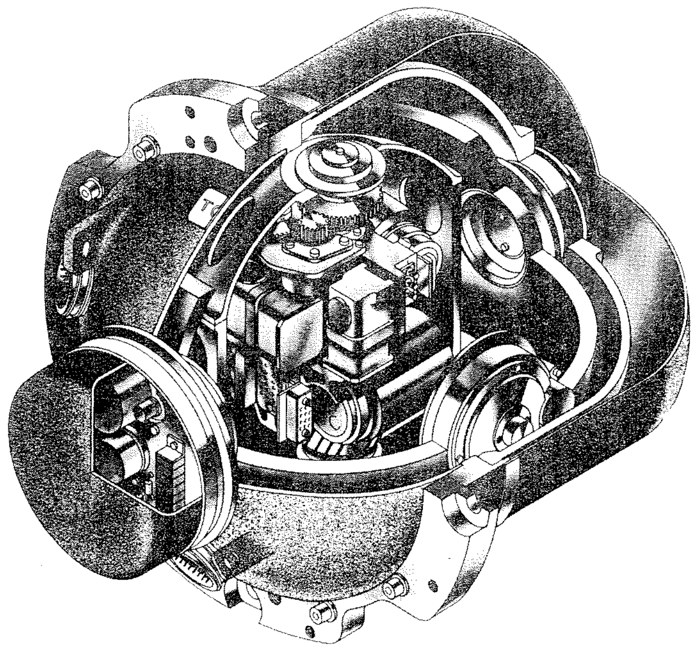
(b) *Semi-analytic:* Gyros and accelerometers all follow the local vertical. Advantages here include simple mechanization, together with inertial sensors that are continually fixed with respect to the earth's gravity field. The disadvantage is the need for considerable accuracy of the gyro torquer and the associated correction circuitry.

(c) *Analytic:* Gyros and accelerometers are fixed in space with no attempt to orient them to local vertical. This system is as mechanically simple as the semi-analytic type. However, the associated computer becomes quite complex.

Gyro stable platforms discussed in the remainder of this section are mechanically configured such that they can be used as semi-analytic or analytic type references.

Essentially a cluster of gyros mounted within gimbals, stable platforms perform the important function of having the gyro outputs control the gimbals by means of a servo loop. Manipulating arrangements of gimbals and gyros produces a great variety of platform types. For illustrative purposes, this publication provides an initial discussion of a "three-gimbal, three single-degree-of-freedom gyro" type of platform. This typical unit serves as a basis of comparison for alternate configurations.

Any gyroscope mounted in gimbals tries to maintain its position fixed in space. In an ideal, frictionless gimbal structure, the gyro has a fixed reference. In reality, frictional torques and accelerating torques cause the gyro to precess or drift, which, in effect, disturbs its spatial reference. In devices which do not have to exhibit extreme accuracy, such as vertical or free gyros, this drift is tolerated. However, increasing flight vehicle performance demands greater and greater accuracy. Today's desired drift rates of 0.001 degree/hour for inertial systems to 0.5 degree per hour for less sophisticated navigation systems can be achieved only by means of stable platforms. These are designed to use the gyro as a sensor to control a null-seeking servo loop which can provide power to generate a counter-torque to the gimbals and maintain the gyro reference fixed in space.



THREE-GIMBAL STABLE PLATFORM

Since auxiliary power is now available to drive the gimbals, a cluster composed of a number of gyros can be built. In a platform using single-degree-of-freedom gyros, three such gyros are mounted together, with their input axes forming a mutually orthogonal triad. Associated with each gyro is a gimbal servo system which maintains the corresponding gyro axis fixed in space.

The three-gyro triad is positioned so that the input axes correspond to the roll, pitch, and yaw axes of the flight vehicle. The frame to which the three gyros are mounted is itself mounted in a succession of gimbals. This frame, free to move in azimuth, is called the azimuth gimbal, and its motion is controlled by the yaw gyro. The azimuth gimbal is mounted within a gimbal called the pitch gimbal which, in turn, is mounted within a gimbal identified as the roll gimbal. Relative motion between pitch and roll gimbals is a measure of pitch motion. The roll gimbal is mounted in a case referred to as the fixed gimbal which is fastened to the airframe. Relative angle between the roll gimbal and the fixed case is a measure of roll motion.

Other sequences of gimbal arrangement are also possible. One common arrangement for platforms used in ballistic missiles employs the outermost axis for pitch and the next innermost axis for roll. Stabilization of the cluster is not affected by gimbal order, but gimbal angles do not correspond to flight vehicle attitude unless computed corrections are provided.

For simplicity of discussion, a single axis platform is analyzed here. It has been previously stated that each gyro controls a corresponding gimbal; hence a three axis platform is, in effect, three single axis platforms with coupling inputs. These coupling inputs, however, are not considered in the single axis analysis.

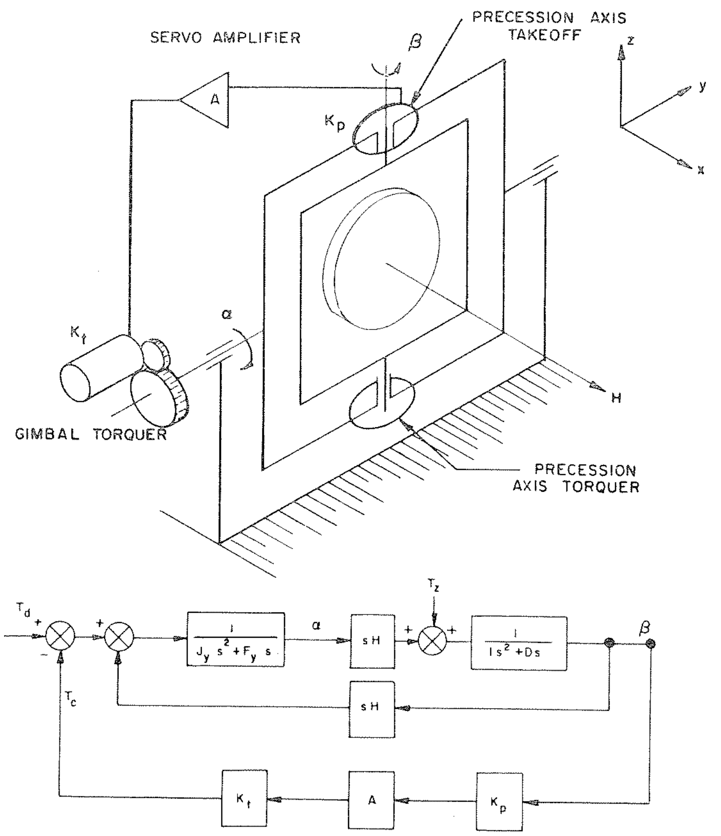
Figure 49 illustrates a typical single axis platform utilizing rate integrating gyros. While it is theoretically possible to build platforms utilizing rate gyros, rate integrating gyros, or free gyros, the latter two are the types most frequently used. Rate gyros are too insensitive to low level angular rates of change. Therefore, the discussion here considers only platforms which use free and rate integrating gyros. LaPlace operator notation is used. The single axis platform is subjected simultaneously to a torque about the input axis (Y) and the output axis (Z). The resultant motion is described by the following equations:

$$T_y = (s^2 J_y + sF_y)\,\alpha + SH\beta \qquad (78)$$

$$T_z = (Is^2 + Ds)\,\beta - sH\alpha \qquad (79)$$

where:

- α = angular motion about input axis
- β = angular motion about output axis
- J_y = moment of inertia of gimbal about input axis
- I = moment of inertia of gyro about output axis
- D = damping coefficient of gyro about output axis
- F_y = damping coefficient of gimbal about input axis
- H = angular momentum of gyro rotor



SINGLE AXIS PLATFORM

Figure 49

If a torque disturbance T_d is applied to the gimbal structure and the gyro input axis, the gyro spin axis precesses about the output axis. The precession motion generates an output signal from the gyro pickoff which is in turn fed to an amplifier driving a gimbal torquer. This gimbal torquer produces a counter-torque to the platform, and the net input torque to the gimbal axis is the difference between the disturbing torque T_d and the counter torque T_c.

$$T_y = T_d - T_c \qquad (80)$$

A condition in which the platform experiences no motion about its input axis is sought. This can be attained if the disturbing torques are removed by the servo system so that T_y becomes zero.

The gimbal servo loop is usually referred to as the alignment loop. Like all loops, it is subject to the same criteria of stability and response over a range of operating frequencies, and a typical analysis is concerned with the transfer function between the disturbing torque T_d and the input angle α, or between T_d and the gyro precession angle β. The basic block diagram does not include compensation networks. In actual practice, compensation networks are required to assure that:

(a) Adequate gain margin is allowed for unusual gain changes.

(b) Adequate phase margin is provided to minimize torque resonance between T_c and T_d.

(c) The phase lead and gain characteristics limit the counter-torque T_c above the system natural frequency.

Regardless of the type of gyro used, the function of gyro stabilization loops in a stable platform is the same. The preceding discussion used a fluid-floated rate integrating gyroscope to illustrate a gimbal stabilization servo loop. Had an air bearing gyro been used instead, the servo loop would have become oscillatory because of the air bearing gyro's negligible damping. To overcome this deficiency, a notch filter would have to be inserted in the loop to provide by electrical means the damping required.

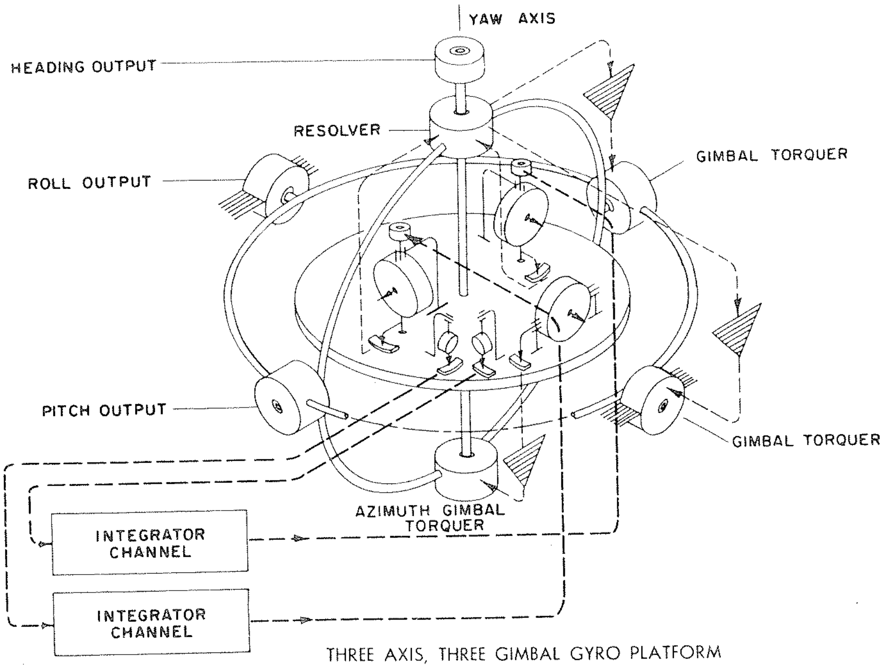
Figure 50 shows a three-axis three gimbal stable platform. Each of the three gimbal alignment servo loops performs in the same manner as described for the single axis platform. In the three axis platform, an azimuth coordinate resolver is required. The inner cluster is free to move relative to the gimbals. Hence, the gyro input axes are not aligned with their corresponding gimbal axes. Outputs of gyros sensing roll and pitch are fed to the azimuth coordinate resolver, and the outputs of the resolver are then fed to the gimbal alignment amplifiers.

The three axis platform analysis must consider cross coupling effects between the loops and the additional degrees of freedom represented. The stable platform is gimbal-mounted for three degrees of freedom, and a torque disturbance about one axis can cause a response about all three axes. Although an extended analysis of a three axis platform is beyond the scope of this work, the effects which must be considered are products of inertia, products of angular motion, and gyro pickoff response. An initial analysis which considers only the gyro pickoff response usually suffices by setting up the three-axis and single-axis platform equations in matrix form.

By utilizing the gyros and gimbal servo loops described above, building a device on which components may be mounted for purposes of stabilization becomes feasible. In inertial navigation or stellar-inertial navigation systems, accelerometers and star-tracker telescopes are stabilized in space.

As yet, the spatial orientations that the gyros assume have not been specified. For space travel some convenient astral body could be chosen. However, the surface of the earth is still the zone of most immediate concern for navigation systems.

Travel on the surface of the earth (and at lower altitudes) implies a reference based on "vertical." A platform fixed in inertial space does not provide an earth's vertical.



THREE AXIS, THREE GIMBAL GYRO PLATFORM

Figure 50

Hence, some means is required to establish this vertical with the platform. One method is to gimbal-mount an independent vertical sensing reference on the platform, but a more convenient and common method applies computed correction signals to the platform. These signals torque the platform so that it maintains vertical with respect to the earth.

A simple pendulous mass indicates vertical when unaccelerated. However, any applied acceleration causes hang-off and indicates a false vertical. Therefore, a more sophisticated means of indicating vertical is required, and the method currently receiving wide-spread application is Schuler tuning.

SCHULER PENDULUM

The direction of vertical on the earth's surface can be obtained by the use of a mass suspended from a fixed support point by a string. Such an arrangement also represents a pendulum. If the mass is displaced a small angle from vertical, it oscillates with a period expressed by

$$T = 2\pi \sqrt{\ell / g} \qquad (81)$$

where ℓ is the length of the string and g is gravity acceleration. If the support point is moved from rest, the mass is deflected from vertical by an angle Θ which is expressed as $\Theta = \tan^{-1} a/g$, where a represents the support point.

Hypothetically assuming that the support string is increased to a length equal to the radius of the earth so that the suspended mass is located at the center of the earth, the support point may move in any manner relative to the earth's center. Therefore, the supporting string always remains vertical regardless of accelerations to which the support may be subjected.

In effect a simple undamped pendulum has been created with a period $t = 2\pi\sqrt{L/g}$, in which L is equal to the radius of the earth. The period of the earth's radius pendulum is 84.4 minutes. Such a device is called a Schuler pendulum, named for its discoverer, Max Schuler.

A compound pendulum having the same period and properties as the simple 84-minute pendulum also has insurmountable mechanical design limitations. However, an analogous system using accelerometers can be made to behave in the same manner as a Schuler pendulum. Consider an accelerometer mounted on a platform that is free from linear motion, and which transmits a signal proportional to the platform's tilt angle. Under these conditions the accelerometer output signal is integrated twice and the resultant signal is used to drive the platform in a manner such that the accelerometer's output signal is reduced to zero. Output of the accelerometer may be expressed as

$$e = g\Theta \qquad (82)$$

(for small angles g sin $\Theta \approx g\Theta$)

$$\Theta = -K \iint e\,dt\,dt = -Kg \iint \Theta\,dt\,dt \qquad (83)$$

Differentiating both sides of equation (83) yields

$$\frac{d^2\Theta}{dt^2} + Kg\Theta = 0 \qquad (84)$$

The resultant differential equation takes the same form as the equation for a simple undamped pendulum. By proper selection of the integrator scale factor, K, the accelerometer-integrator-platform system can behave like a simple pendulum having an oscillatory period of 84.4 minutes.

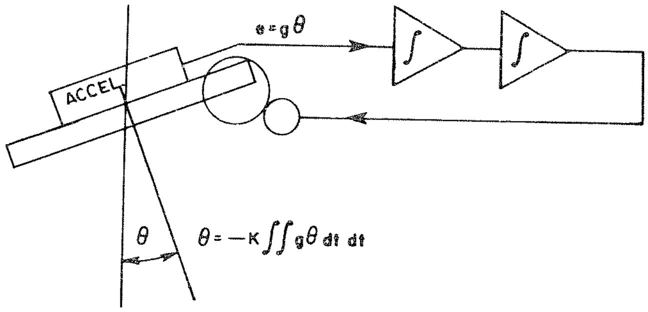


Figure 51

When the system is disturbed by an input acceleration, the error oscillation of the position output will have a period of 84.4 minutes just as in the case in which the mass of the Schuler pendulum is displaced from vertical.

A Schuler-tuned platform system refers to a stable platform whose accelerometers are used in a loop which is an analog of the Schuler pendulum. This loop includes a gyro precession axis torquer. Since the Schuler pendulum maintains an accurate vertical reference, it can be used to generate correction signals to tilt the gyro spin axes to a vertically referenced plane rather than its inertial space reference. The accelerometer output is proportional to aircraft acceleration. If this acceleration signal is integrated with respect to time, it represents the instantaneous velocity of the aircraft. Dividing this velocity by the earth's radius gives the angular velocity at which the platform must rotate to remain tangent to the earth's surface. The Schuler-tuned signal is applied to the gyro precession torquer, properly scaled to indicate the required angular velocity.

A Schuler-tuned system having a period of 84.4 minutes is referred to as an "undamped Schuler mode," for its derivation is based on an undamped simple pendulum. Another type, termed the "damped Schuler mode," also is used. Without damping, a simple pendulum oscillates at its natural frequency and at an amplitude equivalent to its initial input. To eliminate phenomena arising from undesired transients, damping is introduced to accelerate transient decay, and the Schuler mode is de-tuned to some multiple of the Schuler frequency during those intervals when vehicle acceleration is near zero. In a manned aircraft, vehicle acceleration time is generally much shorter than periods of unaccelerated flight. Under such conditions, the undamped Schuler mode is used during periods of vehicle acceleration, while the damped higher frequency mode is used for periods of unaccelerated flight. Switching between the two modes is usually accomplished by an automatic switch actuated by the product of acceleration and time. The degree of de-tuning and the level at which mode switching occurs depends on the aircraft's flight characteristics and the system accuracy desired.

The table below indicates the form and manner in which errors of a Schuler-tuned system propagate. Note that all errors are bounded with the exception of position error caused by time-proportional gyro drift.

ERRORS IN A SCHULER-TUNED SYSTEM

ERROR SOURCE	ERRORS		
	TILT	VELOCITY	POSITION
Gyro Drift	$\frac{\omega_d}{\omega_s} \sin \omega_s t$	$R\omega_d(1-\cos \omega_s t)$	$R\omega_d\left(t - \frac{\sin \omega_s t}{\omega_s}\right)$
Initial Velocity	$\frac{V_o}{R\omega_s} \sin \omega_s t$	$V_o \cos \omega_s t$	$\frac{V_o}{\omega_s} \sin \omega_s t$
Initial Tilt	$\Theta_T \cos \omega_s t$	$\frac{\Theta_T g}{\omega_s} \sin \omega_s t$	$R\Theta_T(1-\cos \omega_s t)$

where

ω_d = gyro drift
ω_s = Schuler frequency
V_o = initial velocity
Θ_T = initial tilt
R = Earth's radius
g = local gravity

Since the value of an oscillating reference may be questioned, system designs use auxiliary signals to provide damping. Effective damping may be provided by data available from air-speed measurements, Doppler radar, or acceleration signals.

Having established a means of maintaining vertical, providing for initial erection of the platform remains. Since gravity is an acceleration, common practice utilizes the accelerometer as a vertical reference. Its output is fed to a gyro precession torquer which actuates the gimbal alignment loop. The gimbals drive until the accelerometer output is at null, indicating vertical. In those cases where accelerometers are not required, vertical erection may be achieved by pendulous reference devices or mercury or electrolytic switches.

PLATFORM GIMBAL TORQUERS

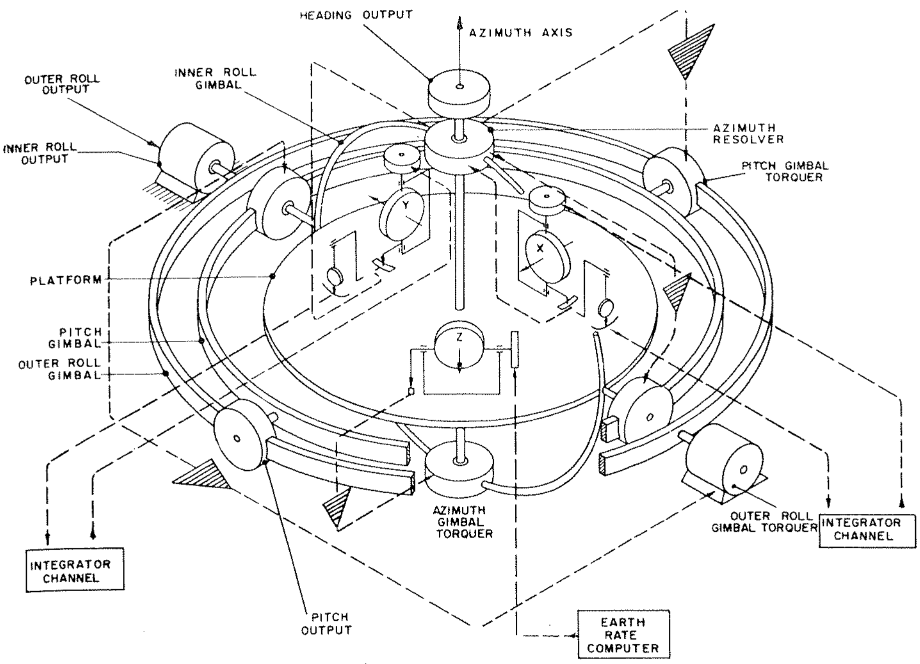
Attention is now given to the actual components which are used in building a platform with emphasis placed on the effect of components on platform performance. Details relating to component design are discussed elsewhere in this publication and in another Kearfott publication, "Technical Information for the Engineer: Servo Motors, Motor Generators, Synchros."

It must be understood that a platform can be no better than its components. Components must be selected which conform to the accuracies desired. Platform drift-rate accuracy depends on the gyros which are used. Acceleration sensitivity and verticality are a function of the accelerometers, and attitude information is usually indicated by synchro transducers whose accuracy and nulls depend on the units selected.

Two kinds of gimbal torquers can be used. One is a servo motor driving through a gear train, and the other is a direct drive torquer. Each has advantages and disadvantages.

DC direct drive torquers are preferred to AC types because of the former's smaller size and greater efficiency. The need for brushes is the penalty paid for DC torquers.

The desirability of using a direct drive torquer is based on several features. Such a torquer possesses a high torque-to-inertia ratio and dispenses with gear train backlash and maintenance. Elimination of backlash nonlinearities improves the reliability of mathematical analysis and a direct coupled torquer has proportionately lower



4 GIMBAL STABLE ELEMENT

Figure 52

friction torque. When the support structure rotates with respect to the stabilized structure in a gear drive, the geared servo motor accelerates the support member movement, producing an additional error. The presence of gears accentuates the effect of an input rate ω_c, which must accelerate and rotate the gimbal. This requires acceleration of the gear train to overcome gear-train friction torque. The applied rate appears as a torque at the shaft of the final gear and is expressed as:

$$T = (J_m S + F_m) N^2 \omega_c \qquad (85)$$

where N is the gear ratio, J_m is moment of inertia of motor, and F_m is the motor damping coefficient.

On the other hand, a gear-drive servo motor also has advantages. Since torque requirements may be changed over a wide range by using different gear ratios, it is a more flexible device. Economically, the gear drive may have the advantage of utilizing a single size servo motor with different gearing instead of three different size direct drives. Providing comparable power, a servo motor gear drive is smaller, uses less power, and has smaller slip rings. Power and size are further reduced by a smaller platform. No brushes are required by the servo motor, and generally, the gear drive is smaller, less costly, more versatile, and more reliable than a comparable direct drive unit in the moderate torque range.

Selection between the two is based on the system performance required. Where extreme accuracies are demanded or where very large drives are needed, selection of the direct drive torquers is indicated. In a large number of other applications, gear drive torquers prove satisfactory. As in all engineering problems, selection depends on careful consideration of all the factors involved.

PLATFORM GIMBAL SYSTEMS

When two gimbal axes of a three-axis system become colinear, gimbal lock occurs. To provide unlimited attitude freedom in a platform, the four-gimbal system has been devised.

The four-gimbal system illustrated in Figure 52 is similar to the three-gimbal system previously described. The difference lies in the gimbal sequence and in the type of control associated with the addition of the fourth or redundant roll gimbal. The gimbal sequence is azimuth, inner roll, pitch, and outer roll. The outer roll gimbal receives its control signal from a pickoff mounted between the inner roll and pitch gimbals. This pickoff maintains perpendicularity between the inner roll and pitch gimbals, obviating the need of a gyro signal to control the outer roll gimbal. As a result, roll motions of an aircraft produce only low level platform disturbances. The outer roll gimbal servo bears most of the burden for roll stabilization. As the pitch angle increases, the outer roll gimbal must move through larger angles to null the inner roll gimbal. When pitch reaches 90°, the inner roll axis is aligned with the azimuth axis, and the outer roll gimbal flips through 180°. Hence, full freedom of motion without gimbal lock is obtained.

COMPARISON OF THE TWO SYSTEMS

Advantages of a Three-Gimbal System

- Smaller size and weight
- Lower cost

Disadvantages of a Three-Gimbal System

- Limited maneuverability of aircraft
- Lower system accuracy
- Higher susceptibility to effects of vibration

Advantages of a Four-Gimbal System

- Unlimited maneuverability of aircraft
- Magnitude of inner roll gimbal deflections are very small (on the order of control signal level)
- Reduction of platform drift due to torque rectification
- Attenuation of effects due to gyro non-linearities
- High system stability

Disadvantages of a Four-Gimbal System

- Greater size and weight due to extra gimbal
- Greater complexity due to additional servo
- Higher overall cost

AIRCRAFT MANEUVERABILITY

The Three-Gimbal System

The use of only three gimbals poses a problem when the pitch angle approaches 90°. In this instance, the azimuth and roll axes coincide, and one degree of freedom is inadvertently lost (referred to as gimbal lock). For this reason, it is necessary to limit the pitch of the aircraft to ±85° for a three-gimbal system.

The Four-Gimbal System

The four-gimbal stable element has 360° of freedom about the roll, pitch, and azimuth axes. This freedom of motion is maintained even when the aircraft pitches 90°, for this occurrence causes only the redundant outer roll gimbal to be inactivated by coincidence with the azimuth axis. Through the use of gimbal axis resolvers, roll stabilization is maintained by the inner roll gimbal. Thus, it is possible to program unlimited aircraft maneuvers, since the four-gimbal platform does not suffer from limitations imposed by gimbal axis orientation.

Reduction of Platform Drift Due to Torque Rectification

Drift rate of a gyroscopic stable element is affected by aircraft flutter. Flutter, a cyclic disturbance, is particularly detrimental when the subsequent gyro displacements are relatively large. This phenomenon is ascribed to torque rectification, which occurs when positive and negative portions of the undulating disturbances are not averaged out, but instead rectified. Resulting biased torque causes the gyro to drift at an appreciable rate, degrading system performance.

The Three-Gimbal System

Torque rectifying characteristics of a three-gimbal system become evident in the presence of yaw oscillation when the aircraft is pitched. In this case, angular oscillation forces the roll gimbal to cycle. Reaction torque, sensed by the platform cluster, produces an undulating gyro precession. During these undulations, a minute component of the momentum vector lies in the plane of the roll gimbal, providing the necessary conditions for a vector product. The sense of the vector product (the uncompensated torque on the gyro) remains unchanged since the projection of the momentum vector and the direction of the axis of roll oscillation are in phase. Thus, the disturbance, oscillatory in nature, is rectified, and a unidirectional gyro drift is, in essence, produced.

The Four-Gimbal System

The four-gimbal system is relatively unaffected by torque rectification because the necessary outer roll gimbal signal is generated by the non-orthogonality of the inner roll and pitch gimbals. Since the orientation of these two gimbals, and not that of the gyros, is involved, gyro excursions are small. Consequently, platform drift due to torque rectification is minimized.

SYSTEM SERVO STABILITY

During aircraft motions involving large pitch angles, the gimbal roll axis (which is parallel to the aircraft roll axis) assumes an angle with respect to the stable cluster which remains level. Under this condition, the gimbal roll axis is not parallel to the plane containing the input axes of the vertical gyros. The respective gyros perceive a component of roll, while the system is required to displace the appropriate gimbals by an amount equal to the full value of roll. This is accomplished by multiplying the gain of the roll servo by the reciprocal of the cosine of the pitch angle.

Overall stability of the roll servo is a function of the open loop gain and the gyro damping feedback torque. For a three-gimbal system, both the open loop gain of the roll axis and the gyro damping feedback torque vary as the cosine of the pitch angle. The cosine is small at high pitch angles. Hence, the gain and damping are also small, and it is evident that a "cone of instability" exists.

Various applications that do not involve the secant expander and the variable gain associated with it are available in four-gimbal systems. Thus, the variable gain necessitated by changing geometric orientation of the pitch gimbal is a problem that is easily resolved by a four-gimbal system. Improved system stability and extended frequency response result.

To recapitulate, in applications which do not essentially require a high degree of maneuverability and precision, a three-gimbal stable element provides the advantages of low cost, low weight, compactness and simplicity.

The four-gimbal platform is recommended when high accuracy and utmost reliability are essential requirements for an inertial platform. Although it is larger in size and weight and more complex than a three-gimbal stable element, the four-gimbal arrangement provides the advantages of unlimited maneuverability, system stability, and the lowest drift rate obtainable.

TWO-GYRO PLATFORMS

Basic operating principles of a gyro stable platform, together with the relative merits inherent in three and four gimbal configurations have heretofore used a single-degree-of-freedom rate integrating gyro sensor in the illustrative examples given. These examples should not be construed as an implication that stable platforms can only be built from single-degree-of-freedom gyroscopes, for it is also possible to build platforms which utilize two-degree-of-freedom gyros.

Since stable platforms are used when a high-accuracy three-axis reference is required, it is obvious that three single-degree-of-freedom gyros are needed to establish a three-axis reference triad. To establish such a reference using two-degree-of-freedom units, only two gyros are needed, a condition which also provides one redundant reference axis. A single-degree-of-freedom rate integrating gyro whose output takes the form of a precession motion about the output axis, is the type used in the three-gyro platform, and since the gyro is used in conjunction with a servo loop, a small angle is required to generate the error signal. Because cross coupling is possible as a consequence of the hangoff angle, very tight alignment loops are used to negate any input resulting from it. A two-degree-of-freedom unit is a displacement gyro whose spin vector remains fixed in space. The alignment loop error signal is essentially the relative motion between the fixed rotor and its case. Under such an arrangement, and because no gain exists, a displacement gyro is less sensitive than a rate integrating type in which high gain can result as a consequence of its output angle equalling $H/B \int \omega dt$. These basic differences determine associated alignment loop behavior in three-gyro and two-gyro platforms.

When properly built, both types can control a platform effectively, but from the standpoint of fabrication it is more difficult to balance a two-axis gyro properly than a single-axis type, for two output axes in the former must be precisely balanced in contrast to one in the latter. The factor best determining the potential of each gyro type, when oriented as in a platform cluster, is in the relative susceptibility of each to mass unbalance effects produced by acceleration. Despite differences in response to a given acceleration input, both types of gyros are capable of generating equally accurate control signals — by inherent orientation in the three-gyro version and by appropriate averaging circuitry in the two-gyro configuration.

The fact that it is possible to build identical gimbal structures in which only the design of the inner gyro cluster differs clearly illustrates the inherent potential of each. Moreover, current practice is such that effective relative volume is the same for three-gyro and two-gyro platforms of equivalent performance. In the latter type, some provision must be made for redundant axis averaging, and it is customary to arrange the gyros in such a platform so that the azimuth axis is the redundant one.

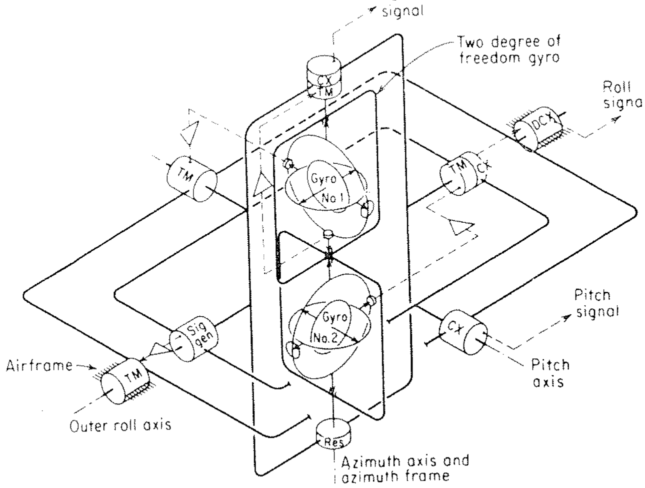


Figure 53

GYROCOMPASSING

An inertial system's usefulness depends largely on the accuracy to which its stable platform can be aligned initially. When used in a fixed-base system such as that in a ballistic missile resting in its silo, the stable platform may be aligned by a precise external reference. Alignment becomes a major problem, however, when the base is moving as in ships, submarines, aircraft, land vehicles, etc. Further, a self-contained alignment capability within the system is desirable, particularly in military applications where speed and accuracy are equally essential. By using the same inertial sensors stabilizing the platform, it is possible to devise a gyrocompassing system which is fundamentally an automatic self-contained method for establishing true heading. A gyro stable platform may be instrumented in a number of ways to perform gyrocompassing. The method described below is a typical means of gyrocompassing with a platform.

In the gyrocompassing alignment loop shown in Figure 54, it is assumed that the base is fixed and that the input axis of one of its orthogonal gyros points east and perpendicular to the earth's rotational axis. When so oriented, the gyro cannot sense earth's rate, and consequently does not precess relative to the earth. Should misalignment occur, the east gyro precesses in relation to the earth, thereby generating an error signal which not only tilts the platform from the vertical, but also throws its north accelerometer out of alignment with the earth's gravity vector. The accelerometer, sensing the tilt, then generates a signal which torques both the east gyro such that the north accelerometer remains level, and the azimuth gyro to maintain the east gyro in its original easterly orientation.

On a moving base, an external velocity reference must be provided to compensate for vehicular motion. Without such reference, the east gyro will assume a null position determined by the vector sum of the earth's rate and motion of the base. In aircraft, Doppler radar can provide the velocity information necessary, while an odometer device could furnish velocity data for a moving land vehicle. At sea, an EM log would be used.

Gyrocompassing is not an instantaneous function but requires time to reach a fair degree of accuracy. Alignment time depends on such factors as initial conditions, alignment loop gain and damping, and base motion characteristics. In the steady state, azimuth misalignment may be reduced to a condition in which the east gyro's drift ω_d balances the earth's rate drift $\omega_e \cos \lambda \sin \theta$. The misalignment angle θ_z then depends on the gyro drift level and the latitude at which gyrocompassing is being performed.

Equation (86) may be used to compute the limiting value of gyrocompassing accuracy by inserting appropriate values for gyro drift rate and latitude. Further, it is obvious that at the terrestrial poles, where λ is 90 degrees, equation (86) is undefined.

$$\theta_z \cong \frac{\omega_d}{\omega_e \cos \lambda} \qquad (86)$$

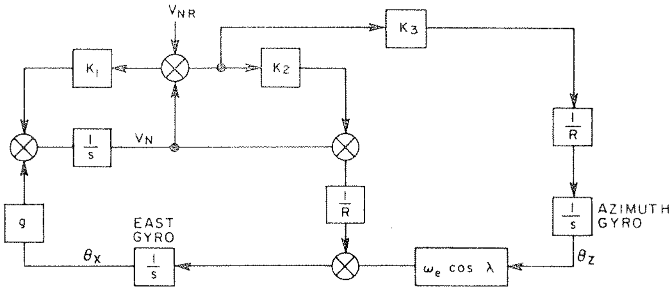


Figure 54

In Figure 54,

ω	=	earth's rate
λ	=	latitude
R	=	radius to earth's center
V_N	=	northerly velocity
V_{NR}	=	external velocity reference
K_1, K_2, K_3	=	loop constants
θ_x	=	east gyro precession angle
θ_z	=	azimuth gyro precession angle

Projecting the sides of this angle to the celestial sphere shows that they pass through the observer's point and the star's substellar point. The observer's point is located on a circle of equal altitude, or line of position (LOP), which is ambiguous in position determination when but one star sight is used. If a sight is taken on a second celestial body at the same time as the first, the observer thus establishes two circles of position intersecting at two points on the earth's surface. Both circles being lines of position, the observer must be located at one of the points of intersection. Usually, these points of intersection are so far apart that there is little doubt regarding which point constitutes the observer's position. Should any doubt persist, however, a sight on a third celestial object can be made to establish still another line of position from which true position can be verified. After making these observations, a navigator consults either his American Nautical Almanac or his Air Almanac to determine declination and Greenwich hour angle for the time of his sightings. These references, published annually, provide this information for all celestial objects commonly used in celestial navigation.

Guidance systems using stellar-inertial measuring units are governed by the principles described above, but with certain differences which are elaborated upon below.

By the time a navigator aboard a high-speed aircraft determines his position through celestial observation and computation, he is a few hundred miles away from his plotted position. Since navigational procedures are time consuming when performed by humans, some method that is far more rapid became mandatory, especially since it is impossible to use such slow methods in the guidance of missiles and other spacecraft. A typical stellar-inertial navigation system, which effectively automates the procedures a navigator uses, consists of an inertial platform, a star tracker, and a computer. The stable platform portion, having an inertial cluster consisting of precision gyroscopes and accelerometers, is the heart of the system, and serves to stabilize the star tracker in space. Usually provided with independent motion about its azimuth and elevation axes, the star tracker still requires a rough approximation of position if star position information is to be of value. Hence, the inertial guidance segment of the system provides basic positional information which is corrected further by stellar data. One might ask, "How does the star tracker know where in the sky to look?" Just as the stable platform replaced the navigator, a digital computer, programed to provide almanac data, replaces one of his tools — the American Nautical Almanac or the Air Almanac mentioned earlier. The quantity of information fed into the digital computer depends upon the threshold magnitude of stars detectable by the star tracker. Once present position is known to inertial accuracy, the computer, in effect, commands the tracker to scan that segment of the sky where a star should be discovered.

Stellar inertial measuring units take many configurations, some of which are considered here. The variety of inertial sensors, too, is as great for a SIMU as it is for conventional inertial platforms. Star trackers, also available in a variety of different types, may be classified according to the type of optics and detectors they use, but are basically telescopic devices having a detector and a scanning mechanism. Telescopes may be either reflecting or refracting types, while detectors are usually photomultipliers, vidicon tubes, or solid state devices. Scanning mechanisms may consist of a mechanical system driving a rotating slit, or they may employ an electronic system such as a flying spot. Despite the inherent differences between these various devices they can all be used in the same way, but a discussion of their relative merits is beyond the scope of this publication.

Because stellar inertial system effectiveness depends largely on vidicon tube or solid state detector characteristics, these devices must be sensitive enough to detect a sufficiently large number of specific stars against the brightness of the sky. Should the detecting device be sensitive only to the very brightest stars, or should the vehicle carrying it be traveling in a part of the sky devoid of useable stars, operational utility is limited. Since an excessively narrow field of view demands ultraprecise telescope aiming, it is better that the detector have a relatively wide field consistent with sensitivity requirements. By so doing, the value of the tracker is increased, since it can more readily perform its intended function by correcting the uncertainties of the inertial navigation system.

A difference of greater degree exists between "single star" and "two-star" stellar-inertial measuring units. The latter, used where position determination is of prime importance, is carried aboard such mobile vehicles as ships, planes, or terrestial vehicles such as missile launchers and the like. In the latter application, where ballistic missile launching is concerned, it is obvious that initial position must be accurately determined if the selected target is to be struck.

Since a single star SIMU is used to determine precise heading rather than position, its employment for this purpose alone does not contradict previous statements regarding the necessity for two star sights in establishing position. Because a ballistic missile flies in an essentially two-dimensional path, it must be accurately oriented in a plane intersecting both its launch point and its ultimate target. To improve the accuracy with which this plane may be established, the single-star tracker is directed to sight on a selected star by a system computer during the missile's boost phase. Inertially generated information selects the line of sight to which the star tracker must adhere. Should the tracker discover that the star selected deviates in position from that predicted, it automatically accepts the actual star position over that preselected, causing the generation of correction signals within the guidance system to change the missile's trajectory plane. In effect, star position information is used to correct for gyro drift.

Because stellar inertial measuring unit accuracy is measured in arc seconds rather than arc minutes, the designer of associated gyroscopic reference devices must deal with extremely critical requirements. Gimbal loop stabilization is one such requirement because excessive gimbal motion induced by severe vibrations causes the star tracker detector image to smear. Hence, particular attention must be paid to both gimbal balance and servo loop behavior so that maximum stabilization may be effected.

Star tracker usage imposes a completely new set of design problems as well. While conventional stable elements can be "blind" except for a small viewing window for visual alignment, star trackers must be able to view virtually any part of the sky, depending on the position of a preselected star. Hence, a SIMU must have a built-in provision for appropriate viewing windows. No single window meets all requirements, however, for the field of view is dictated by both a vehicle's flight trajectory and the SIMU's location within the vehicle. Further, not only must windows be of optical quality to eliminate errors introduced by refraction, but they must be installed where they can perform their intended function in a manner best suited to a particular application. For example, a window may constitute a part of a flight vehicle's outer "skin" in one application, while in another, best results may require the window to be an integral part of the SIMU case. Despite such problems, however, stellar-inertial measuring units have been built and used with success, and their use will undoubtedly become even more widespread as the need for greater guidance accuracy increases.

PRECISION ACCELEROMETERS

The need for high-accuracy guidance systems for aircraft and missiles resulted in the development of three-gyro platforms. Greater inherent accuracy is obtained by clustering a triad of gyroscopes within a gimballed support than is usually achieved by combining conventional directional and vertical gyroscopes.

Early-model platforms were designed to provide pitch, roll, and yaw attitude of relatively slow-speed aircraft, based upon a reference frame established by gravity-sensitive pendulums. These pendulous devices possessed relatively poor accuracy, but were simple in construction and adequately fulfilled the requirements of early platform systems. Advancements in the field of gyrodynamics demanded a corresponding improvement in vertical sensing devices, which could also detect acceleration forces acting upon an aircraft. To satisfy this need the development of a family of high-accuracy accelerometers followed.

Gyrodynamicists realized that greater accuracies in gyros were attainable by utilizing flotation principles to reduce bearing friction. This concept led to a distinctly separate class of high-accuracy gyros which initiated true inertial navigation. High-accuracy, low drift gyros can establish a coordinate measuring system relating the vehicle's spatial motion to some fixed point on the earth's surface. Accelerometers commonly employed in attitude and heading platforms could not provide the greater degree of accuracy needed to sense the most minute vehicle motion with maximum precision. However, force-balance servoed accelerometers and other types of instruments having the required degree of accuracy were developed.

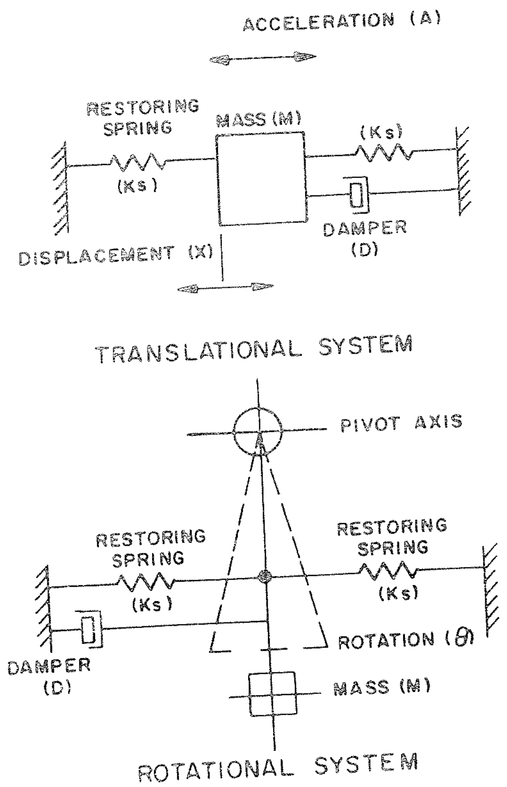


Figure 57

Accelerometers used in platforms fulfill one of two major functions which classify the instrument according to application. Platforms in which the accelerometer's major function is the proper alignment of reference axes are considered basically to be vertical reference devices. In such applications, pitch and roll gyros are continuously slaved to the accelerometer in either the normal erection mode or a Schuler-tuned mode of operation. The gyros thus establish an erected gimbal system which is used as an attitude reference frame.

Platforms in which the accelerometer's major function is the detection and measurement of vehicle accelerations are considered to be basically inertial. In such applications high-accuracy gyroscopes establish a coordinate set of axes along which vehicle accelerations may be detected and measured. Some of the more widely used types of precision accelerometers are described below.

VERTICAL REFERENCE ACCELEROMETERS

Accelerometers in this classification possess a wide range of accuracy standards depending upon system use. In most cases the application does not require the use of a servoed device, although some manufacturers of reference type platforms do use force-balance accelerometers.

When discussing the determination of vertical, it is important to understand clearly what is meant by the term "vertical," for, as stated previously, an acceleration device cannot distinguish the difference between acceleration and gravity. A vertical reference device acting as a plumb line on the rotating non-spherical earth, assumes a direction in which gravity acts on a rotating earth. Since gravity in this case is a resultant between the earth's mass attraction force and its rotational centripetal force, and because the earth itself is not a perfect sphere, geocentric vertical does not coincide with gravity vertical.

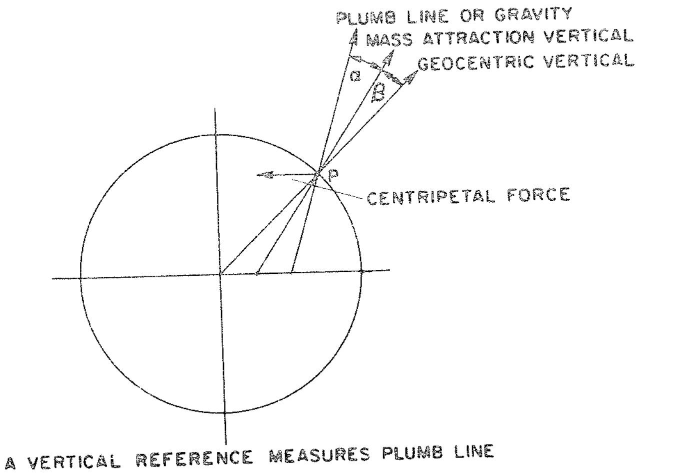


Figure 58

The vertical reference accelerometer is usually a simple seismic device consisting of a damped spring-mass second-order single-degree-of-freedom system as illustrated in Figure 57.

Such systems may be represented in block notation form as shown in Figure 59. They are generally characterized by low resonant frequencies, usually on the order of 15 to 60 cps. The same frequencies in rotational systems lead to cross axis errors of sizable magnitude.

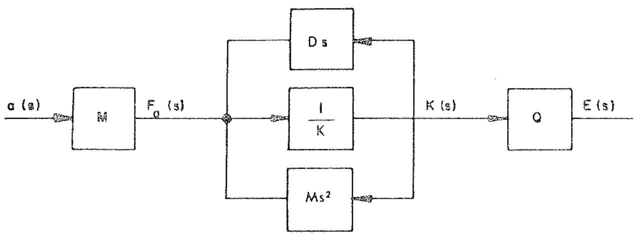


Figure 59

a = input acceleration
M = mass
F_a = acceleration force
D = damping (viscous)
K = spring constant
Q = transducer scale factor
s = LaPlace operator
E = output voltage

$$\text{Overall transfer function} = \frac{E}{a}(s) = \frac{Q}{S^2 + \frac{DS}{M} + \frac{K}{M}} \quad (87)$$

Accelerometers of this type are often classified according to the transducer used to indicate the mass displacement in terms of an analog quantity such as voltage. A differential transformer is commonly used in an accelerometer having an AC output, but a potentiometer is usually used in a DC instrument.

Piezo-electric crystals, strain gage, vibrating wire, and variable reluctance type transducers are also utilized.

INERTIAL ACCELEROMETERS

The indispensable element in any inertial navigation system may be rightly said to be an accelerometer, since it is for the mounting of this device that a gyro-stabilized inertial stable platform is created. A distinction must be made, too, between accelerometers and velocity meters, (sometimes termed integrating accelerometers), for in the latter, an acceleration input produces an output proportional to the integral of the input acceleration. This is an important measurement in ballistic missiles, for the vehicle's guidance system should place it accurately in space at a definite velocity. Hence, the accuracy of acceleration measurement rather than the degree of gyro drift is the limiting factor in the accuracy of a ballistic missile guidance system.

Acceleration measuring devices that are most widely used fall into three major categories, namely, force balance pendulous, vibrating string, and pendulous integrating gyroscope accelerometers.

FORCE BALANCE ACCELEROMETERS

These accelerometers consist essentially of a mass which moves along an acceleration-sensitive axis, and have a pickoff device which detects motion of the mass. Pickoff output resulting from mass motion is fed to a high-gain amplifier whose output current flows, in turn, through a force balance coil, producing the force necessary to return the displaced mass to null. This type of an accelerometer, together with its associated amplifier is, in effect, a high-gain null-seeking servo in which the current flowing through the force balance coil, measured as a voltage across a resistor in series with the coil, is directly proportional to the acceleration applied.

The design and fabrication of a force balance accelerometer is both simple and economical. A limitation frequently encountered, however, and one which can be overcome through the use of very high-gain loops, is the susceptibility of a force balance accelerometer to cross coupling acceleration errors attributable to the hangoff angle required for error signal generation. Since this type of accelerometer always operates about a null, very sensitive pickoffs can be devised for use with force balance coils having a high degree of linear response.

A typical Kearfott accelerometer is a force balance type which employs an inverted pendulous mass together with a flexure type of suspension.

A force balance pendulous accelerometer, utilizing a differential transformer type pick-off, a high gain capture amplifier, and a DC permanent magnet force coil is illustrated in Figure 60. The same design is shown in typical block notation form in Figure 61. These typify many of the high precision force-balance accelerometers currently manufactured.

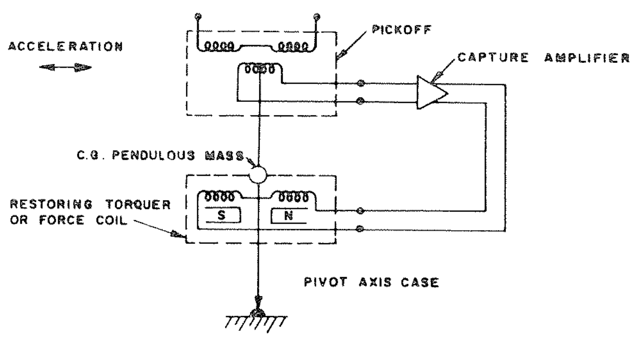


Figure 60

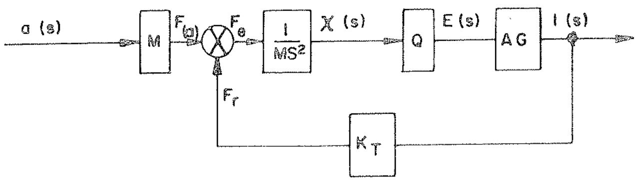


Figure 61

a = acceleration

M = pendulous mass

F_a = force due to acceleration

F_I = restoring force

F_E = error

x = linear displacement of mass

A = amplifier gain

Q = pick-off scale factor

G = transfer function of shaping or stabilization networks and is in the form of a typical lag-lead network such as:

$$G = \frac{(T_1S + 1)(T_2S + 1)}{(T_3S + 1)(T_4S + 1)} \quad (88)$$

where T_1, T_2, T_3 and T_4 are time constants, and
S = LaPlace operator

$$\text{The overall transfer function} = \frac{1}{a}(s) = \frac{QAG}{S^2 + QAG\,K_T/M} \quad (89)$$

PULSE TORQUING

Since instrumentation today trends generally toward signal digitalizing, one type of accelerometer based on a vibrating string exhibits a singular advantage in this respect because its output takes a digital form directly. Similarly, pendulous integrating gyroscope accelerometers are commonly, though not always, constructed so that their readout is digital in nature. Such being the case, one would assume that the closed loop force balance type of accelerometer described earlier might become obsolete, for although the analog dc current flowing through its force-balance coil is a measure of applied acceleration, it is not digital in form.

However, such an analog force-balance type of accelerometer can be modified so that it is pulse-torqued, providing an output which is essentially digital. In operation, current pulses flow through the force-balance coil of such an accelerometer, whose force-balance null-seeking servo loop is so designed that any force produced by applied acceleration and acting on the proof mass is balanced by pulse forces. Should each pulse be the same in energy magnitude, then the number of pulses per unit of time, i.e. the pulse rate, is proportional to the acceleration applied. Effectively, each pulse represents a fixed increment of velocity, so a pulse torqued accelerometer can furnish velocity data simply by counting pulses. Such accelerometers, the development and refinement of which is actively pursued by Kearfott, are potentially even more accurate than their analog counterparts in providing velocity information, for they eliminate the need for analog integrators, thereby also eliminating the error attendant in their use.

Instrumentation of the pulse torquing concept may employ a variety of techniques, one of which uses a pulse generator producing a train of constant-current pulses having constant width and frequency. Pulse polarity is determined by the accelerometer pickoff, which in turn is dependent upon pendulum position. At null, alternate positive and negative pulses are generated. But when the pendulum is deflected by an applied acceleration, the pickoff gates the pulse generator to change pulse polarities in sufficient number to oppose the pendulous torque. Essentially, the summation of asymmetric pulses is proportional to the applied acceleration. Each pulse represents a velocity increment, the exact magnitude of which depends on the system design.

Pulse torquing accelerometers have become possible only because improved transistor devices can operate at very high switching rates. Demand for accelerometers of this type stems not only from the growing use of digital computers in navigation systems, but also from a desire to eliminate analog to digital converters wherever possible.

VIBRATING STRING ACCELEROMETERS

Vibrating string accelerometers, used successfully in ballistic missile guidance systems, consist of a proof mass supported by a pair of vibrating strings (wires) held under tension. Since the natural frequency of a taut wire is a function of its tension, an input acceleration applied along the longitudinal axes of the supporting strings causes an increase in tension in one while decreasing tension in the other. Hence, each string vibrates at a different frequency, a condition which can be expressed as

$$f_1^2 = T_1/4\delta L^2 \quad (90)$$

$$f_2^2 = T_2/4\delta L^2 \quad (91)$$

where f_1 and f_2 represent each of the different frequencies, T represents tension, δ represents mass per unit of wire length and L is supporting wire length.

Recalling that tension difference $(T_1 - T_2)$ is proportional to the product of mass and acceleration, the following equations may be written.

$$f_1^2 - f_2^2 = \frac{(T_1 - T_2)}{4\delta L^2} = \frac{ma}{4\delta L^2} \quad (92)$$

which may be rewritten as

$$(f_1 - f_2)(f_1 + f_2) = \frac{ma}{4\delta L^2} \quad (93)$$

and which is an expression showing that the difference in vibrating string frequency is proportional to applied acceleration.

$$f_1 - f_2 = \left[\frac{m}{(f_1 + f_2)\,4\delta L^2}\right] a \quad (94)$$

where m represents proof mass magnitude.

The sum of the frequencies $(f_1 + f_2)$ can be held constant by a servo loop tied to a reference oscillator.

Vibrating string accelerometers are virtually direct force-to-frequency sensors, for they measure velocity, which is a time integral of acceleration, by counting the difference of cycles in the vibration frequency. However, some practical problems which must be overcome in any selected design include high wire stress that is incompatible with low metal creep, complex associated electronics, and the provision of sufficient radial support by proof mass suspension. Despite such problems, vibrating string accelerometers can be made to perform with a high degree of precision and reliability.

PENDULOUS INTEGRATING GYROSCOPE ACCELEROMETER

A pendulous integrating gyro accelerometer may be considered to be a single axis gyro alignment loop in which an intentional unbalance is created in the gyro rotor. This unbalance, termed pendulosity, is the product of the mass multiplied by its moment arm from the gyro's precession axis. An acceleration along the input axis causes motion about the precession axis because of the mass unbalance. Mounted in a gimbal, the gyro is part of a null-seeking servo loop in which motion about the precession axis is detected by a pickoff whose output, after amplification, is fed to a gimbal torquer to cause relative motion between the gyro gimbal and its case. The torquer, positioned along the gyro's input axis, drives the loop to null, at which time gimbal angular motion is equivalent to the integral of input acceleration, which is velocity.

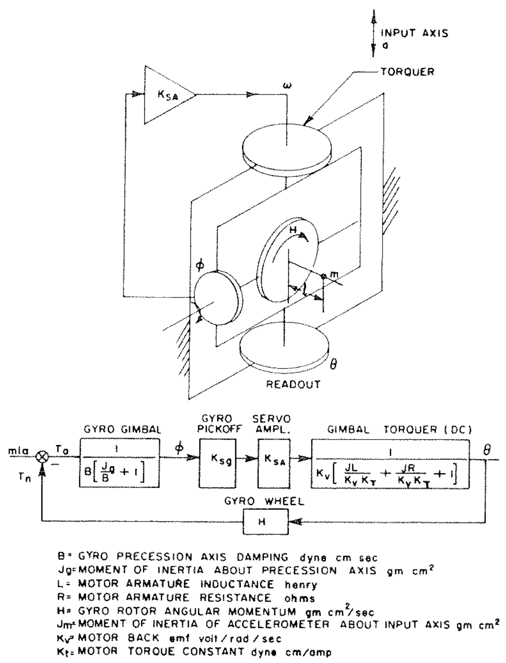


Figure 62

Torque due to acceleration:

$$T_a = m\ell a \tag{95}$$

Restoring Torque:

$$T_r = \omega H \tag{96}$$

$$T_r = T_a \text{ for null.} \tag{97}$$

$$m\ell a = \omega H \tag{98}$$

$$\left(\frac{m\ell}{H}\right) a = \omega \tag{99}$$

$\frac{m\ell}{H}$ is a constant of the device.

Integrating both sides of equation (99):

$$\left(\frac{m\ell}{H}\right) V = \theta \tag{100}$$

Hence the output angle is a direct measure of velocity.

A pendulous integrating gyro accelerometer's unique advantage lies in its freedom from any Hooke's joint type of restraint. Linear unsaturable torque attributable to the pendulous element is balanced by a similarly unsaturable linear torque developed by the gyro's precession. A contributing factor to this accelerometer's overall linearity and accuracy as an integrator is its freedom from an external integration device.

According to prevailing practice, the angular readout of a pendulous integrating gyro accelerometer is preferred in digital form.

BALLISTIC MISSILE APPLICATIONS

Ballistic missiles currently compose the greatest market for inertial guidance system application, and in such systems accurate velocity determination is of prime importance. For example, in an ICBM having a range of about 5000 nautical miles and traveling at a rate of more than 20,000 feet per second, a velocity error of only one foot per second would cause the missile to miss its intended target by approximately one mile. Faced with such a circumstance, it is obvious that conventional accuracies of one part per thousand, or even one part in five thousand are insufficient for highly precise guidance. Hence, great attention is paid to the design, construction, and calibration of precise integrating accelerometers and velocity meters used in ballistic missile guidance systems.

Accelerometer errors may be categorized as those caused by bias shifts (K_O, M_O, N_O), scale factor errors (ΔK_1, M_1, N_1), nonlinearities (K_2, K_3), cross coupling, and nonorthogonality. A mathematical error model for a typical integrating accelerometer may be represented in the form

$$\Delta V = K_O + \Delta K_1 A_i + K_2 A_i^2 + K_3 A_i^3 + \left[M_O A_J + M_1 A_J A_i + N_O A_k + N_1 A_k A_i \right] \tag{101}$$

in which cross axis terms are enclosed in brackets, and in which ΔV is the thrust velocity error, A_i is the thrust acceleration along the input axis, and A_J and A_k are thrust accelerations along axes normal to the input axis. K_n terms are accelerometer error coefficients in which temperature effects are already assumed to be adjusted.

To determine error coefficients of an acceleration sensing device, testing and calibration should be performed under calibrated acceleration inputs. Though this is accomplished to some extent by precision centrifuges when solitary sensor testing is desired, a technique known as sled testing is widely used whenever it is necessary to determine velocity meter behavior in a guidance system.

SLED TESTING

Although sled testing is a means by which detailed accelerometer nonlinearities may be evaluated from a controlled group of sled runs, the apparatus is also used for shakedown and malfunction detection purposes. A precision method providing quantitative data with a high degree of accuracy, sled testing, because of its complexity, is limited to use in the design development phase of accelerometer production.

To sled test an inertial guidance system, a forebody is propelled along a two-rail track by means of a high thrust rocket sled which can develop velocities up to 2000 feet per second, depending upon the load carried and the rockets used. Telemetry aboard the sled transmits pertinent performance data during both acceleration and deceleration, the latter function being performed by aerodynamic forces together with the retarding force of water braking achieved by dragging a scoop through frangible water-filled dams located in a trough between the rails. Dam placement, number of dams, and water depth within them are variables determining deceleration force characteristics. The resulting high thrust, acceleration, deceleration, and vibration test environment compares in magnitude to that experienced during missile flight, except for the extremely limited duration, usually 40 to 50 seconds.

A space-time or Velocity Measuring System (VMS), determines sled velocity with respect to the track by accurate time measurements made as the sled passes precisely positioned precalibrated test stations spaced along the track. Accelerometer analysis, based on velocity comparison between telemetered and VMS measurements is a sophisticated procedure demanding extensive data together with digital computer equipment to handle it.

In determining accelerometer error coefficients quantitatively, sled testing is notably superior and preferable to flight testing because of three essential conditions. First, a VMS provides more accurate data than that obtained in missile tracking, while secondly, a sled run provides both positive and negative acceleration information which facilitates separation of an accelerometer's nonlinear terms. Thirdly, optical instrumentation along the track can provide accurate determination of platform angular position at several points, which permits partial separation of gyro and accelerometer errors. Because a sled run is so brief in duration, however, its value in gyro and platform error coefficient determination is limited.

BASIC PARAMETERS

The most significant basic parameters for both vertical reference and inertial accelerometers are summarized in the following table. To simplify the basis for comparison and to classify an accelerometer according to possible application, Kearfott spring-restrained and inertial instruments are compared.

To achieve the high degree of accuracy associated with inertial type accelerometers, an instrument of extreme precision is required.

These precision instruments must also be sufficiently rugged and durable to operate under high vibrational, shock, and thermal missile environments without performance degradation. Missile applications also demand small, lightweight instruments. Most Kearfott inertial accelerometers currently in production adhere to the pendulous force-balance principle. These simple, rugged, and durable devices are capable of the highest accuracy requirements.

	Vertical Reference Type	Inertial Type
Description	Spring-restrained, damped, single-degree-of-freedom, linear. Differential transformer output (AC).	Force-balance, pendulous permanent magnet torquer, one or two degrees of freedom, DC feedback current is linear measure of acceleration.
Threshold	5×10^{-4}g (most DC potentiometer output accelerometers are limited to 10^{-3}g)	3×10^{-7}g (by actual measure)
Linearity	0.05 % of full scale	5×10^{-5}g to 1g and less than 0.01 % of applied acceleration to higher g's.
Range (Dynamic)	2000	8×10^{7}
Range (Max.)	± 1g (adjustable within dynamic range of 2000)	± 25g (adjustable upward within limits of amplifier)
Zero Uncertainty (from vertical)	± 100 arc seconds	± 10 arc seconds
Null (or zero) stability	± 50 arc seconds	± 5 arc seconds (day to day) ± 2 arc seconds (continuous)
Cross axis error	1.0 % of applied accelerations	0.005 % of applied accelerations.
Undamped natural frequency	9 cps (approx.)	above 300 cps
Damping	5 times critical	0.7 of critical
Output	AC voltage	DC voltage

ACCELEROMETER TESTING

The heart of an inertial guidance system is the accelerometer. In such a high accuracy system accelerometers must sense extremely small vehicle accelerations with minimum error. In addition, they must be capable of measuring high acceleration inputs accurately while operating in severe thermal, shock, and vibrational environments.

In evaluating accelerometer performance it is necessary to investigate thoroughly the effect of operational environments on performance. Since these environments exist in actual applications, it is mandatory that these instruments maintain their static accuracies with a minimum of degradation.

Performance Criteria

When considering the performance of a servo-captured accelerometer, the function of the amplifier is necessarily an integral part of overall instrument performance. Of equal importance are the limitations imposed upon a servo-captured accelerometer by the electronics used in conjunction with it.

Basic Accelerometer Requirements

1. Accelerometers must possess a high degree of sensitivity and a minimum threshold value, even though they are required to operate under high acceleration inputs.

2. Accelerometers must transform these acceleration inputs into usable signals with a high degree of accuracy.

Static testing is not only the most direct means of accelerometer evaluation but in many cases it is the only practical approach.

Error-Causing Forces in a Pendulous Accelerometer

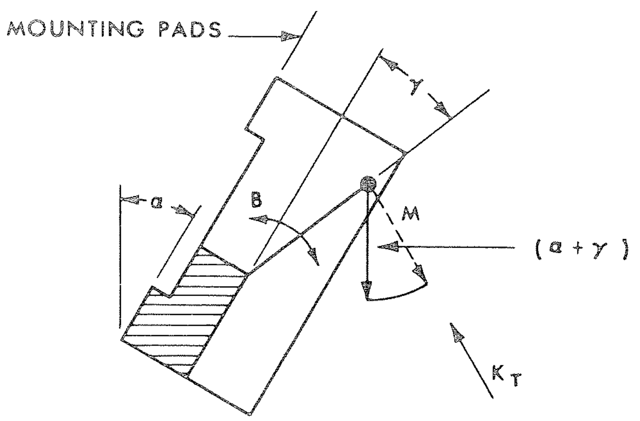


Figure 63.

If an accelerometer had no internal forces acting upon it, electrical output would be exactly zero if the instrument mounting pads were in a vertical plane. Similarly, if the mounting pads were positioned in the one g field (as indicated by maximum accelerometer electrical output), the output would remain fixed in magnitude and would reverse polarity when considering gravity as acting in the two opposite senses. When internal forces are acting, however, the above relationships do not hold. These forces may be broken down into the two following components:

Bias, B, is the amount of residual spring force acting upon the pendulum. The suspension system anchoring the pendulum to the housing possesses a spring rate. Although extremely small, its minute displacement generates a significant error in measured acceleration forces.

Axis error, γ, is the non-coincidence of the center of gravity of the pendulous mass with the hinge axis and the pickoff null position.

$I_\alpha = \frac{Mg}{K_t} \sin(\alpha + \gamma) \pm B$ (to a high degree of approximation)

where:

I_α is the electrical output in response to a tilt angle, α.

M is the pendulous mass.

K_t is the transfer function of the restoring torquer.

γ, B (defined earlier).

g is gravity force.

Therefore the measured restoring or feedback current in any tilt position is composed of that portion caused by the gravity component, plus the component of axis error, plus the bias.

A tilting test as applied to force-balance accelerometers is one in which the accelerometer is tested (or calibrated) against earth's gravity force over the range from -1g to $+1$g by tilting the accelerometer case from the horizontal, so that the resolved component of gravity along the sensitive axis is varied according to the size of the angle of tilt.

Axis error manifiests itself as an angular deflection of the pendulum relative to the mounting pads (or longitudinal axis).

Axis error, like bias, is considered positive if it is in the direction of positive acceleration.

It is often expressed as the angle existing due to the misalignment of the pendulum center of gravity relative to the mounting pads. However, an equally acceptable method expresses bias as an equivalent error in g's or as parts error in 10,000.

The Four Cardinal Points

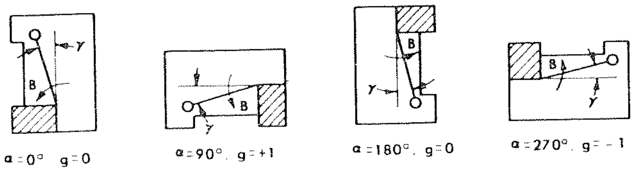


Figure 64. Typical Force-Balance Pendulous Accelerometer in Four Attitudes.

$\alpha = 0°$, and 180° mounting pads in a truly vertical plane. $\alpha = 90°$, and 270° mounting pads are in a truly horizontal plane.

In the $\alpha = 0°$ position, accelerometer output is expressed:

$$I_o = B + S \sin \gamma$$

where $S = \frac{Mg}{K_t}$

and is equivalent to the instrument calibration on scale factor in milliamperes per g

γ, B (defined earlier)

Output in the $\alpha = 180°$ position is expressed:
$I_{180} = B - S \sin \gamma$

From measurements made in the $\alpha = 0°$ and $\alpha = 180°$ positions it is possible to determine both bias and axis error.

Solving for B: $2B = I_o + I_{180}$

$B = \frac{I_o + I_{180}}{2}$ in milliamperes.

$B = \frac{I_o + I_{180}}{2S}$ expressed as equivalent error in g's where I_o and I_{180} are expressed in milliamperes.

Similarly, solving for sin γ

$\frac{I_o - I_{180}}{2S} = \sin \gamma = \gamma$ (small angles) and is expressed as equivalent error in g's when I_o and I_{180} are expressed in milliamperes.

An alternate method for determining bias error is obtained from the $\alpha = 90°$ and 270° positions. Output in the $\alpha = 90°$ position is expressed:
$I_{90} = B + S \sin (90 + \gamma)$

Output in the $\alpha = 270°$ position is:

$I_{270} = B - S \sin (90 + \gamma)$

Then solving for B;

$B = \frac{I_{90} + I_{270}}{2S}$ equivalent error in g's when I_{90} and I_{270} are expressed in milliamperes.

It is sometimes convenient to express axis error in equivalent angular error, in which case:

$\gamma = \frac{I_o + I_{180}}{2S} (10^{-1})$ where I_o and I_{180} are expressed in microamperes and γ is expressed as equivalent error in milliradians.

Error expressed in arc seconds is found by multiplying the above by 19.6 (or approximately 20).

Measurements taken at the four cardinal points are actually the only measurements or test points needed to determine the accuracy of an accelerometer of this type.

Bias Determination

Bias may be determined from output measurements made in the zero and one-g fields. When determined from zero g measurements, it is designated B_1. This value is determined by algebraically adding the restoring current measurements in the two positions. When the bias is determined from the two one-g measurements of restoring current (again added algebraically) it is designated B_2.

In all measurements, bias is considered positive if it tends to increase the output in the + 1g position, or is such that it is in the direction of positive acceleration when determined in the zero-g field.

Bias is expressed by an equivalent error in g's and is usually acceptable if less than 0.0001g (0.0002g if best accuracy is not a requirement). Some inertial platform accelerometers are even acceptable with 0.0005g bias errors. Frequently it is convenient to express the error in parts per 10,000.

Bias Stability Measurement

Bias measurements are made on a day-to-day basis from a cold start using the tilting tests already described. Compilation of this data is best represented by a continuous plot of the actual bias expressed as an equivalent error in g's.

It has already been stated that a system can be designed to incorporate compensation for known bias within an accelerometer. However, this method of correction is valid only if the bias is known to be stable, and therefore such long-term stability measurements are extremely important.

A continuous plot of bias yields a spread of values no greater than an equivalent error of 0.0001g to 0.0002g, when considered over a time duration of one year.

Bias Discrepancy

Since bias is defined as the amount of residual spring force acting on the pendulum, it is immediately apparent that any such force is independent of the accelerometer mounting pads' orientation. Therefore, when considering the data available from the tilting test, proper determination should yield identical bias errors as related to both the zero and one-g position.

Usually the data indicates an apparent discrepancy between two bias measurements, and if it does, the term "bias discrepancy" is used to describe an accelerometer fault, even though it may be totally unrelated to the acting bias force. By definition, bias discrepancy is the algebraic difference between the one-g bias (B_2) and the zero-g bias (B_1), and is expressed in equivalent g's or error parts in 10,000. Satisfactory performance may usually be realized from an accelerometer if bias discrepancy is maintained at less than 0.0001g equivalent error. The expression for bias discrepancy is:

$$BD = B_1 - B_2$$

where B_1 and B_2 are bias errors in equivalent g's.

Actually, a bias discrepancy is caused by a demagnetizing (or magnetizing) effect produced within the permanent magnet circuitry by feedback current flowing in the force coils. Normally, in an accelerometer possessing a basically symmetrical design, two force coils, each located within a field of permanent magnet flux, produce additive force vectors to balance the pendulum. When feedback current flows in the series-connected force coils, an equilibrium with an equal and opposite demagnetizing effect-produced balance exists only if the reluctance paths for the magnetizing flux in each of the magnet circuits are identical. The greater the degree of asymmetry within the accelerometer, the more unequal the reluctance paths, and the more significant is the apparent demagnetization or magnetization of the permanent magnet flux field. For a fixed length of conductor, the force required to balance is proportional to the magnetic flux density and the feedback current. It is apparent then, that in the zero-g field no current flows through the coils (except for a small amount caused by the action of internal forces), while in the one-g field, currents of sufficient magnitude to cause a demagnetizing effect result.

Therefore those bias measurements taken with the accelerometer in the zero-g field represent true bias error, while those taken in a one-g field contain both bias error and the effects of asymmetry. The effects of asymmetry are most significant when an accelerometer is operated within a vibrational environment. Under such conditions the reversing force vectors cause rectification of the steady-state pulsating dc current.

In accelerometer designs utilizing a single magnet it is necessary to provide a compensating coil, so arranged that it sets up flux patterns which compensate exactly for the effects of demagnetization.

Temperature Effects on Bias

Temperature variations may affect bias because of unequal thermal expansion rates between the sensitive element or its vital parts and the instrument case, for example, in the physical shifting of the null position. Applications usually limit the total variation of temperature to a range of about 20 degrees Fahrenheit, such that if temperature variations affect the bias, resultant errors do not seriously degrade the performance associated with a narrow operating temperature range.

Maximum bias temperature effect should not exceed one part in 100,000 per Fahrenheit degree. A twenty-degree Fahrenheit range means that performance is not degraded beyond a factor of two over the operating temperature range.

Zero Uncertainty

Zero uncertainty, sometimes referred to as verticality error or zero offset, is the equivalent angle by which the plane of the accelerometer mounting pads deviates from a truly vertical datum with zero feedback current in the circuit. Actual error is determined by positioning the accelerometer in the zero-g field.

An actual measurement of feedback current is then taken. The error is found from the following equation:

$$V = \frac{I_n (10^3)}{S}$$

where I_n is the measured restoring current in milliamperes, S is scale factor, and V is expressed in milliradians.

The zero-g measurement is always made with the pendulum inverted. In this orientation it is possible to have the bias error partially or entirely compensated for by the axis error even though when taken singly each represents equivalent errors of 0.0001g or greater. Similarly, axis and bias errors having equivalent values less than 0.0001g could be additive in the zero-g field, producing an equivalent verticality error greater than could be tolerated in some high accuracy inertial systems.

Errors greater than 0.1 milliradian (or 20 arc seconds) are generally not acceptable, although 0.2 milliradian may be usable in less accurate inertial systems.

Zero Stability

Zero uncertainty is a maximum condition of error. Within the limits of the uncertainty an accelerometer exhibits a random drift of the actual zero position. This instability can usually be attributed to a variation in the mechanism controlling the bias, although a physical shifting of the pickoff null position or the repositioning of the pendulum mass center may also be a cause.

Data related to zero stability is meaningful only if it is considered over a relatively long time interval. Therefore, day-to-day stability plots of zero position are maintained. Test data are obtained at periodic intervals from the tilting tests of an accelerometer in a cold start condition (i.e., electrical discontinuity between accelerometer and capture circuitry exists before and after the compilation of each set of test data). A typical Kearfott accelerometer tested over a one year period indicates zero stability on the order of ±10 arc seconds, or about one half the maximum error of 20 arc seconds for zero position.

If zero stability is considered over any continuous time interval during which the accelerometer remains operative, improved performance may be realized.

Day-to-day zero stability is best indicated by determining the error by which the accelerometer mounting pads deviate from a truly vertical datum, when considered for any number of consecutive tests. Mechanics of the test procedure are no different than those already described. The test frequency should be on a daily or weekly basis, the results of which are best indicated by a continuous plot of the actual vertical deviation error expressed in equivalent arc seconds.

Stability over a continuous operational interval is best determined by positioning the accelerometer mounting pads within a vertical plane while monitoring the electrical output in a continuous recording, during which step input disturbances to the accelerometer are made. Stability and repeatability to within 2 to 5 arc seconds are a normal order of magnitude when considered over any test interval of a continuous duration and when use is made of a recorder possessing a high degree of sensitivity.

Scale Factor

Scale factor is the numerical value of the entire servo loop's transfer function, including the amplifier. Every attempt is made to correlate actual measured scale factor with a known one-g input. An accelerometer cannot be manufactured with an absolute zero mechanical spring rate, but it can be designed to have a very high electrical-to-mechanical stiffness ratio, so that in the presence of steady acceleration, only a very small portion of the overall restoring force may be attributed to the mechanical spring. For generally acceptable performance, an electrical-to-mechanical stiffness ratio of 2,000 to 1 or greater is considered essential.

In considering scale factor from restoring current measurements (electrical output) taken with the accelerometer in the one-g field, the effects of all the forces acting are as shown below.

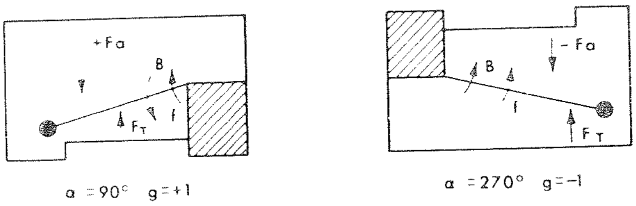


Figure 65.

where F_a = force due to acceleration (+ or − g) acting upon pendulous mass

F_T = force produced by restoring coil (feedback current)

f = restoring force due to mechanical suspension (hinge)

B = bias force as defined earlier

then

$$F_a = F_T + f \pm B \text{ or } F_T = F_a - f \pm B$$

$$I_\alpha = 90 \approx F_{T_\alpha} = 90 = F_a - f + B$$

$$I_\alpha = 270 \approx F_{T_\alpha} = 270 = F_a - f - B$$

$$I_{90} + I_{270} = 2(F_a - f)$$

$$\frac{I_{90} + I_{270}}{2} = F_a - f$$

It is apparent that electrical restoring force does not constitute the entire restoring action. It can be concluded, however, that f would remain constant for a fixed gain system and would contribute to the restoring action equally when the acceleration sense is either positive or negative.

A gain adjustment within the servo loop increasing the electrical-to-mechanical stiffness ratio would necessarily increase the measured scale factor because the effect due to the hinge deflection force f would be less. For example, for a typical Kearfott force balance accelerometer, a change in scale factor of 0.01% is observed when amplifier gain is varied by 10%. For this reason it is imperative, when considering high accuracy measurements, that reasonably close gain control be maintained.

Scale factor may be expressed as:

$$K_a = \frac{I_{90} + I_{270}}{2}$$

where: K_a is the scale factor in milliamperes per g

I_{90} is the measured restoring current at 90° of tilt.

I_{270} is the measured restoring current at 270° of tilt.

Scale Factor Stability

Since many force balance accelerometer designs utilize a permanent magnet restoring torquer, a continuous change in instrument scale factor may be observed due to normal magnet aging. This aging effect can be minimized (through proper design and controlled stabilization) to less than 0.04% rms increase in scale factor when considered over a one-year time interval, and when exposed to normal handling and environmental operating conditions.

Test results determining scale factor stability are best represented by a continuous plot of the data compiled on a day-to-day basis and taken from normal tilting tests.

Temperature Effects on Scale Factor

Accelerometer designs of the force or torque balance type utilizing permanent magnet restoring torquers (such as a D'Arsonval movement) are sensitive to variations in temperature. The temperature effect of permanent magnets may be partially compensated for by means of properly designed temperature compensator shunts. The shunt is usually placed across the flux gap and flattens the temperature effect curve such that the instrument scale factor remains constant within tolerable limits over a desired temperature range. Kearfott accelerometers of the pendulous force-balance type are designed to have a scale factor that remains constant within ±0.01% over pre-selected temperature ranges of approximately 20 Fahrenheit degrees.

By proper selection of shunt material and its configuration, the range over which optimum properties are realized may be extended to +200°F or lowered to practical limits.

Hinge Axis Error

In pendulous type accelerometers, the axis of pendulum rotation defines the orthogonality of the sensitive or measuring axis with respect to the instrument mounting pads or datum. The axis of rotation is often termed the hinge axis. Hinge axis error is defined as the angular deviation between the hinge axis and the mounting pads, and is expressed in milliradians. It represents a condition of orthogonality such that accelerations directed along the cross axis cause a condition of error along the measuring axis proportional to the degree of externally uncorrected misalignment. In this case, the mounting pads are utilized as the primary frame of reference.

Normally, a maximum hinge axis error of 0.05 milliradian can be attained and when needed, may be held to less than 0.02 milliradian.

A satisfactory test method makes use of the tilting test to establish a truly vertical reference plane, as in the 90° tilt position.

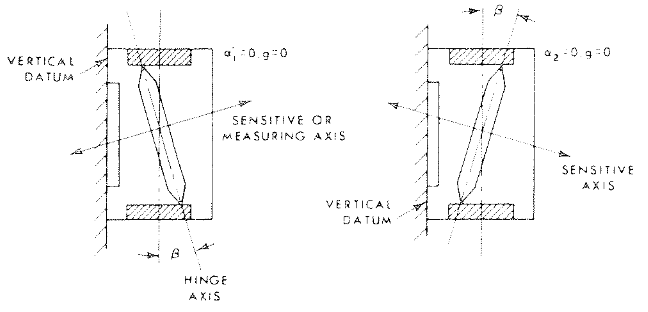


Figure 66. Accelerometer Positioned with Mounting Pads In Vertical Plane.

From the two measurements of restoring current obtained by rotating the accelerometer around its sensitive axis in the zero-g field, hinge axis error may be determined. Then, in the $\alpha'_1 = 0$ position, accelerometer output is:

$$I'_1 = B + S \sin \beta$$

where:

B is the bias
S is the scale factor
β is hinge axis error (angular deviation)
I'_1 is accelerometer electrical output in the $\alpha'_2 = 0$ position.

In the α'_2 position, accelerometer output is:

$$I'_2 = B - S \sin \beta$$

where I'_1 is accelerometer electrical output in the $\alpha'_2 = 0$ position.

Hinge axis error may be determined by algebraically subtracting the two output measurements and solving for β

$$I'_1 - I'_2 = 2S \sin \beta = 2S\beta \text{ (for small angles)}$$

$$\beta = \frac{I'_1 - I'_2}{2S}$$

where β is error in milliradians (I'_1 and I'_2 are measured restoring current in microamperes and is the scale factor in milliamperes per g.

Linearity

Linearity error is defined as the deviation from the best fitted straight line drawn through a plot of the electrical output in response to a known acceleration input. It may be determined conveniently by the tilt method already outlined together with additional test positions within each of the four quadrants considered. When tilting positions are used to determine linearity errors, actual measured values of restoring current are compared to a theoretically correct value calculated from the instrument scale factor and the corresponding tilt angle. This may be expressed as:

$$\sigma = \left(\frac{\left| I_\alpha \right| - \left| S \sin\alpha \right|}{S} \right) \times 100$$

Where σ is linearity error expressed as a percent of one g.
I_α is accelerometer output in response to input angle of tilt (α), expressed in milliamperes
S is accelerometer scale factor in milliamperes per g.

Methods employed in tilting tests to determine bias and axis errors would normally be extended to as many additional points as the test requires, but the four cardinal positions constitute a portion of the total accumulated data. A more critical approach to linearity errors in accelerometers results if errors are considered as deviations from a line connecting zero and one-g measurements.

Consider a source of error encountered in tilting tests, commonly termed two-cycle error. This error is attributable to the effects of pendulum deflector (or hang-off) angle (which is actually the angular deviation between case and pendulum, the magnitude of which is dependent upon the acceleration input).

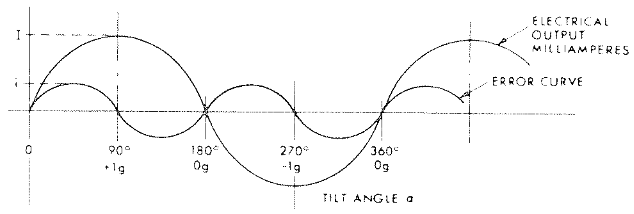


Figure 67

In the diagram above, the error curve is the deviation of the measured electrical output from the theoretically correct value and is seen to have a frequency twice that of the output. This cyclic error has its maximum value at a frequency determined from the relationship:

$$i = K_n \left| \sin(\alpha + bS \sin \alpha) - \sin \alpha \right|$$

Where i is deviation error
S is instrument scale factor
α is the angle of tilt of the accelerometer mounting pads from the vertical
b is a constant dependent upon the servo loop parameters.

Linearity errors are related to three major causes: (1) bias error, (2) component of axis error, and (3) demagnetizing effect as determined by bias discrepancy. The relative effect of each of these may be shown in the following manner.

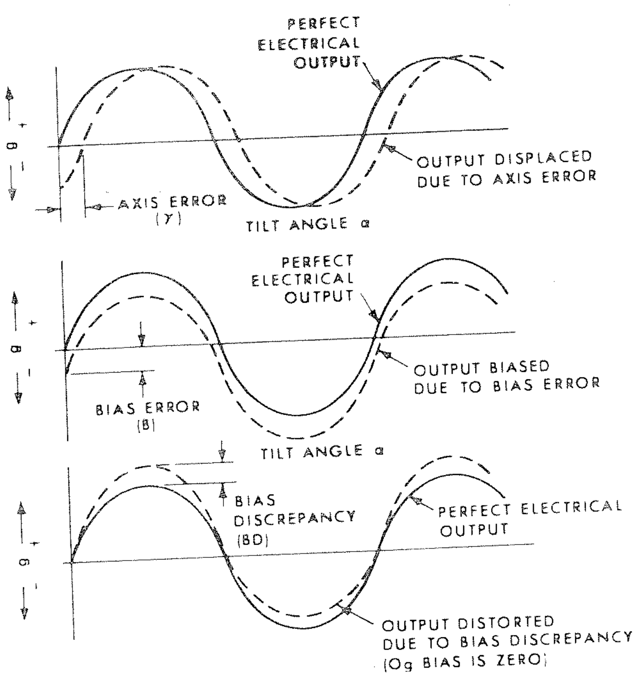


Figure 68.

It is possible and quite practical to correct electrically for both bias and axis error,* since they remain constant within a given accelerometer. If this is done, the curve below results:

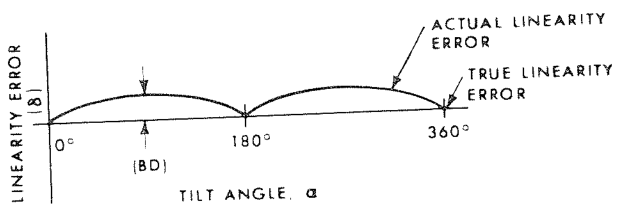


Figure 69.

If bias discrepancy is held to close limits, true linearity error may be correspondingly reduced. This leads to an alternate method of presenting linearity errors in which corrections are made for bias and the component of axis error. If corrections are made for bias (B_1 bias) and axis error, maximum error will always occur at 90° and 270° of tilt angle from vertical, the magnitude of which is numerically equal to bias discrepancy. It is, therefore, unnecessary to perform extension linearity tests to include a number of tilt positions. Measurements made at tilt positions corresponding to one-half g positions are sufficient to assure satisfactory compliance with performance requirements.

Linearity errors are expressed in one of two forms:

A) Corrections for bias and component of axis error.

Actual error is expressed as:

$$\delta = \sigma - (\gamma \sin \alpha) - B_1$$

$$\sigma = \frac{I_\alpha - S \sin \alpha}{S}$$

where

- δ is linearity error in equivalent g's
- γ is axis error (defined earlier)
- B_1 is bias (defined earlier)
- α is the tilt angle
- S is the scale factor in milliamperes per g
- I_α is the measured current corresponding to a specific tilt angle

B) Best-fitted straight line connecting zero and ± one g errors. Test results may be plotted as the equivalent error in g's.

Linearities to High g Inputs

Much has been said concerning the use of gravity and the components of gravity in evaluating accelerometers. This test method has gained wide acceptance when considering acceleration inputs up to one g. Beyond one g, centrifuge machines are often considered for test purposes, but the high accuracy requirements needed rule out most commercially available machines.

A method suitable for testing some accelerometers to higher g inputs is the "added weight" test. This test does not provide the same testing accuracies attainable using tilting tests, but it does present the best method of high-g testing short of those utilizing high accuracy centrifuge machines.

The accelerometer must have its sensitive element reasonably accessible so that with proper instrumentation, weights simulating high-g loading can be attached to the sensitive element in controlled increments. Listed below are items of major concern when considering an added weight test:

1. Weights must be accurately known and should be of a material basically unaffected by the environment (i.e. resistant to any corrosive effects).
2. Weights must be added without effecting a change in attachment method. Any ligament used to tie weights to the sensitive element must lie along a path undisturbed by the incremental addition of weights.
3. Care must be exercised in discerning heat effects due to feedback current in the restoring coil. Prolonged loading of the sensitive element results in errors of 3 or 4 parts in 10,000 in the measured current values, which would be non-existent during high-g loadings of short duration.

Test data are obtained by positioning the instrumented accelerometer in the plus one-g field. With prior knowledge of scale factor, and assuming any errors at one-g loading to extend to two-g loading conditions without amplification, the exact value of incremental mass may be determined.

Threshold

Threshold is defined as the minimum acceleration input causing accelerometer electrical output. Pendulous accelerometers having no rotary or relative motion between adjacent parts possess inherently low threshold values. Usually this limitation in measurement is beyond the ability of even high accuracy dividing heads to reproduce the desired input. For example, dividing heads capable of two-second incremental tilt angles, which are considered to be of extremely high precision, have an angular increment corresponding to an acceleration input of 10^{-5}g, which is still considerably greater than the threshold values expected in high precision instruments.

Threshold may be determined by the loaded beam method, in which a relatively stiff cantilever beam is gradually loaded at its free end by adding incremental weights while observing accelerometer output. Prior knowledge of instrument scale factor extended to small angular inputs (at which one radian is equivalent to one g is a valid approximation), is used to indicate the angular deflection of the beam corresponding to the electrical output. By addition of decreasing load increments to the beam, a point will be reached where the addition of an incremental weight does not cause a change in accelerometer output. Therefore, the previously applied loading effect establishes the limitation of response, while the corresponding measured current yields the threshold in equivalent g.

Sensitivity

Sensitivity is defined as the minimum change in acceleration input causing a change in accelerometer electrical output. It is usually assumed that the incremental change is made from an arbitrary, but relatively large input compared to the incremental change.

Through proper instrumentation of the loaded beam method described earlier, an input having a large component of gravity can be arranged.

Vibration-Induced Errors In Accelerometers

Accelerometers used to measure steady acceleration inputs in the presence of vibratory disturbances may produce output information containing errors of a sizeable order of magnitude. Such performance degradation manifests itself as a change in dc feedback current resulting from a sensed steady acceleration. Although the effect produced may be attributed to several causes, it appears as a pick-off null shift which increases the bias proportionately.

Actual error may be expressed as the change in measured restoring current from the value measured at zero frequency. Measurements can be made only with the accelerometer oriented in one fixed position. Therefore, it is impossible to discern the difference between bias effects, change in scale factor, or demagnetizing effects.

When expressing this condition of degradation, the term "induced error" denotes all the effects producing the errors. Testing is accomplished by positioning the accelerometer on a vibration machine in such a manner that one g steady acceleration is sensed.

Since orientation of the accelerometer within the gravity field does not affect the magnitude of induced errors, it is more advantageous to conduct all tests in the one g field. This arrangement positions the instrument so that it is relatively insensitive to tilting of the vibration head. If tests were conducted in the zero-g field, it would be difficult to discern the difference between actual induced errors and sensed gravity components due to tilting of the vibration head.

Test measurements are made by monitoring feedback current in the same manner as in static tilting tests, during slow scanning of the entire vibration frequency spectrum. Normally, maximum error occurs at the resonant frequency of the servo loop. Acceptable errors in mild vibrational environments usually do not exceed 3 or 4 parts in 10,000 when considered over the full vibrational envelope. For applications in which the vibrational environment is severe, acceptable errors may be extended upward to 10 parts in 10,000 maximum.

Induced errors are caused by any one of three separate effects, resulting in rectification of dc feedback current. These are (1) torque rectification, (2) demagnetizing effects within the force coil circuit, and (3) effects due to anisoelasticity of the structure.

Torque Rectification

Errors resulting from torque rectification may exist in pendulous accelerometers, particularly in wide angle applications, which do not exist in nonpendulous designs.

Sinusoidal vibration applied at an inclined angle to the accelerometer measuring axis causes rectification torques to act on the pendulum. Rectified torque is expressed:

$$T_R = \frac{F_0{}^2 L^2 M}{2K} \operatorname{Sin} \Theta \cos \Theta \, (\cos \Psi)$$

where

- T_R is rectified torque
- F_0 is force of applied sinusoidal input
- L is pendulum length
- Θ is angle inclined to measuring axis along which length F_0 is applied.
- Ψ is phase angle between force and pendulum displacement
- K is electrical spring constant of servo system

The above relation has a maximum torque rectification effect when vibration is applied at 45° to the accelerometer measuring axis. Resultant error is minimized through use of shorter pendulums and higher servo gains.

Calculated errors for a typical Kearfott fluid damped accelerometer having a damping factor of 0.7 are 0.2 parts of 10,000 for ± 1g peak vibration and ±2 parts in 10,000 for ±10g peak vibration.

Demagnetizing Effects

Sinusoidal vibration directly along the accelerometer's measuring axis or at an inclined angle to its measuring axis causes current rectification effects.

In accelerometer designs utilizing a single force coil, compensating windings must be included to create a flux pattern unaffected by force and current reversals during sinusoidal vibratory inputs.

Anisoelasticity Effects

Anisoelastic effects are produced in structures having unequal compliances when subjected to force reversals directed at an angle inclined to coordinate reference axes. Torque produced may be expressed by:

$$T = M^2a^2 (C_1 - C_2) \sin\theta \cos\theta$$

where:

- T is resultant torque
- M is mass of sensing element
- a is peak acceleration due to vibration
- C_1 and C_2 are compliances along the orthogonal axes (datum)
- θ is the angle relative to the datum at which force is applied.

The expression above is a maximum when applied force is directed at an angle of 45°. For pendulous accelerometers there can be no anisoelastic effects since there is no compliance factor along the cross axis. For non-pendulous devices, such as a design utilizing ligament support of a geocentric mass, significant errors may result from unequal mechanical and electrical spring rates.

Frequency Response

A method frequently used to describe servo system performance plots the magnitude of output ratio to input as a function of frequency. Usually the magnitude is expressed in decibels, ($db = 20 \log_{10}$ output/input) and is plotted against the log of frequency applied. The plot itself indicates the frequency range over which the accelerometer responds to input exactly. Over this range, overall servo gain is unity, and is frequently referred to as the flat portion of the frequency response curve.

At the resonant frequency of the servo loop the output to input ratio may take a variety of forms depending upon system damping. In a second order system having a damping factor of 0.7 (70% of critical damping) the frequency response curve shows a drop in the output to input ratio at the resonant frequency, whereas for less damped systems the ratio shows a gain or resonant rise.

Cut-off or resonant frequency and resonant rise must be considered as performance data.

It is advantageous to have as wide a bandwidth and as low a resonant rise as possible (provided transient response is not adversely affected). As frequency increases beyond the cut-off point, the output is slowed because of pendulum inertia (which resists rapid reversals of velocity) such that the curve develops a slope based on the reciprocal of the frequency squared.

When measuring electrical outputs in response to sinusoidal mechanical input vibration it is advantageous to utilize a wave analyzer to eliminate quadrature and harmonic distortion from measured current values which would otherwise be seen in vacuum tube type instruments. Measurements are made by monitoring the voltage across a read-out resistor in series with the feedback circuit.

Capture Limits

The ability of an accelerometer to remain captured and satisfactorily damped is a function of system gain. Over the low frequency range of the spectrum, amplifier limiting determines capture ability. The term "limiting" is synonymous with current or voltage saturation, and is related to the zero frequency value of amplifier current output.

At the resonant or cut-off frequency a loss of gain occurs, resulting in a resonant rise in the frequency response curve. Under this condition the accelerometer sensitive element is required to deflect a greater amount to produce an error voltage sufficient to cause a feedback current capable of capturing the mass.

Capture limits may be determined by increasing the input amplitude at a selected frequency of applied vibration until loss of capture is recognized. A satisfactory method is to observe the wave shape (observed on an oscilloscope) related to the pulsating dc feedback current as it develops a voltage across a readout resistor connected in series with the feedback circuit. The normal sinusoidal wave shape becomes clipped when the pendulum is initially restricted in its motion between mechanical stops. When the pendulum or mass makes contact with the stops, instability results to such an extent that rapid reversals with consequent striking against alternate stops is often clearly audible. Another method consists in monitoring the dc feedback current in a manner similar to the methods employed in making static measurements, as in tilting test methods previously outlined.

Cross Axis Vibration Effects

Most accelerometer designs are based upon freedom of motion along the sensitive axis (or axes) while possessing infinite resistance to deflection along cross axes. Pendulous force balance accelerometers used in high gain servo systems usually have extremely small deflector angles which minimize the effects resulting from acceleration or vibratory forces directed along the pendulum axis. An acceptable order of magnitude for the resultant error is 0.005% per g of the applied acceleration.

Accelerations directed along the second of the two cross axes should not cause performance degradation over static test results. This is a requirement when considering the actual environment in which the instrument is to function.

An adequate indication of performance may be achieved by monitoring the dc feedback current in the same manner as that for static performance testing. A slow frequency scanning rate should be employed to insure that resonant frequencies do not degrade performance even over relatively narrow frequency bands.

Transient Response

The frequency response method is suitable for describing servo system performance, but an alternate and equally suitable method measures the response of a servo system to a suddenly applied input, termed a step input, (in which the input is zero for time less than zero and assumes unity value at time greater than zero). Electrical output (or dc feedback current in Kearfott force balance type accelerometers) may be recorded on a high speed recorder to indicate the form taken by the accelerometer in following a suddenly applied input.

A convenient method of applying the step consists in forcing the sensitive element against its mechanical stops by application of an electrical torque input, after which, at time zero, restraint is removed and the accelerometer allowed to function properly.

If the output is recorded and properly time-scaled, the plot becomes useful in determining response time, settling time, and the degree of damping. Response time is defined as the time required for the output to reach 63% of the applied input, while settling time is the time required for the output to reach and remain within 5% of the applied input. By observing the number of response curve "overshoots" it is possible to gain an indication of relative damping.

TABLE OF LAPLACE TRANSFORM OPERATIONS AND PAIRS

OPERATIONS

$\mathcal{L}\left[f(t) \right] = \int_0^{\infty} f(t)e^{-st}\, dt = F(s)$
Linearity Theorem
$\mathcal{L}\left[af(t) \right] = aF(s)$
$\mathcal{L}\left[f_1(t) \pm f_2(t) \right] = F_1(s) \pm F_2(s)$
Final Value Theorem
$\lim_{s \to 0} sF(s) = \lim_{t \to \infty} f(t)$
Initial Value Theorem
$\lim_{s \to \infty} sF(s) = \lim_{t^2\, 0+} f(t)$

f(t)	F(s)
$\frac{df(t)}{dt}$	$sF(s) - f(0+)$
$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0+) - \frac{df(t)}{dt}(0+)$
$\int f(t)dt$	$\frac{F(s)}{s} + f\frac{(-1)(0+)}{s}$
$\int\int f(t)dt$	$\frac{F(s)}{s^2} + f\frac{(-1)(0+)}{s^2} + f\frac{(-2)(0+)}{s}$

TRANSFORM PAIRS

f(t)	F(s)
Unit step function	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
sin bt	$\frac{b}{s^2+b^2}$
cos bt	$\frac{s}{s^2+b^2}$
e^{-at} sin bt	$\frac{b}{(s+a)^2+b^2}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{(s+a)^{n+1}}$
$t^n\ e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
t cos bt	$\frac{s^2-b^2}{(s^2+b^2)^2}$
t sin bt	$\frac{2bs}{(s^2+b^2)^2}$

INVERSE TRANSFORM PAIRS

F(s)	f(t)
$\frac{1}{1 + Ts}$	$\left(\frac{1}{T}\right) e^{-t/T}$
$\frac{1}{(1 + Ts)^2}$	$\left(\frac{1}{T^2}\right) t e^{-t/T}$
$\frac{1}{s(1 + Ts)}$	$1 - e^{-t/T}$
$\frac{1}{s(1 + Ts)^2}$	$1 - \frac{(T + t)}{T} e^{-t/T}$
$\frac{1}{s^2(1 + Ts)}$	$T\left(e^{-t/T} + \frac{t}{T} - 1\right)$
$\frac{1}{1 + \frac{s^2}{\omega^2}}$	$\omega \sin \omega t$
$\frac{1}{s\left(1 + \frac{s^2}{\omega^2}\right)}$	$1 - \cos \omega t$
$\frac{1}{\left(1 - \frac{s^2}{\omega^2}\right)}$	$- \omega \sin h\omega t$
$\frac{1}{1 + 2\zeta \frac{s}{\omega} + \frac{s^2}{\omega^2}}$	$\frac{\omega}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega t} \sin \omega \left(\sqrt{1 - \zeta^2}\right) t$
$\frac{1}{s\left(1 + 2\zeta \frac{s}{\omega} + \frac{s^2}{\omega^2}\right)}$	$1 + \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega t} \sin\left(\omega \sqrt{1 - \zeta^2}\, t - \psi\right)$ $\psi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{-\zeta}$
$\frac{1}{(1 + T_1 s)(1 + T_2 s)}$	$\frac{1}{T_1 - T_2}\left(e^{-t/T_1} - e^{-t/T_2}\right)$
$\frac{1}{s(1 + T_1 s)(1 + T_2 s)}$	$1 + \frac{1}{T_2 - T_1}\left(T_1 e^{-t/T_1} - T_2 e^{-t/T_2}\right)$
$\frac{1}{s^2(1 + T_1 s)(1 + T_2 s)}$	$t - T_1 - T_2 - \frac{1}{T_1 - T_2}\left[T_2^2 e^{-t/T_2} - T_1^2 e^{-t/T_1}\right]$
$\frac{s}{1 + \frac{2\zeta s}{\omega} + s^2/\omega^2}$	$\omega^2 e^{-\zeta\omega t} \sin\left(\omega \sqrt{1 - \zeta^2}\, t + \psi\right);$ $\psi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{-\zeta}$
$\frac{s}{1 + \frac{s^2}{\omega^2}}$	$\omega^2 \cos \omega t$
$\frac{s}{(1 + T_1 s)(1 + T_2 s)}$	$\frac{1}{T_1 T_2 (T_1 - T_2)}\left(T_1 e^{-t/T_2} - T_2 e^{-t/T_1}\right)$

DENSITIES OF COMMON METALS*

Magnesium	1.74	Copper	8.96
Beryllium	1.84	Silver	10.49
Aluminum	2.70	Lead	11.34
Titanium	4.5	Mercury	13.55
Chromium	7.14	Tantalum	16.6
Zinc	7.14	Uranium	19.05
Tin	7.3	Tungsten	19.3
Iron	7.87	Gold	19.3
Nickel	8.9	Platinum	21.45

* *Values are given in grams per cubic centimeter. Multiply by 62.4 to obtain specific weight in terms of pounds per cubic foot. Divide pounds/ft^3 value by 1728 for specific weight in terms of pound /cubic inch.*

USEFUL CONVERSION FACTORS

Angles and Angular Rates

1 radian = 57.29 degrees
1 degree = 0.01745 radians
1 milliradian = 3.437 arc minutes
1 arc minute = 0.2909 milliradians
1 degree/hour = 4.848 x 10^{-6} radians/second
1 milliradian/second = 206.22 degrees/hour

Conversion Table for Low Level Torques

1000 mg-mm = 98 dyne-cm
980 dyne-cm = 1 gram-cm
.014 in-oz = 1 gram-cm
.001 gram-cm = 10 mg-mm
1 dyne-cm = .0000143 in. oz.

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